From Light-Cone Wave Functions to Generalized Parton Distributions

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Back to the early days of generalized parton distributions

- seminal papers by X Ji and A Radyushkin in 1996–97
- earlier work by the Leipzig group: Geyer et al. 1985–94

Ji’s sum rule for total angular momentum:

\[ J = \int dx \, x \left[ H(x, \xi, t) + E(x, \xi, t) \right]_{t=0} \]

- GPD $H$ and $E$ accessible in deeply virtual Compton scattering and hard exclusive meson production

\[ 2\xi = \text{transfer of longitudinal momentum fraction} \]
\[ t = \text{invariant momentum transfer} \]

Apologies for very incomplete citations
A parenthesis: diffractive meson production

- in the diffractive (small $x$) region, exclusive meson production had earlier been proposed as a way to access the gluon distribution

- $J/\Psi$ photoproduction M Ryskin 1993
- light mesons, large $Q^2$ SJB et al 1994
- high sensitivity because $\sigma \propto [g(x)]^2$

but emphasis was on the “ordinary” gluon PDF not on distributions with information well beyond PDFs

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- rising interest in the community starting 1996 but also much skepticism:
  - measuring DVCS is hopelessly difficult
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- GPD \( H \) and \( E \) accessible in deeply virtual Compton scattering and deeply virtual meson production
- rising interest in the community starting 1996 but also much skepticism:
  - measuring DVCS is hopelessly difficult
    - not quite: first measurements by HERMES and CLAS in 2001
    - many more since: H1, ZEUS, JLab Hall A, COMPASS, ...
  - GPDs are too complicated to handle in practice and to interpret in physical terms

⇒ Need methods to establish and understand properties of GPDs
Idea: connect GPDs with other, more familiar quantities

- relation with elastic proton form factors
  \[
  \sum_q e_q^2 \int dx H_q(x, \xi, t) = F_1(t), \quad \sum_q e_q^2 \int dx E_q(x, \xi, t) = F_2(t)
  \]

- light-cone wave functions: most detailed information on bound state
  - parton densities:
    \[
    f(x) = \sum_n \int \ldots \int \psi_n(x_i, k_i)^2
    \]
    \[
    |\psi|^2 \rightarrow \text{probability interpretation}
    \]
  - form factors:
    \[
    F(t) = \sum_n \int \ldots \int \psi_n(x_i, k_i) \psi_n^*(x_i, k_i')
    \]
    in frame where \( \xi = 0 \)

- connect quantities within bigger picture
- modeling strategies starting from wave fcts

Apologies for even more incomplete citations
GPDs and light-cone wave functions (LCWFs)

▶ extend these ideas to GPDs:

SJB, MD, D-S Hwang 2000

▶ for $|x| > \xi$ (DGLAP region)

$$H(x, \xi, t) = \sum \int \ldots \int \psi_n(\ldots) \psi_n^*(\ldots)$$

with different arguments in $\psi$ and $\psi^*$

→ interference (not probability)
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   → interference (not probability)

► for $|x| < \xi$ (ERBL region)

$$H(x, \xi, t) = \sum \int \ldots \int \psi_{n+1}(\ldots) \psi^*_{n-1}(\ldots)$$

→ different parton numbers
realized earlier for
exclusive semileptonic $B$ decays:
  SJB, D-S Hwang 1998
Lorentz invariance

- $\xi$ independence of integral relations
  \[
  \sum_q e_q^2 \int dx H_q(x, \xi, t) = F_1(t), \quad \sum_q e_q^2 \int dx E_q(x, \xi, t) = F_2(t)
  \]
  \[
  \int dx x \left[ H_q(x, \xi, t) + E_q(x, \xi, t) \right] = J_q(t)
  \]
  follows from Lorentz invariance.

- Extension to higher moments: polynomiality
  \[
  \int dx x^{n-1} \text{GPD}(x, \xi, t) \text{ is polynomial in } \xi^2 \text{ with degree depending on } n
  \]

- DGLAP and ERBL regions must cooperate to fulfill these constraints

- In LCWF representation need nontrivial relations between $\psi_{n-1}, \psi_n$, and $\psi_{n+1}$
  - can check explicitly in perturbation theory
  - can be difficult/impossible to ensure in models

* Not everything on the light-cone is trivial.
Implications of LCWF representation

- exhibit role of orbital angular momentum in GPD $E$
  → in Ji’s sum rule and in Pauli form factor $F_2(t)$
  - in $E = \sum \int \int \psi \psi^*$ have mismatch between helicity of proton and summed helicities of partons in $\psi$ and/or $\psi^*$
    - SJB, D-S Hwang, B-Q Ma, I Schmidt 2000; M Burkardt, G Schnell 2006

- positivity constraints
  - upper bound on interference: $|A_1 A_2^*| \leq |A_1| |A_2|$}
  $\Rightarrow$ upper bound on GPDs in DGLAP region in terms of PDFs
    - B Pire, J Soffer, O Teryaev 1998; . . . ; P Pobylitsa 2001–02
  * derivation very transparent with LCWFs, although possible without

- models for GPDs
  - using $\psi_n \psi_n^*$ overlap for $\xi = 0$ or for $\xi \neq 0$ in the DGLAP region
    → positivity guaranteed by construction
  - covariant models for $\xi \neq 0$ in full $x$ range: challenging
    so far no explicit use of $\psi_{n+1} \psi_{n-1}^*$ overlap in ERBL region
GPD models from LCWFs

► models for $\xi \neq 0$ in full $x$ range:

- may construct models with approximate polynomiality
  
  S Ahmad, H Honkanen, S Liuti, S Taneja 2008

- use $\psi_n \psi_n^*$ overlap in DGLAP region, covariant extension to ERBL region using double distributions and inverse Radon transform
  
  D Müller et al. 2008, 2017; N Chouika et al. 2017

► which LCWFs?

- LCWFs from quark models see e.g. review by S Boffi, B Pasquini 2008

- LCWFs in perturbation theory: use to study generic features of GPDs e.g. correlations between $x, \xi, t$; behavior at $x = \xi$

- use perturbative LCWFs as a template i.e. modify without losing consistency SJB et al. 2000; ...

- LCWFs from AdS/CFT conjecture → talk by G de Teramond

GPDs in light-front holographic QCD: SJB et al. 2018

Apologies for very incomplete citations
From GPDs to impact parameter distributions

- Fourier transform of GPDs at $\xi = 0$ w.r.t. transverse momentum transfer
  → spatial density of partons with mom. fraction $x$  
  M Burkardt 2000
- no limitation by Compton wave length of target
- impact parameter densities and their relation with form factors and PDFs
  had been discussed much earlier  
  D Soper 1977
- extension to $\xi \neq 0$: interference terms instead of probabilities  
  MD 2002
- simple representation in terms of LCWFs $\tilde{\psi}(x_i, b_i)$ in mixed representation
  of mom. fractions $x_i$ and transverse positions $b_i$

- provides interpretation for “imaging” hadrons with hard exclusive processes

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Conclusions

- Generalized parton distributions greatly enrich our possibilities to probe the detailed structure of hadrons. They have become a core part of the physics programs at COMPASS, JLab and at the EIC.
- To describe/model/parameterize/interpret GPDs is challenging.
- Representing GPDs in light-cone momentum and transverse space → imaging hadrons.
- The LCWF representation provides many insights into GPDs and their relation with other quantities.
  
  Stan has made essential contributions in this area.