In tribute to
Stan Brodsky

With warm thanks for lots of inspiring discussions and collaborations, ever since our first joint paper:

Lošjin, Transversity 2011

The Intrinsic Charm of the Proton

S.J. Brodsky (SLAC), P. Hoyer, C. Peterson, N. Sakai (Nordita). Apr 1980. 5 pp.

Published in **Phys.Lett. 93B (1980) 451-455**

Cited by 760 records

Picnic in the Bay Area, 1991

Family get-togethers

At a lake in Finland, 2006

Sightseeing

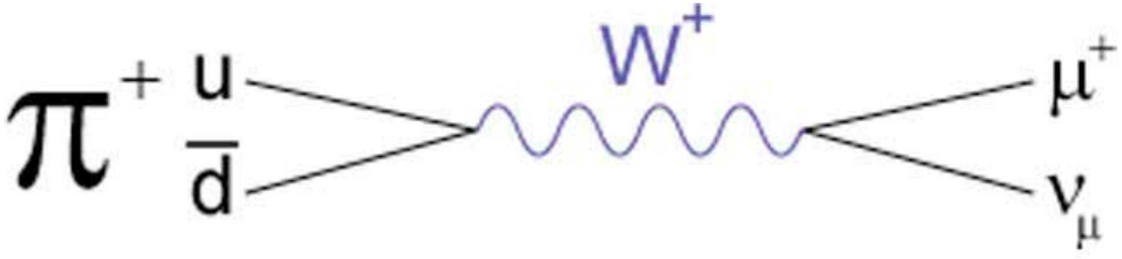
With Galileo Galilei
in Florence, 2004

With H. C. Ørsted in Denmark, 2010

Questioning consensus

Stan told me how he convinced Feynman that hadrons have sizeable Fock states with only valence quarks:

Only the $u\bar{d}$ Fock state in the π^+ contributes to its weak decay:



In any case he convinced me. This inspired the following.

Fock expansion of bound states

CPHI 2020, 6 February 2020

Paul Hoyer
University of Helsinki*

The common statement:

"Hadrons are non-perturbative bound states of QCD"

suggests that

QFT methods for atoms are irrelevant for hadrons.

Is this self-evident?

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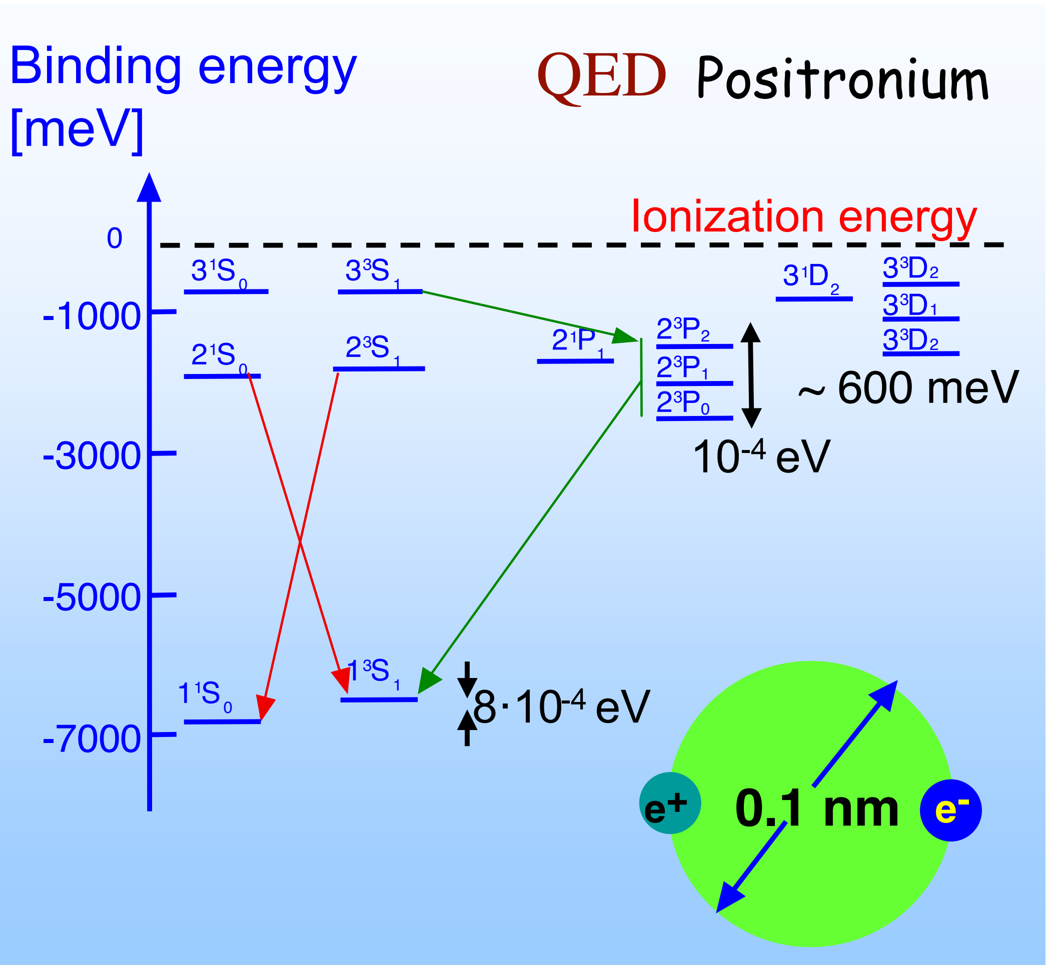
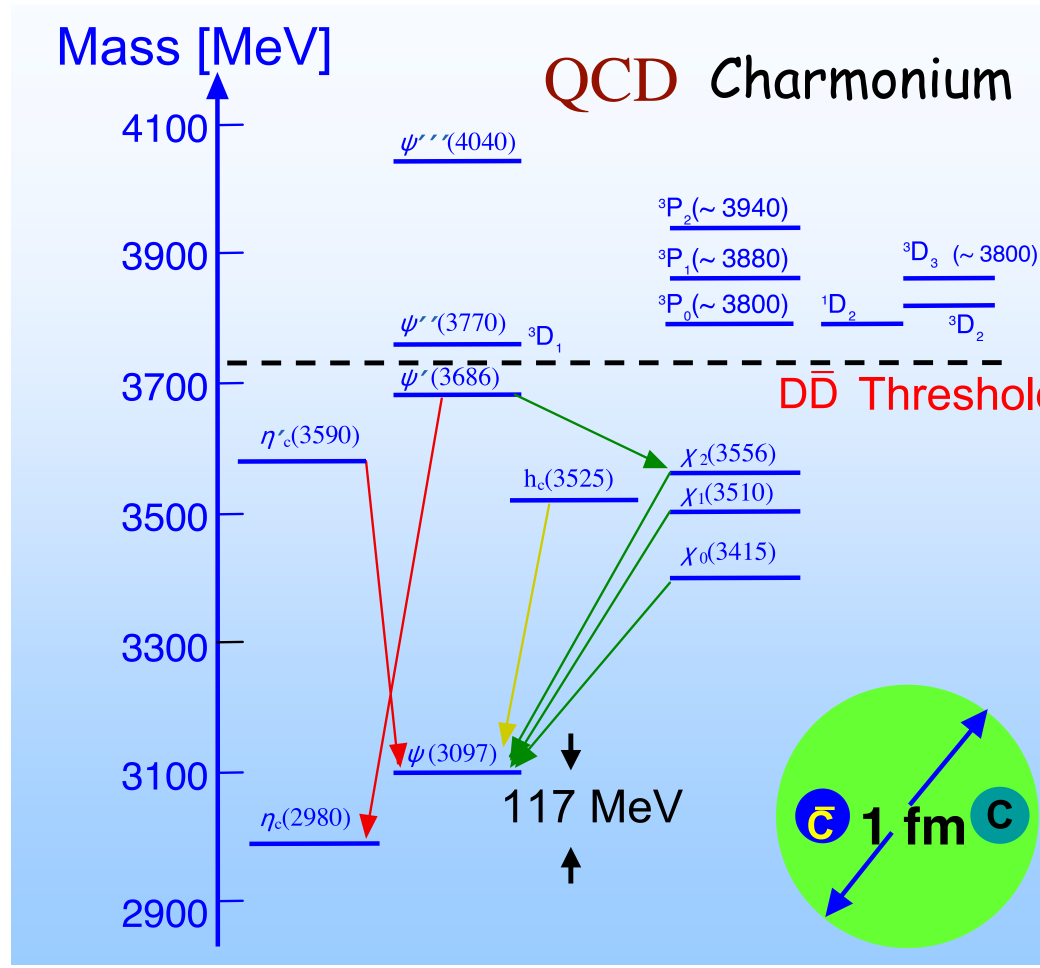
*Presently visiting the University of Pavia.

Quarkonia are like atoms with confinement





$$V(r) = -\frac{\alpha}{r}$$

Classical potential

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r}$$

Cornell potential

A. Einstein: Raffiniert ist der Herrgott aber boshaft ist er nicht.

PQED for atoms ≠ PQED for scattering

The Hydrogen wave function is exponential in α

The binding energy can be expanded in powers of α and $\log\alpha$

The art of atoms

Recoil effects in the hyperfine structure of QED bound states

Rev. Mod. Phys. **57** (1985) 723

G. T. Bodwin

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D. R. Yennie

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

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Instituto de Fisica, Universidade Federal de Rio de Janeiro, Rio de Janeiro, Brazil

"In spite of the statement in the preceding paragraph that bound-state theory is nonperturbative, it is possible to make use of small parameters such as α … to develop expressions in increasing orders of smallness. However, the nonperturbative nature of the expansion shows up in non-analytic dependence on these parameters (such as logarithms). As indicated in the preceding paragraph, there is an art in developing a theoretical expression in this manner."

State of the art: Hyperfine splitting in Positronium

G. S. Adkins,
Hyperfine Interact. **233** (2015) 59

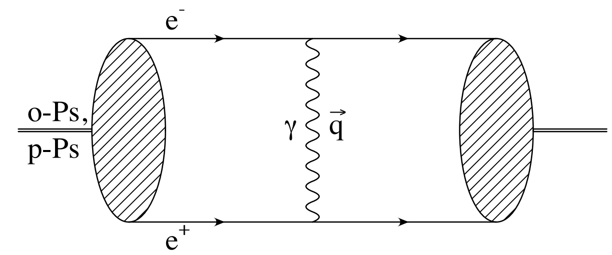
$$\Delta\nu_{QED} = m_e\alpha^4\left\{\frac{7}{12} - \frac{\alpha}{\pi}\left(\frac{8}{9} + \frac{\ln 2}{2}\right) + \frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha + \frac{1367}{648} - \frac{5197}{3456}\pi^2 + \left(\frac{221}{144}\pi^2 + \frac{1}{2}\right)\ln 2 - \frac{53}{32}\zeta(3)\right] - \frac{7\alpha^3}{8\pi}\ln^2\alpha + \frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2 - \frac{217}{90}\right) + \mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}$$

$\Delta\nu_{EXP}$ = 203.394± .002 GHz

The stakes for QCD are high:

Can we do an analogous calculation for the proton?

Atomic PQED uses Feynman Diagrams



A. Czarnecki, hep-ph/9911455

QCD Feynman diagrams expand around

free quarks and gluons at $t = \pm\, \infty$

This excludes confinement.

The absence of confinement in QCD Feynman diagrams does not justify discarding perturbative methods for hadrons.

Bound states are eigenstates of the Hamiltonian

$$H\,|\psi\rangle = E\,|\psi\rangle$$

at an instant of time t (or x^+)

There is no need to consider constituents propagating in time.

The QED Fock expansion

$$|Pos\rangle = \phi_{ee}\,|e^+e^-\rangle + \phi_{ee\gamma}\,|e^+e^-\gamma\rangle + \phi_\gamma\,|\gamma\rangle + \phi_{4e}\,|e^+e^-\,e^+e^-\rangle + \ldots$$

defines a perturbative expansion around $|e^+e^-\rangle$

with higher Fock states given by $H^n_{int}\,|e^+e^-\rangle$

In temporal ($A^0 = 0$) gauge, each Fock state is bound by the instantaneous potential given by Gauss' law.

Hyperfine splitting is verified to $O(\alpha^4)$, as well as boost covariance.

QCD: Add a homogeneous solution of Gauss' law

A single, universal parameter Λ determines

- The isotropic gluon energy density $\propto \Lambda^4$
- The linear $O(\alpha_s^0)$ potential for $q\bar{q}$ Fock states
- The corresponding potentials for qqq, $q\bar{q}g$, gg,… states

The quarkonium phenomenology follows from L_{QCD}.

The method can be applied also to relativistic hadrons.

Confinement is compatible with a perturbative expansion.

See the slides of my Pavia lectures (2020):

https://www.mv.helsinki.fi/home/hoyer/Pavia/200120_Hoyer_Pavia-figs.pdf

and on the arXiv: 1807.05598v2, 1911.07286