SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

CPHI 2020

3-7 February 2020, CERN

Exploring the spin effects in dihadrons with Aram

Hrayr Matevosyan

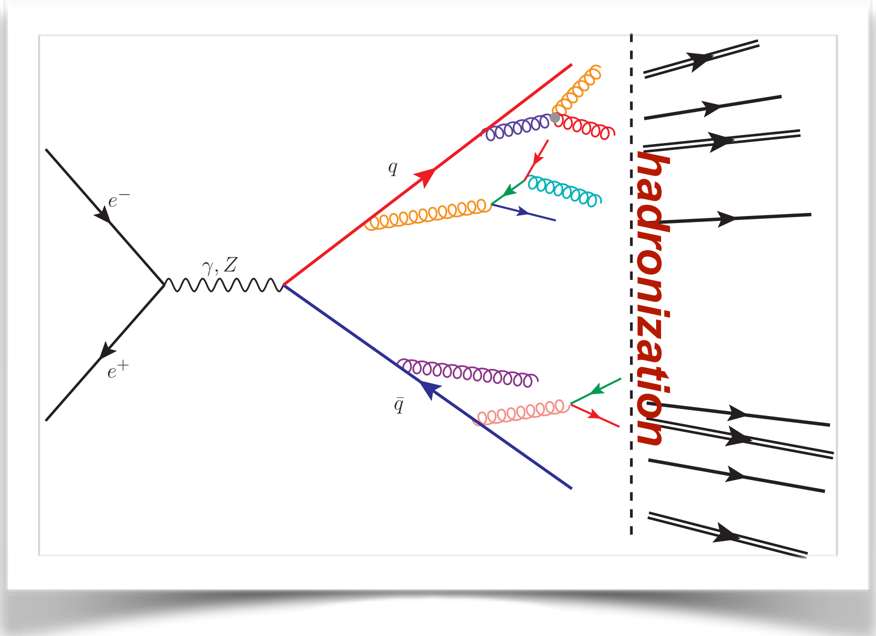
Overview

Collaborated with Aram from 2013:

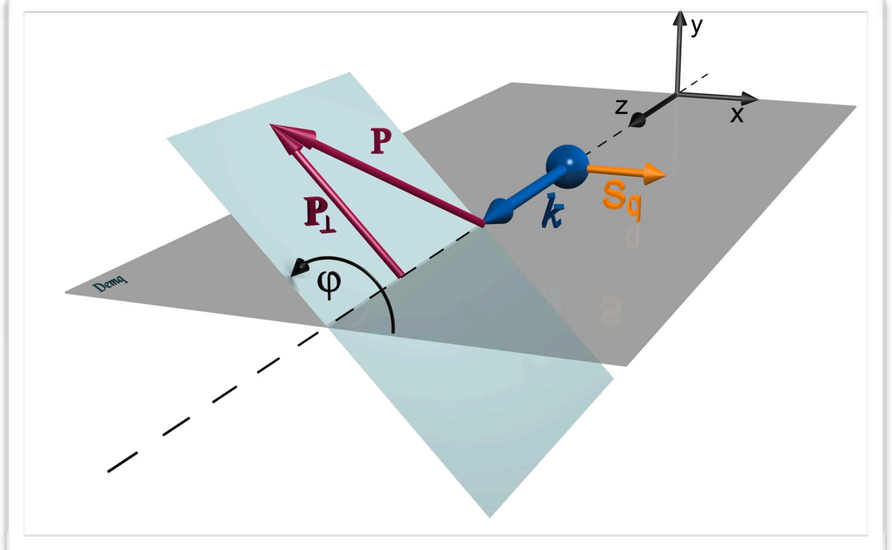
- *13 Publications, including 2 PRLs.*
- *I will talk about some highlights involving spin-induced dihadron observations.*
    - ✦ *Dihadron modulations from Collins effect.*
    - ✦ *Spin in quark-jet hadronization: DiFFs.*
    - ✦ *New measurements of dihadron FFs in e^+e^-.*

Hadronization of a Polairzed Quark

- **Modelling quark hadronization**

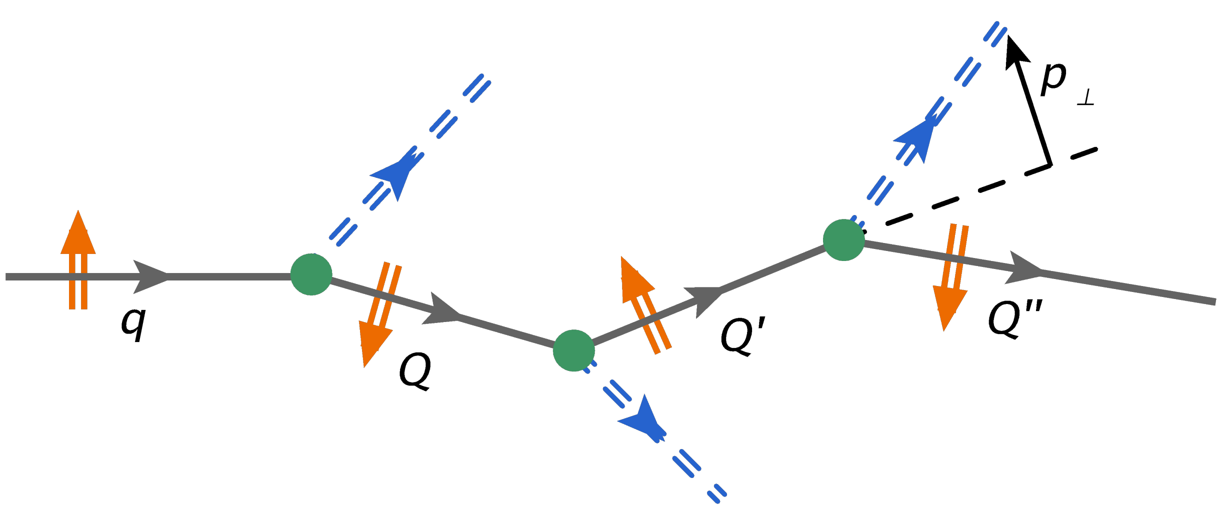


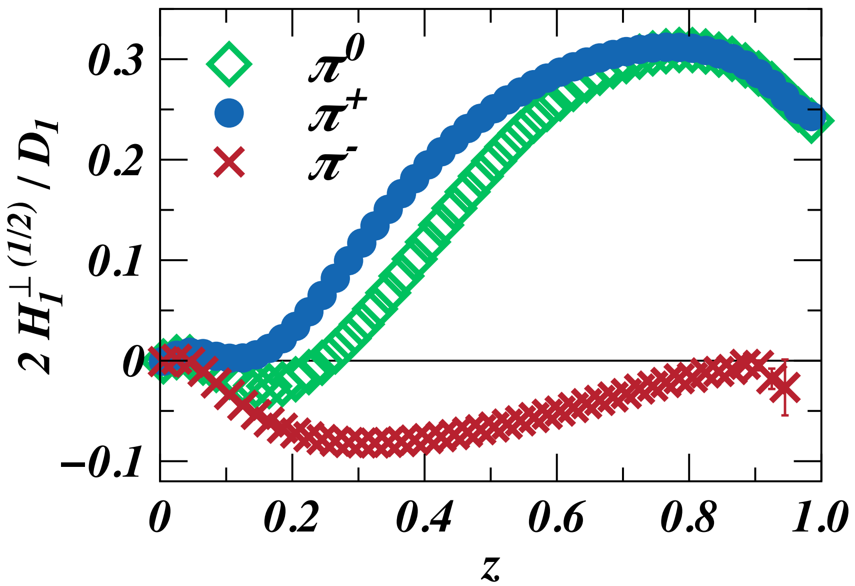
- **Collins Effect**



- **Extended NJL-jet Model: spin non-flip and flip probabilities:**

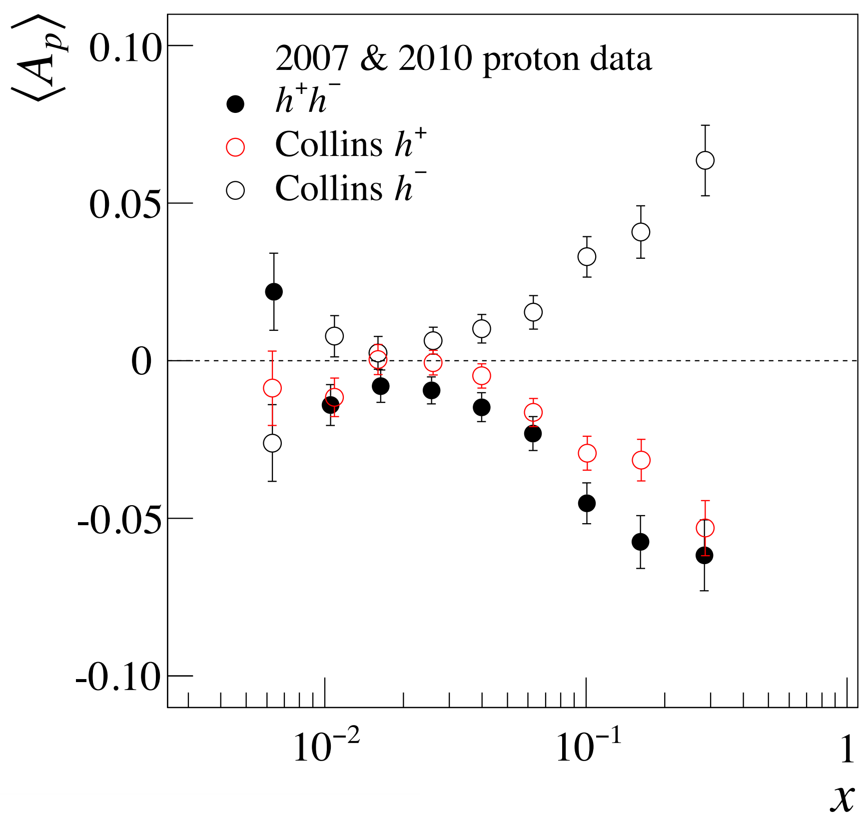
H.M.,Bentz, Thomas, PRD.86:034025, 2012.



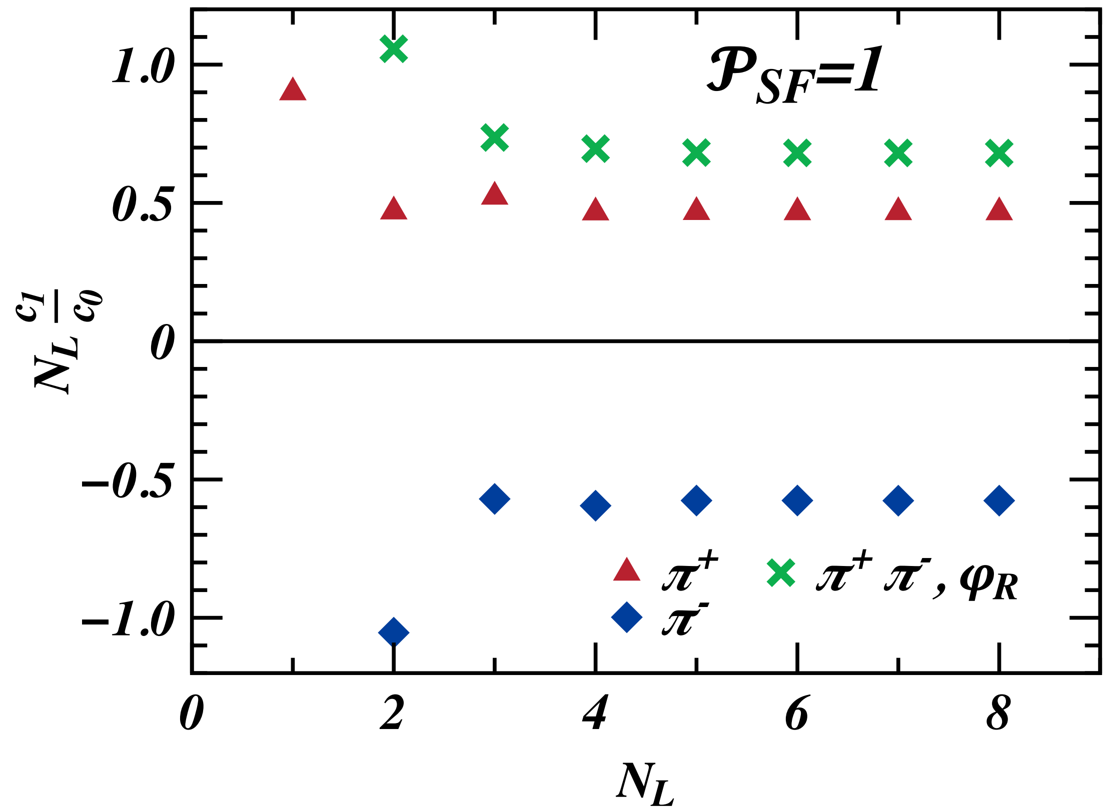


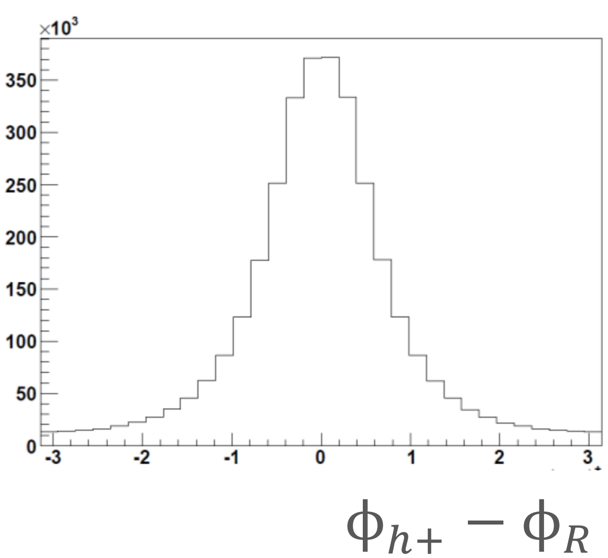
Understanding Collins-IFF Interplay

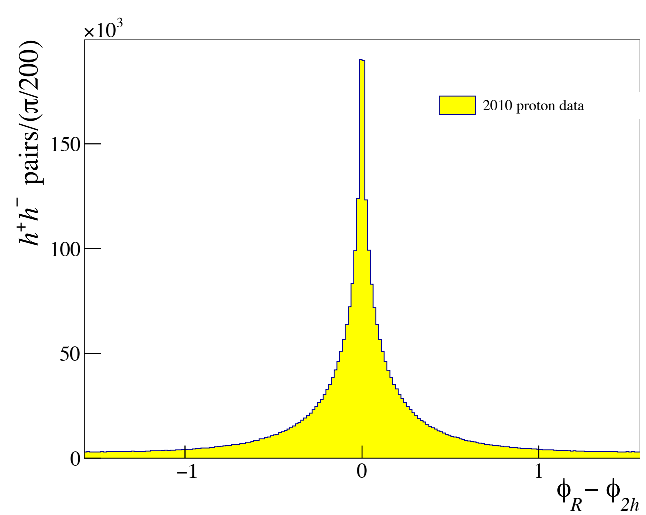
COMPASS
arXiv:1401.7873 (2014).

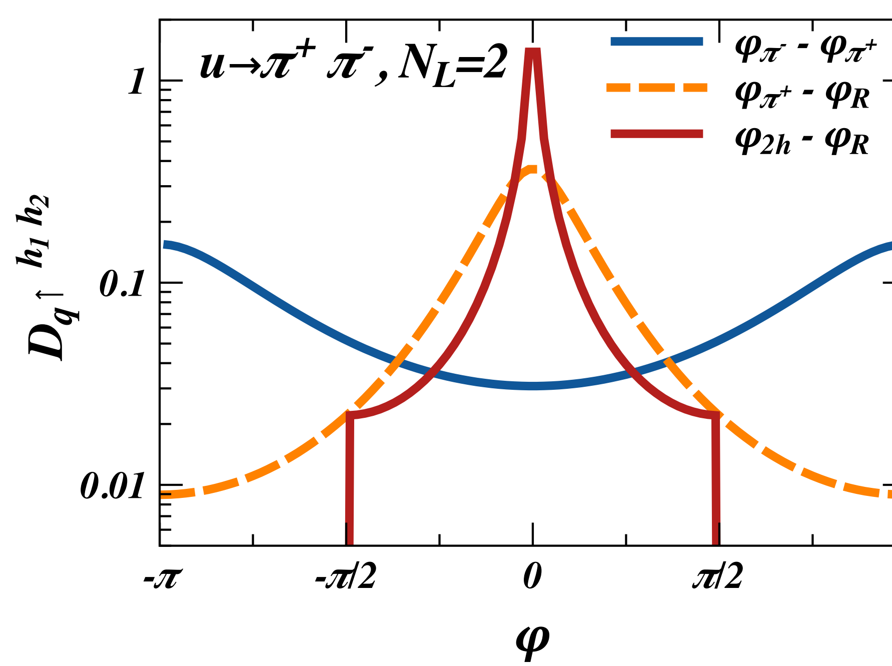


NJL-Jet
PLB 731, 208 (2014).

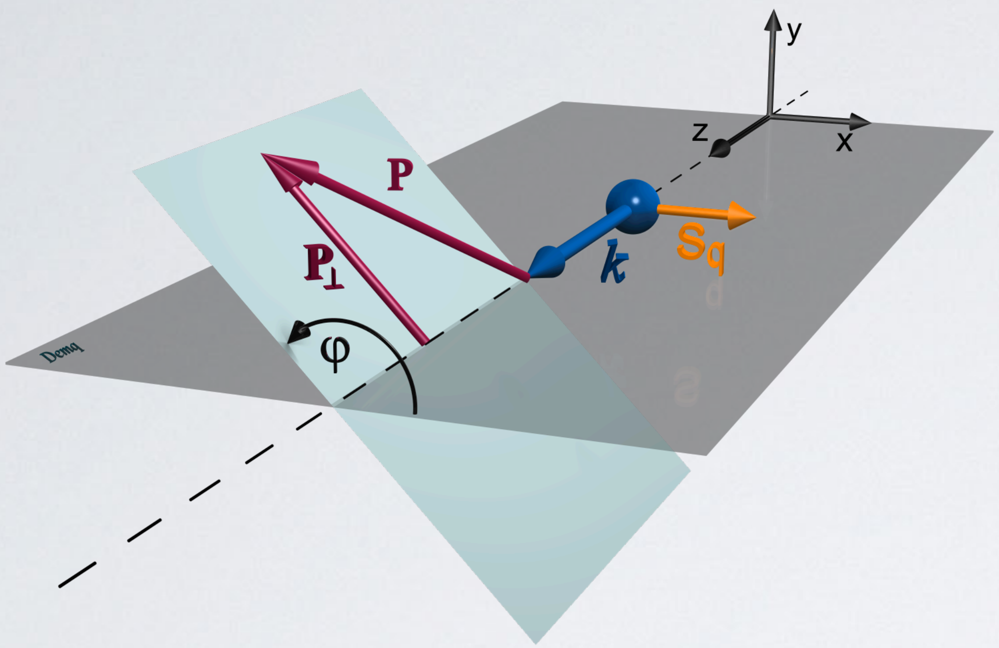


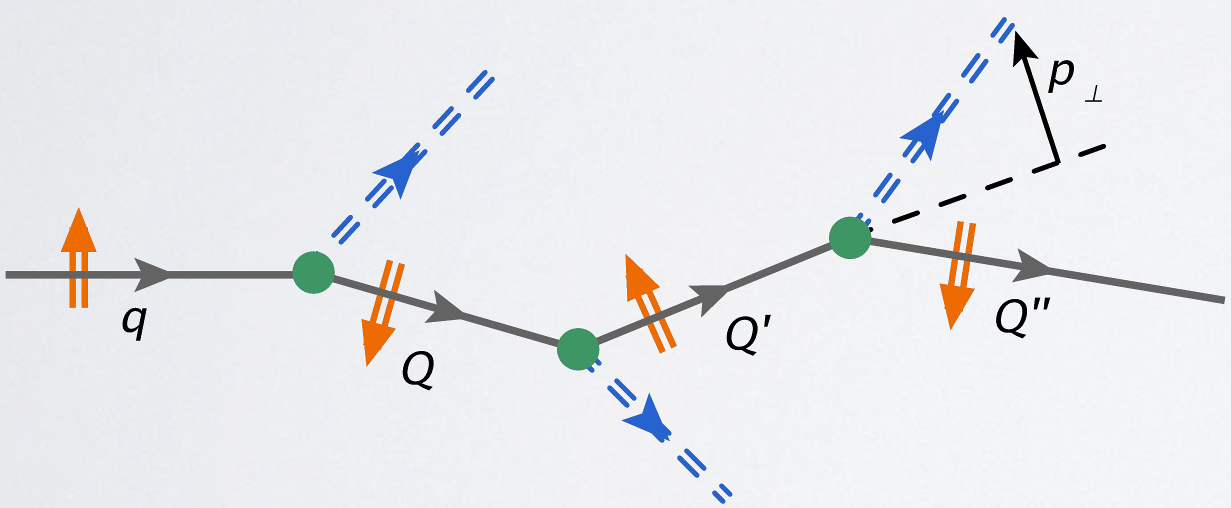




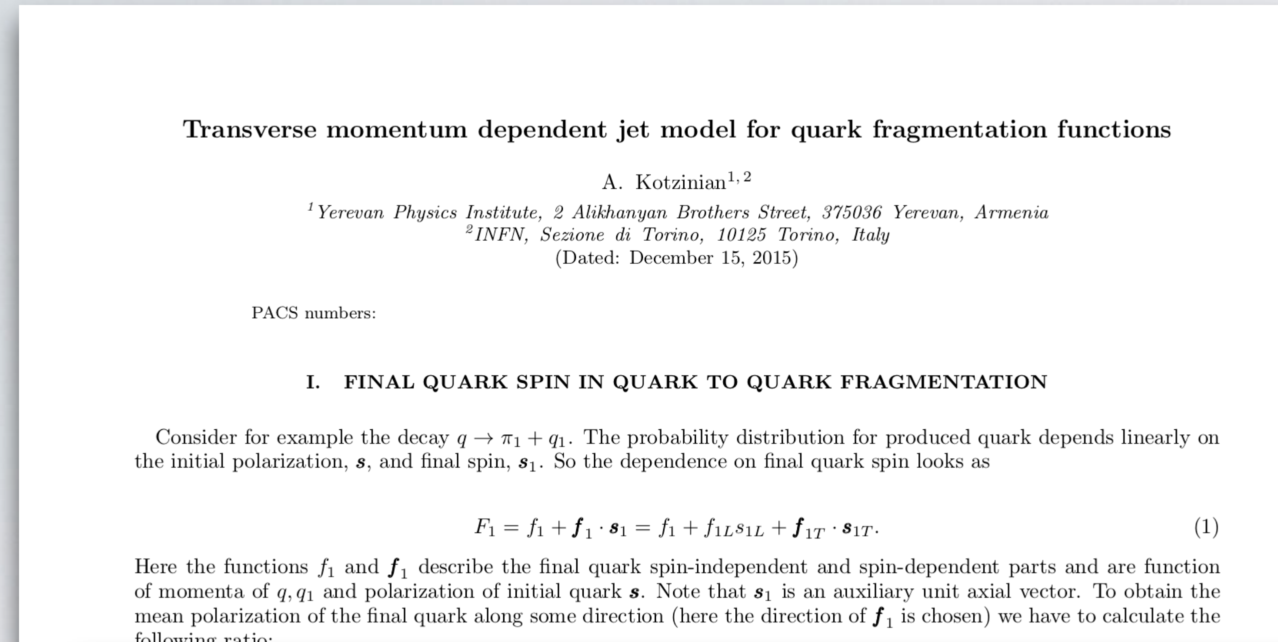


Conquering the SPIN





Conquering the SPIN



Transverse momentum dependent jet model for quark fragmentation functions

A. Kotzinian[1,2]

[1]*Yerevan Physics Institute, 2 Alikhanyan Brothers Street, 375036 Yerevan, Armenia*
[2]*INFN, Sezione di Torino, 10125 Torino, Italy*
(Dated: December 15, 2015)

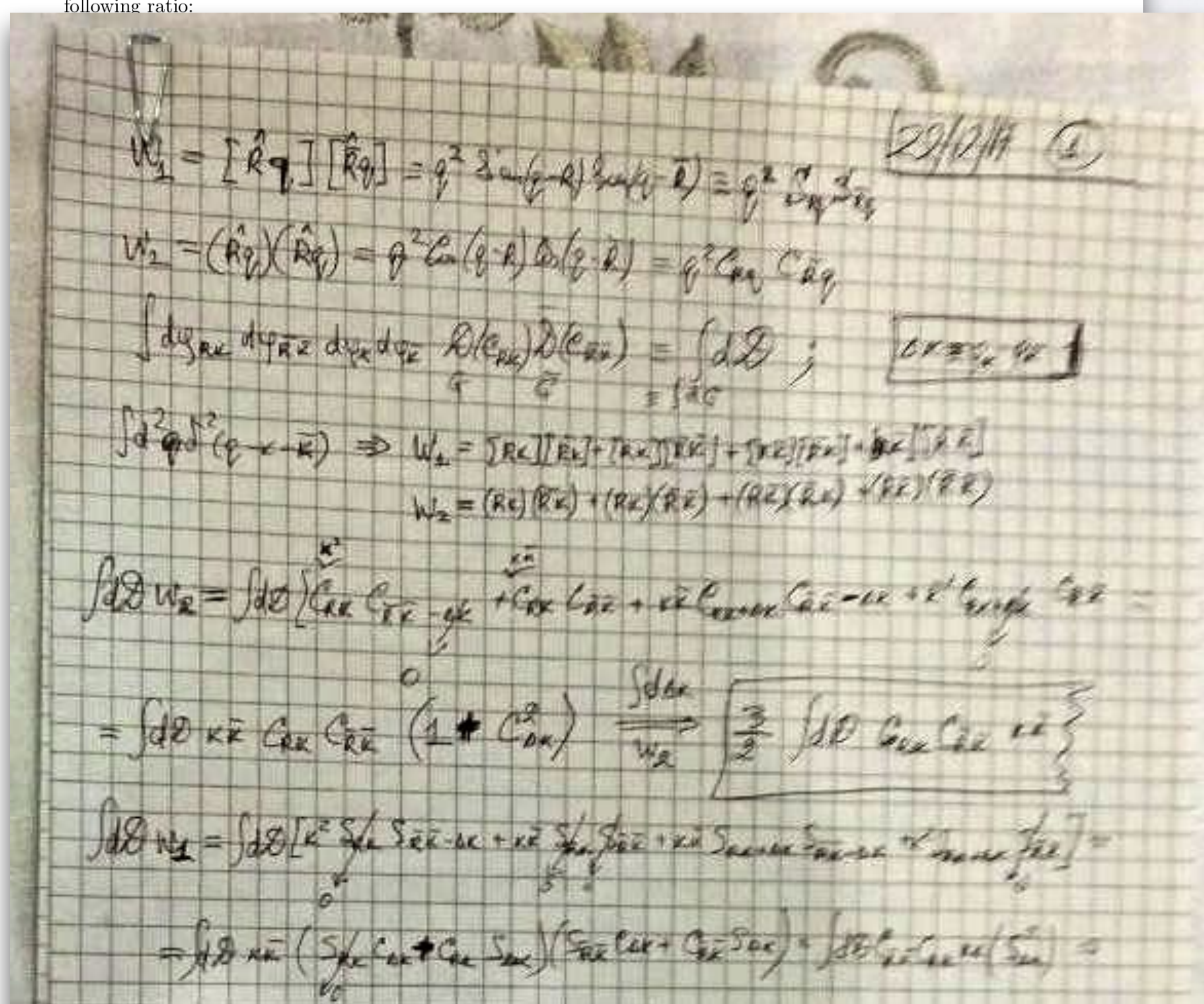
PACS numbers:

I. FINAL QUARK SPIN IN QUARK TO QUARK FRAGMENTATION

Consider for example the decay $q \to \pi_1 + q_1$. The probability distribution for produced quark depends linearly on the initial polarization, $\boldsymbol{s}$, and final spin, $\boldsymbol{s}_1$. So the dependence on final quark spin looks as

$$F_1 = f_1 + \boldsymbol{f}_1 \cdot \boldsymbol{s}_1 = f_1 + f_{1L} s_{1L} + \boldsymbol{f}_{1T} \cdot \boldsymbol{s}_{1T}. \quad (1)$$

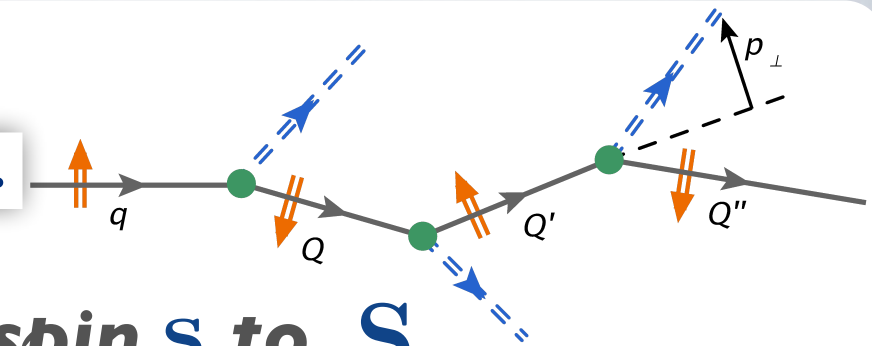
Here the functions f_1 and $\boldsymbol{f}_1$ describe the final quark spin-independent and spin-dependent parts and are function of momenta of q, q_1 and polarization of initial quark $\boldsymbol{s}$. Note that $\boldsymbol{s}_1$ is an auxiliary unit axial vector. To obtain the mean polarization of the final quark along some direction (here the direction of $\boldsymbol{f}_1$ is chosen) we have to calculate the following ratio:



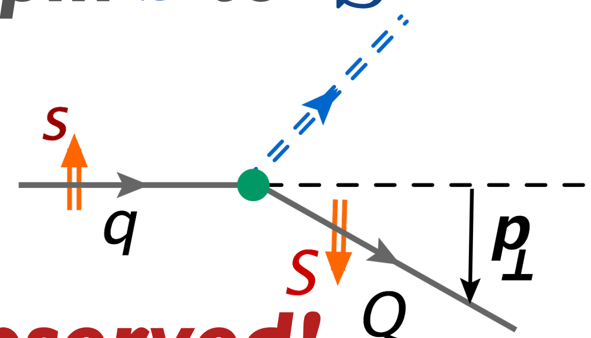
POLARIZATION IN QUARK-JET FRAMEWORK

✦ Extended quark-jet:

Bentz, Kotzinian, *H.M*, Ninomiya, Thomas, Yazaki: PRD 94 034004 (2016).



- ***The probability for the process*** $q \to Q$, ***initial spin*** $\mathbf{s}$ ***to*** $\mathbf{S}$



$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_\mathbf{s} + \boldsymbol{\beta}_\mathbf{s} \cdot \mathbf{S}$$

- ***Intermediate quarks in quark-jet are unobserved!***

We need the induced final state spin $\mathbf{S}'$.

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'} \rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

- ***Remnant quark's*** $\mathbf{S}'$ ***uniquely determined by*** $z, \mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

$$\mathbf{S}' = \frac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$$

- ***Process probability is the same as transition to unpolarized state.***

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$

REMNANT QUARK'S POLARISATION

- **We can express the spin of the remnant quark** $\mathrm{S}' = \frac{\beta_{\mathrm{s}}}{\alpha_{\mathrm{s}}}$ **in terms of *quark-to-quark TMD FFs*.**

$$\alpha_q \equiv D(z,\boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp \times \boldsymbol{s}_T)\cdot\hat{\boldsymbol{z}}\frac{1}{z\mathcal{M}}\, H^\perp(z,\boldsymbol{p}_\perp^2)$$

$$\beta_{q\parallel} \equiv s_L\, G_L(z,\boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T)\frac{1}{z\mathcal{M}}H_L^\perp(z,\boldsymbol{p}_\perp^2)$$

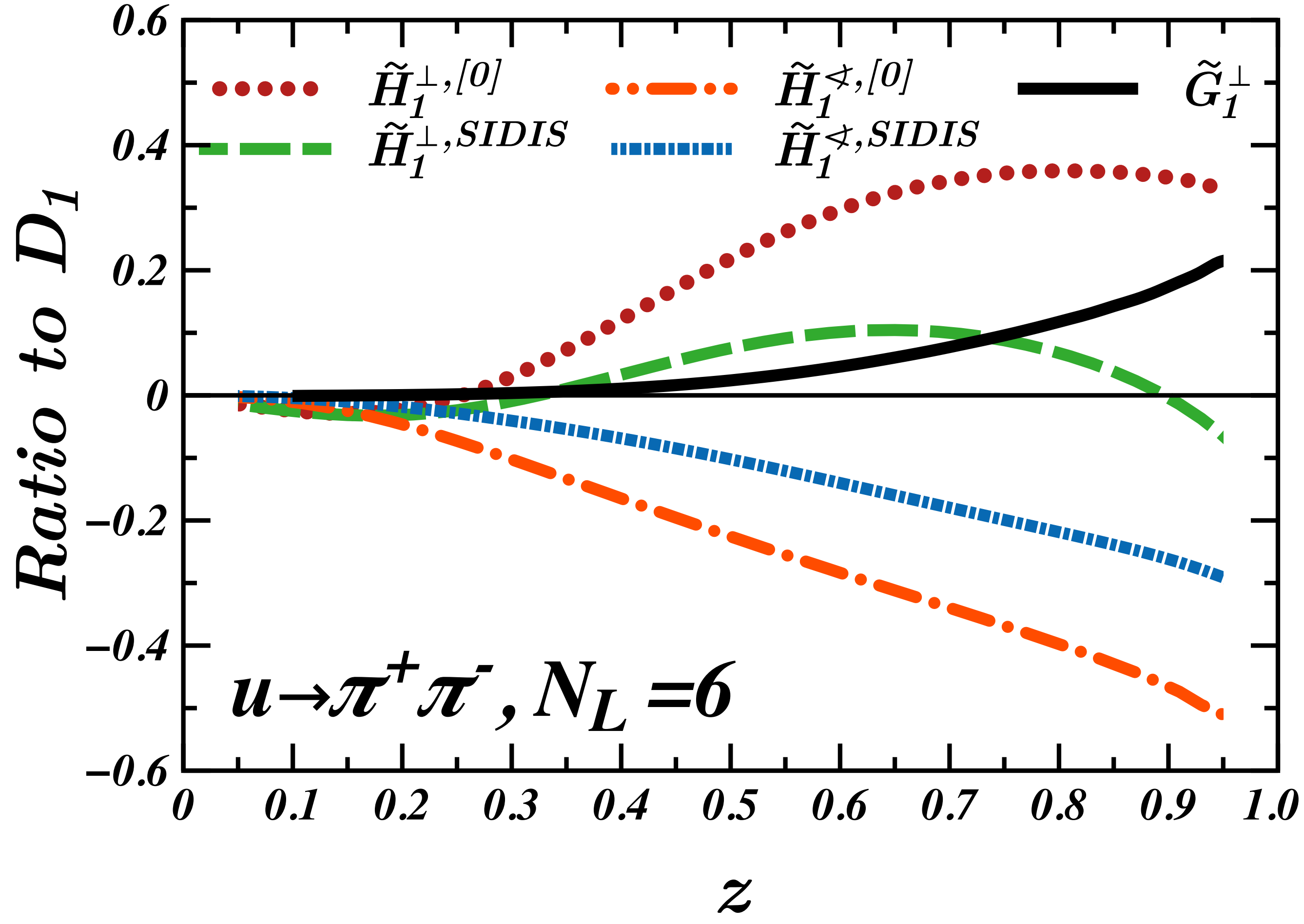
$$\boldsymbol{\beta}_{q\perp} \equiv \boldsymbol{p}'_\perp\frac{1}{z\,\mathcal{M}}D_T^\perp(z,\boldsymbol{p}_\perp^2) - \boldsymbol{p}_\perp\frac{1}{z\mathcal{M}}s_L G_T(z,\boldsymbol{p}_\perp^2) + \boldsymbol{s}_T\, H_T(z,\boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp(\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T)\frac{1}{z^2\mathcal{M}^2}\, H_T^\perp(z,\boldsymbol{p}_\perp^2)$$

$$F^{q\to Q}(z,\boldsymbol{p}_\perp;\boldsymbol{s},\boldsymbol{S})$$

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

Analysing Power for Quark Spin

- **Comparing the analysing powers for all polarized**

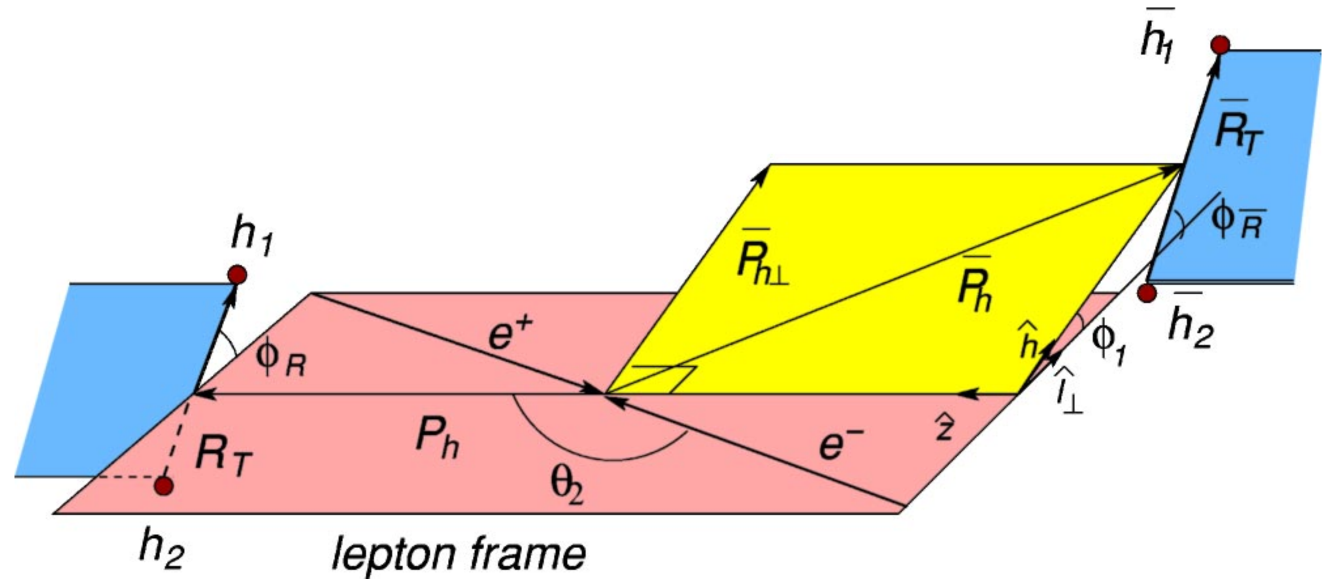


- ***Signals for all possible hadron pairs.***

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- ***Can access both helicity and transverse pol. dependent DiFFs:***

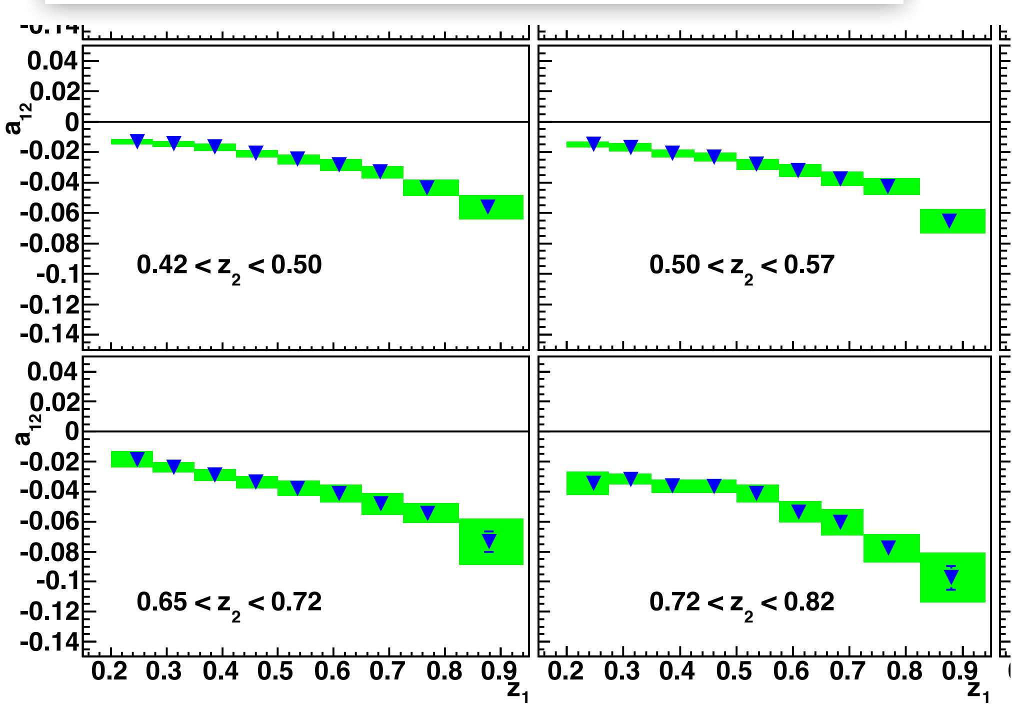


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

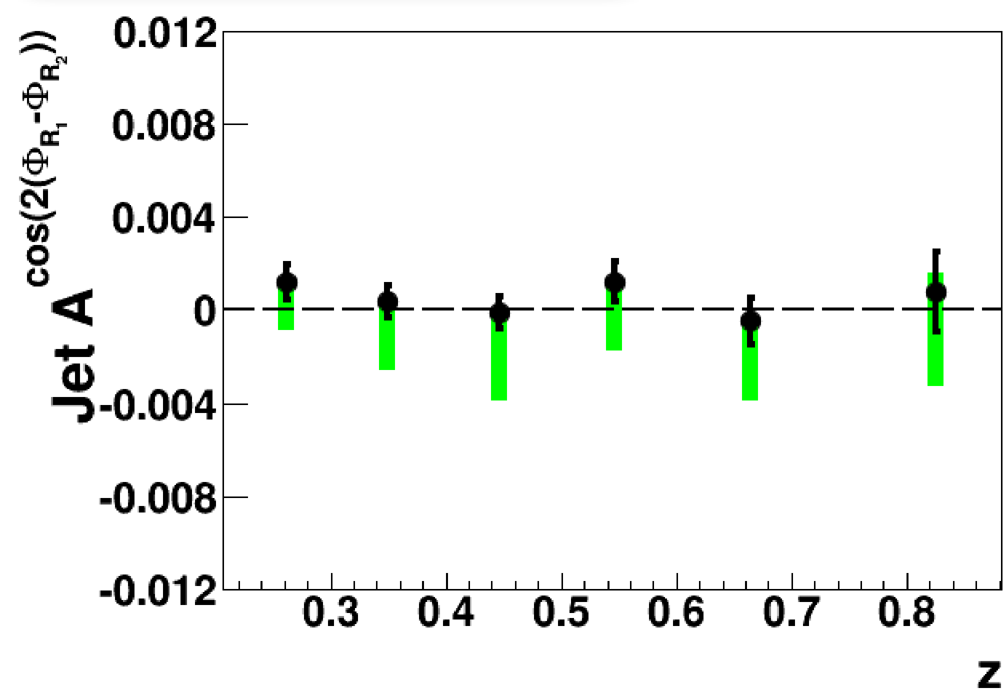
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



PoS DIS2015 (2015) 216



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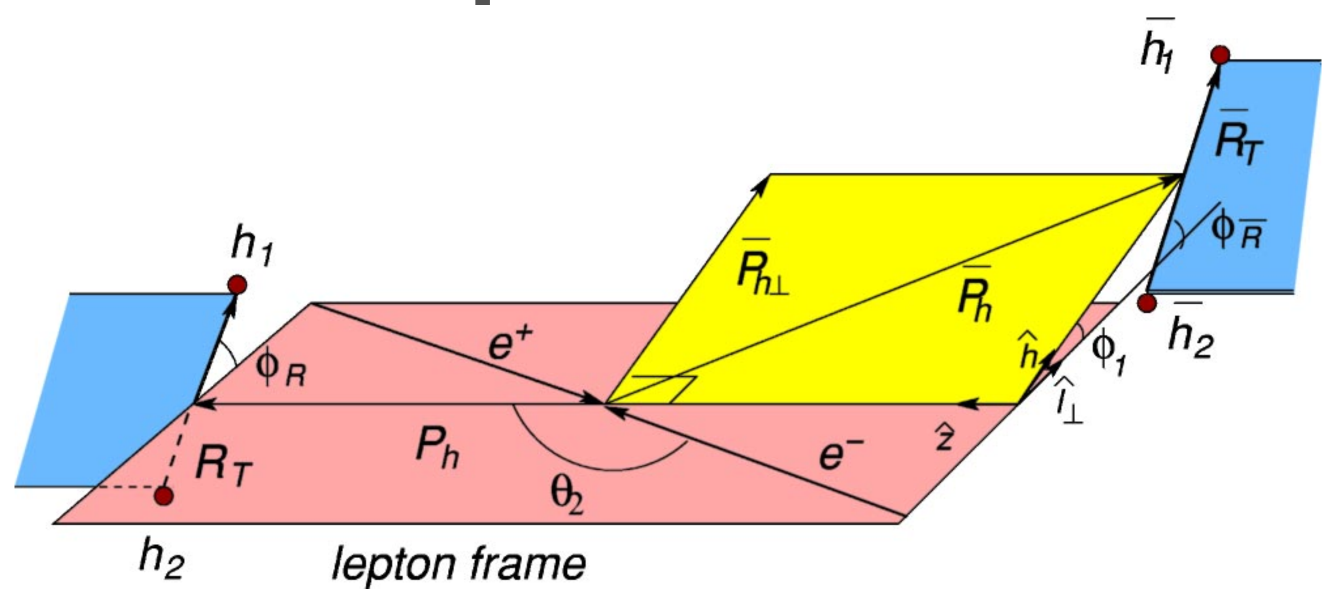

$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

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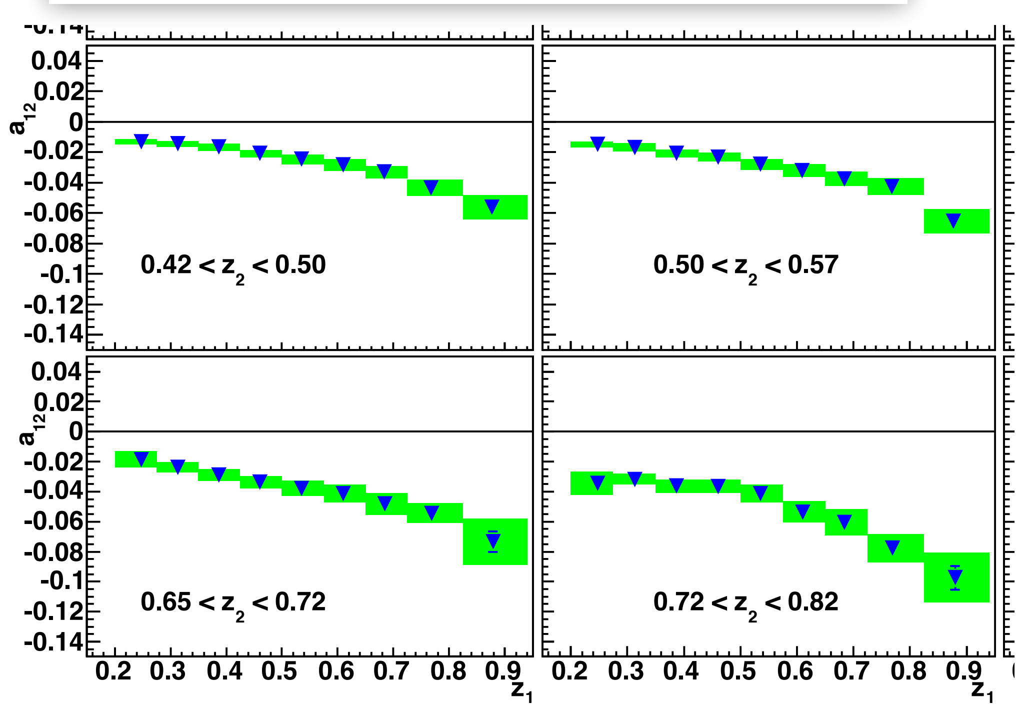
Phys.Rev.Lett. 107 (2011) 072004



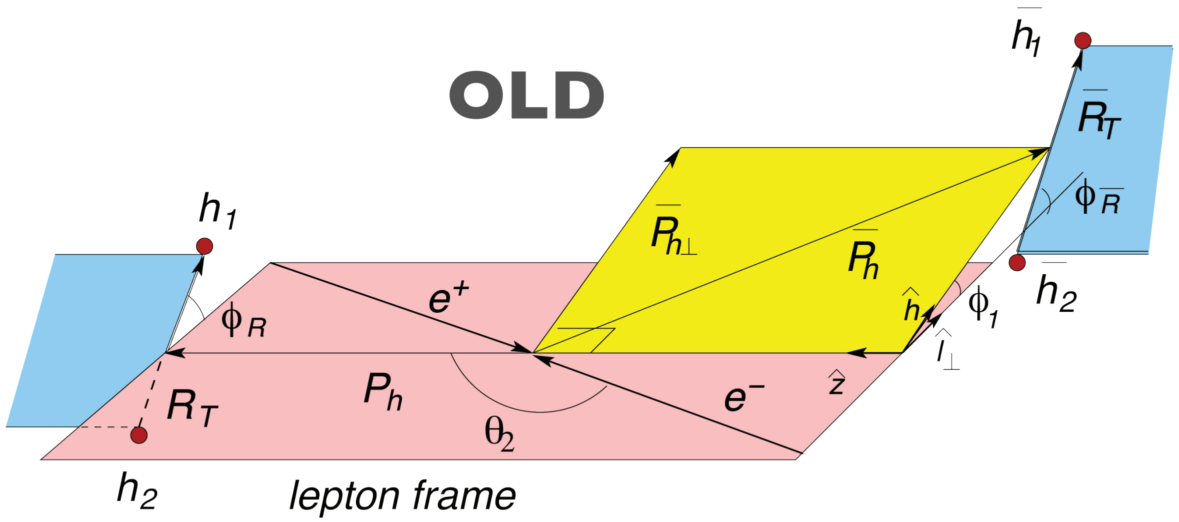
PoS DIS2015 (2015) 216

Re-derived e^+e^- Cross Section

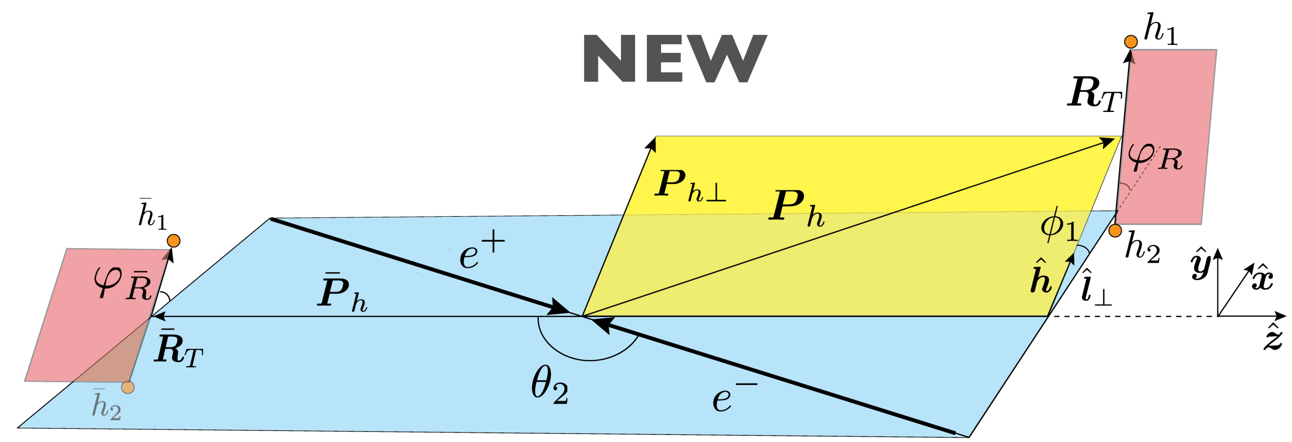
H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***An error in kinematics was found:***

OLD



NEW



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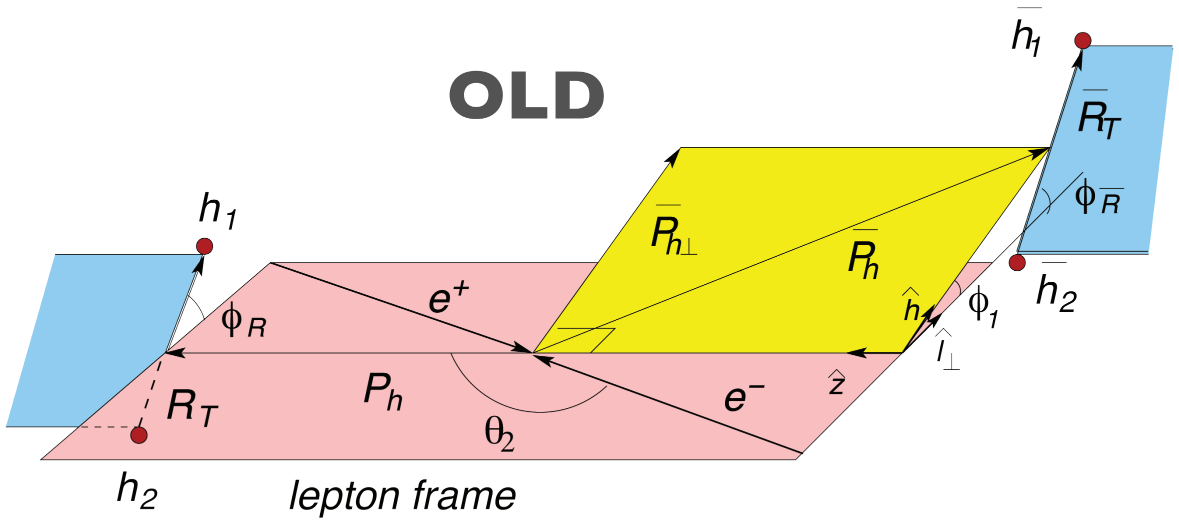
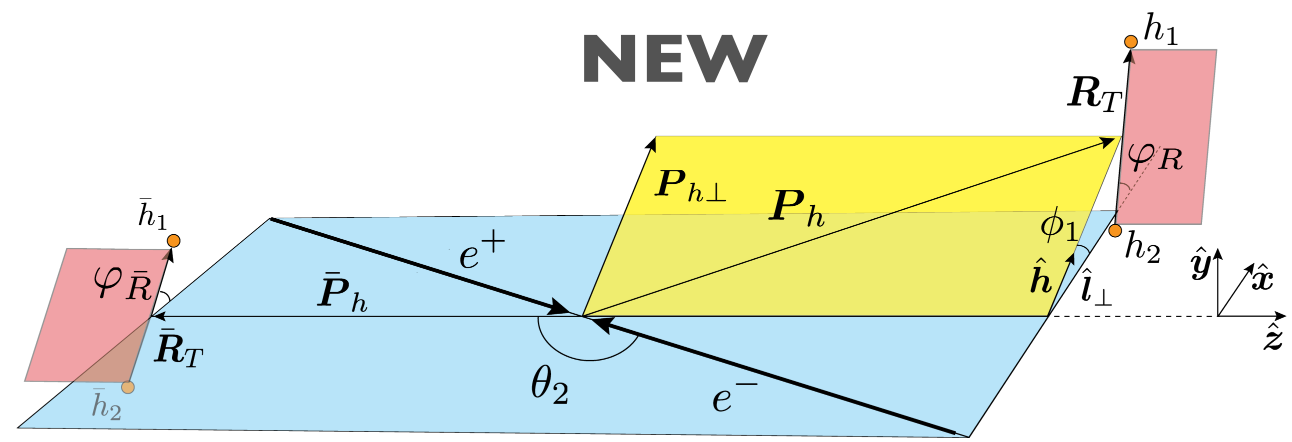


NEW



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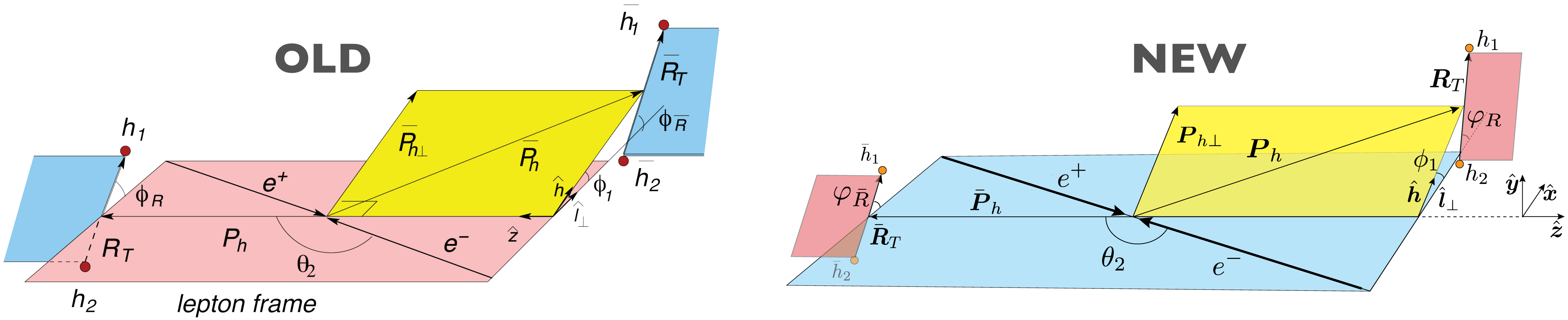


- ***The new fully differential cross-section expression:***

$$
\frac{d\sigma\Big(e^+e^- \to (h_1 h_2)(\bar h_1 \bar h_2)X\Big)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar z d\bar\xi d\varphi_{\bar R} d\bar M_h^2 dy} = \frac{3\alpha^2}{\pi Q^2} z^2 \bar z^2 \sum_{a,\bar a} e_a^2 \Bigg\{ A(y)\mathcal{F}\Big[D_1^a \bar D_1^{\bar a}\Big]
$$

$$
+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar k}) H_1^{\perp a}\bar H_1^{\perp \bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar R}) H_1^{\sphericalangle a}\bar H_1^{\sphericalangle \bar a}\right]
$$

$$
+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar R}) H_1^{\perp a}\bar H_1^{\sphericalangle \bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar k}) H_1^{\sphericalangle a}\bar H_1^{\perp \bar a}\right]
$$

$$
- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar M_h^2}\sin(\varphi_k-\varphi_R)\sin(\varphi_{\bar k}-\varphi_{\bar R}) G_1^{\perp a}\bar G_1^{\perp \bar a}\right]\Bigg\}.
$$

Helicity-dependent DiFF in e+e-

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).

- ***The relevant terms involving*** $G_1^{\perp}$ **:**

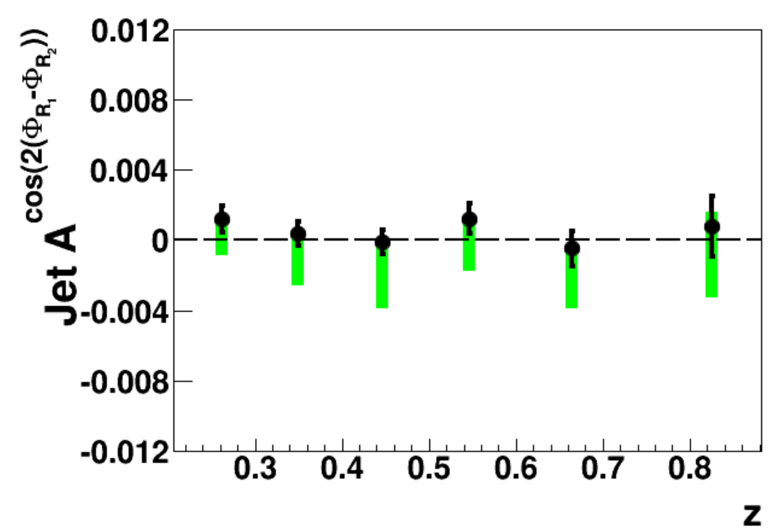
$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

- ***Note:*** ***any azimuthal moment involving only*** $\varphi_R,\ \varphi_{\bar{R}}$ ***is zero.***

$$\langle f(\varphi_R, \varphi_{\bar{R}})\rangle_L = 0$$

- ***The old asymmetry by Boer et. al. exactly vanishes!***
- ***Explains the BELLE results.***

$$A^{\Rightarrow} = \frac{\langle \cos(2(\varphi_R - \varphi_{\bar{R}}))\rangle}{\langle 1 \rangle} = 0!$$



Accessing $G_1^\perp$ DiFF in e+e-

H.M. , Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).

- ***The relevant terms involving $G_1^\perp$:***

$$d\sigma_L \sim \mathcal{F}\left[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\right]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

$$\left\langle \frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h \bar{M}_h} \right\rangle$$

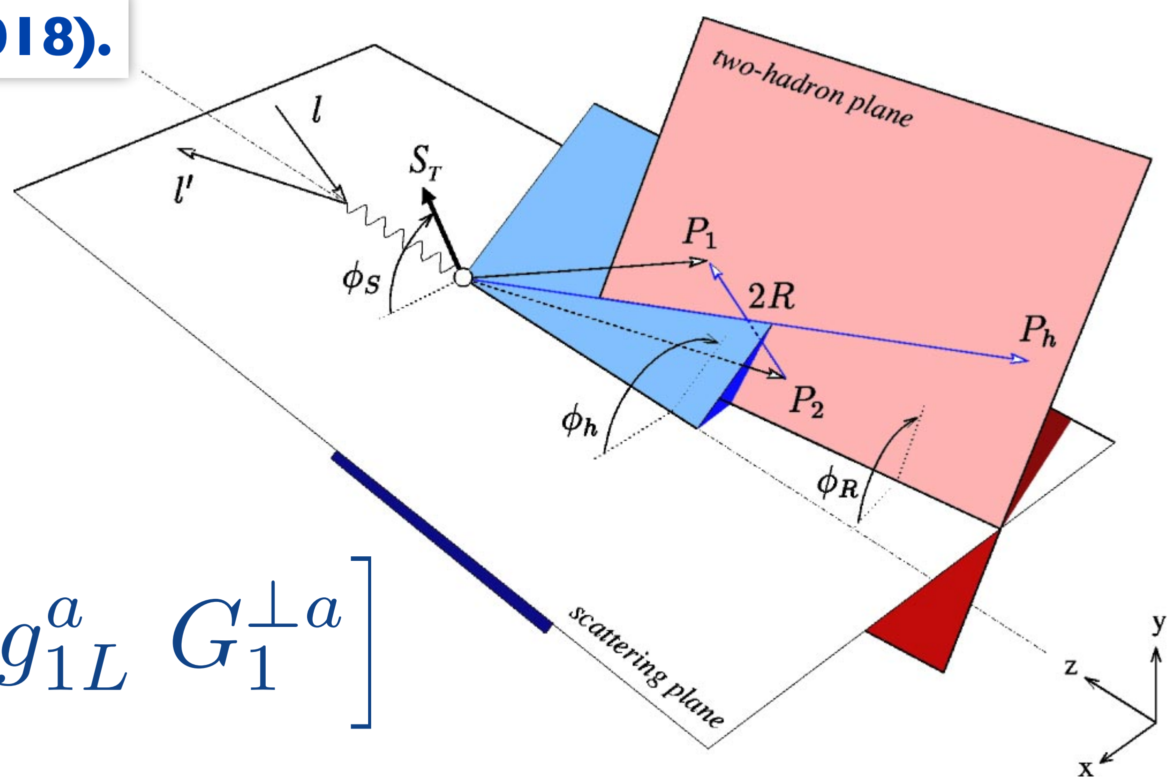
$$= \frac{12\alpha^2 A(y)}{\pi Q^2}\sum_{a,\bar{a}} e_a^2 \left(G_1^{\perp a,[0]} - G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp \bar{a},[0]} - G_1^{\perp \bar{a},[2]}\right),$$

- ***A new asymmetry to access $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$***

$$A_{e^+e^-}^{\Rightarrow}(z,\bar{z},M_h^2,\bar{M}_h^2) = 4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z,M_h^2)\; G_1^{\perp \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z,M_h^2)\; D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}$$

New way to access $G_1^\perp$ DiFF in *SIDIS*

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).



- ***The relevant terms involving*** $G_1^\perp$

$$d\sigma_{UL} \sim S_L \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\ g_{1L}^a\ G_1^{\perp a}\Big]$$

- ***Weighted moment accesses same*** $G_1^\perp$ ***as in e⁺e⁻ .***

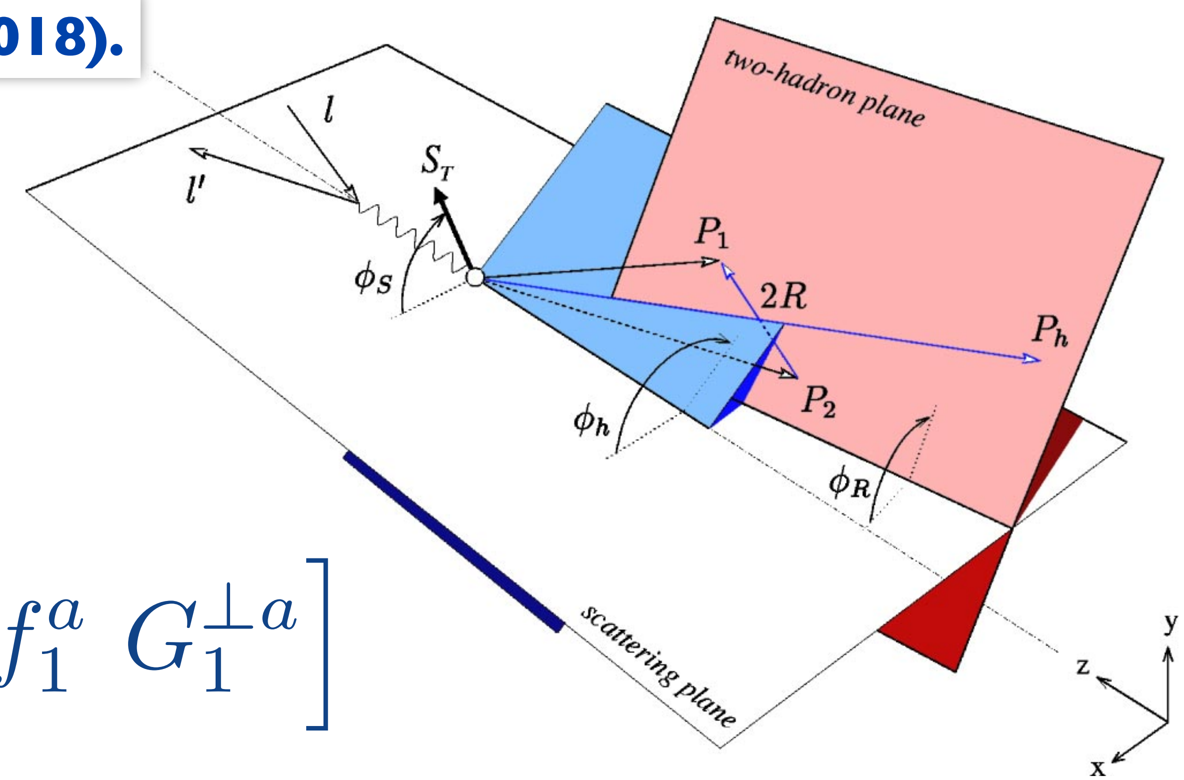
$$\left\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \right\rangle_{UL} \sim S_L \sum_a e_a^2\ g_{1L}^a(x)\ z\ G_1^{\perp a}(z, M_h^2)$$

$$A_{SIDIS}^{\Rightarrow}(x, z, M_h^2) = S_L \frac{\sum_a g_{1L}^a(x)\ z\ G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\ D_1^a(z, M_h^2)}.$$

New way to access $G_1^\perp$ DiFF in *SIDIS: II*

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).

Consider a polarized beam.



- ***The relevant terms involving*** $G_1^\perp$

$$d\sigma_{LU} \sim \lambda_e \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\ f_1^a\ G_1^{\perp a}\Big]$$

- ***Weighted moment accesses same*** $G_1^\perp$ ***as in e+e-.***

$$\left\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \right\rangle_{LU} \sim \lambda_e \sum_a e_a^2\ f_1^a(x)\ z\ G_1^{\perp a}(z, M_h^2)$$

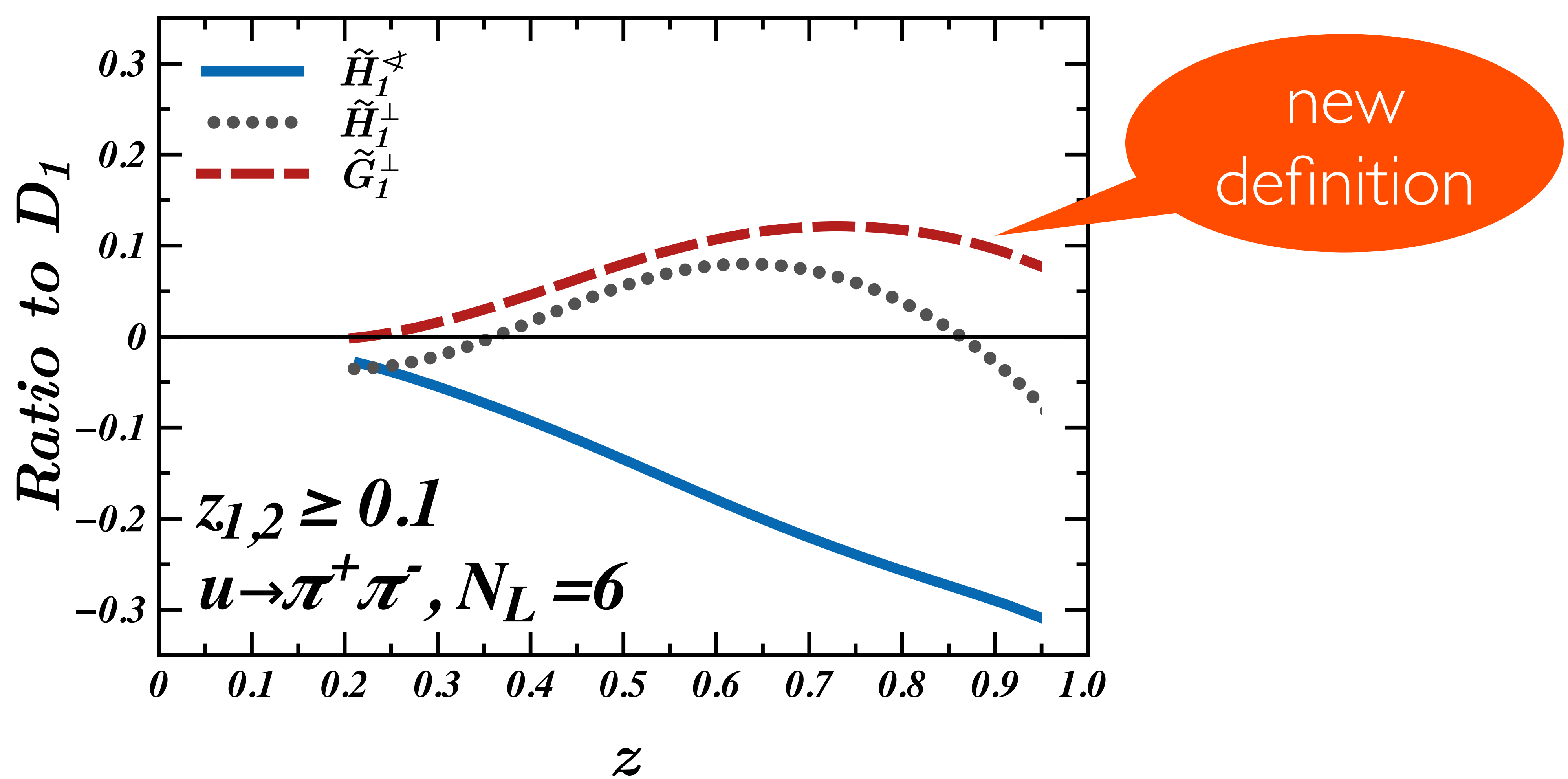
$$A_{SIDIS}^{\hookrightarrow}(x, z, M_h^2) \sim \lambda_e \frac{C'(y)}{A'(y)} \frac{\sum_a f_1^a(x)\ z\ G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\ D_1^a(z, M_h^2)}.$$

Feasibility of new measurements of $G_1^{\perp}$

✦ **The analysing powers of DiFFs from quark-jet framework.**

Phys. Rev. D96 074010, (2017); Phys. Rev. D97, 014019 (2018).

- $G_1^{\perp}$ ***naturally smaller than*** $H_1^{\sphericalangle}$***, but should be measurable!***



✦ **Reanalyze BELLE and COMPASS data.**

✦ **Measure it at BELLE II and JLab 12GeV.**

A NEW MEASUREMENT

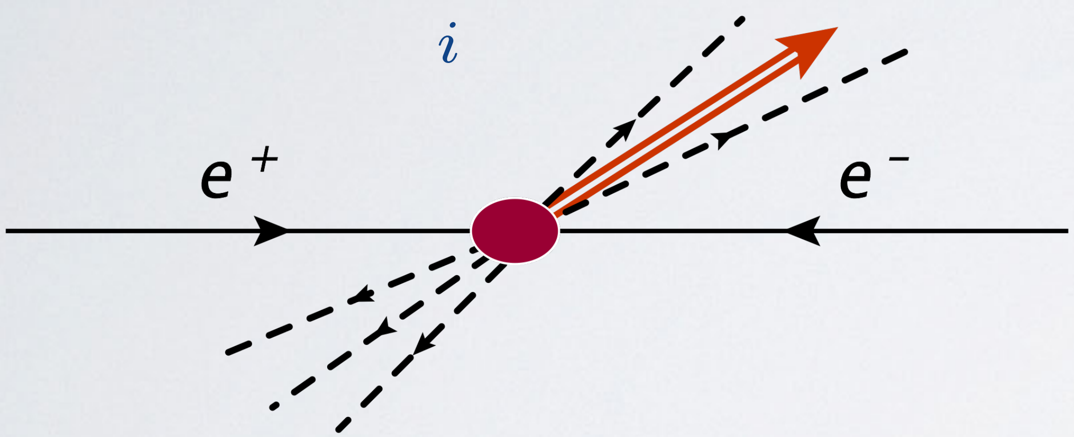
in e^+e^-

FRAGMENTATIONS FROM e+e-

FF

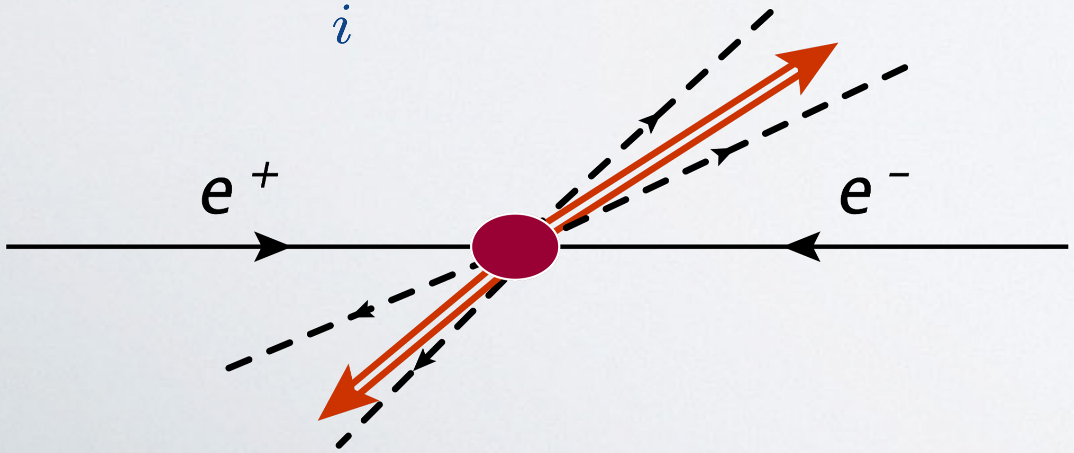
❖ inclusive hadron

$$\sum_i D_i^h$$



❖ back-to-back hadrons

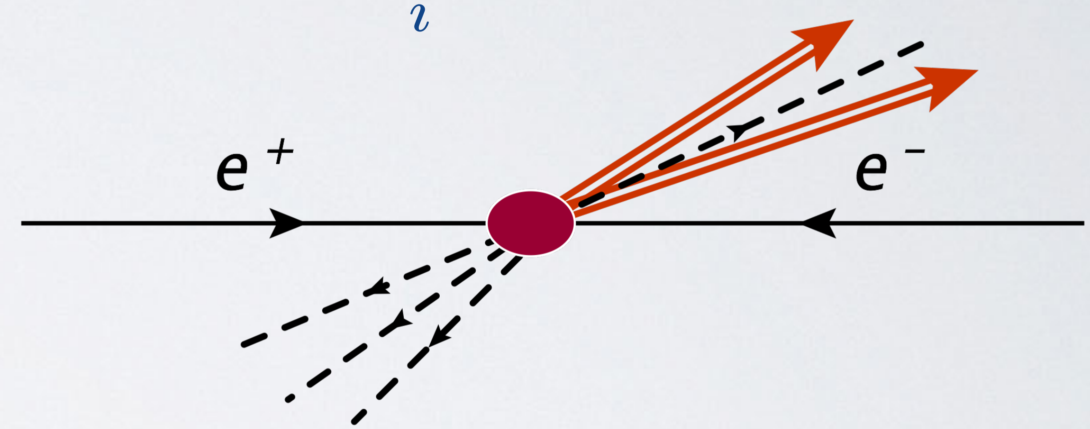
$$\sum_i D_i^{h_1} \otimes D_{\bar{i}}^{\bar{h}_1}$$



DiFF

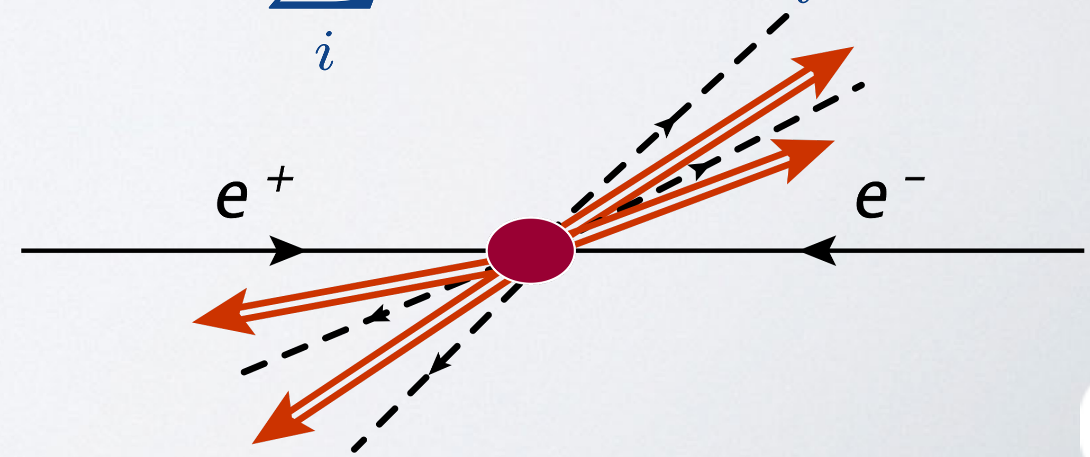
❖ inclusive hadron pair

$$\sum_i D_i^{h_1 h_2}$$



❖ back-to-back hadron pairs

$$\sum_i D_i^{h_1 h_2} \otimes D_{\bar{i}}^{\bar{h}_1 \bar{h}_2}$$

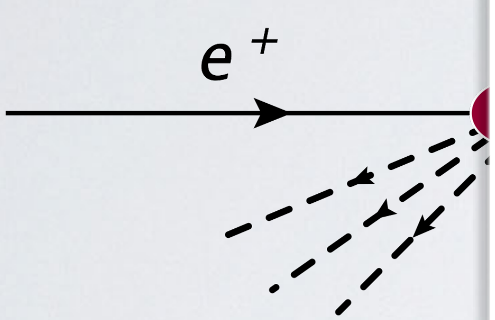


FRAGMENTATIONS FROM e^+e^-

FF

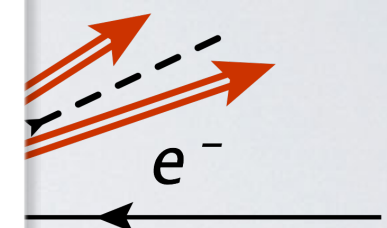
❖ inclusive hadron

$\sum_i$



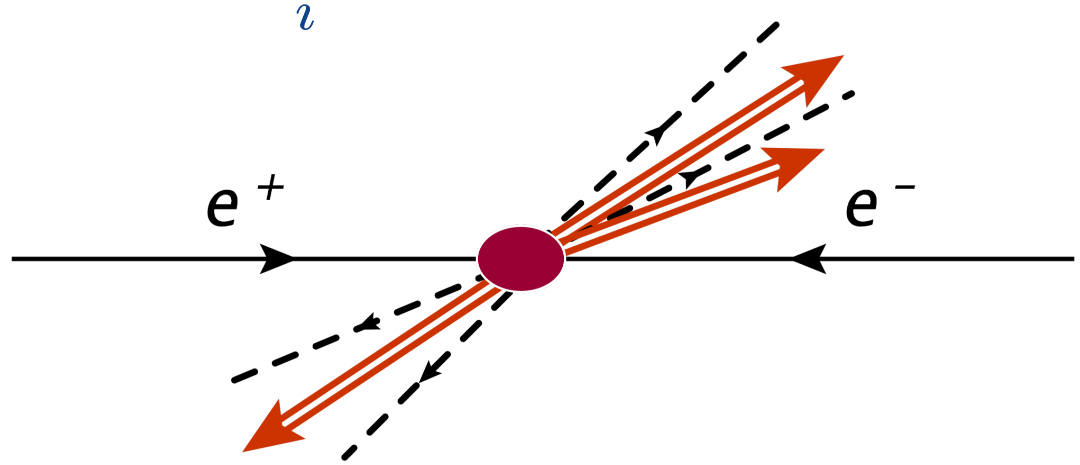
DiFF

❖ inclusive hadron pair



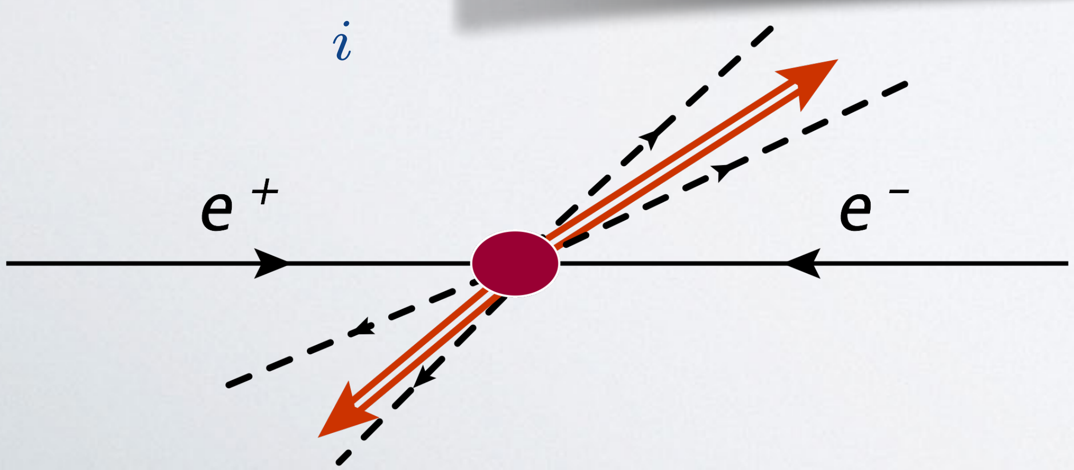
❖ back-to-back hadron pair and a hadron

$$\sum_i D_i^{h_1 h_2} \otimes D_{\bar{i}}^{\bar{h}_1}$$



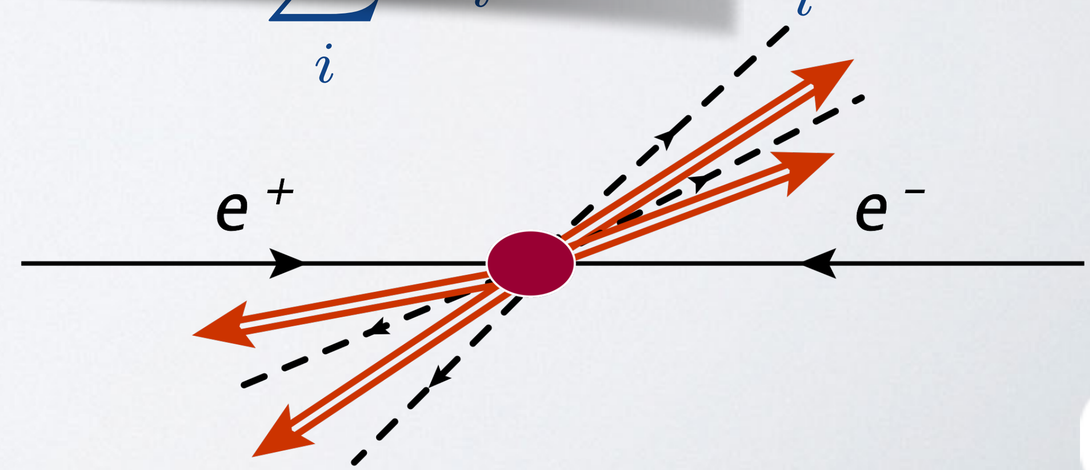
❖ back-to-bac

$\sum_i D$



on pairs

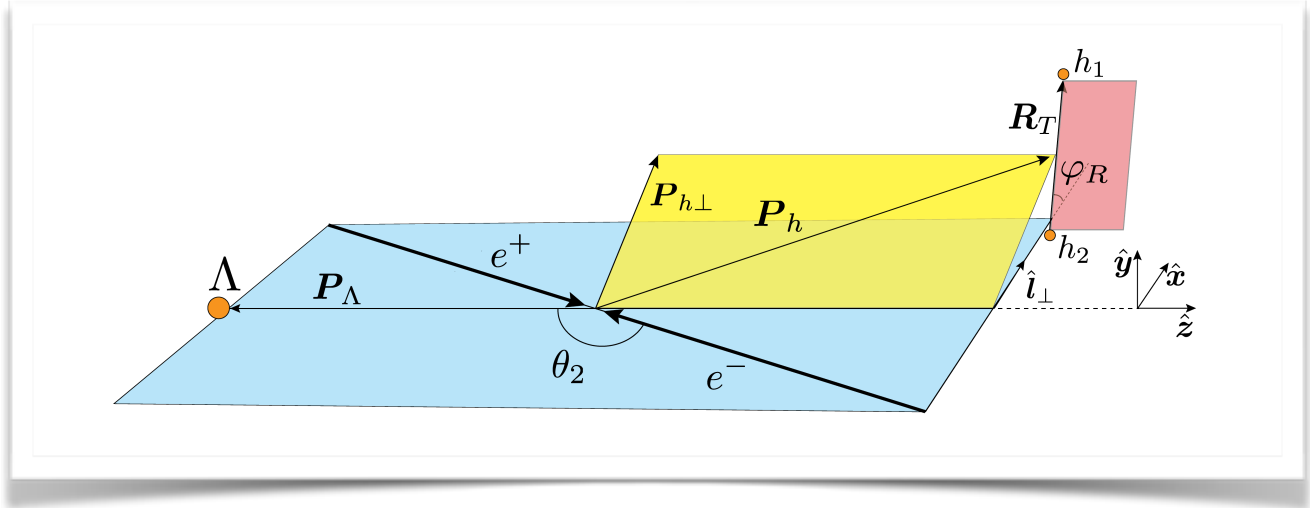
$\sum_i \; D_{\bar{i}}^{\bar{h}_1 \bar{h}_2}$



The Cross Section

H.M. , Kotzinian, Thomas: JHEP 1810 (2018) 008.

- *Use the standard kinematics to derive LO x-sec.*



$$
\begin{aligned}
\frac{d\sigma\Big(e^+e^- \to (h_1 h_2) + \Lambda + X\Big)}{d^2\boldsymbol{q}_T\; dz\; d\varphi_R\; dM_h^2\; d\xi\; d\bar{z}\; dy} = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} z^2 \bar{z}^2 \sum_a e_a^2 \\
\times \Bigg\{ \quad & A(y)\; \mathcal{F}\Big[D_1^{a\to h_1 h_2} D_1^{\bar{a}\to\Lambda} \Big] \\
& - S_T A(y)\; \mathcal{F}\Big[\frac{\bar{k}_T}{M_\Lambda} \sin(\varphi_{\bar{k}} - \varphi_S)\; D_1^{a\to h_1 h_2} D_{1T}^{\perp,\bar{a}\to\Lambda} \Big] \\
& + \lambda_\Lambda A(y)\; \mathcal{F}\Big[\frac{k_T R_T}{M_h^2} \sin(\varphi_k - \varphi_R)\; G_1^{\perp,a\to h_1 h_2} G_{1L}^{\bar{a}\to\Lambda} \Big] \\
& + S_T A(y)\; \mathcal{F}\Big[\frac{k_T R_T}{M_h^2} \sin(\varphi_k - \varphi_R) \frac{\bar{k}_T}{M_\Lambda} \cos(\varphi_{\bar{k}} - \varphi_S)\; G_1^{\perp,a\to h_1 h_2} G_{1T}^{\bar{a}\to\Lambda} \Big] \\
& + S_T B(y)\; \mathcal{F}\Big[\Big(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_S) H_1^{\perp,a\to h_1 h_2} \\
& \qquad + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_S) H_1^{\sphericalangle,a\to h_1 h_2} \Big)\; H_{1T}^{\bar{a}\to\Lambda} \Big] \\
& + \lambda_\Lambda B(y)\; \mathcal{F}\Big[\Big(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h_1 h_2} \\
& \qquad + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1 h_2} \Big)\; \frac{\bar{k}_T}{M_\Lambda} H_{1L}^{\perp,\bar{a}\to\Lambda} \Big] \\
& + S_T B(y)\; \mathcal{F}\Big[\Big(\frac{k_T}{M_h} \sin(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h_1 h_2} \\
& \qquad + \frac{R_T}{M_h} \sin(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1 h_2} \Big)\; \frac{\bar{k}_T^2}{M_\Lambda^2} \cos(\varphi_{\bar{k}} - \varphi_S) H_{1T}^{\perp,\bar{a}\to\Lambda} \Big] \\
& + \quad B(y)\; \mathcal{F}\Big[\Big(\frac{k_T}{M_h} \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h_1 h_2} \\
& \qquad + \frac{R_T}{M_h} \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1 h_2} \Big)\; \frac{\bar{k}_T}{M_\Lambda} H_1^{\perp,\bar{a}\to\Lambda} \Big] \quad \Bigg\},
\end{aligned}
$$

Flavor Decomposition of DiFFs

❖ Integrated cross section

$$\frac{d\sigma\Big(e^+e^- \to (h_1h_2) + \Lambda + X\Big)}{dz\ dM_h^2\ d\bar{z}\ dy} = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2\ D_1^{a\to h_1h_2}(z, M_h^2)\ \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z}),$$

❖ Isospin symmetry

$$D_1^{u\to\pi^+\pi^-} = D_1^{\bar{u}\to\pi^+\pi^-} \approx D_1^{d\to\pi^+\pi^-} = D_1^{\bar{d}\to\pi^+\pi^-},$$

$$D_1^{s\to\pi^+\pi^-} = D_1^{\bar{s}\to\pi^+\pi^-}.$$

❖ One pair inclusive: cannot disentangle the flavor dependence

$$d\sigma(e^+e^- \to (h_1h_2) + X) \sim \sum_q e_q^2\ D_1^{q\to\pi^+\pi^-} \approx \frac{5}{9} D_1^{u\to\pi^+\pi^-}(z) + \frac{1}{9} D_1^{s\to\pi^+\pi^-}(z)$$

❖ New process: use the knowledge of single hadron FFs!

$$d\sigma(e^+e^- \to (h_1h_2) + \pi^+ + X) \sim \frac{5}{9} D_1^{u\to\pi^+\pi^-}(z) D_1^{u^+\to\pi^+}(\bar{z}) + \frac{1}{9} D_1^{s\to\pi^+\pi^-}(z) D_1^{s^+\to\pi^+}(\bar{z}),$$

$$D_1^{q^+\to h}(\bar{z}) \equiv D_1^{q\to h}(\bar{z}) + D_1^{\bar{q}\to h}(\bar{z}).$$

Weighted Asymmetries.

❖ Unpolarized hadrons: Accessing Collins x IFF.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \frac{B(y)}{M_\Lambda^2 M_h}$$
$$\times \sum_a e_a^2 \int d\xi \int d\varphi_R \int d^2\boldsymbol{q}_T \int d^2\boldsymbol{k}_T \int d^2\bar{\boldsymbol{k}}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) q_T \cos(\varphi_q + \varphi_R)$$
$$\times \left[\left(k_T \bar{k}_T \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a\to h_1 h_2} + R_T \bar{k}_T \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle, a\to h_1 h_2} \right) H_1^{\perp, \bar{a}\to\Lambda} \right],$$

❖ Momentum weighting helps to disentangle TM convolutions.

$$\int d^2\boldsymbol{q}_T \; \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) \; q_T \cos(\varphi_q + \varphi_R) = (k_T \cos(\varphi_k + \varphi_R) + \bar{k}_T \cos(\varphi_{\bar{k}} + \varphi_R)).$$

❖ Resulting moment and the asymmetry.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\perp \bar{a},[1]}(\bar{z}),$$

$$A^{Coll} = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\perp \bar{a},[1]}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}.$$

Weighted Asymmetries.

❖ Unpolarized hadrons: Accessing Collins x IFF.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \frac{B(y)}{M_\Lambda^2 M_h}$$

$$\times \sum e^2 \int d\xi \int d\varphi_R \int d^2\boldsymbol{q}_T \int d^2\boldsymbol{k}_T \int d^2\bar{\boldsymbol{k}}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) q_T \cos(\varphi_q + \varphi_R)$$

$$\ldots R_T \bar{k}_T \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle, a\to h_1 h_2} \Big) H_1^{\perp, \bar{a}\to\Lambda} \Big],$$

A. M. Kotzinian and P. J. Mulders
Phys. Rev. D 54, 1229 (1996)

❖ Momentum weighting helps to disentangle TM convolutions.

$$\int d^2\boldsymbol{q}_T\ \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T)\ q_T \cos(\varphi_q + \varphi_R) = (k_T \cos(\varphi_k + \varphi_R) + \bar{k}_T \cos(\varphi_{\bar{k}} + \varphi_R)).$$

❖ Resulting moment and the asymmetry.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2\ H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2)\ H_1^{\perp \bar{a},[1]}(\bar{z}),$$

$$A^{Coll} = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2\ H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2)\ H_1^{\perp \bar{a},[1]}(\bar{z})}{\sum_a e_a^2\ D_1^{a\to h_1 h_2}(z, M_h^2)\ \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}.$$

Weighted Polarized Asymmetries: L

❖ Accessing Helicity DiFF

$$\langle \beta_L \rangle_{G_1^\perp G_{1L}} = \left\langle \frac{q_T}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2 \; G_1^{\perp, a \to h_1 h_2}(z, M_h^2) \; G_{1L}^{\bar{a} \to \Lambda}(\bar{z}).$$

$$\langle s_L \rangle^{\sin(\varphi_q - \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{\sum_a e_a^2 \; G_1^{\perp, a \to h_1 h_2}(z, M_h^2) \; G_{1L}^{\bar{a} \to \Lambda}(\bar{z})}{\sum_a e_a^2 \; D_1^{a \to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a} \to \Lambda}(\bar{z})},$$

Nonzero measurements of longitudinal Λ polarization at ALEPH!

❖ Combination of IFF with Kotzinian-Mulders type FF:

$$\langle \beta_L \rangle_{H_1^\sphericalangle H_{1L}^\perp} = \left\langle \frac{q_T}{M_\Lambda} \sin(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a \to h_1 h_2}(z, M_h^2) \; H_{1L}^{\perp \bar{a}, [1]}(\bar{z}),$$

$$\langle s_L \rangle^{\sin(\varphi_q + \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a \to h_1 h_2}(z, M_h^2) \; H_{1L}^{\perp \bar{a}, [1]}(\bar{z})}{\sum_a e_a^2 \; D_1^{a \to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a} \to \Lambda}(\bar{z})}.$$

Happy Birthday Aram!