A GNSS measurement for High School Students of EEE Project

Marina Trimarchi
Museo Storico della Fisica e Centro Studi e Ricerche “Enrico Fermi”, Roma
Dipartimento MIFT, Università degli Studi di Messina
INFN – Sezione di Catania
EEE Meetings for Students

- **Coordination Meetings**
- **Students report about their activity on EEE**
- **Status and Maintenance of the Telescopes, Data Analysis, Questions and Discussions**
- **General Lessons from EEE researchers**
- **Hardware, Software, Organization of events**
- **Conferences of EEE project**
- **Students describe their activities**
- **General Lessons and Masterclasses**
- **Students in groups perform measurements and data analysis and report about their work**
- **Prizegiving for best contributions and analysis**
How a GNSS works

• 4 satellites in view
• Solve 4 non linear equations to get position and time
• Distance measurements
  -- through the time of flight of a signal sent by a transmitter on the satellite
    • $\Delta t$ between clock on satellite and on receiver

Global Navigation Satellite System
Simulation of GNSS functioning by measuring the range between local representative receiver and satellites
Exercise for students:

- Put 3 simulated satellites on a cartesian reference
  - Simple positions: on the intersections between walls
- Use a laser distance meter to measure their positions
  - \((x_A, y_A, z_A), (x_B, y_B, z_B), (x_C, y_C, z_C)\)
- Measure your distances from the 3 satellites
  - \(d_A, d_B, d_C\)

The laser distance meter uses the time of flight of a laser to compute distances

\(\rightarrow\) Then the signal is transmitted and received by the same instrument

\(\rightarrow\) As the receiver and the satellite clocks are synchronized \(\rightarrow\) Only 3 range measurements required

- Compute your position by inverting the equations of distances
- Check the results with an “actual measurement” \(\rightarrow\) rule, laser meter, etc.

\[
R_A = \sqrt{(x - x_A)^2 + (y - y_A)^2 + (z - z_A)^2}
\]
\[
R_B = \sqrt{(x - x_B)^2 + (y - y_B)^2 + (z - z_B)^2}
\]
\[
R_C = \sqrt{(x - x_C)^2 + (y - y_C)^2 + (z - z_C)^2}
\]
Example of Results

- Python Code developed by Dr. V. Pettiti (INRIM)
  - 2 points → P1 and P2

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | Distanze misurate | Posizione stimata | Posizione misurata | Scostamento |
| 2 | $d_A$ | $d_B$ | $d_C$ | $x$ | $y$ | $z$ | $x$ | $y$ | $z$ | $\Delta x$ | $\Delta y$ | $\Delta z$ |
| 3 | [m] | [m] | [m] | [m] | [m] | [m] | [m] | [m] | [m] | [m] | [m] | [m] |
| 4 | 5.54 | 4.29 | 4.37 | 1.46 | 2.50 | 0.38 | 1.45 | 2.51 | 0.45 | -0.01 | 0.01 | 0.07 |
| 5 | 4.12 | 4.72 | 5.55 | 3.13 | 1.74 | 0.48 | 3.18 | 1.78 | 0.45 | 0.05 | 0.04 | -0.03 |
| 6 | 4.13 | 4.78 | 5.56 | 3.18 | 1.79 | 0.46 | 3.18 | 1.78 | 0.45 | 0.00 | -0.01 | -0.01 |
| 7 | 4.12 | 4.75 | 5.54 | 3.16 | 1.78 | 0.48 | 3.18 | 1.78 | 0.45 | 0.02 | 0.00 | -0.03 |
| 8 | 4.13 | 4.74 | 5.55 | 3.15 | 1.76 | 0.47 | 3.18 | 1.78 | 0.45 | 0.04 | 0.02 | -0.02 |
| 9 | 4.13 | 4.75 | 5.55 | 3.15 | 1.77 | 0.47 | 3.18 | 1.78 | 0.45 | 0.03 | 0.01 | -0.02 |

- Compare
  - Estimated Coordinates (given by the code)
  - Measured Coordinates (obtained by an instrument)

- Max observed deviation → 5 cm
  - Uncertainties of the same order of magnitude in satellite positions
  - Not complete orthogonality of the walls

A good method for didactic experiment!
Useful links

• **General lessons:**
  – https://agenda.centrofermi.it/event/120/contributions/1041/
  – https://agenda.centrofermi.it/event/120/contributions/1042/

• **Python code:**
  • ***If you plan to use code, please cite author: “Valerio Pettiti (INRIM)”***
    – https://indico.cern.ch/event/855335/contributions/3627925/attachments/1953452/3244196/Simula_GPS_Pettiti_EEE.py