An overview of Quantum Cosmology

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Overview

Quantize gravity and quantum cosmology

General features of quantum gravity

General features of quantum cosmology

Quantum cosmology: applications

Why should quantize gravity?

- It is not obvious at all... gravity is spacetime
- Is quantum nature more fundamental? In which sense?
- Quantum mechanics (QFT) defined on Hilbert space but based on Minkowski S-T
- Notwithstanding, it might be a reasonable idea...

Why should quantize gravity?

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- Quantum mechanics (QFT) defined on Hilbert space but based on Minkowski S-T
- Notwithstanding, it might be a reasonable idea...
  - Theoretical unification
  - Source term in GR ($T^{\mu\nu}$) is quantum
  - Black Hole Thermodynamics (entropy) suggests $\exists$ microstates
  - Singularity theorems of GR
  - Initial conditions in cosmology (what's the I.C. of the universe?)
  - Problem of time (external in QFT, arbitrary in GR)

What means quantum gravity?

- Several approaches
- semiclassical schemes (QFT in curved S-T)
- Supergravity
- String Theory
- Asymptotic safety
- Causal dynamical triangulations
- Noncommutative geometry
- Canonical quantization
  - Quantum geometrodynamics: Wheeler-DeWitt (WDW)
  - Loop quantum gravity (LQG)

Conceptual issues in quantum cosmology

Quantum cosmology is an attempt to take into account quantum effects in models of the universe.

- Patching models $\times$ the wave function of the universe

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- Patching models × the wave function of the universe

Issues to be addressed:

- Is quantum mechanics consistent for closed systems? (how happens measurements?)
- Emergence of classical spacetime
  - * WKB approximation
    peaked about classical spacetimes — nonclassical spacetimes

    $$\Psi \approx e^{iS} \qquad \qquad \Psi \approx e^{-I}$$

  - * decoherence to avoid quantum interference
- How to define probability (WDW is not positive similar to KG)
- Singularity avoidance depends on the interpretation of quantum mechanics

Minisuperspace is quantum cosmology for the FLRW universe.

Cosmological Standard Model

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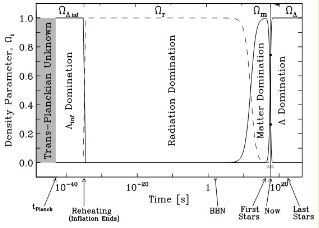


General Properties

- homogeneous and isotropic > 200 Mpc
- exist for at least 13, 7 billions of years
- Expanding: denser and hotter in the past
- 3 eras: radiation / dust / dark energy

$T \backsim$ $0.25\,eV - 1\,eV - 50\,KeV - 0.5\,MeV$

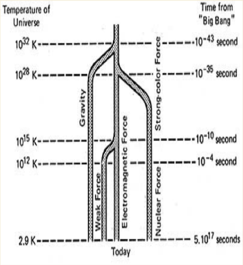
$T \backsim$ $1\,MeV$ $[0.2\,sec]$ neutrino decoupling

$T \backsim$ $200\,MeV$ $\left[10^{-5} sec\right]$ baryogenesis

$T \backsim$ $100\,GeV - 10\,TeV$ $\left[10^{-10} - 10^{-14} sec\right]$
particle Standard Model still valid

$T \backsim$ $10\,TeV - 10^{19} GeV$ $\left[10^{-14} - 10^{-44} sec\right]$
beyond SM but still classical gravity

$T >$ $10^{19} GeV$ what comes next?



General features of quantum gravity

Gauge theories as constrained systems

- more dynamical variables than physical degrees of freedom
- Covariant form of Electromagnetism: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

- In Flat S-T, the symmetry group is given by the Poincaré group

$$\begin{aligned} \vec{E}' &= \gamma\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) - \frac{\gamma^2}{\gamma+1}\left(\frac{\vec{v}.\vec{E}}{c}\right)\frac{\vec{v}}{c} \\ \vec{B}' &= \gamma\left(\vec{B} - \frac{\vec{v}}{c} \times \vec{E}\right) - \frac{\gamma^2}{\gamma+1}\left(\frac{\vec{v}.\vec{B}}{c}\right)\frac{\vec{v}}{c} \end{aligned}$$

- gauge transformation: $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$ but $F_{\mu\nu} \rightarrow F_{\mu\nu}$
- what's an observable? (deterministic dynamics)

Constrained Dynamical Systems

- Lagrangian system $S = \int \mathrm{d}t \, L\left(\vec{q}, \dot{\vec{q}}\right) \qquad \mathrm{i} = 1, \ldots, n$

Euler-Lagrange: $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial L}{\partial \dot{q}^i}\right) - \dfrac{\partial L}{\partial q^i} = 0 \quad \Longrightarrow \quad \mathbf{W} \cdot \ddot{\vec{q}} = \vec{V}$

with $\quad W_{ik}\left(\vec{q}, \dot{\vec{q}}\right) \equiv \dfrac{\partial^2 L}{\partial \dot{q}^k \, \partial \dot{q}^i} \qquad V_i\left(\vec{q}, \dot{\vec{q}}\right) \equiv \dfrac{\partial L}{\partial q^i} - \dfrac{\partial^2 L}{\partial q^k \, \partial \dot{q}^i} \dot{q}^k$

If exists $\mathbf{W}^{-1}$,

$\ddot{\vec{q}} = \mathbf{W}^{-1} \cdot \vec{V} \quad \Longrightarrow \quad \ddot{q}_k = \ddot{q}_k\left(\vec{q}, \dot{\vec{q}}\right)$ regular systems

If det $\mathbf{W} = 0$,

$\ddot{\vec{q}}$ { cannot be described by $\left(\vec{q}, \dot{\vec{q}}\right)$; evolution is not univocally determined } singular systems

Hamiltonian for Constrained Systems

- Legendre transformation defines the momenta

$$p_k \equiv \frac{\partial L}{\partial \dot{q}^k}\left(\vec{q}\,,\dot{\vec{q}}\right)$$

- Main issue.... cannot invert the acceleration in terms of $\ddot{\vec{q}} = \ddot{\vec{q}}\,(\vec{q},\vec{p})$
- let assume all m constraints are independent

$$\phi_j\left(\vec{q}\,,\dot{\vec{q}}\right) \approx 0 \qquad \text{primary constraints}$$

- $H(\vec{q}\,,\vec{p}) = \vec{p}\cdot\dot{\vec{q}} - L\left(\vec{q}\,,\dot{\vec{q}}\right)$ assumes E-L eq.'s
- Need to extend the Hamiltonian

$$H_T \equiv H + \sum_j \mathscr{U}^j\,\phi_j$$

$\mathscr{U}^k$ are arbitrary functions of (q^k, p_j)

GR Hamiltonian Formalism

- It is a constrained hamiltonian system
  Symmetry group identified with the covariance group: $\mathscr{M}\mathscr{M}\mathscr{G}$

- Einstein equation has no evolutionary parameter

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \Rightarrow \quad g_{\mu\nu}\left(x^0, x^1, x^2, x^3\right)$$

Basic variables:

- Arnowitt-Deser-Misner (ADM) formalism: Geometrodynamics
  The tri-metric h_{ij} is the dynamic variable

- Loop quantum gravity:
  The densitized triads E_i^a and connection $A_i^a = \Gamma_i^a + \gamma K_i^a$

ADM Formalism

- Suppose a topology $R \otimes \Sigma^3$, hence $\exists$ time-like congruence $\vec{e}_0$
  The subspace Σ can be expanded by $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$

$$\vec{e}_\mu \cdot \vec{e}_\nu \equiv g\left(\vec{e}_\mu, \vec{e}_\nu\right) = \eta_{\mu\nu}$$

- Given an arbitrary (t, x^i), the vector $\vec{t} \equiv \frac{\partial}{\partial t} = N\,\vec{e}_0 + N^k\,\vec{e}_k$

Projecting the metric

$$\begin{aligned} h_{ij} &\equiv g_{\alpha\beta}\, e^\alpha{}_i\, e^\beta{}_j \\ g_{0i} &\equiv g_{\alpha\beta}\, t^\alpha\, e^\beta{}_i = N^a\, h_{ai} = N_i \\ g_{00} &\equiv g_{\alpha\beta}\, t^\alpha\, t^\beta = N^2 - N^a N_a \end{aligned} \qquad g_{\mu\nu} = \begin{pmatrix} N^2 - N^a N_a & -N_i \\ -N_j & -h_{ij} \end{pmatrix}$$

- how h_{ij} is immersed: extrinsic curvature

$$K_{\mu\nu} \doteq -\frac{1}{2} \perp_{\alpha\mu} \perp_\nu{}^\beta\, \nabla_{(\beta} e^\alpha{}_{0)} = -\frac{1}{2} \perp_\mu{}^\alpha \perp_\nu{}^\beta\, \mathscr{L}_{\vec{e}_0}\left(g_{\alpha\beta}\right)$$

$$K_{ab} = -N\,\Gamma^0_{ab} \qquad K_{0b} = N^a\, K_{ab} \qquad K_{00} = N^a N^b\, K_{ab}$$

ADM Formalism

- The action reads

$$S = \int d^4x \ \sqrt{-g}\, R = \int dt d^3x \, N \sqrt{h} \left(K^{ij} K_{ij} - K^2 + {}^{(3)}R \right)$$

- Equations of motion

$$\frac{\delta S}{\delta N} = 0 \Rightarrow \quad \sqrt{h} \left(K^{ij} K_{ij} - K^2 - {}^{(3)}R \right) = 0 \qquad G_{\mu\nu}\, t^\mu t^\nu = 0$$

$$\frac{\delta S}{\delta N_i} = 0 \Rightarrow \quad 2\sqrt{h}\, \nabla_j \left(K_i{}^j - \delta_i{}^j K \right) = 0 \qquad G_{\mu\nu}\, t^\mu \perp^\nu{}_\alpha = 0$$

$$\frac{\delta S}{\delta h_{ij}} = 0 \Rightarrow \qquad G_{\mu\nu} \perp^\mu{}_\beta \perp^\nu{}_\alpha = 0$$

$$\Rightarrow \quad \dot{K}_{ij} = -N \left[{}^{(3)}R_{ij} + K K_{ij} - 2 K_i^m K_{mj} \right] + \nabla_j \nabla_i N - \nabla_{(i} N^m K_{j)m} - N^m \nabla_m \nabla_j K_i$$

ADM Formalism

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- The momenta

$$P = \frac{\partial \mathscr{L}}{\partial \dot{N}} = 0 \quad , \quad P^i = \frac{\partial \mathscr{L}}{\partial \dot{N}_i} = 0 \quad , \quad \Pi^{ij} = \frac{\partial \mathscr{L}}{\partial \dot{h}_{ij}} = -\sqrt{h}\left(K^{ij} - h^{ij} K\right)$$

- The total hamiltonian

$$\mathscr{H}_T = \int dt\, d^3x \left(N\, \mathscr{H}_0 + N_i\, \mathscr{H}^i + \lambda\, P + \lambda_i\, P^i\right)$$

$$\mathscr{H}_0 = G_{ijkl}\, \Pi^{ij}\, \Pi^{kl} - {}^{(3)}R\, \sqrt{h} \qquad \text{(super-hamiltoniana)}$$

$$\mathscr{H}^i = -2\nabla_j\, \Pi^{ij} \qquad \text{(super-momentum)}$$

$$G_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

- Preservation of constraints

$$\dot{P} \approx 0 \quad \Rightarrow \quad \mathscr{H}_0 \approx 0 \qquad\qquad \dot{P}^i \approx 0 \quad \Rightarrow \quad \mathscr{H}^i \approx 0$$

- Dynamics

$$\dot{N} = \lambda \qquad \dot{N}^i = \lambda^i \qquad \dot{h}_{ij} = \ldots \qquad \dot{\Pi}_{ij} = \ldots$$

Quantizing Constrained Systems

- Non-relativistic dynamics reads

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = H(\hat{q},\hat{p})\,\psi(x,t)$$

- Canonical quantization: dynamical variable $\rightarrow$ operator
- Poisson brackets goes to commutation relations

$$[\hat{q},\hat{p}] = i\hbar$$

- We cannot translate the constraints as relations for operators

  Example: $\phi(q,p) = p \quad \longrightarrow \quad \phi(\hat{q},\hat{p}) = 0 \quad \Rightarrow \quad \hat{p} = 0$
  But

$$[\hat{q},\hat{p}] = i\hbar = \hat{q}\hat{p} - \hat{p}\hat{q} = 0 \qquad \Rightarrow \qquad i\hbar = 0! \quad \text{inconsistency}$$

- Constraints restricts the possible physical states, hence

$$\boxed{\phi(\hat{q},\hat{p})\psi = 0} \qquad \text{must annihilate the wave function}$$

General features of quantum cosmology

Quantum Cosmology

There are a few issues....

- What is the meaning to quantize spacetime?
- The problem of time in quantum gravity

$$i\hbar \frac{\delta}{\delta\tau}\Psi\left(\varphi, A_\mu, h_{\mu\nu}\right) = \hat{H}\,\Psi\left(\varphi, A_\mu, h_{\mu\nu}\right) = 0 \qquad \text{no evolution?}$$

Quantum cosmology issues....

- Scale factor is defined in the half-line $\mathbb{R}^+$
- How to make predictions? (Interpretations of quantum mechanics)

Quantization on the half-line

- The scale factor is defined on the half-line $a \in (0, \infty)$
- Thus, operators generically are no longer self-adjoint
- The most important example: unitary evolution requires self-adjoint hamiltonians
- We need to impose boundary conditions.... example $\hat{H} = \hat{P}_a^2 + k\hat{a}^2$

$$\left\langle \psi_1 \middle| \hat{H}\psi_2 \right\rangle = \left\langle \hat{H}\psi_1 \middle| \psi_2 \right\rangle \qquad \text{symmetric condition}$$

$$\int_0^\infty \mathrm{d}a\, \psi_1^* \frac{\mathrm{d}^2\psi_2}{\mathrm{d}a^2} = \int_0^\infty \mathrm{d}a\, \psi_2 \frac{\mathrm{d}^2\psi_1^*}{\mathrm{d}a^2}$$

square integrable

$$\left.\left(\psi_1^* \frac{\mathrm{d}\psi_2}{\mathrm{d}a} - \psi_2 \frac{\mathrm{d}\psi_1^*}{\mathrm{d}a}\right)\right|_{a=\infty} = \left.\left(\psi_2 \frac{\mathrm{d}\psi_1^*}{\mathrm{d}a} - \psi_1^* \frac{\mathrm{d}\psi_2}{\mathrm{d}a}\right)\right|_{a=0} \Rightarrow \boxed{\left.\frac{\mathrm{d}\psi}{\mathrm{d}a}\right|_{a=0} = r\,\psi\Big|_{a=0} \quad r \in \mathbb{R}}$$

Interpretations of Quantum Mechanics

- Copenhagen interpretation doesn't work for cosmology (cannot be applied!)
- Many-worlds interpretation
- Consistent histories interpretation
- Bohm-De Broglie Interpretation

Quantum cosmology: applications

Quantum Cosmology with Scalar Field

- Ashtekar *et.al* claim WDW is singular $\left(\text{Prob.} = 1\right)$
- ***Free Scalar Field using Consistent History*** - Craig and Singh

Quantum Cosmology with Scalar Field

- Ashtekar *et.al* claim WDW is singular $\left(\text{Prob.} = 1\right)$
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- Change of variables: $\boxed{\alpha \equiv \log a}$

$$\text{WdW eq.} \quad \pm i\partial_\phi \Psi(\alpha.\phi) = \sqrt{\hat{\Theta}}\,\Psi(\alpha,\phi) \quad \text{with} \quad \Theta := -\frac{4\pi G}{3}\partial_\alpha^2$$

- Only positive frequency sector, evolution reads $U(\phi - \phi_0) = e^{i\sqrt{\Theta}(\phi-\phi_0)}$
- The probability for a bounce is

$$p(h_{B-B}) = \|C^\dagger_{B-B}|\Psi\rangle\|^2 = \lim_{\phi\to\infty}\langle\Psi_L|P_{\overline{\Delta\alpha_2}}(\phi)|\Psi_L\rangle = 0$$

- For two times, every solution is singular
- Three times there is no decoherence (nothing can be said!)

Quantum Cosmology with Scalar Field

- ***Wheeler-DeWitt Quantization can Solve the Singularity Problem***
  N. Pinto-Neto, etal. - PRD 86 (2012) 063504.
- ***The de Broglie-Bohm theory applied to quantum cosmology***

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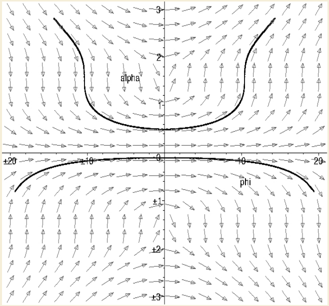
- but there is no continuity equation

$$\frac{\partial}{\partial\phi}\left(R^2\frac{\partial S}{\partial\phi}\right)-\frac{\partial}{\partial\alpha}\left(R^2\frac{\partial S}{\partial\alpha}\right)$$
$$\neq \frac{\partial R^2}{\partial\phi}-\frac{\partial}{\partial\alpha}\left(R^2\frac{d\alpha}{d\phi}\right)$$

- no probability for bohmian trajectories



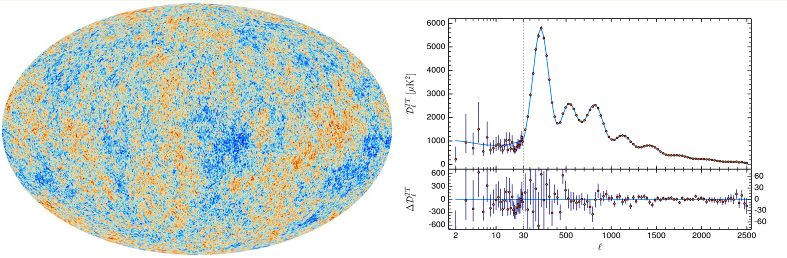
Concluding remarks

- Consistent histories: have probabilities but unable to investigate singularity for more than two times.
- de Broglie-Bohm: know the entire evolution, but looses probability

Connecting with observations

Testing Primordial Universe (Observations)

- While waiting for primordial gravitational waves....
- The farthest observable is the CMB



- Three main observables:

scalar spectral index   tensor-to-scalar ratio   non-gaussianity

$$n_s = \frac{\mathrm{d}\log \mathrm{P}_{\mathscr{R}}}{\mathrm{d}\log k} \qquad r = \frac{\mathrm{P}_{\mathscr{R}}}{\mathrm{P}_{\mathscr{T}}} \qquad f_{NL}$$

- but there is also

B-mode polarization,   tensor spectral index,   running, etc...

Testing Primordial Universe (Observations)

Connection with observations[1] : Correlation functions

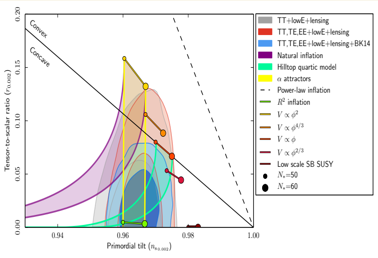
two point $$\langle \zeta^*(\vec{k}_1)\,\zeta(\vec{k}_2)\rangle = \frac{(2\pi)^2}{k^3}\delta(\vec{k}_1 - \vec{k}_2)P_\zeta(k_1)$$

bispectrum $$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^7\delta\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right)\frac{P_\zeta^2}{\Pi\, k_i^3}\mathscr{A}\left(\vec{k}_1, \vec{k}_2, \vec{k}_3\right)$$

- Power spectrum parametrization

$$\mathrm{P}_{\mathscr{R}} = A_s\left(\frac{k}{k_*}\right)^{n_s - 1} \qquad n_s - 1 = \frac{\mathrm{d}\ln \mathrm{P}_{\mathscr{R}}}{\mathrm{d}\ln k}$$

$$\mathrm{P}_{\mathscr{T}} = A_t\left(\frac{k}{k_*}\right)^{n_t} \qquad k_* = 0.05\mathrm{Mpc}^{-1}$$



- Tensor-to-scalar ratio

$$r \equiv \frac{\mathrm{P}_{\mathscr{T}}}{\mathrm{P}_{\mathscr{R}}} \qquad k_* = 0.002\mathrm{Mpc}^{-1}$$

$$n_s = 0.9649 \pm 0.0042\,, \quad r_{0.002} < 0.064$$

[1] Planck 2018 results. X. Constraints on inflation arXiv:1807.06211v1 [astro-ph.CO]
Planck2018 results. VI. Cosmological parameters arXiv:1807.06209v1 [astro-ph.CO]

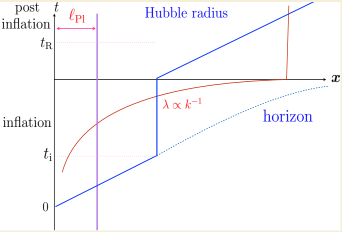
Primordial Universe Alternatives

Shortcomings of the Standard Model

- physical mechanism to produce the expansion
- matter / anti-matter asymmetry
- cosmic singularity in a finite past time
- particle horizon
- flatness of spatial sections
- initial condition for structure formation

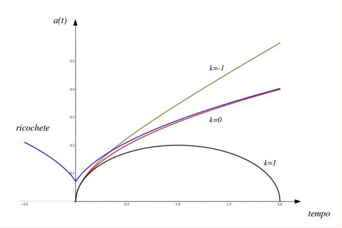
Inflation

accelerated expansion phase



Bounce

contraction → expansion



Quasimatter Bounce Equivalent to Starobinsky Inflation

L. F. Guimarães , F. T. Falciano , G. Brando

Physical Review D 99, 103515 (2019)
ArXiv:1902.05031 [gr-qc]

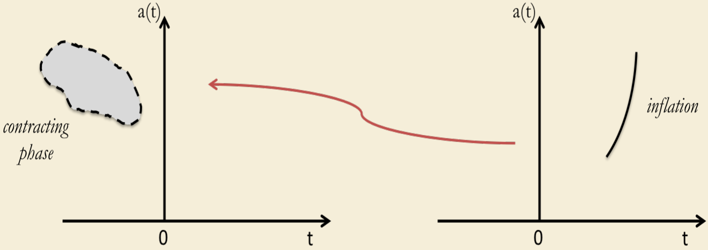
Mimicking Starobinsky Inflation

What's the embedding dynamics?

- GR with perfect fluid / scalar field / ...
- F(R) without matter content / with conventional matter / ...

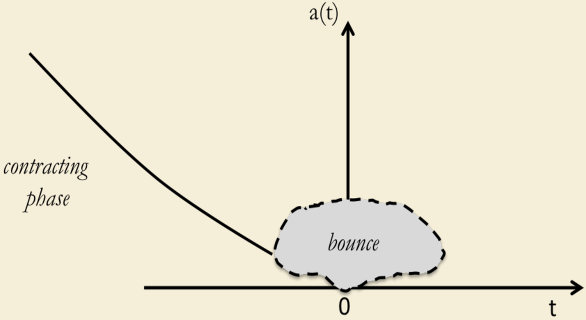
- Collapsing with scalar field: $z_s^B = a_B\,\dot{\varphi}/H$
- Quasi-de Sitter should be mapped into a quasi-matter dominated universe

$$\dot{\varphi}^2 \simeq 2V \qquad \Rightarrow \qquad \dot{\varphi} \simeq \sqrt{3}H \qquad \Rightarrow \qquad z_s^B(\eta) \approx a_s^B(\eta)$$



Quantum Bounce

- Quantum mechanisms: Wheeler-deWitt, Superstring motivated (Ekpyrotic), LQC



Overview

- Starobinsky Inflation

$$n_s - 1 = -\frac{2}{N} \quad , \quad r = \frac{12}{N^2} \quad \Rightarrow \quad r = 3\ (n_s - 1)^2$$

Consistency (single field slow-roll): $n_t = -r/8 \ \Rightarrow \ n_t = -\frac{3}{8}\ (n_s - 1)^2$

- Our model

$$n_s - 1 = -2\epsilon_c\ , \quad n_t = -2\epsilon_c\ , \quad r = \frac{8}{3}\epsilon_c^2 \quad \Rightarrow \quad r = \frac{2}{3}\ (n_s - 1)^2$$

- we are a factor $2/9$ smaller!

Convolution of two contributions

The mapping: inflation $\longrightarrow$ bounce

Starobinsky: $(z_s/z_T)^2 = \frac{Q_s}{F} \approx \frac{3}{2}M_{\rm Pl}^2\epsilon_c^2$

matter bounce: $(z_s/z_T)^2 = 3M_{\rm Pl}^2$

This gives a factor $2/\epsilon_c^2$

The bounce

extra factor of $\epsilon_c^2/9$

Thanks for your attention!