Power corrections at moderate q_T from Reggeized partons

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Introduction

The talk is based on [M. A. N., V. A. Saleev, Phys.Lett. B790 (2019) 551; PoS DIS2019 (2019) 193; J.Phys.Conf.Ser. 1435 (2020) no.1, 012024].

Cross-section for the un-polarized Drell-Yan process ($S = (P_1 + P_2)^2$, $Q^2 = q^2 = (k_1 + k_2)^2$):

$$p(P_1) + p(P_2) \to \gamma^\star(q) + X \to l^+(k_1) + l^-(k_2) + X,$$

can be decomposed over *helicity structure functions* (HSFs) $F_{UU}^{(1,\dots)}$ as follows ($x_{A,B} = Qe^{\pm Y}/\sqrt{S}$):

$$\begin{aligned} \frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)} \cdot \left(1 + \cos^2\theta\right) + F_{UU}^{(2)} \cdot \left(1 - \cos^2\theta\right) + \\ &+ F_{UU}^{(\cos\phi)} \cdot \sin(2\theta)\cos\phi + F_{UU}^{(\cos 2\phi)} \cdot \sin^2\theta\cos(2\phi)\Big], \end{aligned}$$

TMD-factorization

The TMD-factorization for structure functions:

$$
\begin{aligned}
F_{UU}^{(1,2,\ldots)}(x_A, x_B, \mathbf{q}_T) &= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1} + \mathbf{q}_{T2} - \mathbf{q}_T) \times \\
&\times \mathcal{F}_q(x_A, \mathbf{q}_{T1}) \mathcal{F}_{\bar{q}}(x_B, \mathbf{q}_{T2}) \times f_{q\bar{q}}^{(1,2,\ldots)}(\mathbf{q}_{T1}, \mathbf{q}_{T2}) \\
&+ Y_{UU}^{(1,2,\ldots)}
\end{aligned}
$$

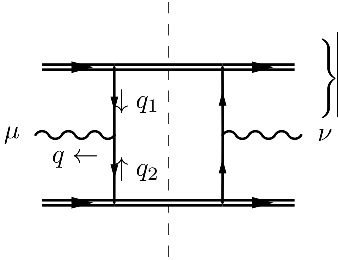
- Factorization for "TMD-term" is proven at leading power in q_T/Q
- "Y-term" is responsible for large q_T-behavior
- Typically "Y-term" is computed in *Collinear Parton Model* (CPM)
- *Does such a prescription correctly include all $O(q_T/Q)$ power corrections missing in "TMD-term"?*
- Possible problem is related with (lack of commonly accepted) **QED** gauge-invariant definition of "TMD-term" at $q_T \neq 0$.

TMD Parton-model

Leptonic ($L_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensors:

$$d\sigma \sim L^{\mu\nu} W_{\mu\nu},$$

Parton model for hadronic tensor:



Decomposition for quark correlatior (un-polarized protons, $P_1^\mu = P_1^+ n_-^\mu/2$):

$$\Phi_{\bar{q}-}^{\alpha\beta} = \frac{q_1^+}{2}\left(\hat{n}_-^{\alpha\beta} f_1^{(q)} + \frac{q_{T1}^i \epsilon_{ij}}{\Lambda}\left(i\sigma^{-j}\gamma_5\right)^{\alpha\beta} h_1^{(\perp q)}\right)$$

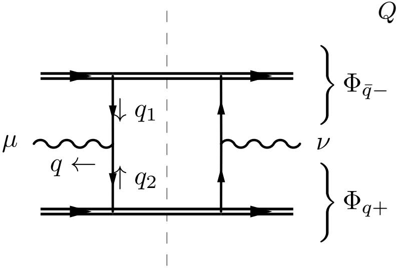
where $f_1^{(q)}(x, \mathbf{q}_T)$ – TMD quark number density, $h_1^{(\perp q)}(x, \mathbf{q}_T)$ – Boer-Mulders function [D. Boer, P. Mulders, 1998].

Idea: decompose Dirac structure of quark correlator in the proton rest frame (16=4+4+6+1+1):

$$\Phi_{\alpha\beta} = f_1^\mu \gamma_{\alpha\beta}^\mu + f_2^\mu (\gamma^\mu\gamma^5)_{\alpha\beta} + f_3^{\mu\nu}\left(i\sigma^{\mu\nu}\gamma^5\right)_{\alpha\beta} + f_4 \delta_{\alpha\beta} + f_5\left(i\gamma^5\right)_{\alpha\beta},$$

then apply boost: $f_1^\mu \sim q_1^+ n_-^\mu$, $f_3^{\mu\nu} \sim q_1^+ n_-^\mu k_{T1}^\nu$, where $k_{T1}^i = \epsilon^{ij}(q_{T1}^j/\Lambda)$ – $\perp$ pseudo-vector, f_2, f_4, f_5 – drop-out in un-polarized case.

TMD Parton-model



$$
\begin{aligned}
\frac{W_{\mu\nu}}{Q^2} &= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1}+\mathbf{q}_{T2}-\mathbf{q}_T) \\
&\times \frac{1}{Q^2}\,\mathrm{tr}\left[\Phi_{q+}\gamma_\mu\Phi_{\bar q-}\gamma_\nu\right] + O(|\mathbf{q}_T|/Q) \\
&= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1}+\mathbf{q}_{T2}-\mathbf{q}_T) f_1^{(q)} f_1^{(q)} \\
&\times \underbrace{\frac{1}{4N_c Q^2}\,\mathrm{tr}\left[\left(\frac{q_2^-}{2}\hat n_+\right)\gamma_\mu\left(\frac{q_1^+}{2}\hat n_-\right)\gamma_\nu\right]}_{w_{\mu\nu}} \\
&+ (\mathrm{Boer-Mulders}) + O(|\mathbf{q}_T|/Q)
\end{aligned}
$$

$$F_{UU}^{(1)} = f_1^{(q)}(x_A,\mathbf{q}_{T1})\otimes f_1^{(\bar q)}(x_B,\mathbf{q}_{T2}),\ \ F_{UU}^{(2,\cos\phi)} \sim O(q_T^2/Q^2)$$

$$F_{UU}^{(\cos 2\phi)} = h_1^{(\perp q)}(x_A,\mathbf{q}_{T1})\otimes h_1^{(\perp\bar q)}(x_B,\mathbf{q}_{T2})\otimes\frac{2(\mathbf{q}_T\mathbf{q}_{T1})(\mathbf{q}_T\mathbf{q}_{T2})-\mathbf{q}_T^2(\mathbf{q}_{T1}\mathbf{q}_{T2})}{\mathbf{q}_T^2\Lambda^2}$$

Problem: partonic tensor doesn't satisfy Ward identity at $q_T \neq 0$:

$$\boxed{q^\mu w_{\mu\nu} = O(q_T/Q),}$$

formally, GI is restored by $O(|\mathbf{q}_T|/Q)$-corrections to $w_{\mu\nu}$.

Curing the Ward-identity

How to restore Ward identity on the level of **partonic tensor** for the **number-density** contribution:

$$w_{\mu\nu} = \frac{1}{4N_c Q^2} \mathrm{tr}\left[\hat{\tilde{q}}_2 \gamma_\mu \hat{\tilde{q}}_1 \gamma_\nu\right],$$

Naively, one can just put momenta $\tilde{q}_{1,2}$ on-shell ($\tilde{q}_{1,2}^2 = 0$), retaining $\tilde{q}_1 + \tilde{q}_2 = q$. This can be done in two ways ("Quasi on-shell schemes"):

1 Without explicit dependence on $\mathbf{q}_{T1,2}$ [Collins, 2011]:

$$(\tilde{q}_1^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa+1)n_-^\mu + q^-(\kappa-1)n_+^\mu\right) + \frac{q_T^\mu}{2},$$
$$(\tilde{q}_2^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa-1)n_-^\mu + q^-(\kappa+1)n_+^\mu\right) + \frac{q_T^\mu}{2},$$

where $\kappa = \sqrt{Q_T^2/Q^2}$ and $q^\pm = Q_T e^{\pm Y}$. Then

$$\boxed{f^{(1)}_{\mathrm{QOS}-1} = 1,\ f^{(2)}_{\mathrm{QOS}-1} = f^{(\cos\phi)}_{\mathrm{QOS}-1} = f^{(\cos 2\phi)}_{\mathrm{QOS}-1} = 0.}$$

Curing the Ward-identity

2 With explicit dependence on $\mathbf{q}_{T1,2}$:

$$(\tilde{q}_1^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(q_1^+ n_-^\mu + \frac{\mathbf{q}_{T1}^2}{q_1^+} n_+^\mu\right) + q_{T1}^\mu,$$

$$(\tilde{q}_2^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(\frac{\mathbf{q}_{T2}^2}{q_2^-} n_-^\mu + q_2^- n_+^\mu\right) + q_{T2}^\mu,$$

where $q_1^+ = (Q_T^2 + t_1 - t_2 + \sqrt{D})/(2q^-)$, $q_2^- = (Q_T^2 - t_1 + t_2 + \sqrt{D})/(2q^+)$ and $D = (Q_T^2 - t_1 - t_2)^2 - 4t_1 t_2$.

Then $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$ get nonzero contribution from **number density**:

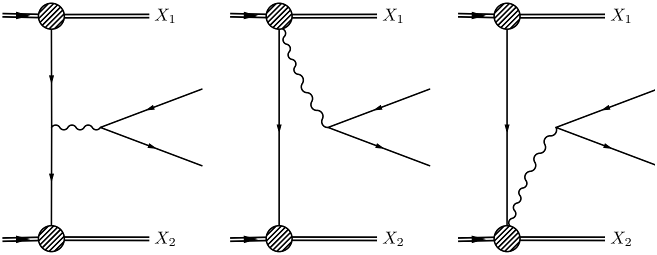
$$f_{\mathrm{QOS}-2}^{(1)} = 1 - \frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2Q^2} + \frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{2Q_T^2 Q^2},\ f_{\mathrm{QOS}-2}^{(2)} = \frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{Q^2} - \frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{Q_T^2 Q^2},$$

$$f_{\mathrm{QOS}-2}^{(\cos\phi)} = \sqrt{\frac{Q^2 D}{q_T^2}}\frac{\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2}{Q_T^2 Q^2},\ f_{\mathrm{QOS}-2}^{(\cos 2\phi)} = -\frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2Q^2} + \frac{Q^2+Q_T^2}{2Q_T^2}\frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{q_T^2 Q^2}.$$

So answer for HSFs $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$ at $O(q_T^2/Q^2)$ depends on the way how one restores gauge-invariance of the hadronic tensor. What is the right way?

Full amplitude

In the full theory, not only t-channel ("Parton model") diagram but also diagrams with direct interaction of the photon with the proton and it's remnants are needed to restore gauge-invariance:



(a) (b) (c)

Is the contribution of non-Parton-model diagrams completely out of control or it can be factorized in some limit?

Let's consider the High-Energy limit:

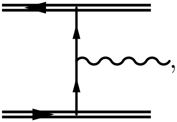
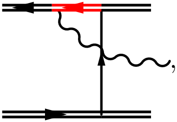
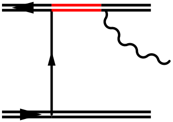
$$\boxed{S \gg Q^2, q_T^2}$$

Spectator model

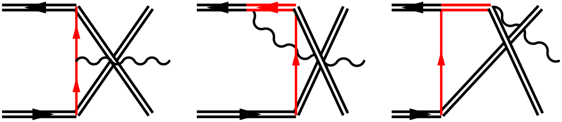
Let's consider the question of factorization in a concrete field-theoretic model, which includes (massless) proton fields, quarks, gluons and *spectator* fields of mass M_s. Protons, quarks and spectators carry $U(1)$ charge.
Let's consider the process:

$$\bar{p}(P_1) + p(P_2) \to \gamma^\star(q) + s(P_1') + s(P_2').$$

Most interesting diagrams in High-Energy limit (+ 2 similar diags.):

$\mathcal{D}_1 =$ , $\mathcal{D}_2 =$ , $\mathcal{D}_3 =$ 

Crossed-diagrams are doubly-suppressed:



Fadin-Sherman vertex

In the leading power in $\sqrt{S} = P_1^+ = P_2^-$, diags. 2 and 3 give:

$$\mathcal{D}_2^\mu \quad \propto \quad e_p \bar{v}(P_1)\gamma^\mu \frac{\hat{P}_1 - \hat{q}}{(P_1 - q)^2} \simeq e_p \bar{v}(P_1) \frac{P_1^+ \gamma^\mu \hat{n}_-}{2(-P_1^+ q^-)} = \bar{v}(P_1) \frac{i\hat{q}_1}{q_1^2} \left[i e_p \frac{\hat{q}_1 n_-^\mu}{q_-} \right],$$

$$\mathcal{D}_3^\mu \quad \propto \quad e_s \frac{(2P_1 + 2q_2 - q)^\mu}{(P_1 + q_2)^2} \bar{v}(P_1) \simeq \frac{P_1^+ n_-^\mu}{P_1^+ q^-} \bar{v}(P_1) = \bar{v}(P_1) \frac{i\hat{q}_1}{q_1^2} \left[-i e_s \frac{\hat{q}_1 n_-^\mu}{q_-} \right].$$

Collecting the contributions of all diagrams one obtains

$$\mathcal{M}_\mu \simeq (-\lambda_{spq}^2) \bar{v}(P_1) \frac{i\hat{q}_1}{q_1^2} \left(-i\Gamma_\mu(q_1, q_2) \right) \frac{-i\hat{q}_2}{q_2^2} u(P_2),$$

where **Fadin-Sherman vertex** [Fadin, Sherman, 1976]:

$$\Gamma_\mu(q_1, q_2) = e_q \gamma_\mu - (e_p - e_s)\hat{q}_1 \frac{n_\mu^-}{q_-} - (e_p - e_s)\hat{q}_2 \frac{n_\mu^+}{q_+},$$

depends only on e_q, since $\boxed{e_p - e_s = e_q}$ and it satisfies Ward identity: $\boxed{q^\mu \Gamma_\mu(q_1, q_2) = 0}$.

Parton Reggeization Approach

This suggests the following **gauge-invariant** factorization for the hadronic tensor of the TMD parton model:

$$\boxed{W_{\mu\nu} \sim \mathrm{tr}\left[\Phi_{q+}\Gamma_{\mu}(q_1, q_2)\Phi_{\bar{q}-}\Gamma_{\nu}(q_1, q_2)\right],}$$

where Φ_q is decomposed in a standard way over number density and Boer-Mulders TMD PDFs. *Of course, this proposal requires further perturbative tests beyond tree-level in QCD interactions.*

Helicity structure functions in PRA

The partonic tensor for **number-density** contribution in above-proposed *Parton Reggeization Approach* (PRA) reads:

$$w_{\mu\nu}^{\mathrm{PRA}} = \frac{1}{4N_c Q^2}\mathrm{tr}\left[\left(\frac{q_2^-}{2}\hat{n}^+\right)\Gamma_\mu(q_1,q_2)\left(\frac{q_1^+}{2}\hat{n}^-\right)\Gamma_\nu(q_1,q_2)\right],$$

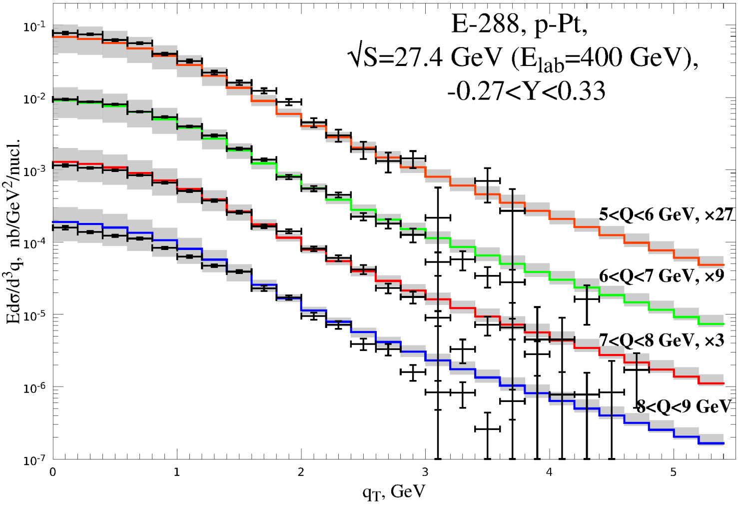
and it leads to the following partonic HSFs:

$$f_{\mathrm{PRA}}^{(1)} = 1 + \frac{q_T^2}{2Q^2},\ f_{\mathrm{PRA}}^{(2)} = \frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{Q^2},$$

$$f_{\mathrm{PRA}}^{(\cos\phi)} = \sqrt{\frac{Q^2}{q_T^2}}\frac{\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2}{Q^2},\ f_{\mathrm{PRA}}^{(\cos 2\phi)} = \frac{q_T^2}{2Q^2}.$$

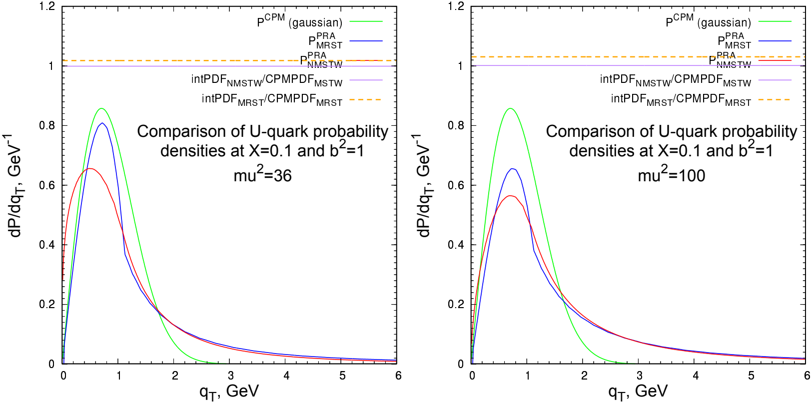
As an **estimate** for number-density TMD PDF, we use the KMR [Kimber-Martin-Ryskin, 2001] formula, which allows one to obtain the unintegrated PDFs from the collinear ones (MSTW-2008 in our case), and resums the $(\alpha_s \log^2 \mathbf{q}_{T1,2}^2/\mu^2)^n$-corrections in Leading Logarithmic Approximation.

Description of E-288 q_T-spectra



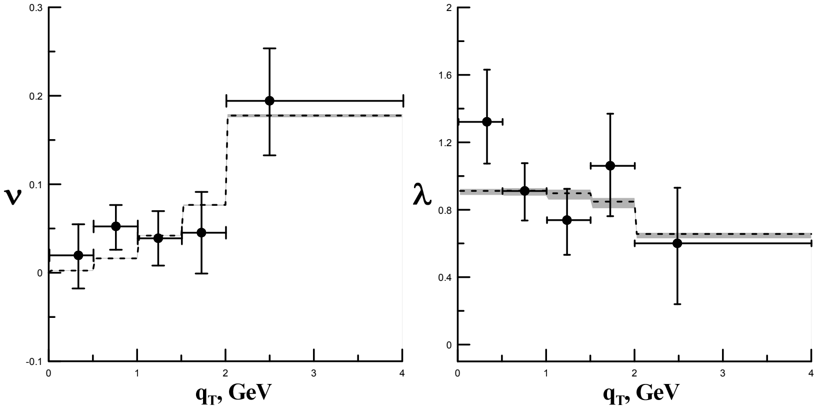
Constant NLO K-factor ~ 1.8 from $\alpha_s \pi^2$-corrections is included.

Comparison of quark UPDFs with Gaussian intrinsic k_T-model



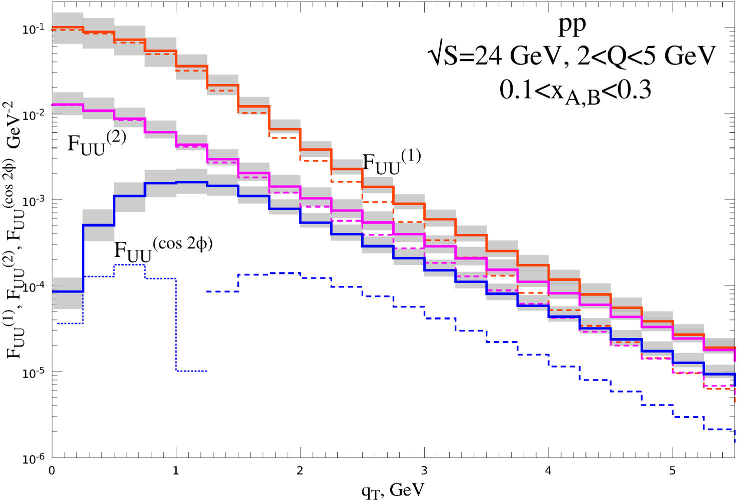
Description of NuSea data ($\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV) on angular coefficients

See [Nefedov, Nikolaev, Saleev, 2013]:



$$\nu \sim F_{UU}^{\cos(2\phi)}.$$

Predictions for HSFs at JINR-NICA ($\sqrt{S} = 24$ GeV)



Solid lines – PRA predictions, dashed lines – $QOS-2$-predictions. At small q_T, the $F_{UU}^{\cos 2\phi}$ HSF in $QOS-2$ is negative.

Including Boer-Mulders contribution

Taking into account Boer-Mulders functions, results for HSFs read:

$$F_{UU}^{(1)} = \sum_{q,\bar{q}} e_q^2 \left[f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2Q^2 + \mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} + \right.$$

$$\left. + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2(\mathbf{q}_T \mathbf{q}_{T1})(\mathbf{q}_T \mathbf{q}_{T2}) - \mathbf{q}_T^2 (\mathbf{q}_{T1} \mathbf{q}_{T2})}{2\Lambda^2 (Q^2 + \mathbf{q}_T^2)} \right],$$

$$F_{UU}^{(2)} = \sum_{q,\bar{q}} e_q^2 \left[f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{(\mathbf{q}_{T1} - \mathbf{q}_{T2})^2}{Q^2 + \mathbf{q}_T^2} + \right.$$

$$\left. + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2\mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2 - (\mathbf{q}_{T1} \mathbf{q}_{T2})(\mathbf{q}_{T1}^2 + \mathbf{q}_{T2}^2)}{2\Lambda^2 (Q^2 + \mathbf{q}_T^2)} \right],$$

$$F_{UU}^{(\cos\phi)} = \sum_{q,\bar{q}} \frac{e_q^2}{Q^2 + \mathbf{q}_T^2} \sqrt{\frac{Q^2}{\mathbf{q}_T^2}} \left[f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) (\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2) - \right.$$

$$\left. - h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) (\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2) \frac{(\mathbf{q}_{T1} \mathbf{q}_{T2})}{\Lambda^2} \right],$$

$$F_{UU}^{(\cos 2\phi)} = \sum_{q,\bar{q}} e_q^2 \left[f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{\mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} + \right.$$

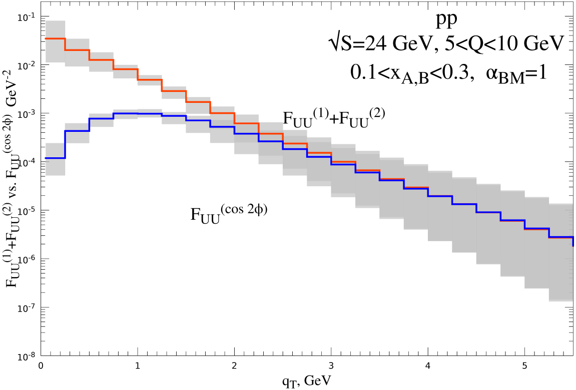
$$\left. + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2Q^2 + \mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} \frac{2(\mathbf{q}_T \mathbf{q}_{T1})(\mathbf{q}_T \mathbf{q}_{T2}) - \mathbf{q}_T^2 (\mathbf{q}_{T1} \mathbf{q}_{T2})}{\Lambda^2 \mathbf{q}_T^2} \right].$$

Numerical estimates

For the estimate we take a simple model:

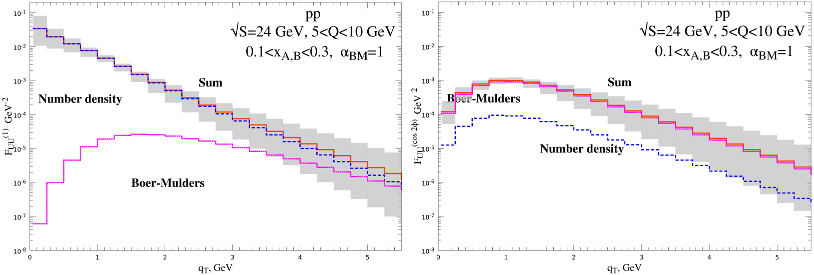
$$h_1^{\perp q}(x, \mathbf{q}_T) = \alpha_{BM} \times f_1^q(x, \mathbf{q}_T),$$

where $\alpha_{BM} \leq 1$ due to condition $F_{UU}^{(\cos 2\phi)} \leq F_{UU}^{(1)} + F_{UU}^{(2)}$, following from positivity of angular distribution:



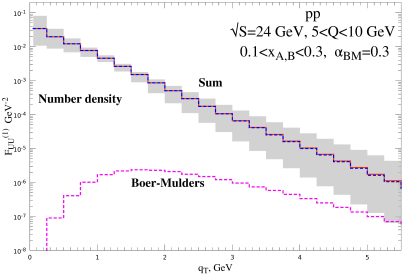
Numerical estimates for NICA

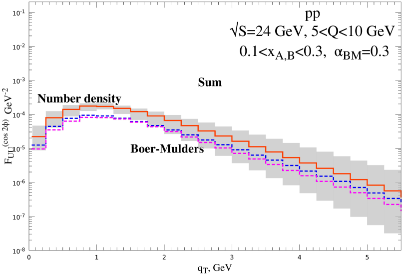
$$\alpha_{BM} = 1$$



Numerical estimates for NICA

$$\alpha_{BM} = 0.3$$





Conclusions

- Gauge-invariance of hadronic tensor is important for Helicity Structure Functions in Drell-Yan process. **Power-suppressed corrections are not small for the existing and planned experiments!**
- Factorization holds in high-energy limit $S \gg Q^2 \geq q_T^2$, but with modified hard-scattering part $\Rightarrow$ PRA.
- Structure-function $F_{UU}^{\cos 2\phi}$ gets contribution not only from Boer-Mulders TMD PDF but also from number-density TMD PDF. Well-established factorization formula is required to separate them.
- PRA predictions with simple KMR unPDF reproduce existing data rather well, however polarization information is still limited. New experiments are needed, COMPASS, NICA-SPD, RHIC, ...
- Approach is to be extended to the case of polarized protons.

Thank you for your attention!