From Light-Cone Wave Functions to Generalized Parton Distributions

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Deutsches Elektronen-Synchrotron DESY

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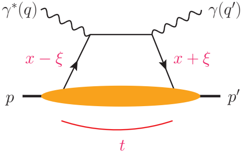
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

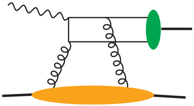
Back to the early days of generalized parton distributions

- seminal papers by X Ji and A Radyushkin in 1996–97
  earlier work by the Leipzig group: Geyer et al. 1985–94
- Ji's sum rule for total angular momentum:

$$J = \int dx\, x \left[H(x,\xi,t) + E(x,\xi,t) \right]_{t=0}$$

- GPD H and E accessible in deeply virtual Compton scattering and hard exclusive meson production





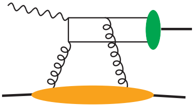
$2\xi =$ transfer of longitudinal momentum fraction
$t =$ invariant momentum transfer

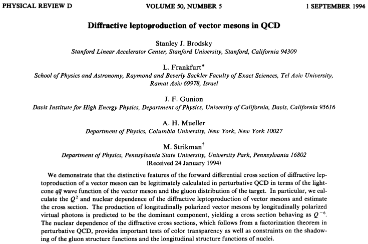
Apologies for very incomplete citations

A parenthesis: diffractive meson production

- in the diffractive (small x) region, exclusive meson production had earlier been proposed as a way to access the gluon distribution

- J/Ψ photoproduction M Ryskin 1993
- light mesons, large Q^2 SJB et al 1994
- high sensitivity because $\sigma \propto \big[g(x) \big]^2$





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Diffractive leptoproduction of vector mesons in QCD

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We demonstrate that the distinctive features of the forward differential cross section of diffractive leptoproduction of a vector meson can be legitimately calculated in perturbative QCD in terms of the light-cone $q\bar{q}$ wave function of the vector meson and the gluon distribution of the target. In particular, we calculate the Q^2 and nuclear dependence of the diffractive leptoproduction of vector mesons and estimate the cross section. The production of longitudinally polarized vector mesons by longitudinally polarized virtual photons is predicted to be the dominant component, yielding a cross section behaving as Q^{-6}. The nuclear dependence of the diffractive cross sections, which follows from a factorization theorem in perturbative QCD, provides important tests of color transparency as well as constraints on the shadowing of the gluon structure functions and the longitudinal structure functions of nuclei.

- but emphasis was on the "ordinary" gluon PDF
  not on distributions with information well beyond PDFs

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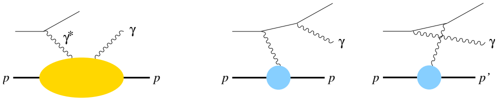
$$J = \int dx\, x \left[H(x,\xi,t) + E(x,\xi,t)\right]_{t=0}$$

- GPD H and E accessible in deeply virtual Compton scattering and deeply virtual meson production
- rising interest in the community starting 1996
  but also much skepticism:
  - measuring DVCS is hopelessly difficult



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- GPD H and E accessible in deeply virtual Compton scattering and deeply virtual meson production
- rising interest in the community starting 1996
  but also much skepticism:
  - measuring DVCS is hopelessly difficult
    not quite: first measurements by HERMES and CLAS in 2001
    many more since: H1, ZEUS, JLab Hall A, COMPASS, ...
  - GPDs are too complicated to handle in practice
    and to interpret in physical terms

⇒ Need methods to establish and understand properties of GPDs

Idea: connect GPDs with other, more familiar quantities

- relation with elastic proton form factors X Ji 1996

$$\sum_q e_q^2 \int dx\, H_q(x,\xi,t) = F_1(t), \qquad \sum_q e_q^2 \int dx\, E_q(x,\xi,t) = F_2(t)$$

- light-cone wave functions: most detailed information on bound state

  - parton densities:

    $$f(x) = \sum_n \int \ldots \int \ldots \left|\psi_n(x_i, \boldsymbol{k}_i)\right|^2$$

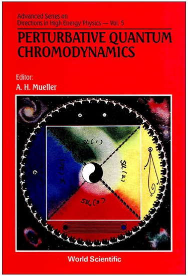
    $|\psi|^2 \;\to\;$ probability interpretation

  - form factors:

    $$F(t) = \sum_n \int \ldots \int \ldots \times \psi_n(x_i, \boldsymbol{k}_i)\, \psi_n^*(x_i, \boldsymbol{k}_i')$$

    in frame where $\xi = 0$

review: SJB, P Lepage in



- connect quantities within bigger picture
- modeling strategies starting from wave fcts

Apologies for even more incomplete citations

GPDs and light-cone wave functions (LCWFs)

- extend these ideas to GPDs:
  SJB, MD, D-S Hwang 2000
  MD, T Feldmann, R Jakob, P Kroll 1998-2000
- for $|x| > \xi$ (DGLAP region)

$$H(x,\xi,t) = \sum_n \int \ldots \int \ldots \psi_n(\cdots)\, \psi_n^*(\cdots)$$

with different arguments in ψ and ψ^*
$\rightarrow$ interference (not probability)

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Light-cone wavefunction representation of deeply virtual Compton scattering ☆

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Abstract

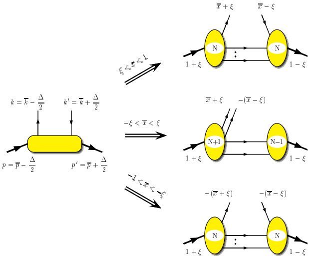
We give a complete representation of virtual Compton scattering $\gamma^* p \to \gamma p$ at large initial photon virtuality Q^2 and small momentum transfer squared t in terms of the light-cone wavefunctions of the target proton. We verify the identities between the skewed parton distributions $H(x,\zeta,t)$ and $E(x,\zeta,t)$ which appear in deeply virtual Compton scattering and the corresponding integrands of the Dirac and Pauli form factors $F_1(t)$ and $F_2(t)$ and the gravitational form factors $A_q(t)$ and $B_q(t)$ for each quark and anti-quark constituent. We illustrate the general formalism for the case of deeply virtual Compton scattering on the quantum fluctuations of a fermion in quantum electrodynamics at one loop.

PACS: 12.20.-m; 12.39.Ki; 13.40.Gp; 13.60.Fz

GPDs and light-cone wave functions (LCWFs)

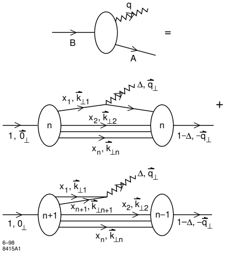
- extend these ideas to GPDs:
  SJB, MD, D-S Hwang 2000
  MD, T Feldmann, R Jakob, P Kroll 1998-2000

- for $|x| > \xi$ (DGLAP region)
  $$H(x,\xi,t) = \sum_n \int \ldots \int \ldots \psi_n(\cdots)\, \psi_n^*(\cdots)$$
  with different arguments in ψ and ψ^*
  $\to$ interference (not probability)



- for $|x| < \xi$ (ERBL region)
  $$H(x,\xi,t) = \sum_n \int \ldots \int \ldots \psi_{n+1}(\cdots)\, \psi_{n-1}^*(\cdots)$$
  $\to$ different parton numbers

  realized earlier for
  exclusive semileptonic B decays:
  SJB, D-S Hwang 1998



Lorentz invariance

- ξ independence of integral relations

$$\sum_q e_q^2 \int dx\, H_q(x,\xi,t) = F_1(t), \qquad \sum_q e_q^2 \int dx\, E_q(x,\xi,t) = F_2(t)$$
$$\int dx\, x \left[H_q(x,\xi,t) + E_q(x,\xi,t) \right] = J_q(t)$$

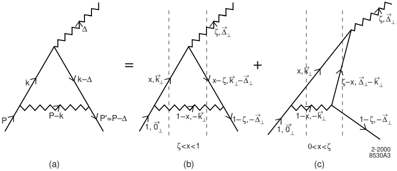
  follows from Lorentz invariance.

- extension to higher moments: polynomiality

  $\int dx\, x^{n-1}$ GPD(x,ξ,t) is polynomial in ξ^2 with degree depending on n

- DGLAP and ERBL regions must cooperate to fulfill these constraints

- In LCWF representation need nontrivial relations between ψ_{n-1}, ψ_n, and ψ_{n+1}
  - can check explicitly in perturbation theory
  - can be difficult/impossible to ensure in models *



* Not everything on the light-cone is trivial.

Implications of LCWF representation

- exhibit role of orbital angular momentum in GPD E
  $\to$ in Ji's sum rule and in Pauli form factor $F_2(t)$
  - in $E = \sum \int \int \psi \psi^*$ have mismatch between helicity of proton and summed helicities of partons in ψ and/or ψ^*

    SJB, D-S Hwang, B-Q Ma, I Schmidt 2000; M Burkardt, G Schnell 2006
- positivity constraints
  - upper bound on interference: $|\mathcal{A}_1 \mathcal{A}_2^*| \le |\mathcal{A}_1|\,|\mathcal{A}_2|$
    $\Rightarrow$ upper bound on GPDs in DGLAP region in terms of PDFs *

    B Pire, J Soffer, O Teryaev 1998; ...; P Pobylitsa 2001–02

  * derivation very transparent with LCWFs, although possible without
- models for GPDs
  - using $\psi_n\, \psi_n^*$ overlap for $\xi = 0$ or for $\xi \neq 0$ in the DGLAP region
    $\to$ positivity guaranteed by construction
  - covariant models for $\xi \neq 0$ in full x range: challenging
    so far no explicit use of $\psi_{n+1}\, \psi_{n-1}^*$ overlap in ERBL region

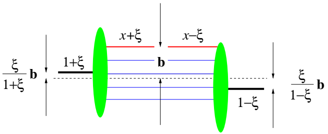
GPD models from LCWFs

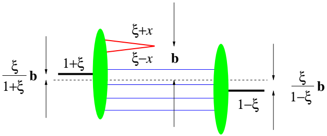
- models for $\xi \neq 0$ in full x range:
  - may construct models with approximate polynomiality
    S Ahmad, H Honkanen, S Liuti, S Taneja 2008
  - use $\psi_n \, \psi_n^*$ overlap in DGLAP region, covariant extension to ERBL region using double distributions and inverse Radon transform
    D Müller et al. 2008, 2017; N Chouika et al. 2017
- which LCWFs?
  - LCWFs from quark models see e.g. review by S Boffi, B Pasquini 2008
  - LCWFs in perturbation theory: use to study generic features of GPDs
    e.g. correlations between x, ξ, t; behavior at $x = \xi$
  - use perturbative LCWFs as a template
    i.e. modify without losing consistency SJB et al. 2000; ...
  - LCWFs from AdS/CFT conjecture $\rightarrow$ talk by G de Teramond
    GPDs in light-front holographic QCD: SJB et al. 2018

Apologies for very incomplete citations

From GPDs to impact parameter distributions

- Fourier transform of GPDs at $\xi = 0$ w.r.t. transverse momentum transfer $\rightarrow$ spatial density of partons with mom. fraction x M Burkardt 2000
  - no limitation by Compton wave length of target
- impact parameter densities and their relation with form factors and PDFs had been discussed much earlier D Soper 1977
- extension to $\xi \neq 0$: interference terms instead of probabilities MD 2002
- simple representation in terms of LCWFs $\widetilde{\psi}(x_i, \boldsymbol{b}_i)$ in mixed representation of mom. fractions x_i and transverse positions $\boldsymbol{b}_i$





- provides interpretation for "imaging" hadrons with hard exclusive processes

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Conclusions

- Generalized parton distributions greatly enrich our possibilities to probe the detailed structure of hadrons.
  They have become a core part of the physics programs at COMPASS, JLab and at the EIC.
- To describe/model/parameterize/interpret GPDs is challenging.
- Representing GPDs in light-cone momentum and transverse space $\rightarrow$ imaging hadrons.
- The LCWF representation provides many insights into GPDs and their relation with other quantities.

  Stan has made essential contributions in this area.