Aram’s no-nonsense TMDs

Piet Mulders

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Memories over 25 out of 70 years

Aram’s no-nonsense TMDs

My first encounter with Aram’s work

Our work

Ideas and discussions
First of all: I clearly admire Aram
Surviving in collinear correlators $\Phi(x)$ and including flavor index

\[ f_1^q(x) \equiv q(x) \quad \Delta q(x) \quad \delta q(x) \]

Including twist three part (only collinear)

\[ \Phi(x) = \left\{ f_1(x) + S_L g_1(x) \gamma_5 + h_1(x) S_T \right\} \frac{P}{2} + \left\{ e(x) + g_T(x) S_T \gamma_5 + S_L h_L(x) \gamma_5 \right\} \frac{M}{2} \]
- quark-quark correlator for free quark ensemble
  \[ u_i(k)\bar{u}_j(k) \rightarrow \]
  \[ u^{(s)}(k)\mathcal{P}_{ss'}(k)u^{(s')}(k) = \mathcal{P}(k)(k' + m) \left( \frac{1 + \gamma_5 \not{k}(k)}{2} \right) \]

- no-nonsense result (Kotzinian and Mulders-Tangerman)
  \[ P(x, k^2_T) = f_1(x, k_T) \]
  \[ P(x, k^2_T) s_L(x, k^2_T) = g_{1L}(x, k_T) S_L + g_{1T}(x, k_T) \frac{k_T \cdot S_T}{M} \]
  \[ P(x, k^2_T) s_T(x, k^2_T) = h_{1T}(x, k_T) S_T + \frac{k_T}{M} \left[ h_{1L}^{+}(x, k^2_T) S_L + h_{1T}^{+}(x, k^2_T) \frac{k_T \cdot S_T}{M} \right] \]

- and relations like
  \[ e = \frac{m}{Mx} f_1 \]
  \[ g_T = \frac{g_{1T}}{x} + \frac{m}{Mx} h_1 \]
  \[ h_L = -\frac{2 h_{1L}^{(1)}}{x} + \frac{m}{Mx} g_1 \]

**LIGHT FRONT language**

**GOOD**

**CHIRAL**

**CHIRAL ODD**

### TMDs (with gauge links)

**PARTON SPIN**

<table>
<thead>
<tr>
<th>TARGET SPIN</th>
<th>QUARKS</th>
<th>(\gamma^+)</th>
<th>(\gamma^+\gamma_5)</th>
<th>(\gamma^+\gamma^\alpha\gamma_5)</th>
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<tbody>
<tr>
<td><strong>U</strong></td>
<td>(f_1)</td>
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<tr>
<td><strong>L</strong></td>
<td></td>
<td>(g_1)</td>
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<tr>
<td><strong>T</strong></td>
<td>(f_{1T})</td>
<td>(g_{1T})</td>
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\[ k = xP + k_T + ... \]
\[ s(k) = s_L\left(\frac{k}{m}\right) + s_T(k) + ... \]
Competitors in 1994/1995

- J.P. Ralston and D.E. Soper, NPB 152 (1979) 109
  - transverse momenta in Drell-Yan at nonzero $Q_T$
  - collinear functions, polarization, leading and subleading twist, notation $f, g, h$
  - early work in my group with e.g. Joachim Levelt, ...
  - TMD functions, polarization, leading, notation $g_{1T}, h_{1L}, ...$, used in Drell-Yan
  - hep-ph/9408305 for subleading functions $g_T$ and $h_L$
  - Includes the omitted term in Ralston-Soper (first: Levelt-Tangerman $\rightarrow$ Pretzelosity)
  - no-nonsense TMD’s, used in leptonproduction
New quark distributions and semi-inclusive electroproduction on polarized nucleons

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Received 23 December 1994; revised 9 February 1995; accepted 23 February 1995

Abstract

The quark-parton model calculation including the effects of intrinsic transverse momentum and of all six twist-two distribution functions of quarks in polarized nucleons is performed. It is demonstrated that new twist-two quark distribution functions and polarized quark fragmentation functions can be investigated in semi-inclusive DIS at leading order in $Q^2$. The general expression for the cross-section of semi-inclusive DIS of polarized leptons on polarized nucleons in terms of structure functions is also discussed.

1Now a visitor at CERN, PPE-Division, CH-1211, Geneva 23, Switzerland.
General consideration of the quark DF in a polarized nucleon in the case of nonvanishing $k_T$ has been done by Ralston and Soper [1] and recently by Tangerman and Mulders [18]. They have found that at the leading twist one needs six independent DFs depending on $x$ and $k_T^2$: $f_1, g_{1L}, g_{1T}, h_{1T}, h_{1L}^+, \text{and } h_{1T}^+$. The distributions $s_L^{(\text{in})}(x, k_T)$ and $s_T^{(\text{in})}(x, k_T)$ are given by [18]

\begin{align*}
\mathcal{P}_N^q(x, k_T^2) &= f_1(x, k_T^2), \\
\mathcal{P}_N^q(x, k_T^2) s_L^{(\text{in})}(x, k_T) &= g_{1L}(x, k_T^2) S_L + g_{1T}(x, k_T^2) \frac{k_T \cdot S_T}{m_D}, \\
\mathcal{P}_N^q(x, k_T^2) s_T^{(\text{in})}(x, k_T) &= h_{1T}(x, k_T^2) S_T \\
&+ \left[ h_{1L}^+(x, k_T^2) S_L + h_{1T}^+(x, k_T^2) \frac{k_T \cdot S_T}{m_D} \right] \frac{k_T}{m_D}, \quad (27)
\end{align*}

where $m_D$ is an unknown mass parameter, $S_L$ and $S_T$ are the nucleon longitudinal and transverse polarization with respect to its momentum. The “new” DFs have clear physical interpretation: for example, $g_{1T}$ describes the quark longitudinal polarization in a transversely-polarized nucleon. It is important to notice that due to this DF even the initial quark longitudinal spin distribution in a polarized nucleon exhibits an azimuthal asymmetry.
Probing transverse quark polarization in deep-inelastic lepton production

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Editor: P.V. Landshoff

Abstract

The azimuthal dependence of hadrons produced in lepton scattering off a polarized hadron probes the quark transverse-spin distributions. In the calculation of the asymmetries, transverse momenta of quarks in the distribution and fragmentation functions must be incorporated. In addition to the $\sin(\phi + \phi_5)$ asymmetry for transversely polarized hadrons, known as the Collins effect, we find a $\sin(3\phi - \phi_5)$ asymmetry. Furthermore, we find a $\sin 2\phi$ angular dependence for longitudinally polarized hadrons.
Competitors in 1994/1995

- J.P. Ralston and D.E. Soper, NPB 152 (1979) 109
  - transverse momenta in Drell-Yan at nonzero $Q_T$
  - collinear functions, polarization, leading and subleading twist, notation $f$, $g$, $h$
  - early work in my group with e.g. Joachim Levelt, ...
  - TMD functions, polarization, leading, notation $g_{1T}$, $h_{1L}$, ..., used in Drell-Yan
  - hep-ph/9408305 for subleading functions $g_T$ and $h_L$
  - no-nonsense TMD’s, used in leptoproduction

Followed up with ‘complete analysis’ (Rik Tangerman), phenomenology (Aram Kotzinian) and study of T-odd functions (Daniel Boer)
The complete tree-level result up to order $1/Q$ for polarized deep-inelastic lepton production

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Abstract

We present the results of the tree-level calculation of deep-inelastic lepton production, including polarization of target hadron and produced hadron. We also discuss the dependence on transverse momenta of the quarks, which leads to azimuthal asymmetries for the produced hadrons.
Longitudinal quark polarization in transversely polarized nucleons

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(Received 27 November 1995)

Accounting for transverse momenta of the quarks, a longitudinal quark spin asymmetry exists in a transversely polarized nucleon target. The relevant leading quark distribution \( g_{1T}(x,k_T^2) \) can be measured in semi-inclusive deep-inelastic scattering. The average \( k_T^2 \) weighted distribution function \( g_{1T}^{(1)} \) can be obtained directly from the inclusive measurement of \( g_2 \). [S0556-2821(96)03813-1]

Based on EOM relations

\[
g_T = \frac{g_{1T}^{(1)}}{x} + \frac{m}{M} \frac{h_1}{x} + \tilde{g}_T = g_1 + \frac{d}{dx} g_{1T}^{(1)}.
\]

\[
g_2(x) = - \left[ g_1(x) - \int_x^1 dy \frac{g_1(y)}{y} \right]
\]

\[
h_L = \frac{m}{M} \frac{g_1}{x} - 2 \frac{h_{1L}^{(1)}}{x} + \tilde{h}_L = h_1 - \frac{d}{dx} h_{1L}^{(1)}
\]

FIG. 2. The function \( g_{1T}^{(1)WW}(x) \) as obtained from E143 data using Eq. (22) or from the BBS parametrizations for \( g_1 \) using Eq. (25).
Many follow-up developments

- T-odd functions (Dennis Sivers, John Collins, Daniel Boer, ...)
- Phenomenology (Aram Kotzinian, Elena Boglione, Rainer Jakob, ...)
- Extensions including higher spin (Alessandro Bacchetta), gluons (Joao Rodrigues, Andreas Metz, ...)

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<thead>
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- Links to off-forward GPD’s, ...
- Operator structure including gauge links and gluonic poles, factorization breaking
  - Ted Rogers, John Collins, ...
  - Fetze Pijlman, Cedran Bomhof, Maarten Buffing, ...
- Analytic structure, bounds and spectator models, low x, ...
  - Leonard Gamberg, Asmita Mukherjee, Sabrina Cotogno, Tom van Daal, Elena Petreska, ...
Since then we met at many meetings, Aram always modest.
... but he certainly deserves to be in the center like here!

and today

Congratulations!
Happy birthday

AND AGED TO PERFECTION
HAPPY BIRTHDAY