

Aram's no-nonsense TMDs

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European Research Council



Memories over 25 out of 70 years

Aram's no-nonsense TMDs

My first encounter with Aram's work

Our work

Ideas and discussions



First of all: I clearly admire Aram

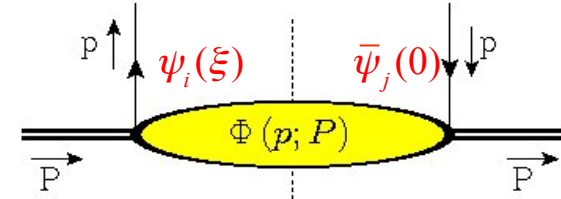
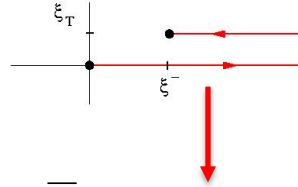


TMDs (with gauge links)

- quark-quark TMD correlator

$$u_i(k)\bar{u}_j(k) \implies$$

$$\Phi_{ij}^{[U]}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) U_{[0, \xi]} \psi_i(\xi) | P, S \rangle \Big|_{\xi \cdot n = 0}$$



- Parametrization

$$\Phi^{[U]}(x, k_T) = \left\{ f_1^{[U]}(x, k_T^2) + i h_1^{\perp [U]}(x, k_T^2) \frac{k_T}{M} \right\} \frac{\not{P}}{2}$$

$$\Phi(x) = f_1(x) \frac{\not{P}}{2}$$

$$\Phi_L^{[U]}(x, k_T) = \left\{ S_L g_1^{[U]}(x, k_T^2) \gamma_5 + S_L h_{1L}^{\perp [U]}(x, k_T^2) \frac{\gamma_5 k_T}{M} \right\} \frac{\not{P}}{2}$$

$$\Phi_L(x) = S_L g_1(x) \gamma_5 \frac{\not{P}}{2}$$

$$\Phi_T^{[U]}(x, k_T) = \left\{ g_{1T}^{[U]}(x, k_T^2) \frac{k_T \cdot S_T}{M} \gamma_5 + f_{1T}^{\perp [U]}(x, k_T^2) \frac{k_T \times S_T}{M} + h_1^{[U]}(x, k_T^2) \gamma_5 \not{S}_T + h_{1T}^{\perp [U]}(x, k_T^2) \frac{k_T^{\alpha\beta} S_{T\alpha} \gamma_{\beta} \gamma_5}{M^2} \right\} \frac{\not{P}}{2}$$

$$\Phi_T(x) = h_1(x) \gamma_5 \not{S}_T \frac{\not{P}}{2}$$

- Surviving in collinear correlators $\Phi(x)$ and including flavor index

$$f_1^q(x) \equiv q(x) \quad g_1^q(x) = \Delta q(x) \quad h_1^q(x) = \delta q(x)$$

- Including twist three part (only collinear)

$$\Phi(x) = \left\{ f_1(x) + S_L g_1(x) \gamma_5 + h_1(x) \not{S}_T \right\} \frac{\not{P}}{2} + \left\{ e(x) + g_T(x) \not{S}_T \gamma_5 + S_L h_L(x) \gamma_5 \right\} \frac{M}{2}$$

TMDs (with gauge links)

- quark-quark correlator for free quark ensemble

$$u_i(k)\bar{u}_j(k) \implies$$

$$u^{(s)}(k)\mathcal{P}_{ss'}(k)u^{(s')}(k) = \mathcal{P}(k)(\not{k} + m) \left(\frac{1 + \gamma_5 \not{\epsilon}(k)}{2} \right)$$

- no-nonsense result (Kotzinian and Mulders-Tangerman)

$$P(x, k_T^2) = f_1(x, k_T^2)$$

$$P(x, k_T^2) s_L(x, k_T^2) = g_{1L}(x, k_T^2) S_L + g_{1T}(x, k_T^2) \frac{k_T \cdot S_T}{M}$$

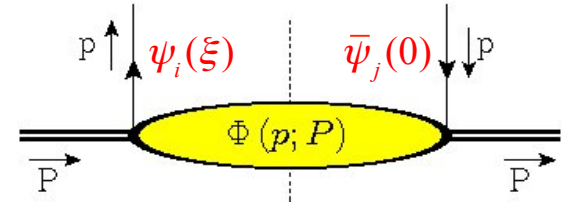
$$P(x, k_T^2) s_T(x, k_T^2) = h_{1T}(x, k_T^2) S_T + \frac{k_T}{M} \left[h_{1L}^\perp(x, k_T^2) S_L + h_{1T}^\perp(x, k_T^2) \frac{k_T \cdot S_T}{M} \right]$$

- and relations like

$$e = \frac{m}{Mx} f_1$$

$$g_T = \frac{g_{1T}^{(1)}}{x} + \frac{m}{Mx} h_1$$

$$h_L = -\frac{2h_{1L}^{\perp(1)}}{x} + \frac{m}{Mx} g_1$$



$$k = xP + k_T + \dots$$

$$s(k) = s_L(k) \frac{k}{m} + s_T(k) + \dots$$

LIGHT FRONT language
GOOD
CHIRAL
CHIRAL ODD

		PARTON SPIN		
QUARKS		γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^a\gamma_5$
TARGET SPIN	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_{1T}^\perp

Competitors in 1994/1995

- J.P. Ralston and D.E. Soper, NPB 152 (1979) 109
 - transverse momenta in Drell-Yan at nonzero Q_T
- R.L. Jaffe and X.D. Ji, PRL 67 (1991) 552
 - collinear functions, polarization, leading and subleading twist, notation f, g, h
 - early work in my group with e.g. Joachim Levelt, ...
- R.D. Tangerman and P.J. Mulders, hep-ph/9403227, PRD 51 (1995) 3357
 - TMD functions, polarization, leading, notation g_{1T}, h_{1L}, \dots , used in Drell-Yan
 - hep-ph/9408305 for subleading functions g_T and h_L
 - Includes the omitted term in Ralston-Soper (first: Levelt-Tangerman \rightarrow Pretzelosity)
- A. Kotzinian, hep-ph/9412283, NPB 441 (1995) 234
 - no-nonsense TMD's, used in lepton production
- R.D. Tangerman and P.J. Mulders, hep-ph/9501202, PLB 352 (1995) 129



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Nuclear Physics B 441 (1995) 234–256

NUCLEAR
PHYSICS B

New quark distributions and semi-inclusive electroproduction on polarized nucleons

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Abstract

The quark-parton model calculation including the effects of intrinsic transverse momentum and of all six twist-two distribution functions of quarks in polarized nucleons is performed. It is demonstrated that new twist-two quark distribution functions and polarized quark fragmentation functions can be investigated in semi-inclusive DIS at leading order in Q^2 . The general expression for the cross-section of semi-inclusive DIS of polarized leptons on polarized nucleons in terms of structure functions is also discussed.

¹ Now a visitor at CERN, PPE-Division, CH-1211, Geneva 23, Switzerland.

General consideration of the quark DF in a polarized nucleon in the case of nonvanishing k_T has been done by Ralston and Soper [1] and recently by Tangerman and Mulders [18]. They have found that at the leading twist one needs six independent DFs depending on x and k_T^2 : $f_1, g_{1L}, g_{1T}, h_{1T}, h_{1L}^\perp$, and h_{1T}^\perp . The distributions $s_L^{(in)}(x, k_T)$ and $s_T^{(in)}(x, k_T)$ are given by [18]

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$$\begin{aligned}
 \mathcal{P}_N^q(x, k_T^2) &= f_1(x, k_T^2), \\
 \mathcal{P}_N^q(x, k_T^2) s_L^{(in)}(x, k_T) &= g_{1L}(x, k_T^2) S_L + g_{1T}(x, k_T^2) \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_D}, \\
 \mathcal{P}_N^q(x, k_T^2) s_T^{(in)}(x, k_T) &= h_{1T}(x, k_T^2) S_T \\
 &\quad + \left[h_{1L}^\perp(x, k_T^2) S_L + h_{1T}^\perp(x, k_T^2) \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_D} \right] \frac{\mathbf{k}_T}{m_D}, \quad (27)
 \end{aligned}$$

where m_D is an unknown mass parameter, S_L and S_T are the nucleon longitudinal and transverse polarization with respect to its momentum. The “new” DFs have clear physical interpretation: for example, g_{1T} describes the quark longitudinal polarization in a transversely-polarized nucleon. It is important to notice that due to this DF even the initial quark longitudinal spin distribution in a polarized nucleon exhibits an azimuthal asymmetry.



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8 June 1995

Physics Letters B 352 (1995) 129–133

PHYSICS LETTERS B

Probing transverse quark polarization in deep-inelastic leptonproduction

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Editor: P.V. Landshoff

Abstract

The azimuthal dependence of hadrons produced in lepton scattering off a polarized hadron probes the quark transverse-spin distributions. In the calculation of the asymmetries, transverse momenta of quarks in the distribution and fragmentation functions must be incorporated. In addition to the $\sin(\phi + \phi_S)$ asymmetry for transversely polarized hadrons, known as the Collins effect, we find a $\sin(3\phi - \phi_S)$ asymmetry. Furthermore, we find a $\sin 2\phi$ angular dependence for longitudinally polarized hadrons.

Competitors in 1994/1995

- J.P. Ralston and D.E. Soper, NPB 152 (1979) 109
 - transverse momenta in Drell-Yan at nonzero Q_T
- R.L. Jaffe and X.D. Ji, PRL 67 (1991) 552
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 - TMD functions, polarization, leading, notation g_{1T}, h_{1L}, \dots , used in Drell-Yan
 - hep-ph/9408305 for subleading functions g_T and h_L
- A. Kotzinian, hep-ph/9412283, NPB 441 (1995) 234
 - no-nonsense TMD's, used in lepton production
- R.D. Tangerman and P.J. Mulders, hep-ph/9501202, PLB 352 (1995) 129

- Followed up with 'complete analysis' (Rik Tangerman), phenomenology (Aram Kotzinian) and study of T-odd functions (Daniel Boer)



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Nuclear Physics B 461 (1996) 197–237

NUCLEAR
PHYSICS B

The complete tree-level result up to order $1/Q$ for polarized deep-inelastic leptonproduction

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Received 18 October 1995; accepted 1 December 1995

Abstract

We present the results of the tree-level calculation of deep-inelastic leptonproduction, including polarization of target hadron and produced hadron. We also discuss the dependence on transverse momenta of the quarks, which leads to azimuthal asymmetries for the produced hadrons.

Includes subleading $1/Q$

Discusses 'staple' links

Use of $g_{1T}^{(1)}$, etc.

No T-odd stuff
(came in 1997)

Longitudinal quark polarization in transversely polarized nucleons

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(Received 27 November 1995)



Accounting for transverse momenta of the quarks, a longitudinal quark spin asymmetry exists in a transversely polarized nucleon target. The relevant leading quark distribution $g_{1T}(x, k_T^2)$ can be measured in semi-inclusive deep-inelastic scattering. The average k_T^2 weighted distribution function $g_{1T}^{(1)}$ can be obtained directly from the inclusive measurement of g_2 . [S0556-2821(96)03813-1]

Based on EOM relations

$$g_T = \frac{g_{1T}^{(1)}}{x} + \frac{m}{M} \frac{h_1}{x} + \tilde{g}_T = g_1 + \frac{d}{dx} g_{1T}^{(1)}$$



$$g_2(x) = - \left[g_1(x) - \int_x^1 dy \frac{g_1(y)}{y} \right]$$

$$h_L = \frac{m}{M} \frac{g_1}{x} - 2 \frac{h_{1L}^{\perp(1)}}{x} + \tilde{h}_L = h_1 - \frac{d}{dx} h_{1L}^{\perp(1)}$$

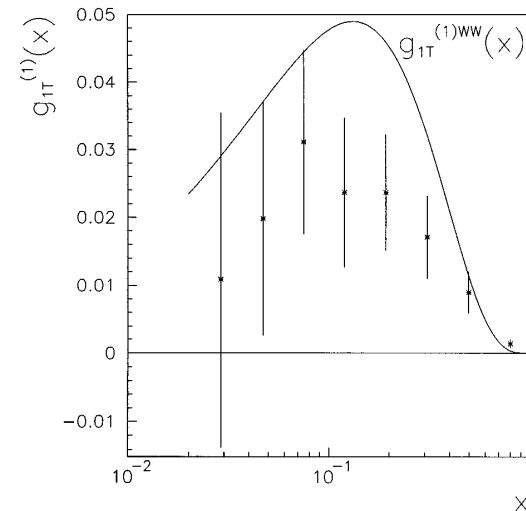


FIG. 2. The function $g_{1T}^{(1)}(x)$ as obtained from E143 data using Eq. (22) or from the BBS parametrizations for g_1 using Eq. (25).

Many follow-up developments

- T-odd functions (Dennis Sivers, John Collins, Daniel Boer, ...)
- Phenomenology (Aram Kotzinian, Elena Boglione, Rainer Jakob, ...)
- Extensions including higher spin (Alessandro Bacchetta), gluons (Joao Rodrigues, Andreas Metz, ...)

		PARTON SPIN		
QUARKS		γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^\alpha\gamma_5$
TARGET SPIN	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	$h_1^\perp h_{1T}^\perp$
	LL	f_{1LL}		h_{1LL}^\perp
	LT	f_{1LT}	g_{1LT}	$h_{1LT}^\perp h_{1LT}^\perp$
	TT	f_{1TT}	g_{1TT}	$h_{1TT}^\perp h_{1TT}^\perp$

		PARTON SPIN		
GLUONS		$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \dots$
TARGET SPIN	U	f_1^g		$h_1^{\perp g}$
	L		g_1^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g h_{1T}^{\perp g}$
	LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
	LT	f_{1LT}^g	g_{1LT}^g	$h_{1LT}^g h_{1LT}^{\perp g}$
	TT	f_{1TT}^g	g_{1TT}^g	$h_{1TT}^g h_{1TT}^{\perp g} h_{1TT}^{\perp\perp g}$

- Links to off-forward GPD's, ...
- Operator structure including gauge links and gluonic poles, factorization breaking
 - Ted Rogers, John Collins, ...
 - Fetze Pijlman, Cedran Bomhof, Maarten Buffing, ...
- Analytic structure, bounds and spectator models, low x , ...
 - Leonard Gamberg, Asmita Mukherjee, Sabrina Cotogno, Tom van Daal, Elena Petreska, ...

Since then we met at many meetings, Aram always modest

**3D Parton Distributions:
path to the LHC**



INFN-Laboratori Nazionali di Frascati
Bruno Touschek Auditorium
November 29th - December 2nd 2016

Topics

- QCD issues associated with 3D nucleon structure
- Dynamics of twist-3
- Framework for 3D PDFs extraction
- Medium modifications of multidimensional PDFs
- Parton orbital motion and correlations
- Measurements for future experiments

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3DSFN Argonne UCONN



... but he certainly deserves to be in the center like here!



and today

Congratulations!

Happy birthday

