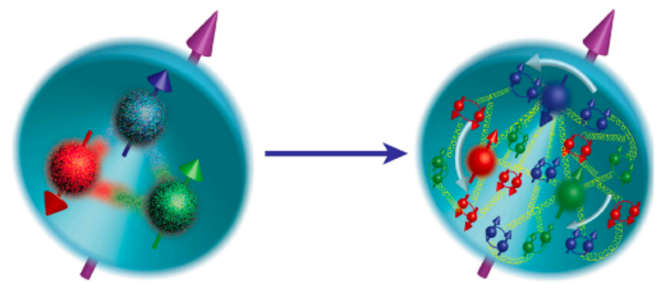
Using & relating TMDs and CTW3 functions in studies of nucleon Structure



Leonard Gamberg

Feb 6th, 2020

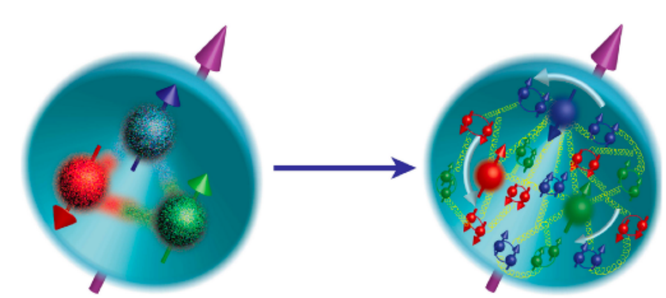
Correlations in Partonic and Hadronic Interactions - 2020 (CPHI-2020)

PennState Berks

Overview comments

- Quite some history on topic: from theoretical studies relating TMD & collinear twist 3 factorization, to phenomenological studies of TSSAs
  - Alexei's talk for most recent pheno study relies on universality of parton correlation functions (TMDs & CTW3 functions)
  - Will revisit some early studies on weighted asymmetries & pheno of TSSAs
  - Important to address given new COMPASS studies on weighted asymmetries
  - Remains an outstanding challenge to put such relations on firm theoretical foundation

TSSAs
central observables to
study 3-D structure



Some insight provided

- Consider the application of the improved Collins-Soper-Sterman (iCSS) factorization formalism for the case of polarized observables, such as the Sivers effect in SIDIS

PRD (2016) Collins, Gamberg, Prokudin, Rogers, Sato, Wang
PLB (2018) Gamberg , Metz, Pitonyak, Prokudin
INT Proc. (2019) Gamberg , Metz, Pitonyak, Prokudin

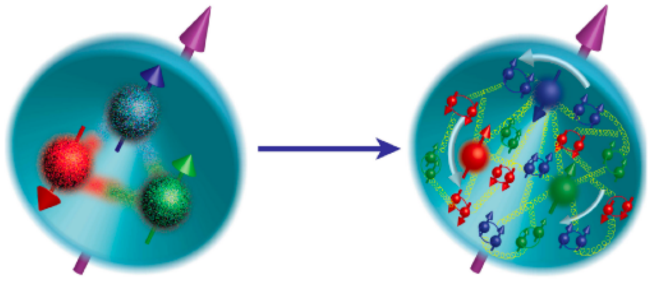
- Discuss how this study extended beyond LO; has impact on issues of relating/ matching TMDs onto collinear twist-3 functions such as between the weighted TMD Sivers function and the collinear twist-3 Qiu–Sterman function

$$T_F(x,\mu) \sim f_{1T}^{\perp(1)}(x,\mu) \quad @ \quad \mu$$

- And generalization

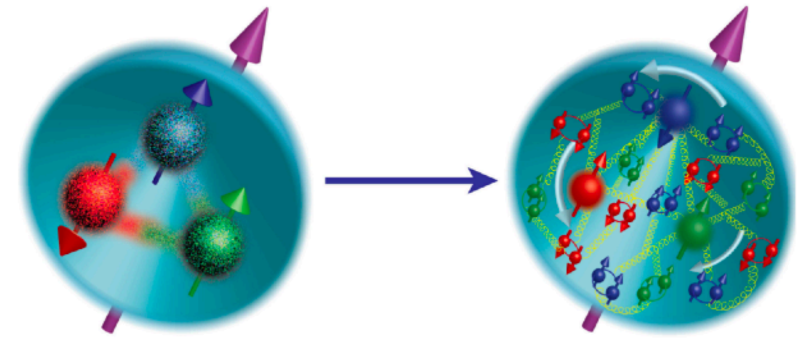
INT Proc. (2019) Gamberg, Metz, Pitonyak, Prokudin

- First down on impact on and for Pheno such relations



arXiv:1212.1701 Accardi et al.

PR picture



EIC White Paper

$W(x, b_T, k_T)$

Wigner distributions

$\int d^2 b_T$ $\int d^2 k_T$

$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

$\int d^2 k_T$ $\int d^2 b_T$

?TMD to collinear

$f(x)$

parton densities

inclusive and semi–inclusive processes

A caution from Davison Soper DIS 2015

Transverse momentum

- The partons in a proton carry momentum components transverse to the beam direction.
- Thus there are transverse momentum dependent (TMD) parton distributions

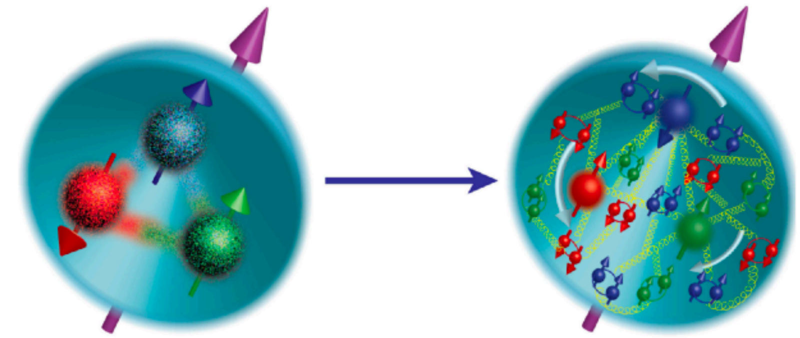
$$f_{a/A}(x, \boldsymbol{k}_\perp, Q^2)$$

- If you are going into the woods, you have to be careful: there are some subtle issues in the definitions of these.
- On an intuitive level

$$f_{a/A}(x, Q^2) \sim \int d\boldsymbol{k}_\perp \; f_{a/A}(x, \boldsymbol{k}_\perp, Q^2)$$

arXiv:1212.1701 Accardi et al.

PR picture



EIC White Paper

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$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

nb CSS TMD factorisation carried out coordinate space: can we can shed some light through CSS ?

$\int d^2 k_T$ $\int d^2 b_T$

?TMD to collinear

$f(x)$

parton densities

inclusive and semi–inclusive processes

Must consider UV, rapidity & IR,Divergences and TMD evolution,

CS NPB1982… CSS 1985, Ji,Ma,Yuan PRD 2004/5, Collins 2011 Cambridge Press, Aybat,Rogers 2011 PRD, Echevarria, Idilbi, Scimemi JHEP 2012, w/Polarization, Boer,Piljman,Mulders 2003, Kang,Yuan, Xaio 2011, Aybat,Collins, Qiu, Rogers 2012

Moments of TMDs and collinear pdfs

Naive connection of moments of TMDs and collinear pdfs based on matrix elements and a <u>**Parton Model picture of "factorization"**</u>

TMD — kinematical CT3 — dynamical CT3

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \; f_{1T}^{\perp}(x, k_T) \;=\; f_{1T}^{\perp(1)}(x) \;=\; -\frac{T_F(x,x)}{2M}$$

Qiu & Sterman 1991

Boer, Mulder, Pijlman (2003); Meissner (2009); ...

Bacchetta Como Proc. (2005)

TMD — kinematical CT3

$$\int d^2\vec{p}_T \, \frac{\vec{p}_T^2}{2z^2 M_h^2} \; H_1^{\perp}(z, p_T) \;=\; H_1^{\perp(1)}(z)$$

Consider the less exotic case

"Parton Model"

	TMD		CT2
$\int d^2\vec{k}_T$	$f_1(x, k_T)$	$=$	$f_1(x)$
		$\vdots$	
	TMD		CT2
$\int d^2\vec{p}_T$	$D_1(z, p_T)$	$=$	$D_1(z)$
		$\vdots$	

Ignore UV div. & effects from soft gluons

Underlies Model building
w/ and w/o evolution using TMD
and collinear evolution approach
Spectator models, GPM, …

Parton model factorisation of the "W" TMD term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2q_T W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Early impact on my work

Relation of moments of TMDs and collinear pdfs based on matrix elements and the Generalized Parton Model framework (GPM) of "factorization"

TMD — kinematical CT3 — dynamical CT3

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, f_{1T}^{\perp}(x, k_T) = f_{1T}^{\perp(1)}(x) = -\frac{T_F(x,x)}{2M}$$

Qiu & Sterman 1991

Boer, Mulder, Pijlman (2003); Meissner (2009); ...

Bacchetta Como Proc. (2005)

TMD — kinematical CT3

$$\int d^2\vec{p}_T \, \frac{\vec{p}_T^2}{2z^2 M_h^2} \, H_1^{\perp}(z, p_T) = H_1^{\perp(1)}(z)$$

Kotzinian and Mulders 1997 twist 2 & twist 3 !

$$A_1(x,y,z) = \frac{2(1-y)}{y(2-y)} \, \frac{h_{1L}^{\perp(1)u}(x) H_1^{\perp(1)u}(z)}{g_1^u(x) D_1^u(z)}.$$

$$A_T(x,y,z,|S_T|) = -|S_T| \, \frac{2(1-y)}{1+(1-y)^2} \, \frac{h_1^u(x) H_1^{\perp(1)u}(z)}{f_1^u(x) D_1^u(z)}$$

Early impact on my work

Appeal of P_{hT} weighting of cross sections results—deconvolution in terms of k_T weighted moments of TMDs many of which are twist 3 !

Mulders & Tangerman 1996

Kotzinian and Mulders 1997 twist 2 & twist 3 !

$$A_1(x,y,z) = \frac{2(1-y)}{y(2-y)} \frac{h_{1L}^{\perp(1)u}(x) H_1^{\perp(1)u}(z)}{g_1^u(x) D_1^u(z)}.$$

$$A_T(x,y,z,|S_T|) = -|S_T| \frac{2(1-y)}{1+(1-y)^2} \frac{h_1^u(x) H_1^{\perp(1)u}(z)}{f_1^u(x) D_1^u(z)}$$

Boer & Mulders 1997 PRD many more Weighted Asymmetries
Collins, Sivers "naive T-odd" functions, Sivers, Boer Mulders ...

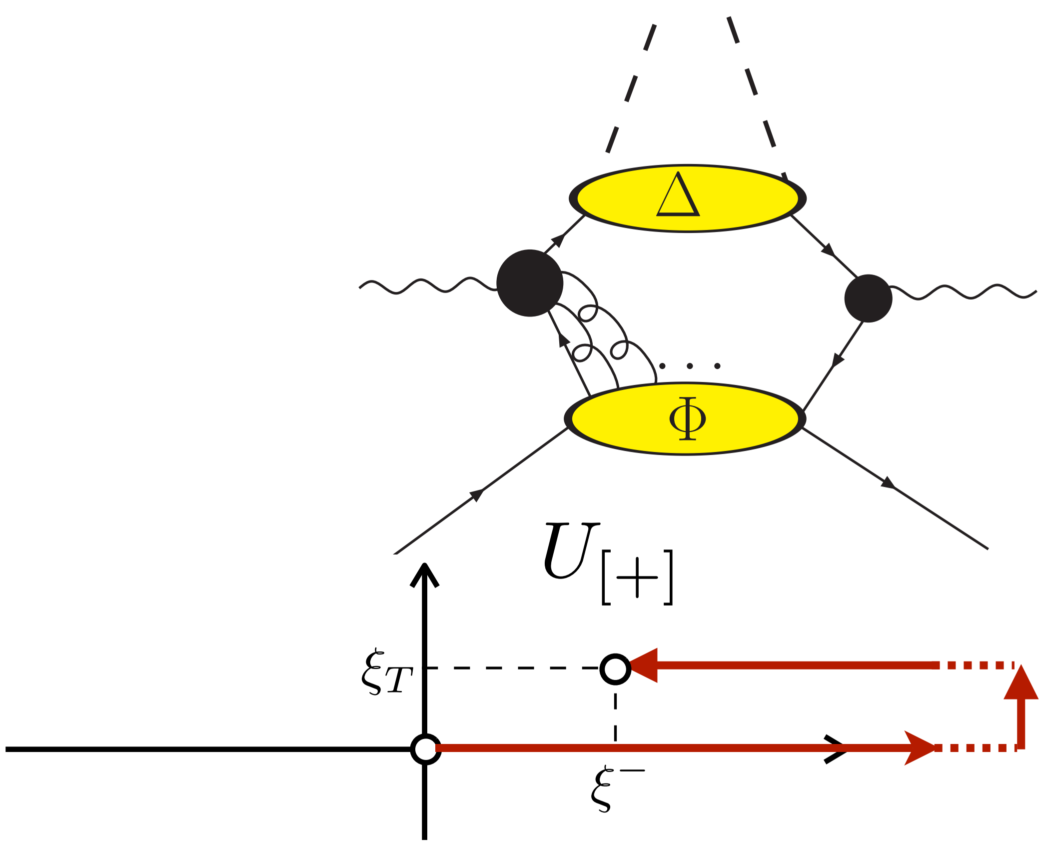
Naive Time reversal odd Effects from Color Gauge invariance via Wilson Lines

Universality is broken: Processes dependence Fund. Prediction of TMD factorization in QCD

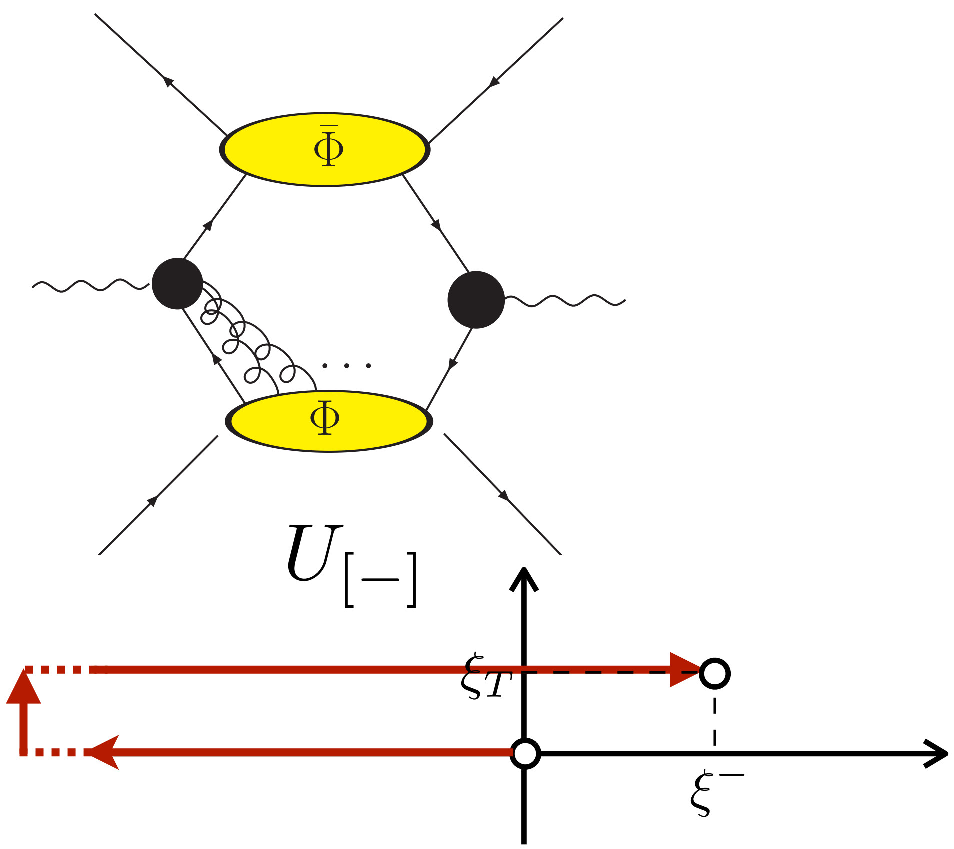
$$f_{1T_{SIDIS}}^{\perp}(x,k_T) = -f_{1T_{DY}}^{\perp}(x,k_T)$$

Process Dependence Collins PLB 02, Brodsky, Hwang, Schmidt NPB 02

$$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$$



P&T $\longleftrightarrow$



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

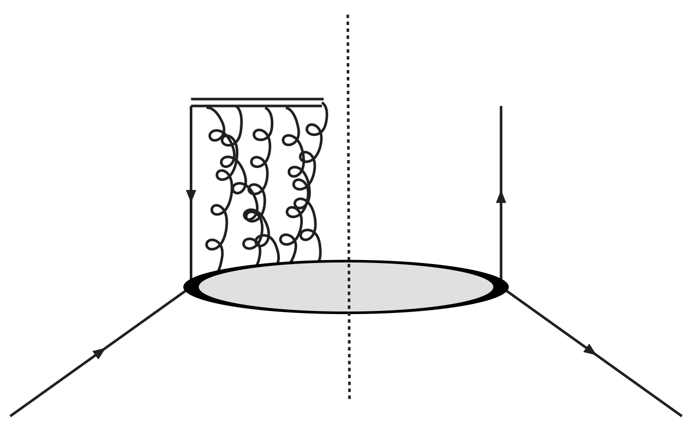
Sivers 1990 PRD
Collins 2002 PRD

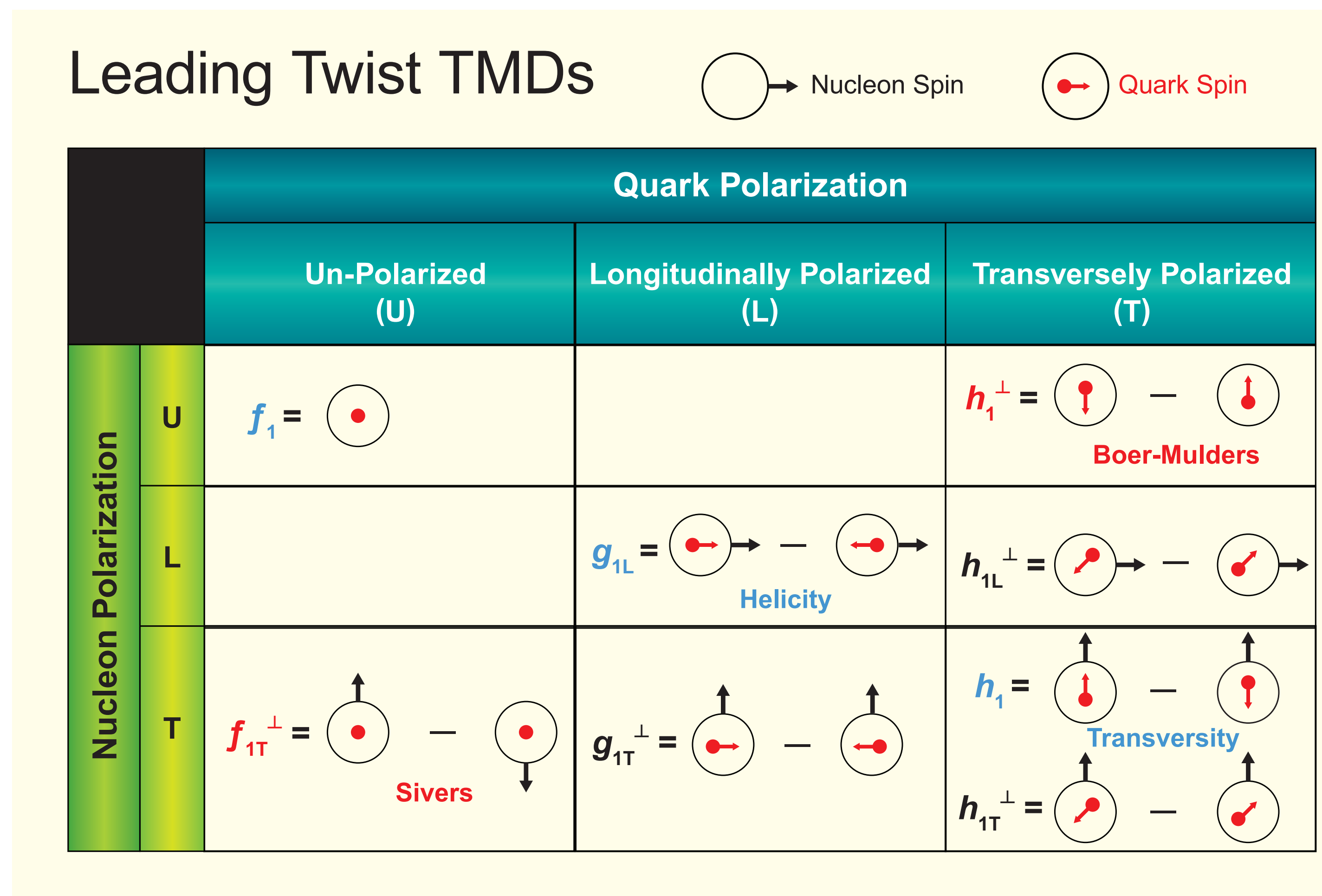
With FSIs richer polarization correlations
6 TMDs to 8

$$\mathcal{C}\big[w f D\big] = x \sum e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big], \qquad F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right], \qquad F_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$





Pheno studies of weighted asymmetries began in earnest

Early Work Weighted Asymmetries predictions

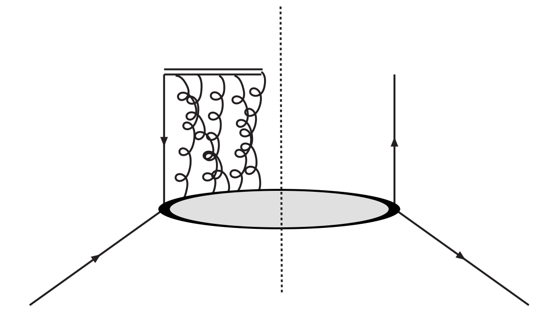
PHYSICAL REVIEW D **67**, 071504(R) (2003)

Novel transversity properties in semi-inclusive deep inelastic scattering

Leonard P. Gamberg
Division of Science, Penn State-Berks Lehigh Valley College, Reading, Pennsylvania 19610

Gary R. Goldstein
Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155

Karo A. Oganessyan*
INFN-Laboratori Nazionali di Frascati, via Enrico Fermi 40, I-00044 Frascati, Italy
(Received 12 January 2003; revised manuscript received 4 February 2003; published 29 April 2003)



Weighted Boer Mulders & Collins functions

$$\left\langle \frac{|P_{h\perp}^2|}{MM_h}\cos 2\phi \right\rangle_{UU} = \frac{\int d^2P_{h\perp} \frac{|P_{h\perp}^2|}{MM_h}\cos 2\phi \;\, d\sigma}{\int d^2P_{h\perp} \;\, d\sigma}$$

A FAVORITE OF STAN

$$= \frac{8(1-y)\sum_q e_q^2 h_1^{\perp(1)}(x) z^2 H_1^{\perp(1)}(z)}{(1+(1-y)^2)\sum_q e_q^2 f_1(x) D_1(z)}$$

Weighted Sivers

$$\left\langle \frac{|P_{h\perp}|}{M}\sin(\phi-\phi_S) \right\rangle_{UT} = \frac{\int d^2P_{h\perp} \frac{|P_{h\perp}|}{M}\sin(\phi-\phi_S) d\sigma}{\int d^2P_{h\perp} \;\, d\sigma} = \frac{(1+(1-y)^2)\sum_q e_q^2 f_{1T}^{\perp(1)}(x) z D_1^q(z)}{(1+(1-y)^2)\sum_q e_q^2 f_1(x) D_1(z)}, \qquad (10)$$

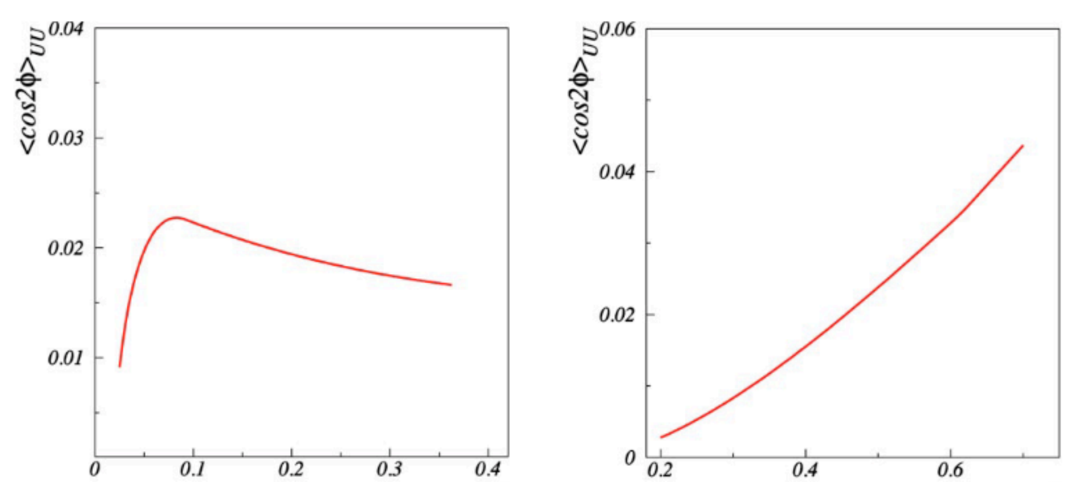


FIG. 1. Left panel: The $\langle \cos 2\phi \rangle_{UU}$ asymmetry for π^+ production as a function of x. Right panel: The $\langle \cos 2\phi \rangle_{UU}$ asymmetry for π^+ production as a function of z.

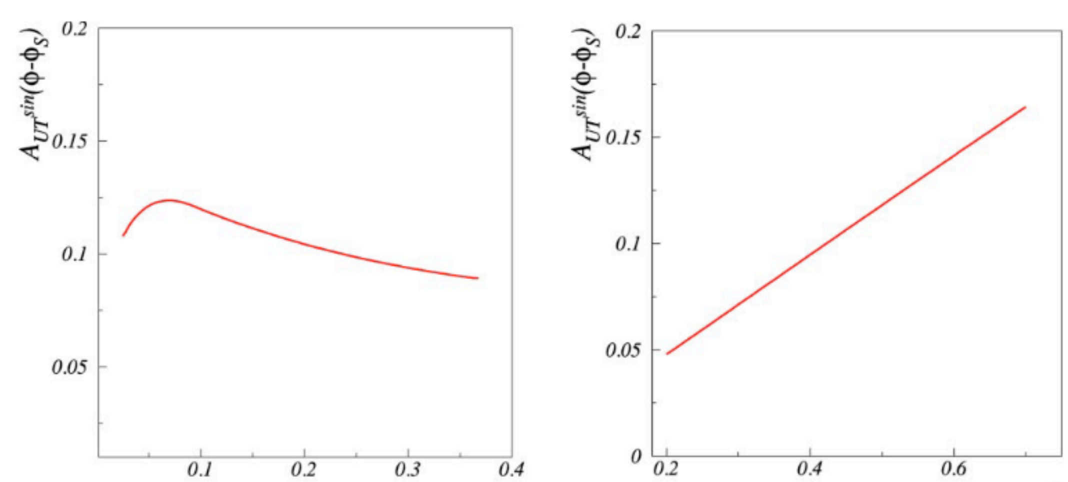


FIG. 2. Left panel: The $A_{UT}^{\sin(\phi-\phi_S)}$ x dependent Sivers asymmetry. Right panel: The $A_{UT}^{\sin(\phi-\phi_S)}$ z dependent Sivers asymmetry.

PHYSICAL REVIEW D **68**, 051501(R) (2003)

A mechanism for the *T*-odd pion fragmentation function

Leonard P. Gamberg
Division of Science, Penn State–Berks Lehigh Valley College, Reading, Pennsylvania 19610, USA

Gary R. Goldstein
Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA

Karo A. Oganessyan
INFN–Laboratori Nazionali di Frascati, I-00044 Frascati, via Enrico Fermi 40, Italy
and DESY, Deutsches Elektronen Synchrotron Notkestrasse 85, 22603 Hamburg, Germany
(Received 21 July 2003; published 11 September 2003)

Weighted Collins Asymmetry

PHYSICAL REVIEW D **68**, 051501(R) (2003)

$$\left\langle \frac{P_{h\perp}}{M_\pi}\sin(\phi+\phi_s)\right\rangle_{UT} = \frac{\int d\phi_s \int d^2P_{h\perp} P_{h\perp}/M_\pi \sin(\phi+\phi_s)(d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_s \int d^2P_{h\perp}(d\sigma^\uparrow + d\sigma^\downarrow)} = |S_T| \frac{2(1-y)\sum_q e_q^2 h_1(x) z H_1^{\perp(1)}(z)}{(1+(1-y)^2)\sum_q e_q^2 f_1(x) D_1(z)}. \qquad (16)$$

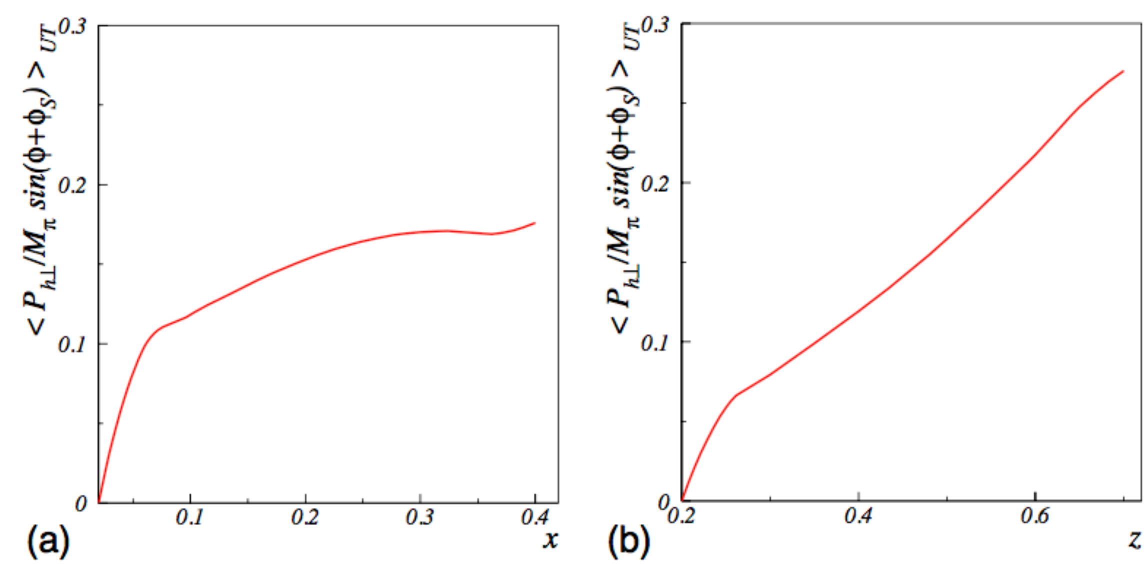


FIG. 4. (a) The $\langle P_{h\perp}/M_h \sin(\phi+\phi_s)\rangle_{UT}$ asymmetry for π^+ production as a function of x. (b) The $\langle P_{h\perp}/M_h \sin(\phi+\phi_s)\rangle_{UT}$ asymmetry for π^+ production as a function of z.

Transversity properties of quarks and hadrons in SIDIS and Drell-Yan

Leonard P. Gamberg (Penn State U., Berks-Lehigh Valley), Gary R. Goldstein (Tufts U., Inst. of Cosmology). Sep 2005. 4 pp.

Published in **AIP Conf.Proc. 792 (2005) no.1, 941-944**
DOI: 10.1063/1.2122192
Conference: C05-04-27 Proceedings
e-Print: **hep-ph/0509312** | PDF

$$A_{UT}^{\sin(\phi+\phi_s)} \approx \frac{M_\pi}{\langle P_{h\perp}\rangle}\left\langle \frac{P_{h\perp}}{M_\pi}\sin(\phi+\phi_s)\right\rangle \quad \text{and} \quad A_{UT}^{\sin(\phi-\phi_s)} \approx \frac{M}{\langle P_{h\perp}\rangle}\left\langle \frac{P_{h\perp}}{M}\sin(\phi\pm\phi_s)\right\rangle$$

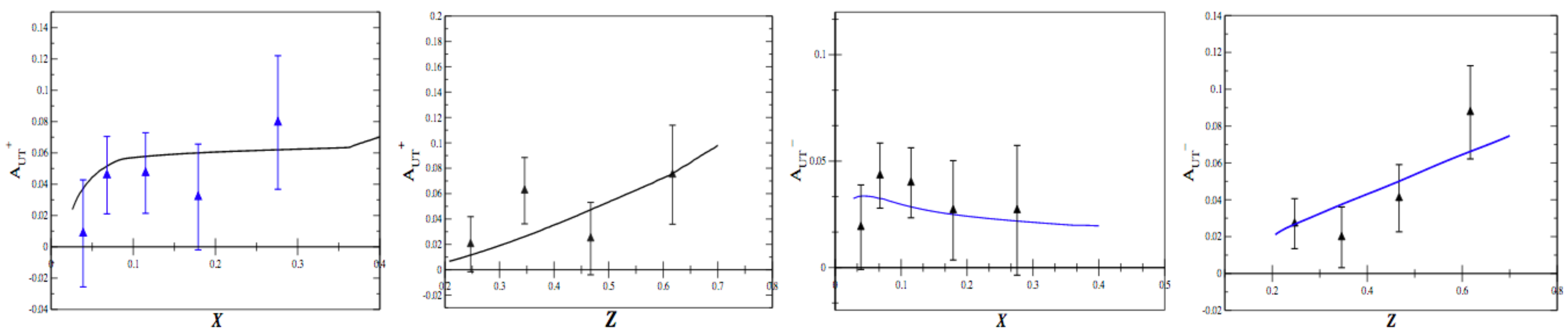


FIGURE 1. Left two Panel: The $\langle\sin(\phi+\phi_s)\rangle_{UT}$ asymmetry for π^+ production as a function of x and z compared to the HERMES data [15]. Right two Panels: The $\langle\sin(\phi-\phi_S)\rangle_{UT}$ as a function of x and z.

Eur. Phys. J. A **45**, 373–388 (2010)
DOI 10.1140/epja/i2010-11016-y

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Weighted azimuthal asymmetries in a diquark spectator model

A. Bacchetta[1,2,a], M. Radici[2,b], F. Conti[1,2,c], and M. Guagnelli[2,d]

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Received: 19 March 2010 / Revised: 8 June 2010
Published online: 5 August 2010 – © Società Italiana di Fisica / Springer-Verlag 2010
Communicated by A. Schäfer

Abstract. We analytically calculate weighted azimuthal asymmetries in semi-inclusive lepton-nucleon deep-inelastic scattering and Drell-Yan processes, using transverse-momentum–dependent partonic densities obtained in a diquark spectator model. We compare the asymmetries with available preliminary experimental data, in particular for the Collins and the Sivers effect. We make predictions for other cases of interest in running and planned experiments.

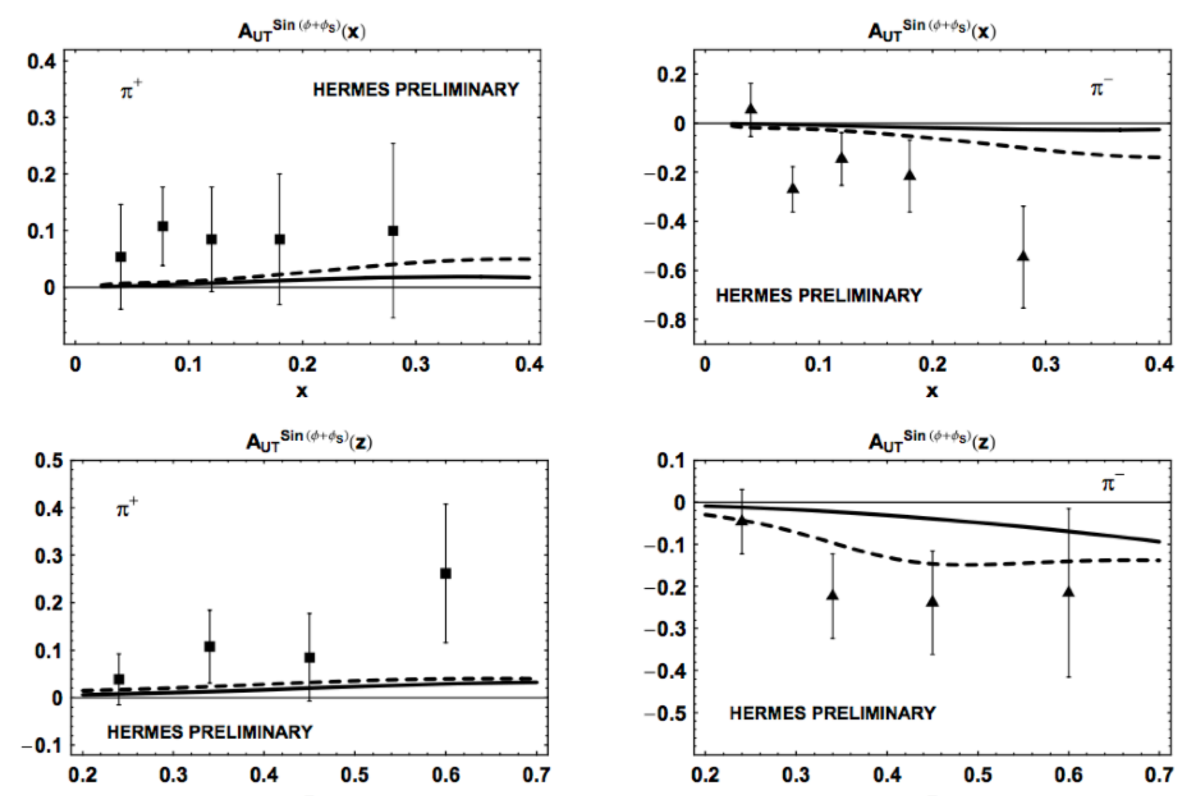

Fig. 8. The weighted single-spin asymmetry $A_{UT}^{\frac{Q_T}{M}\sin(\phi_h+\phi_S)}$ of eq. (12) in the SIDIS kinematics of HERMES (Collins effect) with emission of π^+ (left) and π^- (right), as a function of x (above) and z (below). Experimental data from ref. [15]. The dashed line for the result of the spectator model at its scale $Q_0^2 = 0.3\,\mathrm{GeV}^2$, the solid line for the result at the experimental scale $Q^2 = 2.5\,\mathrm{GeV}^2$ (see text for details about evolution).

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Eur. Phys. J. A **45**, 373–388 (2010)
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THE EUROPEAN
PHYSICAL JOURNAL A

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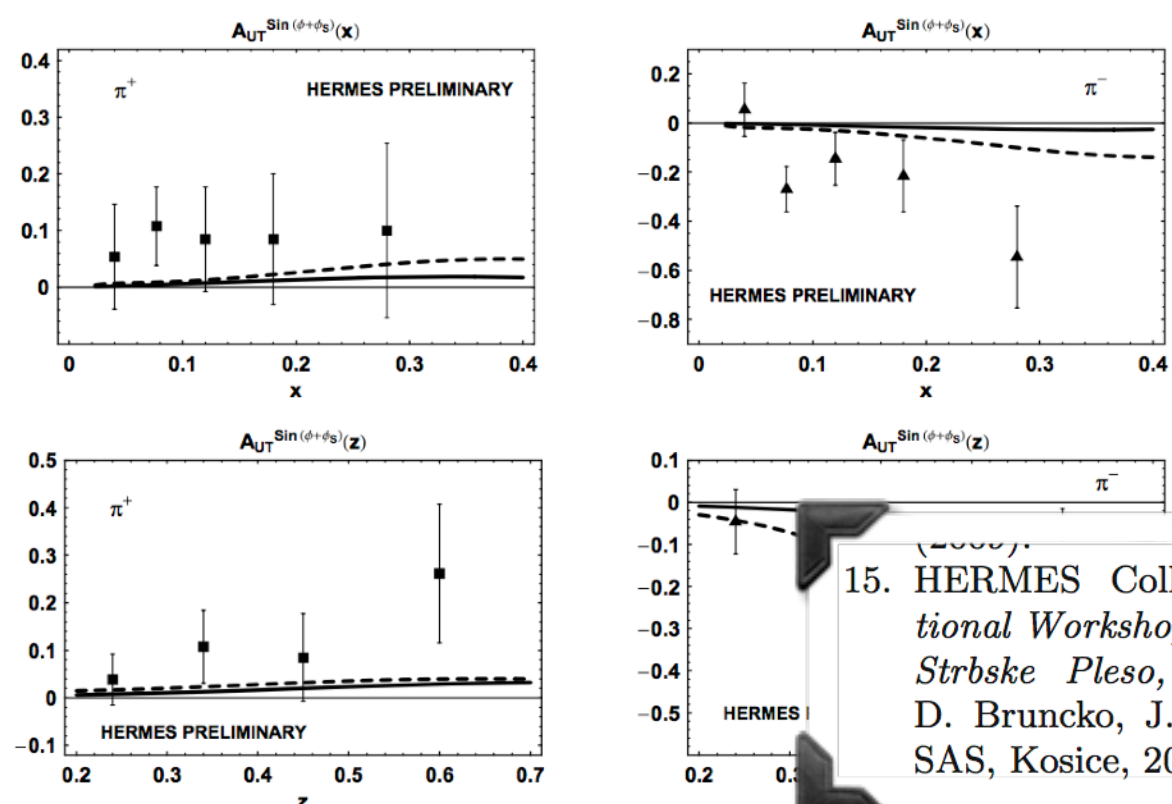


ELUSIVE DATA !

15. HERMES Collaboration (R. Seidl), in *12th International Workshop on Deep Inelastic Scattering (DIS 2004), Strbske Pleso, Slovakia, 14-18 Apr 2004*, edited by D. Bruncko, J. Ferencei, P. Strizenec (Inst. Exp. Phys. SAS, Kosice, 2004) pp. 1140–1143.

!!

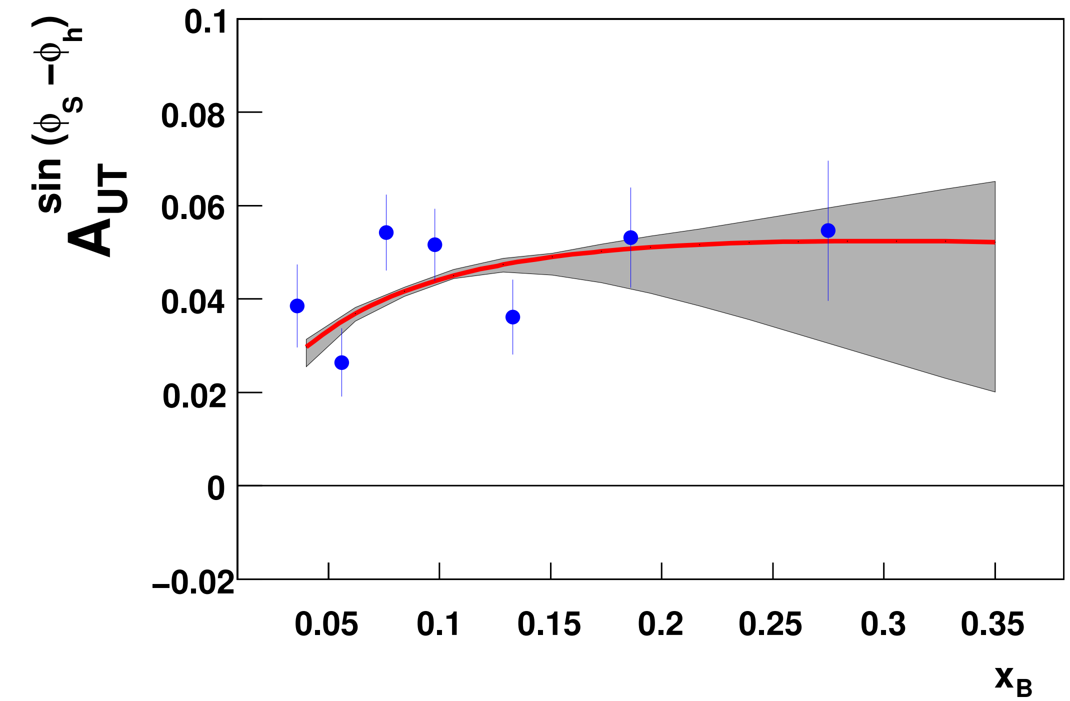
Fig. 8. The weighted single-spin asymmetry $A_{UT}^{\frac{Q_T}{M}\sin(\phi_h+\phi_S)}$ of eq. (12) in the SIDIS kinematics of HERMES (Collins effect) with emission of π^+ (left) and π^- (right), as a function of x (above) and z (below). Experimental data from ref. [15]. The dashed line for the result of the spectator model at its scale $Q_0^2 = 0.3\,\mathrm{GeV}^2$, the solid line for the result at the experimental scale $Q^2 = 2.5\,\mathrm{GeV}^2$ (see text for details about evolution).

Impact/Use in Global analysis of TSSAs
Attempts to demonstrate
process dependence of Sivers effect

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$
$$= -2M f_{1T}^{\perp(1)}(x)$$

Calculate polarized cross section for $P^{\uparrow}P \rightarrow Jet\ X$

Indication on the process-dependence of the Sivers effect
L. Gamberg, Z. Kang, A. Prokudin, **Phys. Rev. Lett. 110, 232301 (2013)**

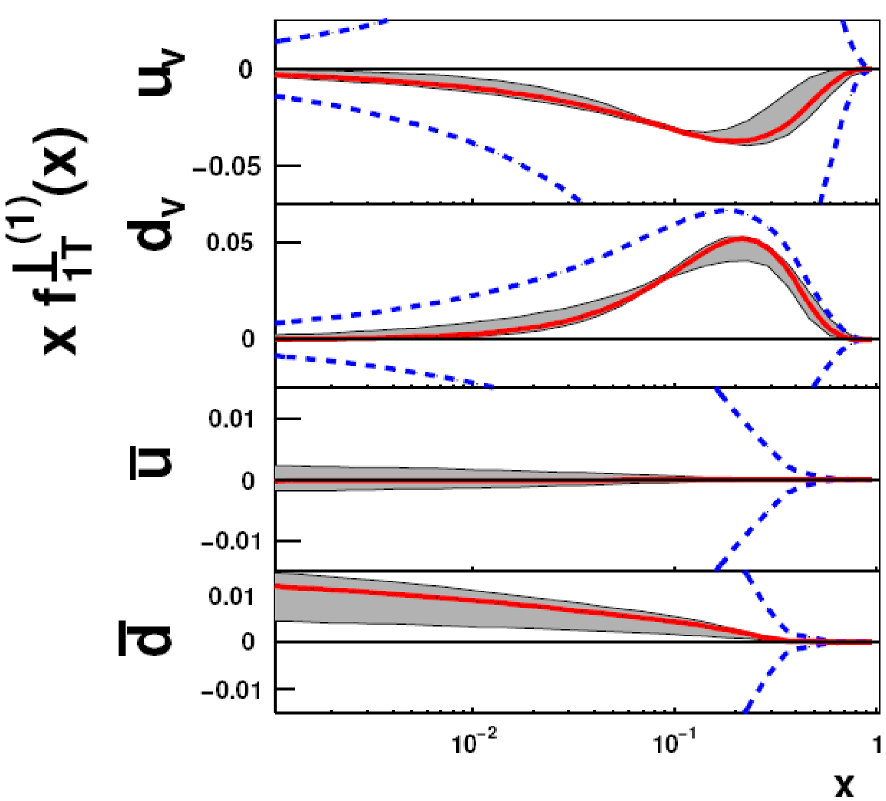


CGI-GPM

Torino Model parametrization but w/ color factors *ie Gauge links*

Twist-3 TMD relation

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x, k_T^2)$$
$$= -2M f_{1T}^{\perp(1)}(x)$$



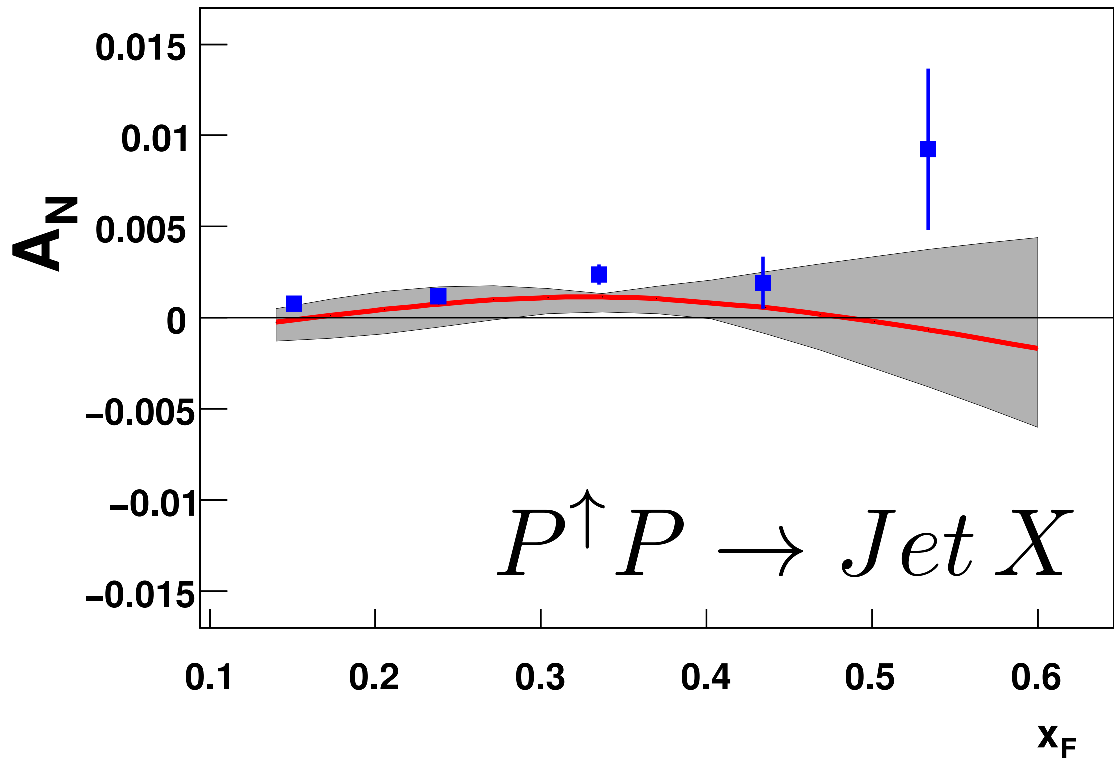
We calculate jet A_N in twist-3:

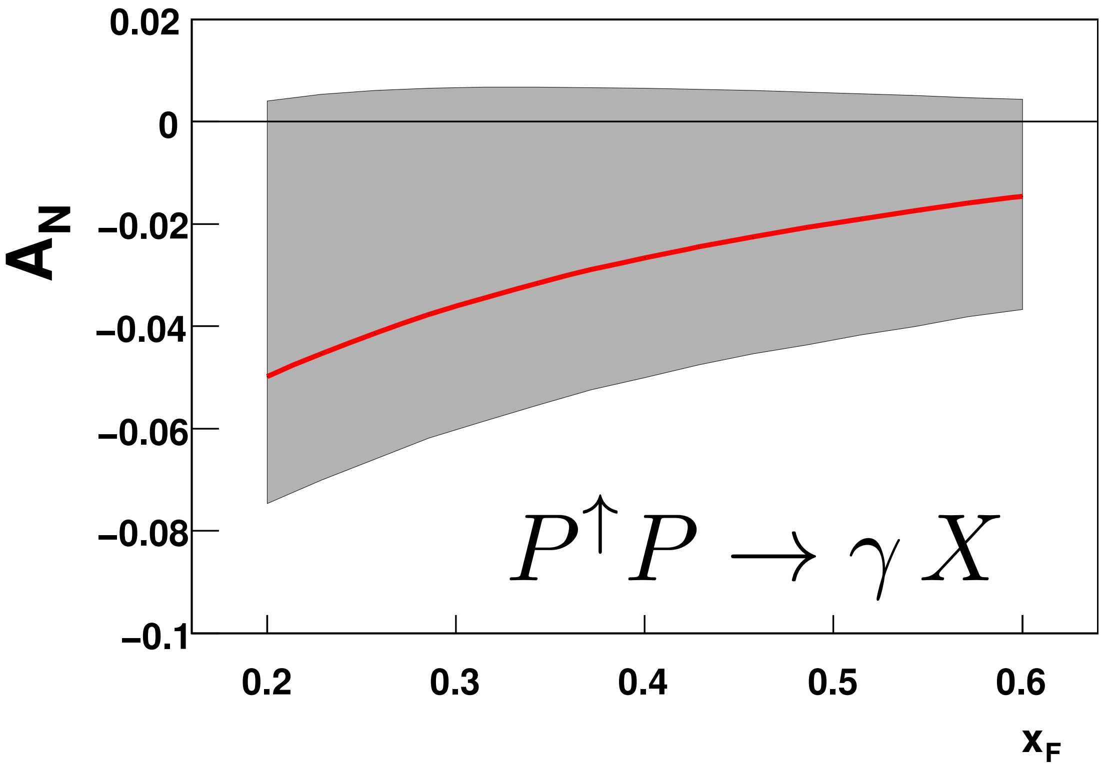
$$E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} = \epsilon_{\alpha\beta} s_\perp^\alpha P_{J\perp}^\beta \frac{\alpha_s^2}{s} \sum_{a,b} \int \frac{dx}{x} \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \left[T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \right]$$
$$\times \frac{1}{\hat{u}} H_{ab\rightarrow c}^{\text{Sivers}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM

L. Gamberg, Z. Kang, A. Prokudin,
Phys. Rev. Lett. 110 (2013)

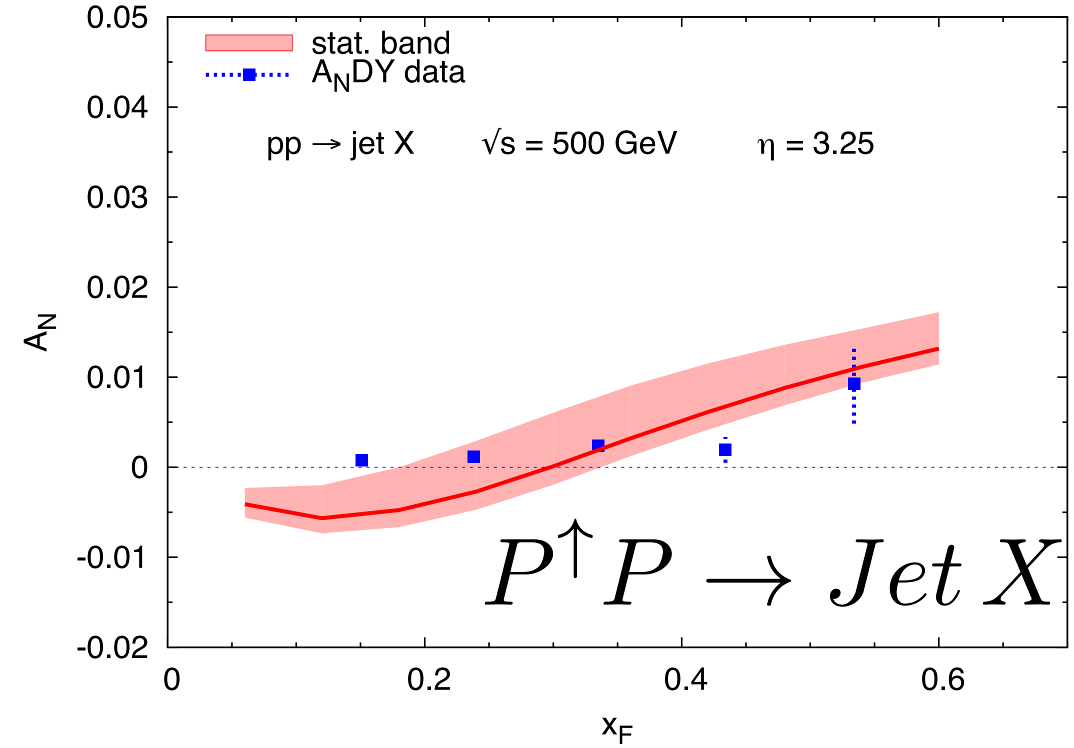
w/ color factors

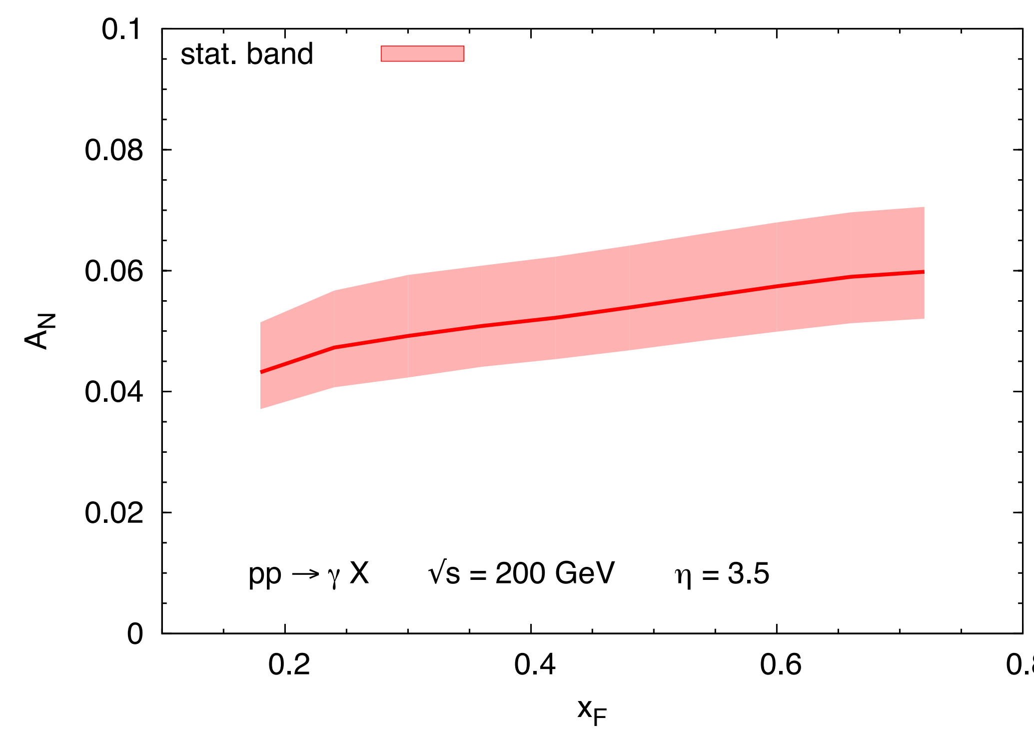




Anselmino et al.
Phys.Rev. D88 (2013)

w/o color factors





Used in analyses of inclusive & semi-inclusive processes as <u>common mechanism</u> for TSSA ?

Left-right spin asymmetry in $\ell N^\uparrow \to hX$

Leonard Gamberg, Zhong-Bo Kang, Andreas Metz, Daniel Pitonyak, and Alexei Prokudin
Phys. Rev. D **90**, 074012 – Published 9 October 2014

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}S_{P\perp}^{\mu}P_{h\perp}^{\nu}\sum_q e_q^2 \int_{z_{\rm min}}^1 \frac{dz}{z^3}\frac{1}{S+T/z}\frac{1}{x}\Bigg\{-\frac{\pi M}{\hat{u}}D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}h_1^q(x)\Bigg\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\,\hat{u}}{\hat{t}^2}\right]$$
$$+\frac{1}{z}H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\hat{t}^3}\right]\Bigg\}\Bigg\},$$

$$\pi\, F_{FT}^q(x,x) = \int d^2\vec{k}_\perp\,\frac{\vec{k}_\perp^{\,2}}{2M^2}\,f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2})\Big|_{\rm SIDIS}$$

$$\hat{H}^{h/q}(z) = z^2\int d^2\vec{p}_\perp\,\frac{\vec{p}_\perp^{\,2}}{2M_h^2}\,H_1^{\perp h/q}(z, z^2\vec{p}_\perp^{\,2})$$

& EOMs

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1).$$

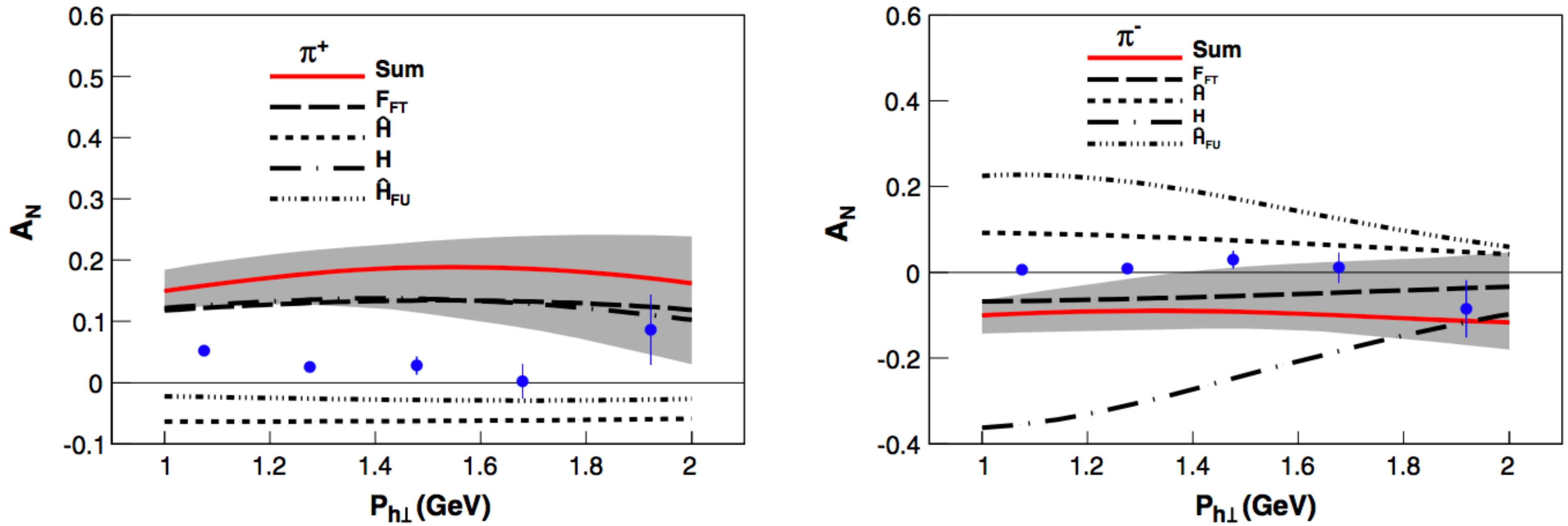


FIG. 3 (color online). A_N as function of $P_{h\perp}$ for π^+ (left panel) and π^- (right panel) production for lepton-proton collisions at $0.1 < x_F^{\rm H} < 0.2$ ($\langle x_F^{\rm H}\rangle \simeq 0.15$) and $\sqrt{S} = 7.25$ GeV. The data are from Ref. [32]. The description of lines is the same as in Fig 2.

Physics Letters B 770 (2017) 242–251

Phenomenological constraints on A_N in $p^\uparrow p \to \pi X$ from Lorentz invariance relations

Leonard Gamberg [a], Zhong-Bo Kang [b,c,d], Daniel Pitonyak [a,*], Alexei Prokudin [a,e]

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}}$$

$$\times\, h_1^a(x)\, f_1^b(x')\left\{\left[H_1^{\perp(1),\pi/c}(z) - z\frac{dH_1^{\perp(1),\pi/c}(z)}{dz}\right]\tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),\pi/c}(z) + \frac{1}{z}\tilde{H}^{\pi/c}(z)\right]\tilde{S}^i_H\right\}$$

Fragmentation contribution from "Collins like" piece drives A_N

We are able to provide a constraint on the fragmentation pieces that remain from the lesser known dynamical functions.

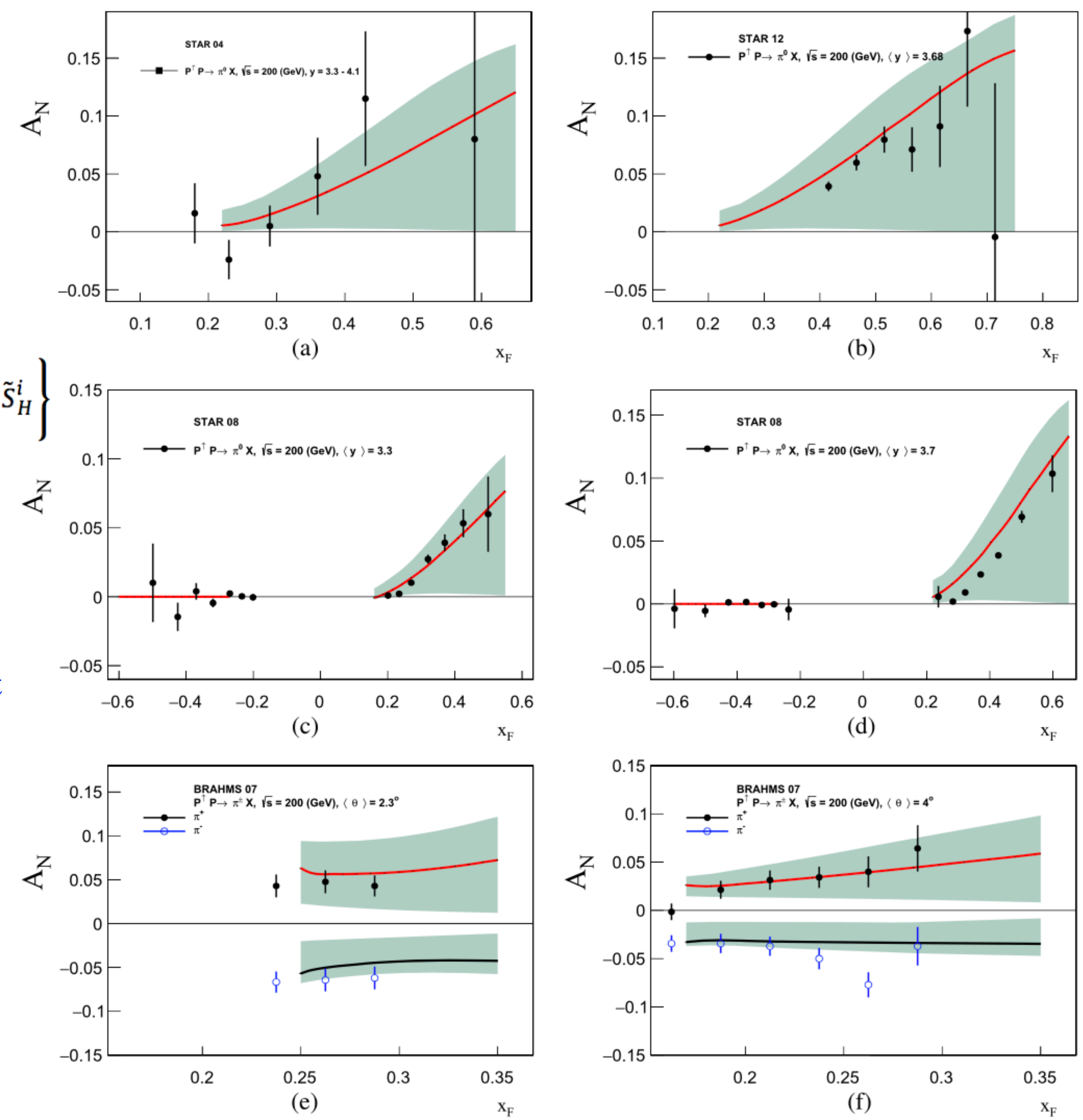
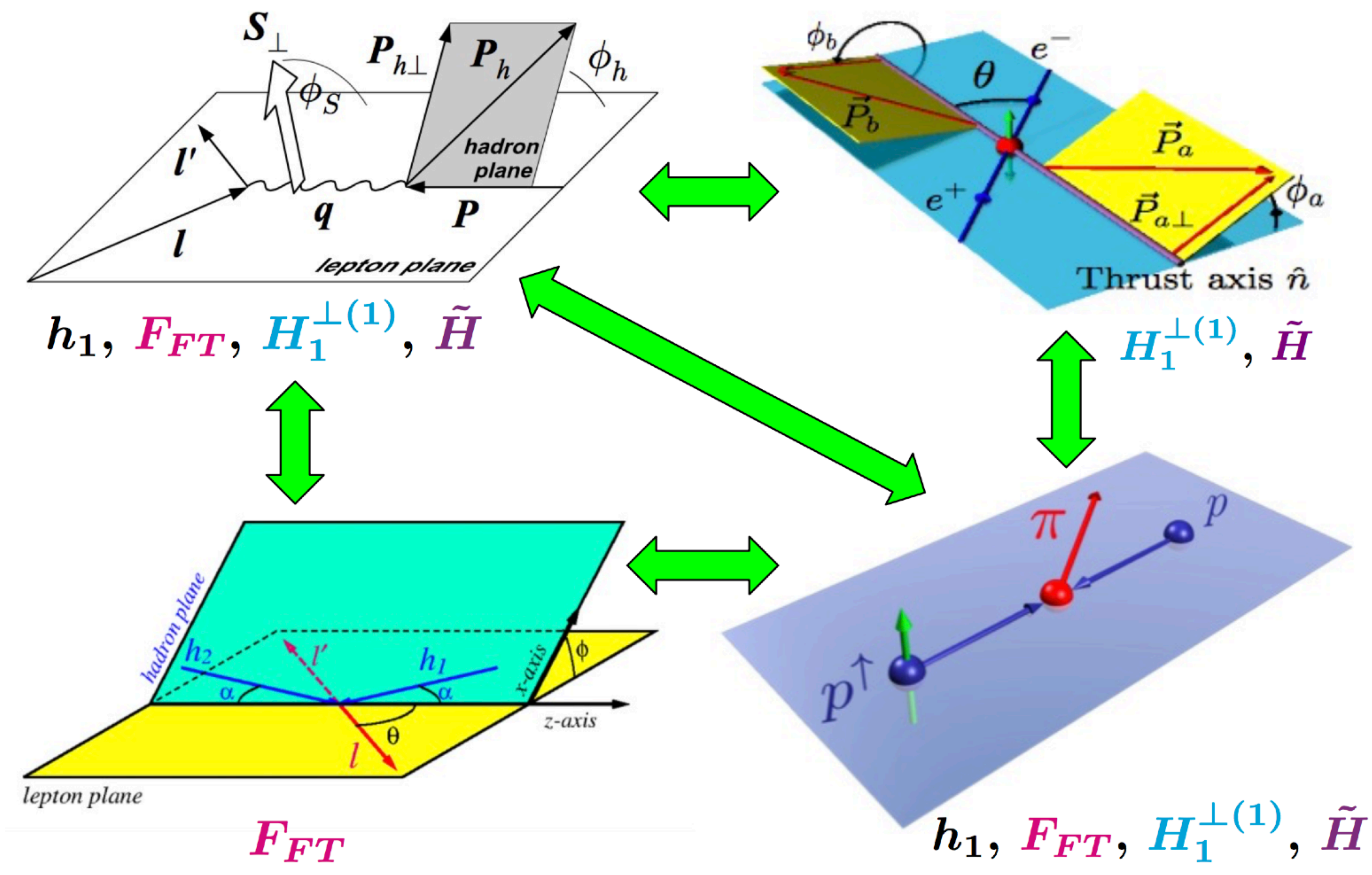


Fig. 1. Calculations of the contribution to A_N from Eq. (16) (using the approximation (20)) compared to (a) STAR Collaboration 2004 data [35] on A_N for π^0, (b) STAR Collaboration 2012 data [36] on A_N for π^0 at $\langle y\rangle = 3.68$, (c) STAR Collaboration 2008 data [39] on A_N for π^0 at $\langle y\rangle = 3.3$, (d) STAR Collaboration 2008 data [39] on A_N for π^0 at $\langle y\rangle = 3.7$, (e) BRAHMS Collaboration 2007 data [35] on A_N for $\pi^\pm$ (black closed data π^+, blue open data π^-) at $\langle\theta\rangle = 2.3^o$, (f) BRAHMS Collaboration 2007 data [35] on A_N for $\pi^\pm$ (black closed data π^+, blue open data π^-) at $\langle\theta\rangle = 4^o$. The solid lines correspond to the results using the central parameters from Ref. [82] for $h_1(x)$ and $H_1^{\perp(1)}(z)$. The shaded regions correspond to an estimate of 90% C.L. error band from Ref. [82] due to uncertainties in $h_1(x)$ and $H_1^{\perp(1)}(z)$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

PDFs and Universality



Gamberg, Kang, Pitonyak, Prokudin Phys.Lett. B770 (2017) 242 & in progress …

TMD PDFs (x, k_T)

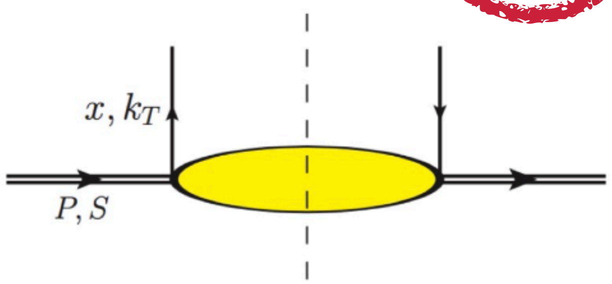
H pol. \ q pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

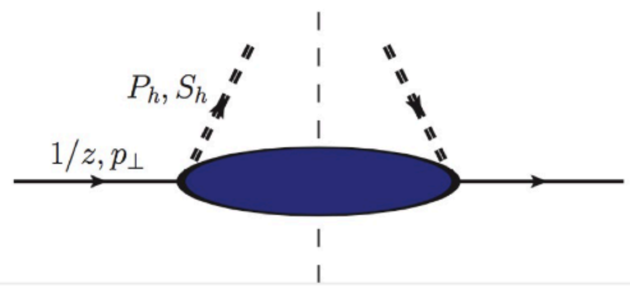
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs $(z, p_\perp)$

H pol. \ q pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))

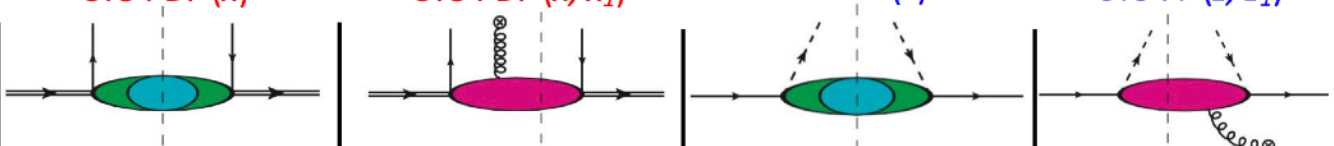




Pheno is based on these identifications of twist 2 TMDs, moments of TMDs and CTW3 fncts

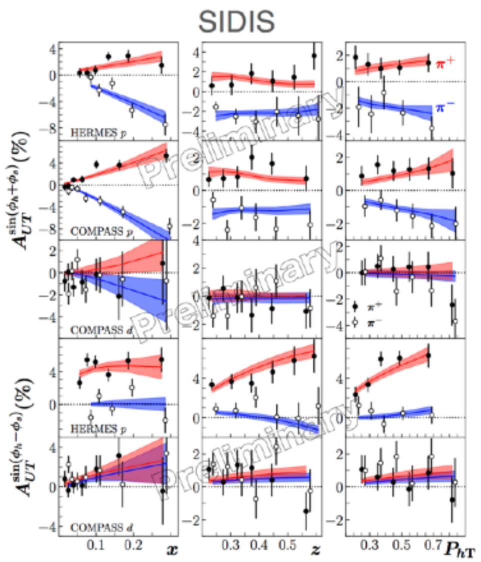
A. Metz and D. Pitonyak, Phys. Lett. B723, 365

The circled functions participate
Inclusive pion production, SIDIS, SIA, DY



Hadron Pol.	CT3 PDF (x) intrinsic	CT3 PDF (x) kinematical	CT3 PDF (x, x_1) dynamical	CT3 FF (z) intrinsic	CT3 FF (z) kinematical	CT3 FF (z, z_1) dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\mathfrak{R},\mathfrak{I}}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\mathfrak{R},\mathfrak{I}}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\mathfrak{R},\mathfrak{I}}, \hat{G}_{FT}^{\mathfrak{R},\mathfrak{I}}$

Postage Stamp of global fit of TSSAs see Alexei's talk

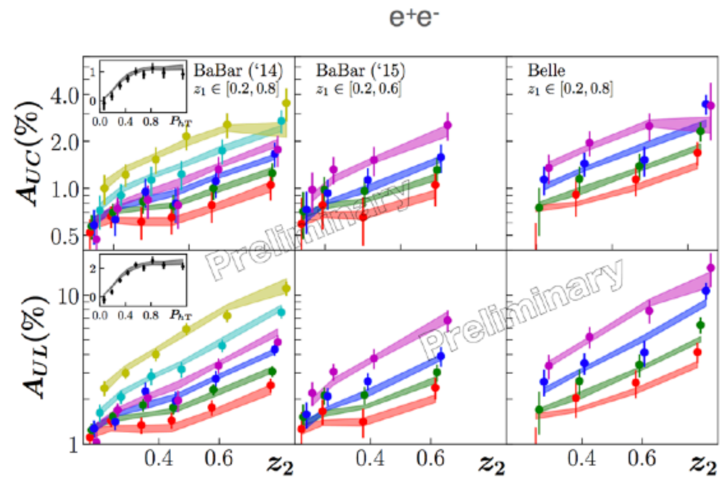


Collins asymmetry

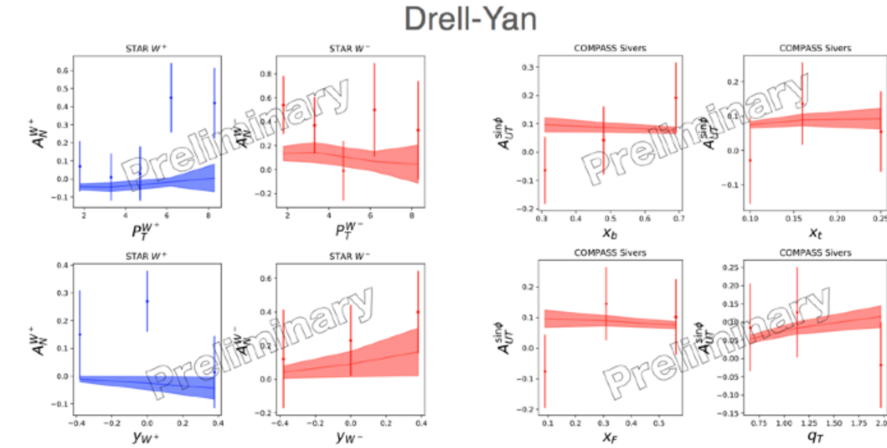
$$\frac{\chi^2}{npoints} = \frac{107.1}{126} = 0.85$$

Sivers asymmetry

$$\frac{\chi^2}{npoints} = \frac{85.4}{88} = 0.97$$

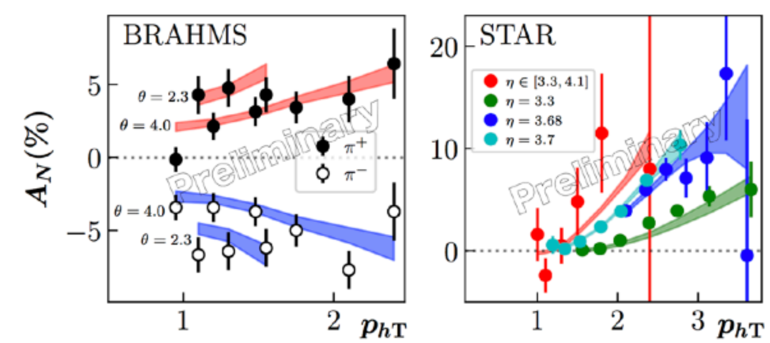


$$\frac{\chi^2}{npoints} = \frac{149.6}{176} = 0.85$$



$$\frac{\chi^2}{npoints} = \frac{29.8}{17} = 1.75 \qquad \frac{\chi^2}{npoints} = \frac{7.6}{12} = 0.63$$

proton-proton A_N



$$\frac{\chi^2}{npoints} = \frac{72.0}{60} = 1.2$$

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

Transversity

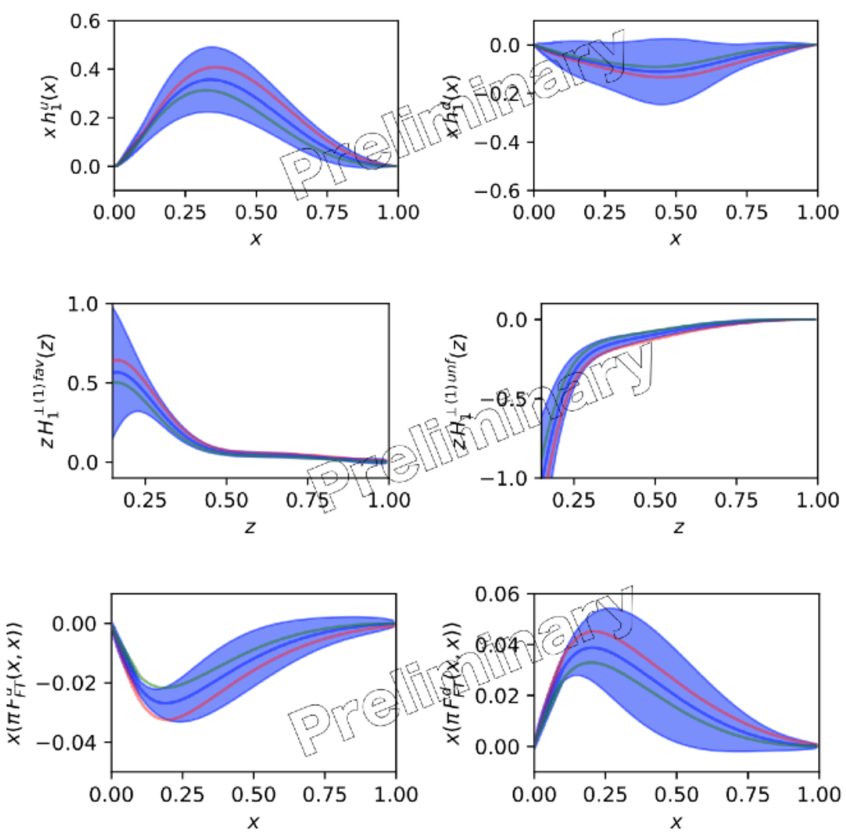
$h_1(x)$

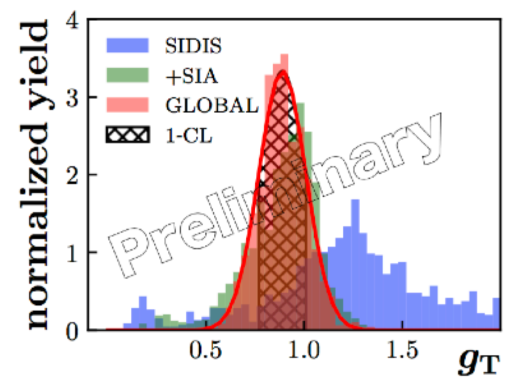
Collins FF

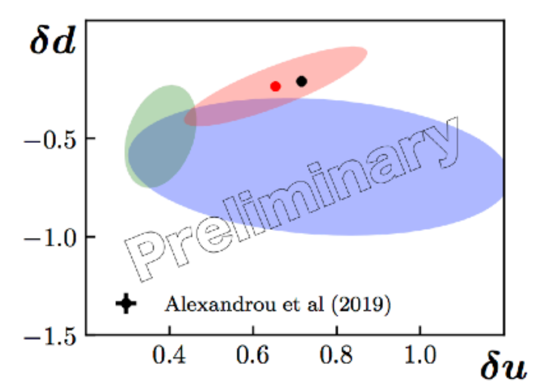
$H_1^{\perp(1)}(z)$

Sivers

$f_{1T}^{\perp(1)}(x)$







- Isovector tensor charge $g_T = \delta u - \delta d$
  $g_T = 0.89 \pm 0.12$ compatible with lattice results

- Tensor charge from up and down quarks is constrained and compatible with lattice results

δu and δd Q^2=4 GeV²

δu= 0.65 ± 0.22

δd= -0.24 ± 0.2

Beyond parton model

Important theoretical work to unify TSSAs in twist 2 and twist 3 processes beyond the GPM: a series of papers Ji, Qiu, Vogelsang, Yuan, 2006 on unified framework to study Sivers effect in DY & SIDIS

Utilized

$$\int d^2\vec{k}_\perp q(z_1, k_\perp) = q(z_1), \qquad \int d^2\vec{k}_\perp \bar{q}(z_2, k_\perp) = \bar{q}(z_2).$$

$$\frac{1}{M_P}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x, k_\perp) = T_F(x, x).$$

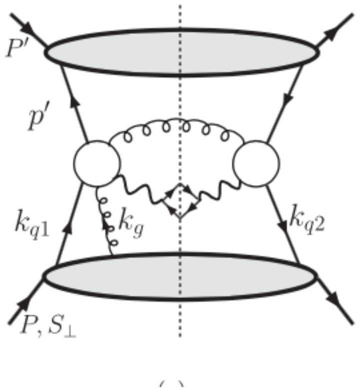
Fixed order polarized CS PQCD

$$\frac{d^4\Delta\sigma(S_\perp)}{dQ^2dyd^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_{\perp\alpha}q_{\perp\beta}\frac{\alpha_s}{2\pi^2}\int\frac{dx}{x}\frac{dx'}{x'} \times\sum_q e_q^2[(H_q^s+H_q^h)\bar{q}(x')+(H_g^s+H_g^h)g(x')]$$

TMD factorization pol. CS

$$\frac{d^4\Delta\sigma(S)}{dQ^2dyd^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_{\perp\alpha}q_{\perp\beta}\frac{1}{M_P}\int d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}d^2\vec{\lambda}_\perp \times\frac{\vec{k}_{1\perp}\cdot\vec{q}_\perp}{q_\perp^2}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{\lambda}_\perp-\vec{q}_\perp) \times q_T(z_1,k_{1\perp},\zeta_1)\bar{q}(z_2,k_{2\perp},\zeta_2)H(Q^2)\times(S(\lambda_\perp))^{-1}$$

Ji, Qiu, Vogelsang, Yuan, 2006



$q_\perp \ll Q$

$k_\perp \gg \Lambda_{\rm QCD}$

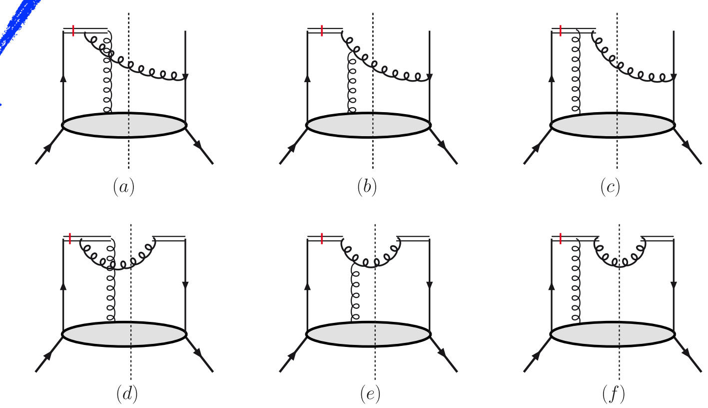


FIG. 10 (color online). Same as Fig. 9, but for the hard-pole contributions.

Factorized Cross sections match in the overlap region

Calculated the "asymptotic term" from FO & TMD

$$\Lambda_{\rm QCD} \ll q_\perp \ll Q$$

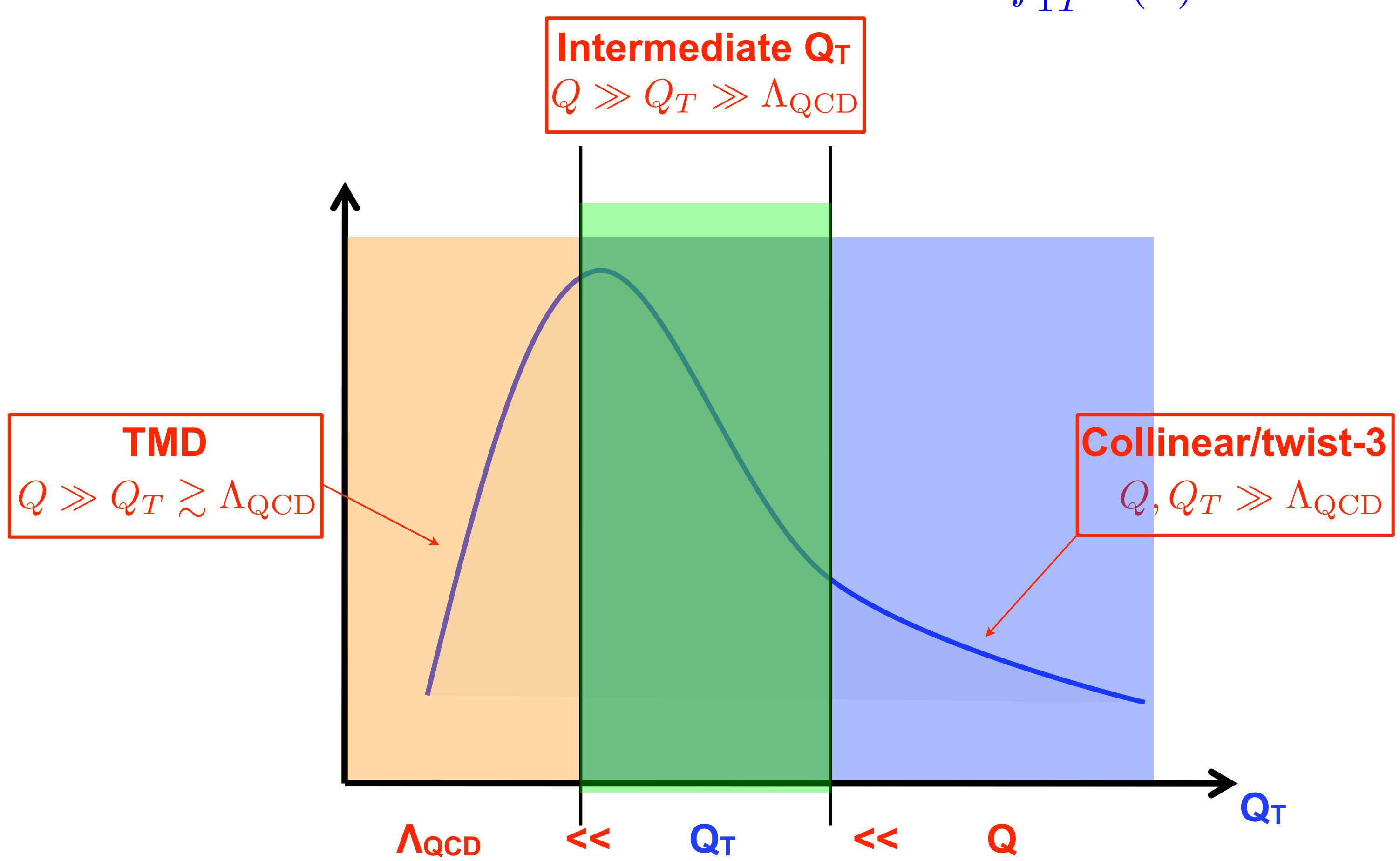
We have given the first-order perturbative result for the Sivers function in Eq. (39). As was shown in [19] (see also [22,23]), its $k_\perp^2$-moment is related to the twist-three quark-gluon correlation function defined in Eq. (3) of Sec. II:

$$\frac{1}{M_P}\int d^2\vec{k}_\perp\vec{k}_\perp^2 q_T(x,k_\perp) = T_F(x,x). \qquad (44)$$

Generalization of Subtraction formalism of CSS to transverse polarization process in SIDIS

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) - AY(q_T, Q) + FO(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

WITH, $$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x, k_T^2)$$
$$= -2M f_{1T}^{\perp(1)}(x)$$



Ji, Qiu, Vogelsang, Yuan, 2006 on unified framework to study Sivers effect in DY & SIDIS

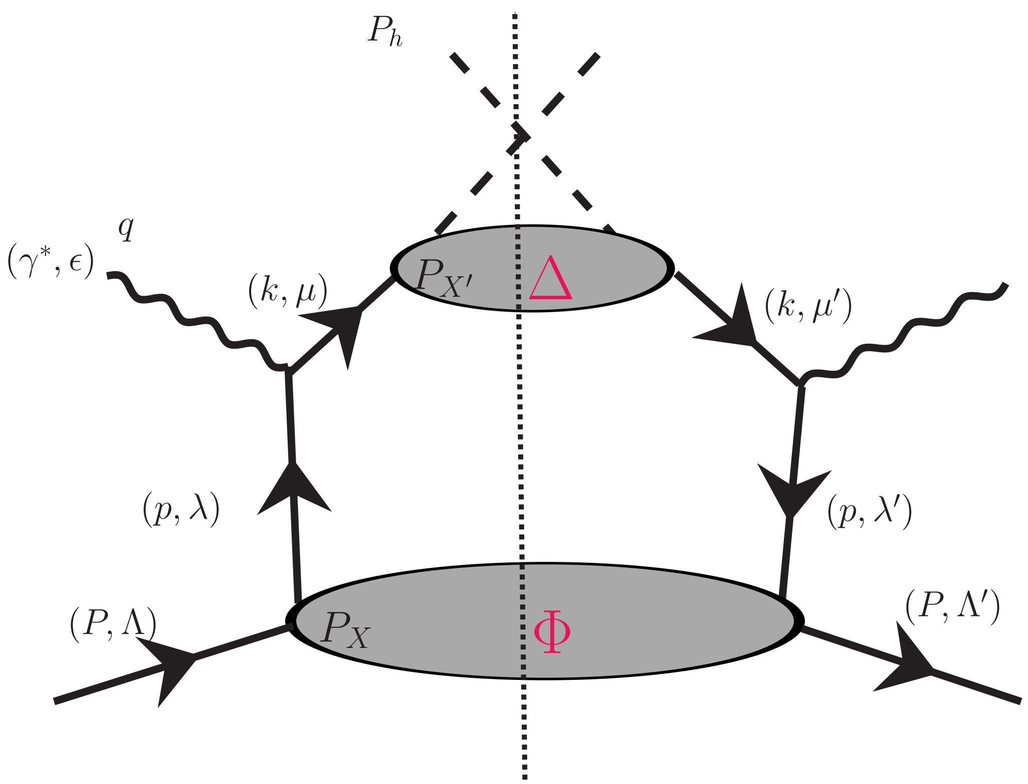
Matching thru TMD factorization CSS

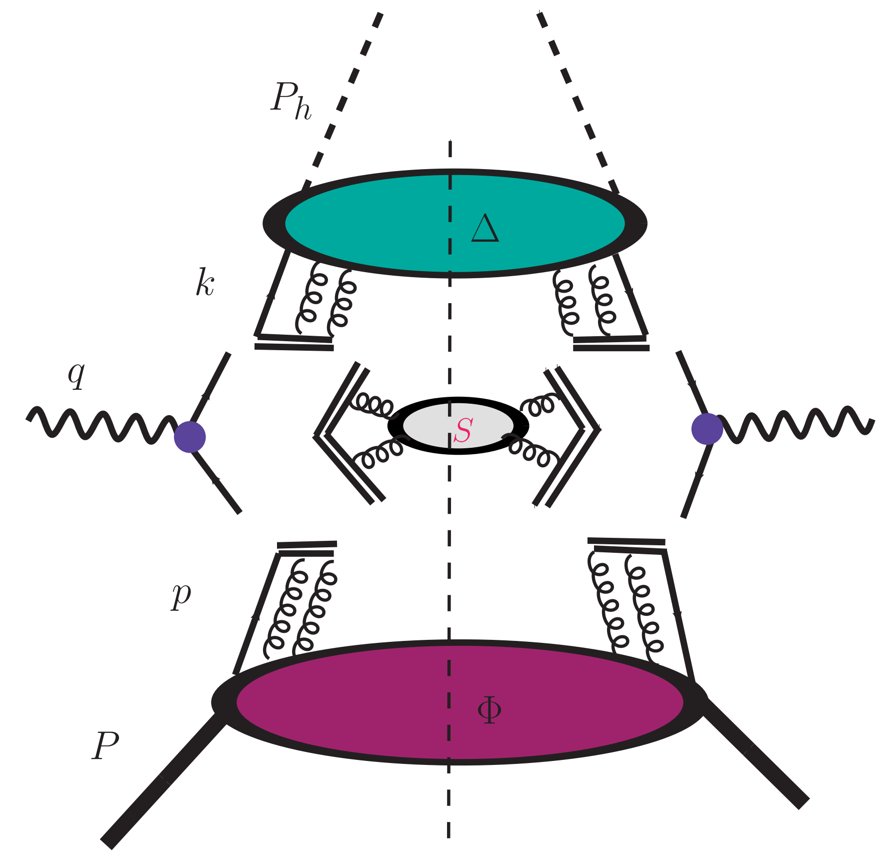
Collins & Soper(1982), & Sterman NPB 1985
Collins 2011 Cambridge Press

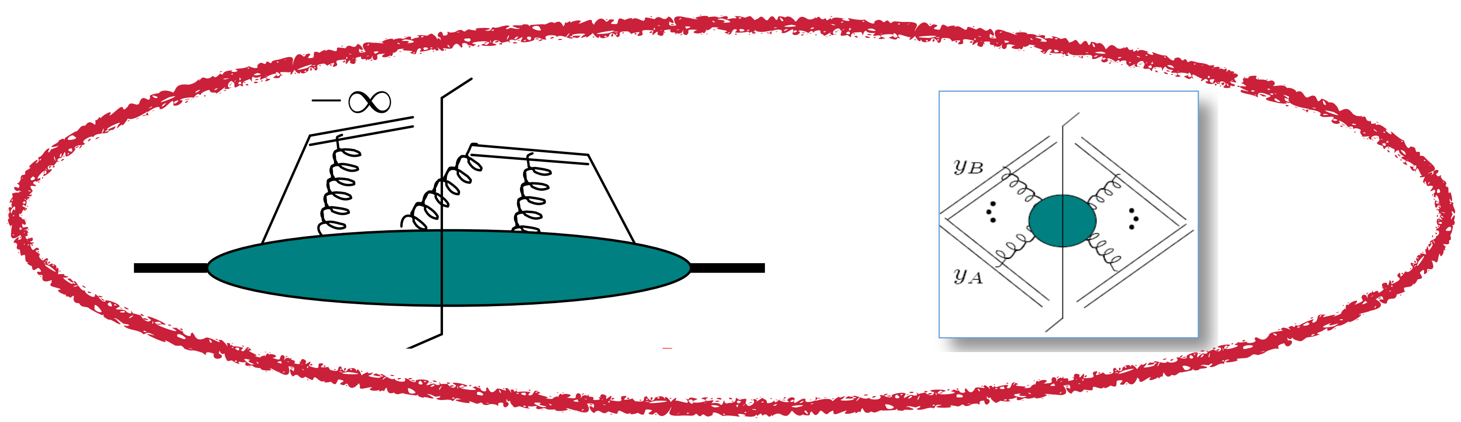
Solution to evolution equations sheds a bit more light on the relation between TMDs and collinear pdfs e.g. Sivers and Qiu Sterman function

PRD (2016) Collins, Gamberg, Prokudin, Rogers, Sato, Wang
PLB (2018) Gamberg , Metz, Pitonyak, Prokudin
INT Proc. (2019) Gamberg , Metz, Pitonyak, Prokudin

TMD factorization/evolution CSS in b space region analysis & Ward Identities



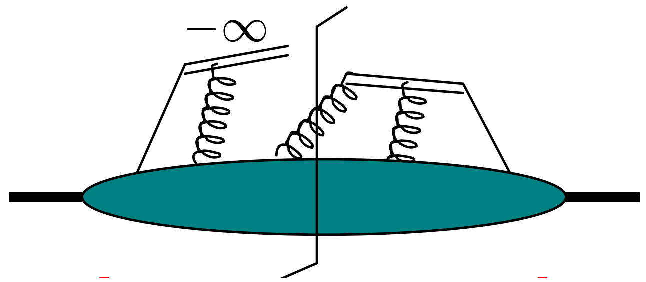


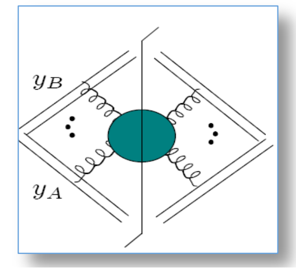


- TMDs w/Gauge links: color invariant
- TMD PDFs & Soft factor have rapidity/LC divergences
- Rapidity regulator introduced to regulate these divergences

Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13), Echevarria, Idilbi, Scimemi JHEP 2012, Collins Rogers 2015, Echevarria, Scimemi, Vladimirov JHEP 2016 …

Elements TMD Evolution/Factorization Soft factor





$$\tilde{f}^{\rm sub}_{j/H}(x,b_T;\mu,y_n) = \lim_{\substack{y_A\to+\infty \\ y_B\to-\infty}} \underbrace{\tilde{f}^{\rm unsub}_{j/H}(x,b_T;\mu,y_P-y_B)} \sqrt{\frac{\tilde{S}(b_T;y_A,y_n)}{\tilde{S}(b_T;y_A,y_B)\tilde{S}(b_T;y_n,y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{f}^{\rm unsub}_{j/H}(x,b_T;\mu,y_P-y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+\mathcal{U}_{[0,b]}\psi(b)|P\rangle\big|_{b^+=0}$$

JCC Soft factor further "repartitioned"
This is done to

1) cancel LC divergences in "unsubtracted" TMDs
2) separate "right & left" movers i.e. full factorization
3) remove double counting of momentum regions

TMD Factorization & Evolution

$$\frac{d\sigma}{d\boldsymbol{q}_{\mathrm{T}}^2 dQ^2 \ldots} = W(q_{\mathrm{T}}, Q) + Y(q_{\mathrm{T}}, Q) + O\left(\frac{m}{Q}\right)^{c} \frac{d\sigma}{d\boldsymbol{q}_{\mathrm{T}}^2 dQ^2 \ldots}$$

$$W_{UU}(q_{\mathrm{T}}, Q) = \sum_{jj'} H_{jj'}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\ \tilde{f}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2)$$

In full QCD, the auxiliary parameters μ and ζ are arbitrary
and this is reflected in the the Collins-Soper (CS) equations
for the TMD PDF, and the renormalization group (RG) equations

Along with .... Renormalization group Equations

Collins-Soper Equation:

$$-\quad \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln \tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

RGE:
get anomalous
for $\tilde{F}$ & $\tilde{K}$

Solve Collins Soper & RGE eqs. to obtain "evolved TMDs"

TMD Evolution-
Solve RGE and CS equation for unpolarised TMD

With $\mu_b = C_1/b_*$ as hard scale, the b dependence of TMDs is calculated in perturbation theory & related to collinear parton distribution (PDFs) thru OPE

$$\tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Collins (2011); ...

Aybat Rogers PRD 2011

Note Unpolarized & Sivers evolve in same way

Recall the correlator in *b*-space Bessel Transform

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) \ = \ \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\, \epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)$$

Boer, Gamberg, Musch, Prokudin JHEP 2011

See lattice studies of Engelhardt et al , Musch 2009- …

Obeys CS Equation, thus unpolarised and Sivers evolve "similarly"

$$\frac{\partial \tilde{\phi}^i_{f/P}(x, \mathbf{b}_{\mathrm{T}}; \mu, \zeta_F) \epsilon_{ij} S^j_T}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \tilde{\phi}^i_{f/P}(x, \mathbf{b}_{\mathrm{T}}; \mu, \zeta_F) \epsilon_{ij} S^j_T.$$

Idilbi,Ji,Ma,Yuan PRD 2004
Aybat Rogers Collins Qiu PRD 2012
Kang Yuan Xiao PRL 2011

TMD Evolution-Solution for unpolarised & Sivers

- TMD/CSS Evolution/Factorization carried out in *b*-space "Bessel transforms"

Boer, Gamberg, Musch, Prokudin 2011 JHEP,
Collins Aybat Rogers Qiu 2012 PRD

$$\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) \;=\; \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)$$

Correlator obeys CSS equation solutions

$$\tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x},\boldsymbol{b_T};\boldsymbol{Q^2},\boldsymbol{\mu_Q}) \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1}(\hat{\boldsymbol{x}};\boldsymbol{\mu_{b_*}})\right)$$

Collins (2011); ...

$$\times\;\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

Qiu & Sterman PRL 1991

$$\tilde{\boldsymbol{f}}_{\boldsymbol{1T}}^{\boldsymbol{\perp(1)}}(\boldsymbol{x},\boldsymbol{b_T};\boldsymbol{Q^2},\boldsymbol{\mu_Q}) \quad \sim \quad \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1,\hat{x}_2, b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{T_F}(\hat{\boldsymbol{x}}_{\boldsymbol{1}},\hat{\boldsymbol{x}}_{\boldsymbol{2}};\boldsymbol{\mu_{b_*}})\right)$$

$$\times\;\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\right]$$

Kang, Xiao, Yuan PRL 2011

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

Regulating small *b* - Modification CSS *FT*-TMD

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$

$$\times \; \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

<u>perturbative Sudakov factor</u>

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

<u>non-perturbative Sudakov factor</u>

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

<u>**Note:**</u> $b_*(0) = 0$ and $(\mu_{b_*})_{b_* \to 0} = \infty$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

b-Dependence driven by perturbative part of ev. Kernel

$$\exp\left[\int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - 2\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right]$$

$$\tilde{W}(b_T \to 0, Q) \sim \exp\left[\frac{C_F}{\pi\beta_0}\int_{\ln\mu_b^2}^{\ln\mu_Q^2} \ln\mu'^2\right] = \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{\mu_b^2}{\mu_Q^2}\right)\right]$$

$$= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{C_1^2}{b_T^2\mu_Q^2}\right)\right]$$

$$= b_T^a \quad \text{where, } a = 2C_F/(\pi\beta_0) > 0$$

$$\to 0$$

Must regulate the large logs in $b_T Q$

A modification to CSS that tames large logs in *W* term sheds some light on matching TMDs to collinear pdfs

- We introduced a regulator for divergence in the *W* -term evolution kernel

Otherwise interpretation collinear expansion at "lowest order" lost

- Parton Model (expectation) W-term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2 k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2 q_T \, W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building
w/ and w/o evolution using TMD
and collinear evolution approach
Anselmino et al. 2005-2016

- Standard CSS W-term

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

Otherwise interpretation collinear expansion at "lowest order" lost

$$f_{CSS}(k_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{ik_T \cdot b_T} \tilde{f}_{CSS}(b_T, Q)$$

$$\int d^2 k_T f_{CSS}(k_T, Q) = \int \delta^2(b_T)\, b_T^a \times \text{logarithmic corrections}$$

$$\int d^2 k_T f_{CSS}(k_T, Q) = 0$$

$$\neq f(x, \mu) \quad !$$

J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang PRD 2016

Gamberg , Metz, Pitonyak, Prokudin PLB 2018

Modification to CSS W Term

B.C. Introduce small b-cuttoff

Similar to Catani et al. NPB 2006 &

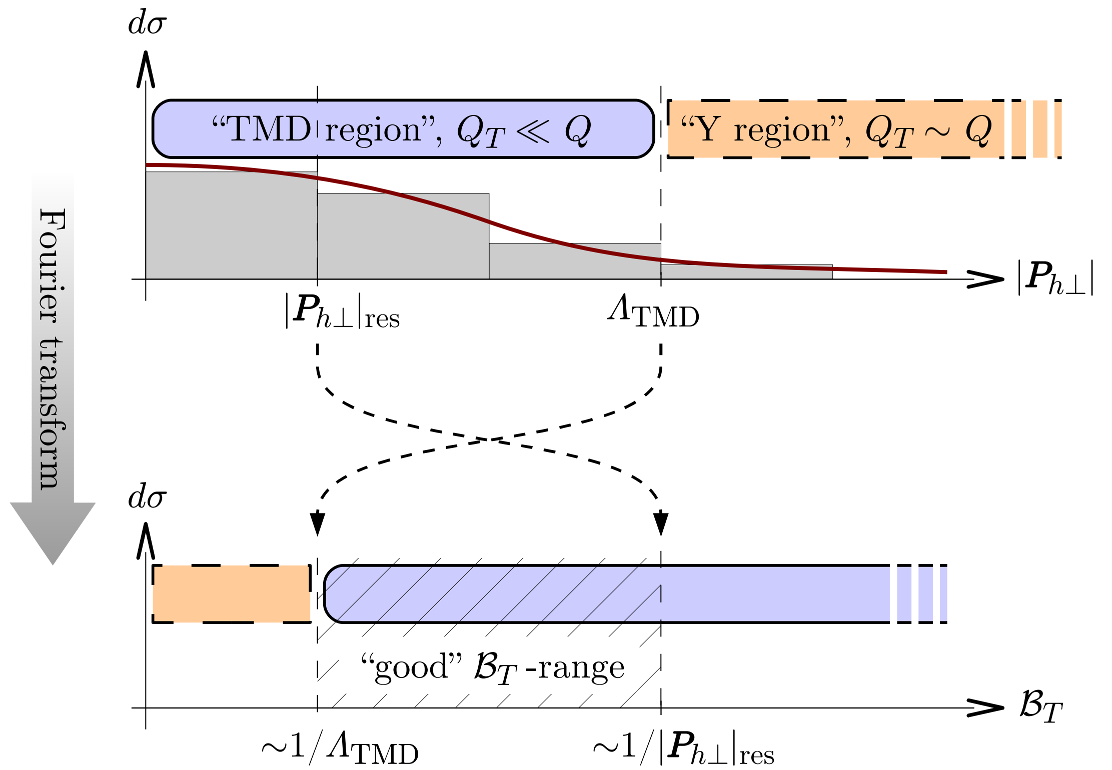
"Also discussed in Bessel Weighting" ppr. Boer LG Musch Prokudin JHEP 2011

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)^2} \Longrightarrow b_c(0) \sim 1/Q$$

Regulate unphysical divergences from in *W* term

Generalized B.C.

$$b_*(b_c(b_\mathrm{T})) \longrightarrow \begin{cases} b_\mathrm{min} & b_\mathrm{T} \ll b_\mathrm{min} \\ b_\mathrm{T} & b_\mathrm{min} \ll b_\mathrm{T} \ll b_\mathrm{max} \\ b_\mathrm{max} & b_\mathrm{T} \gg b_\mathrm{max} \,. \end{cases}$$



Modified *F.T.*-TMDs enhanced CSS

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$\Rightarrow$ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{\boldsymbol{f}}_{\mathbf{1}}(\boldsymbol{x}, \boldsymbol{b_c(b_T)}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f}_{\mathbf{1}}(\hat{\boldsymbol{x}}; \bar{\boldsymbol{\mu}})\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, **Phys. Lett B (2018)**)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{\boldsymbol{f}}_{\mathbf{1}}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) - iM\epsilon^{ij} b_T^i S_T^j \tilde{\boldsymbol{f}}_{\mathbf{1T}}^{\perp(\mathbf{1})}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q})$$

b_T -> $b_c(b_T)$

NO *b_T -> $b_c(b_T)$* replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

Unpolarized and Sivers W term & TMDs

- Phys. Lett B (2018) Gamberg , Metz, Pitonyak, Prokudin
- Phys.Rev. D (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang

$$\tilde{W}_{\mathrm{UU}}(b_c(b_T), Q) = \sum_j H_j(\mu_Q, Q)\, \tilde{f}_1^j(x, b_c(b_T); Q^2, \mu_Q)\, \tilde{D}_1^{h/j}(z, b_c(b_T); Q^2, \mu_Q)\,,$$

$$\tilde{W}_{\mathrm{UT}}^{\mathrm{siv}}(b_c(b_T), Q) = \sum_j H_j(\mu_Q, Q)\, \tilde{f}_{1T}^{\perp(1)j}(x, b_c(b_T); Q^2, \mu_Q)\, \tilde{D}_1^{h/j}(z, b_c(b_T); Q^2, \mu_Q)\,.$$

In particular Sivers *FT* TMD or 1$^{\mathrm{st}}$ Bessel moment

$$\tilde{f}_{1T}^{\perp(1)j}(x, b_c(b_T); Q^2, \mu_Q) = -\frac{1}{2M_P} \sum_{j'} \int_x^1 \frac{d\hat{x}_1}{\hat{x}_1} \frac{d\hat{x}_2}{\hat{x}_2}\, \tilde{C}_{j/j'}^{\mathrm{siv}}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu}))\, T_F^{j'}(\hat{x}_1, \hat{x}_2; \bar{\mu})$$
$$\times \exp\left\{ \tilde{K}(b_*(b_c(b_T)); \bar{\mu}) \ln\left(\frac{Q}{\bar{\mu}}\right) + \int_{\bar{\mu}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

Taking small *b* limit in CSS recover “leading order" but …

- **Relies on modification of *W+Y* construction**
- **Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD 2016**

$$\frac{d\sigma}{dxdyd\phi_S\,dz} \equiv 2z^2 \int d^2\boldsymbol{q}_{\rm T}\,\Gamma(\boldsymbol{q}_{\rm T}, Q, S) = 2z^2\,\tilde{W}^{\rm OPE}_{\rm UU}(b'_{min}, Q)_{\rm LO} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$= \frac{2\alpha_{em}^2}{yQ^2}(1-y+y^2/2)\sum_j e_j^2\, f_1^j(x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

- **Gamberg , Metz, Pitonyak, Prokudin PLB 2018**

$$\frac{d\langle P_{h\perp}\,\Delta\sigma(S_T)\rangle}{dxdydz} = -4\pi z^3 M_P\,\tilde{W}^{\rm siv,OPE}_{\rm UT}(b'_{min}, Q)_{\rm LO} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$= \frac{2\pi\, z\,\alpha_{em}^2}{yQ^2}(1-y+y^2/2)\sum_j e_j^2\, T_F^j(x,x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Agrees with collinear twist-3 result at leading order Kang,Vitev, Xing, PRD(2013)

Relationship between moments of regularised TMDs and collinear pdfs LO result-done in b-space w/ the OPE - small b region

Relies on the small b limit with $b_{\rm min}$ cutoff $\quad b_{min} \propto \frac{1}{Q}$

$$\int d^2\boldsymbol{k}_{\rm T}\, f_1^j(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_1^j(x, b'_{min}; Q^2, \mu_Q) = f_1^j(x; \mu_c) + O(\alpha_s(Q))$$

$$z^2 \int d^2\boldsymbol{p}_{\rm T}\, D_1^j(z, p_T; Q^2, \mu_Q; C_5) = z^2 \tilde{D}_1^{h/j}(z, b'_{min}; Q^2, \mu_Q) = D_1^{h/j}(z; \mu_c) + O(\alpha_s(Q))\,,$$

Because the evolution kernel is same …

$$\int d^2\boldsymbol{k}_{\rm T} \frac{k_T^2}{2M_P^2}\, f_{1T}^{\perp j}(x, k_T; Q^2, \mu_Q; C_5) = \tilde{f}_{1T}^{\perp(1)j}(x, b'_{min}; Q^2, \mu_Q) = \frac{-1}{2M_P} T_F^j(x, x; \mu_c) + O(\alpha_s(Q))$$

$$\int d^2\boldsymbol{p}_{\rm T}\, \frac{p_T^2}{z^2 2M_h^2}\, H_1^{\perp j}(z, p_T; Q^2, \mu_Q; C_5) = \tilde{H}_1^{\perp(1)j}(z, b'_{min}; Q^2, \mu_Q) = \tilde{H}_1^{\perp(1)j}(z, \mu_c) + O(\alpha_s(Q))$$

✦ **Phys. Lett B (2018) Gamberg , Metz, Pitonyak, Prokudin**

Investigate at NLO in particular for Sivers

Calculation of coefficient function

P. Sun and F. Yuan, PRD (2013), Kang, Yuan, Xaio PRL 2011
Scimemi, Tarasov Vladimirov JHEP 2019—most uptodate complete

$$\tilde{C}^{\mathrm{siv}[1]}_{j/j'}(x_1, x_2, b_*(b_c(b_T); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu}))$$

Input to

$$\tilde{f}_{1T}^{\perp(1)j}(x, b_c(b_T); Q^2, \mu_Q) = -\frac{1}{2M_P}\sum_{j'}\int_x^1 \frac{d\hat{x}_1}{\hat{x}_1}\frac{d\hat{x}_2}{\hat{x}_2}\,\tilde{C}^{\mathrm{siv}}_{j/j'}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu}))\, T_F^{j'}(\hat{x}_1, \hat{x}_2; \bar{\mu})$$
$$\times \exp\left\{\tilde{K}(b_*(b_c(b_T)); \bar{\mu}) \ln\left(\frac{Q}{\bar{\mu}}\right) + \int_{\bar{\mu}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

Since the l.h.s. is independent of the renormalization scale we can obtain at NLO in the strong coupling the "generalized DGLAP" equation by taking the derivative with respect to the intrinsic scale $\overline{\mu}$ in the limit $\mu_c \equiv \lim_{b_T \to 0} \bar{\mu} \approx C/b_{\min}$

$$\left.\frac{\partial T_F(x, x; \bar{\mu})}{\partial \ln \bar{\mu}^2}\right|_{\bar{\mu} \to \mu_c} = \frac{\alpha_s(\mu_c)}{2\pi}\mathcal{P} \otimes \mathcal{T}$$

See Kang & Qiu PLB 2012 for non closed evolution kernel $\mathcal{T}$

It is worth mentioning that this procedure implies a specific normalization for matching the collinear PDFs and the TMDs which is fixed by TMD factorization

Comments

- With the redefined *W* term allowed us to construct a relationship between TMDs and collinear PDFs: we observe that a natural renormalization scale is introduced through the " $b_{\min}$" prescription
- Also, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used
- **We have applied to transverse polarized phenomena**
- We are able to recover the well-known relations between TMD and collinear quantities expected from the leading order parton model picture operator definition
- At NLO we are able to obtain the generalized DGLAP evolution equations for the Qiu Sterman function from the RG invariance of the first moment of the Sivers function

Happy Birthday to Aram & Stan !!!