

3ª Escola do Programa de Pós-Graduação em Física da UERJ

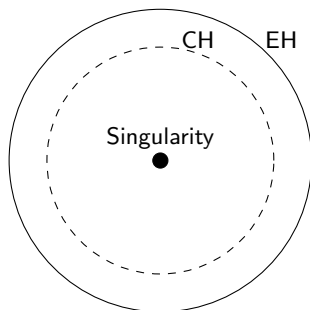
Massive scalar wave packet emission by a charged Black Hole and Cosmic Censorship Conjecture Violation

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Rio de Janeiro, BRASIL

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Introduction



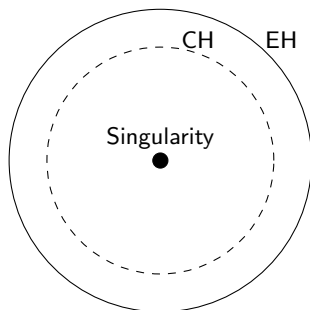
$$M^2 \geq Q^2 + (J/M)^2$$

Black Hole: M, Q, J

$$r_{CH} = M - \sqrt{M^2 - Q^2 - (J/M)^2}$$

$$r_{EH} = M + \sqrt{M^2 - Q^2 - (J/M)^2}$$

The CCC



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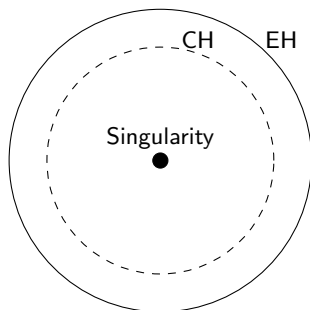
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Particle absorption



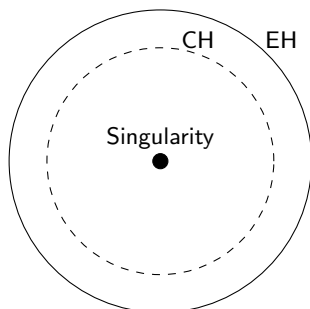
particle: m_w, q, j
 ← •

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After absorption



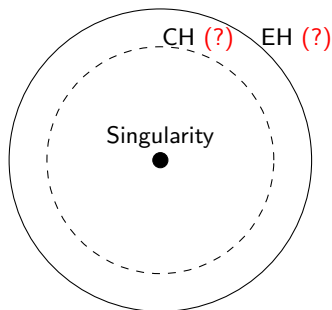
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Singularity



$$M' = M + m_w, \quad Q' = Q + q, \quad J' = J + j$$

Does $M'^2 \geq Q'^2 + (J'/M')^2$ uphold?

If not, then r_{CH} and r_{EH} becomes imaginary,
 exposing the singularity!

Black Hole: M', Q', J'

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Massive scalar field scattering

Static, charged black hole:

$$ds^2 = -\left(1 - \frac{r_s}{r} + \frac{r_q^2}{r^2}\right) dt^2 + \left(1 - \frac{r_s}{r} + \frac{r_q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad (1)$$

Klein-Gordon equation:

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi(t, r) \right) - \mu^2 \Phi(t, r) = 0 \quad (2)$$

$R(r)$ and $\Theta(\theta)$:

$$r^2 \left(1 - \frac{r_s}{r} - \frac{r_q^2}{r^2}\right) \frac{d^2 R}{dr^2} + r \left(2 - \frac{r_s}{r}\right) \frac{dR}{dr} + \left(\frac{\omega^2 r^2 + [(r_s - r)r + r_q^2] \mu^2}{1 - \frac{r_s}{r} - \frac{r_q^2}{r^2}} - \mathcal{A} \right) R = 0, \quad (3)$$

$$\frac{d^2 \Theta}{d\theta^2} + \cot \theta \frac{d\Theta}{d\theta} + \left(\mathcal{A} - \frac{m^2}{\sin^2 \theta} \right) \Theta = 0, \quad (4)$$

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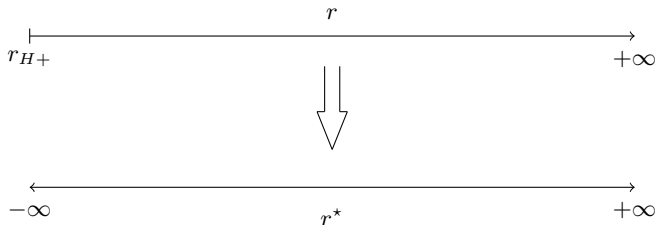
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"Detour"

Tortoise coordinate:

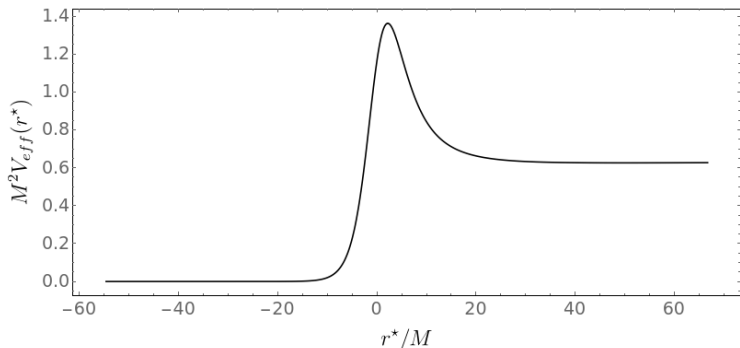
$$\frac{dr}{dr^*} := 1 - \frac{r_s}{r} + \frac{r_q^2}{r^2} \quad (5)$$



Schrödinger? Just like it.

$$\left[\frac{d^2}{dr^{*2}} + \omega^2 - V_{eff}(r) \right] u_{\omega\ell m}(r) = 0,$$

$$V_{eff}(r) := \left(1 - \frac{r_s}{r} + \frac{r_q^2}{r^2} \right) \left(\frac{\ell(\ell+1)}{r^2} + \frac{r_s}{r^3} - \frac{2r_q^2}{r^4} + \mu^2 \right) \quad (6)$$



Issues

- The effective potential in equation (6) is not analytically integrable to find $u_{\omega\ell m}(r)$.
- Most common approaches: Low (or high) frequency regimes; numerical analysis.
- Our approach: Toy model

$$V_{toy}(r^*) = \begin{cases} V_1(r^*) := b_1 \operatorname{sech}^2[a_1(r^* - c_1)], & r^* < r_0^* \\ V_2(r^*) := (b_2 - \mu^2) \left\{ \left[1 - e^{-a_2(r^* - c_2)} \right]^2 - 1 \right\}, & r^* \geq r_0^* \end{cases} \quad (7)$$

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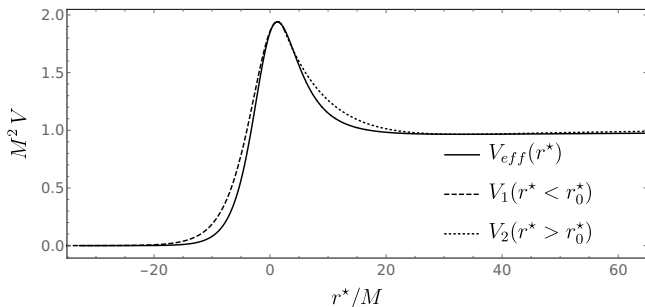
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Asymptotic limit and plane waves

In the asymptotic limit $r^* \pm \rightarrow \infty$, we recover plane waves.

$$u_{lm}(r^*) \approx \begin{cases} t_{\omega lm} e^{-i\omega r^*}, & r^* \rightarrow -\infty \\ k_{\omega lm} e^{-\varpi r^*} + r_{\omega lm} e^{+\varpi r^*}, & r^* \rightarrow +\infty \end{cases} \quad (8)$$

with $\varpi := \sqrt{\mu^2 - \omega^2}$.

Transmission and Reflection rates

$$T_w := \left| \frac{t}{k} \right|^2, \quad R_w := \left| \frac{r}{k} \right|^2 \quad (9)$$

Conservation of flux

$$R_w + \left| \frac{\omega}{\varpi} \right| T_w = 1 \quad (10)$$

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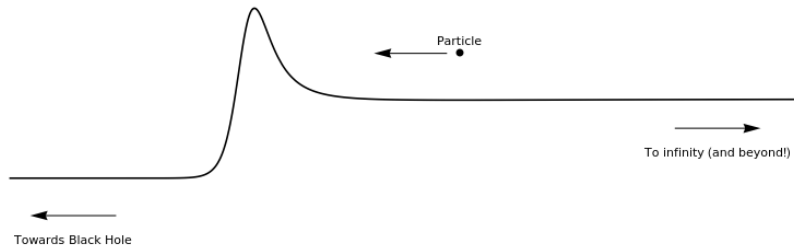
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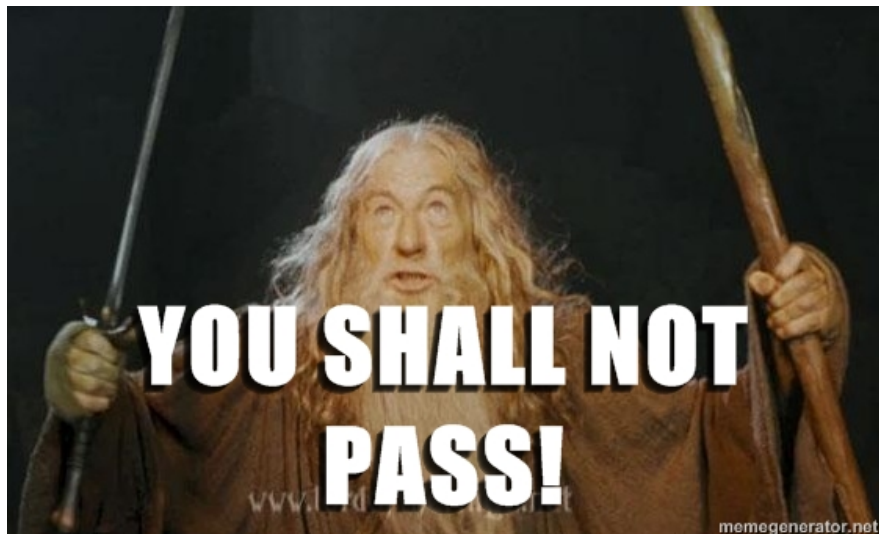
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Low energy...

First case: Energy lower than the potential barrier.

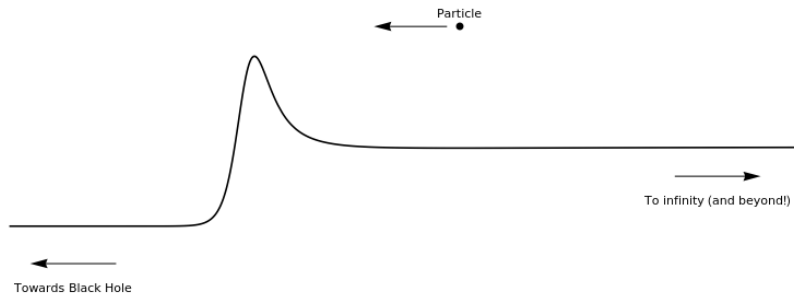


The "censor"

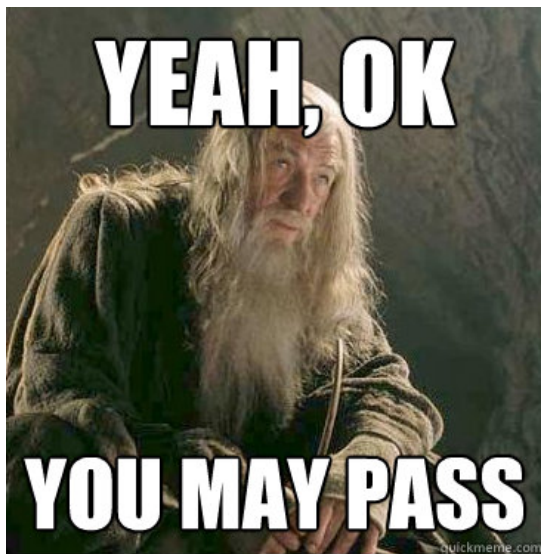


High as fly!

Second case: Energy greater than the potential barrier.

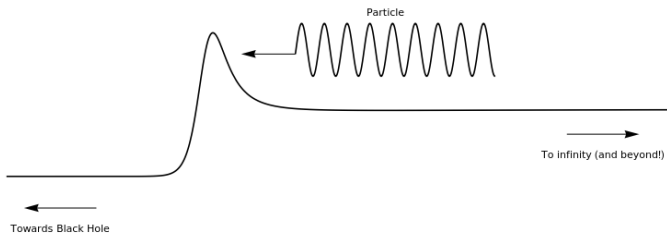


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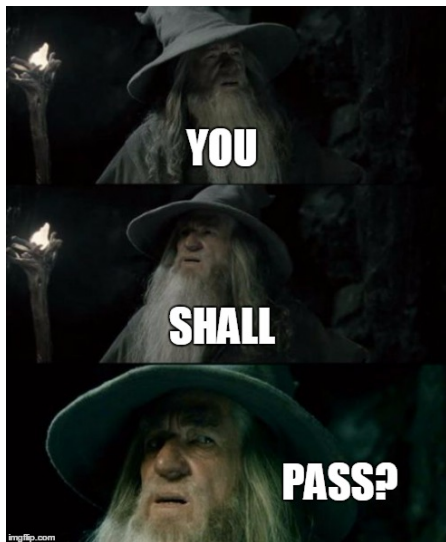


And what if...?

Third case: propagating wave



The "censor"



Quantum Tunneling



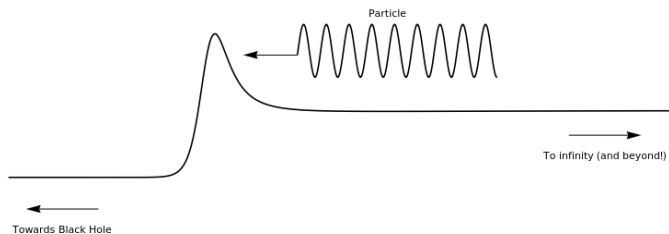
Gandalf was unaware of Quantum tunnelling!

powered by Young Minds Naples

"...the 'cosmic censor' may be oblivious to process involving quantum effects"

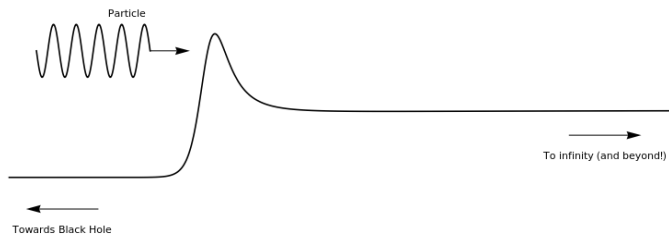
MATSAS, 2009 © PRD79

If absorption...



Particle absorbed \implies new black hole with mass $M' = M + m_w$

If emission...



Particle emitted \implies new black hole with mass $M' = M - m_w$

Problem?

- Waves are eternal

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 - Backreaction

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 - No geodesic motion; Superradiance

Semiclassical approach

Solutions:

- Build a (gaussian) wave packet from the incoming plane waves

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- Build a (gaussian) wave packet from the incoming plane waves
 - Packet is localized in the position space
- Still no charge \implies no superradiance; geodesic motion

Wave packet

Gaussian wave packet from the incoming waves

$$\psi_i(t, r^*) := \int_{\omega_0 - 2\tilde{\xi}}^{\omega_0 + 2\tilde{\xi}} k_{\omega\ell m} \tilde{\psi}(\omega) e^{-i\omega(t+r)} d\omega, \quad \tilde{\psi}(\omega) := \frac{1}{(2\pi\tilde{\xi}^2)^{1/4}} e^{-\frac{(\omega - \omega_0)^2}{4\tilde{\xi}^2}} \quad (11)$$

Reflected and transmitted packets

$$\psi_r(t, r^*) := \int_{\omega_0 - 2\tilde{\xi}}^{\omega_0 + 2\tilde{\xi}} r_{\omega\ell m} \tilde{\psi}(\omega) e^{i\omega(t-r)} d\omega, \quad \psi_t(t, r^*) := \int_{\omega_0 - 2\tilde{\xi}}^{\omega_0 + 2\tilde{\xi}} t_{\omega\ell m} \tilde{\psi}(\omega) e^{-i\omega(t+r)} d\omega \quad (12)$$

Reflection and Transmission rates

$$R := \frac{\int_{r_p^* - 2\xi}^{r_p^* + 2\xi} |\psi_r|^2 dr^*}{\int_{r_p^* - 2\xi}^{r_p^* + 2\xi} |\psi_i|^2 dr^*}, \quad T := \frac{\int_{r_p^* - 2\xi}^{r_p^* + 2\xi} |\psi_t|^2 dr^*}{\int_{r_p^* - 2\xi}^{r_p^* + 2\xi} |\psi_i|^2 dr^*} \quad (13)$$

Conservation of flux ($\omega \approx \omega_0$)

$$R + \left| \frac{\omega_0}{\sqrt{\mu^2 - \omega_0^2}} \right| T = 1 \quad (14)$$

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The quantity f_ϵ

Ratio between the transmission rates for the packet for two configurations of the black hole

$$f_\epsilon := \log \left[\frac{T(\mathcal{M}_2) + \epsilon}{T(\mathcal{M}_1) + \epsilon} \right] \implies \begin{cases} f_\epsilon > 0 \implies \text{No CCC Violation} \\ f_\epsilon < 0 \implies \text{CCC Violation} \\ f_\epsilon = 0 \implies \text{Indiferent} \end{cases} \quad (15)$$

$$\left. \begin{array}{l} \text{Config. 1: } \mathcal{M}_1 := M, \quad \mathcal{Q} := M - \delta Q \\ \text{Config. 2: } \mathcal{M}_2 := M + \delta M, \quad \mathcal{Q} := M - \delta Q \end{array} \right\} \text{Near-extremal BH: } \mathcal{Q} \lesssim \mathcal{M} \quad (16)$$

$$\text{CCC: } \mathcal{M} \geq \mathcal{Q} \implies M + \delta M - m_w \geq M - \delta Q \implies \delta M + \delta Q \geq m_w \quad (17)$$

Scale: $M\mu = 1$. For $M \approx 10M_\odot \implies m_w \approx 10^{-11} \text{ eV}/c^2 \implies$ axion-like particle

- First set: $\delta Q = 10^{-15} M$
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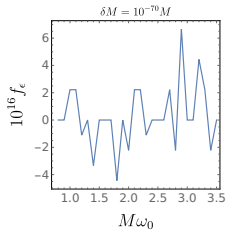
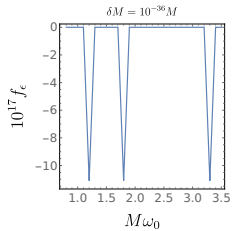
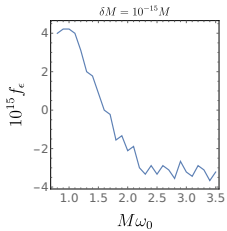
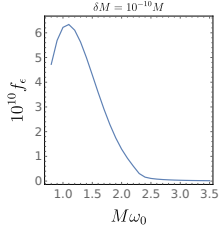
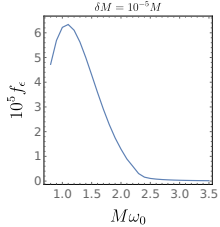
$$f_\epsilon := \log \left[\frac{T(\mathcal{M}_2) + \epsilon}{T(\mathcal{M}_1) + \epsilon} \right] \implies \begin{cases} f_\epsilon > 0 \implies \text{No CCC Violation} \\ f_\epsilon < 0 \implies \text{CCC Violation} \\ f_\epsilon = 0 \implies \text{Indiferent} \end{cases} \quad (15)$$

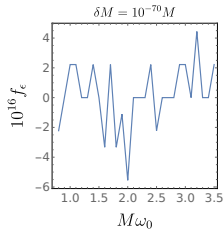
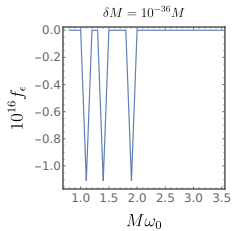
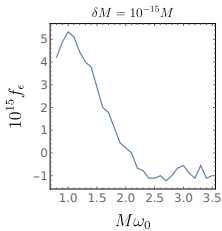
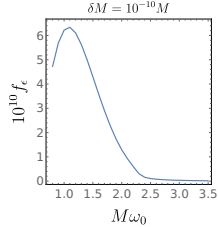
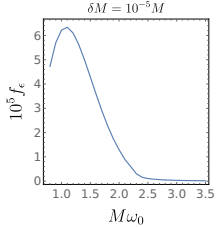
$$\left. \begin{array}{l} \text{Config. 1: } \mathcal{M}_1 := M, \quad \mathcal{Q} := M - \delta Q \\ \text{Config. 2: } \mathcal{M}_2 := M + \delta M, \quad \mathcal{Q} := M - \delta Q \end{array} \right\} \text{Near-extremal BH: } \mathcal{Q} \lesssim \mathcal{M} \quad (16)$$

$$\text{CCC: } \mathcal{M} \geq \mathcal{Q} \implies M + \delta M - m_w \geq M - \delta Q \implies \delta M + \delta Q \geq m_w \quad (17)$$

Scale: $M\mu = 1$. For $M \approx 10M_\odot \implies m_w \approx 10^{-11} \text{ eV}/c^2 \implies$ axion-like particle

- First set: $\delta Q = 10^{-15} M$
- Second set: $\delta Q = 10^{-70} M$





Conclusion

The “censor” seems to know its limits once the CCC is close to be violated.



Fim!

Muito obrigado!

Grazie mille!