SMEFT anomalous dimensions from the S-matrix

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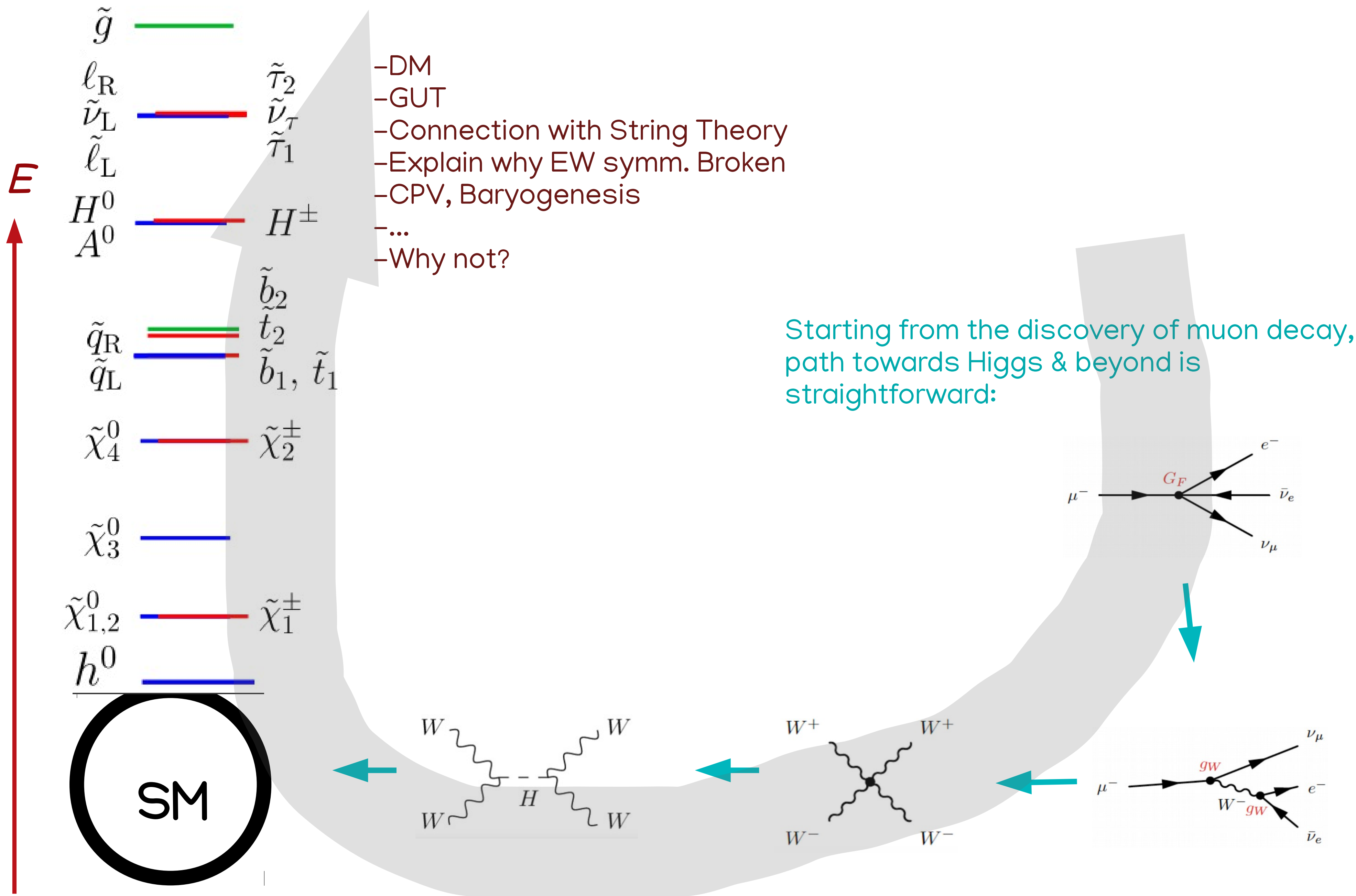
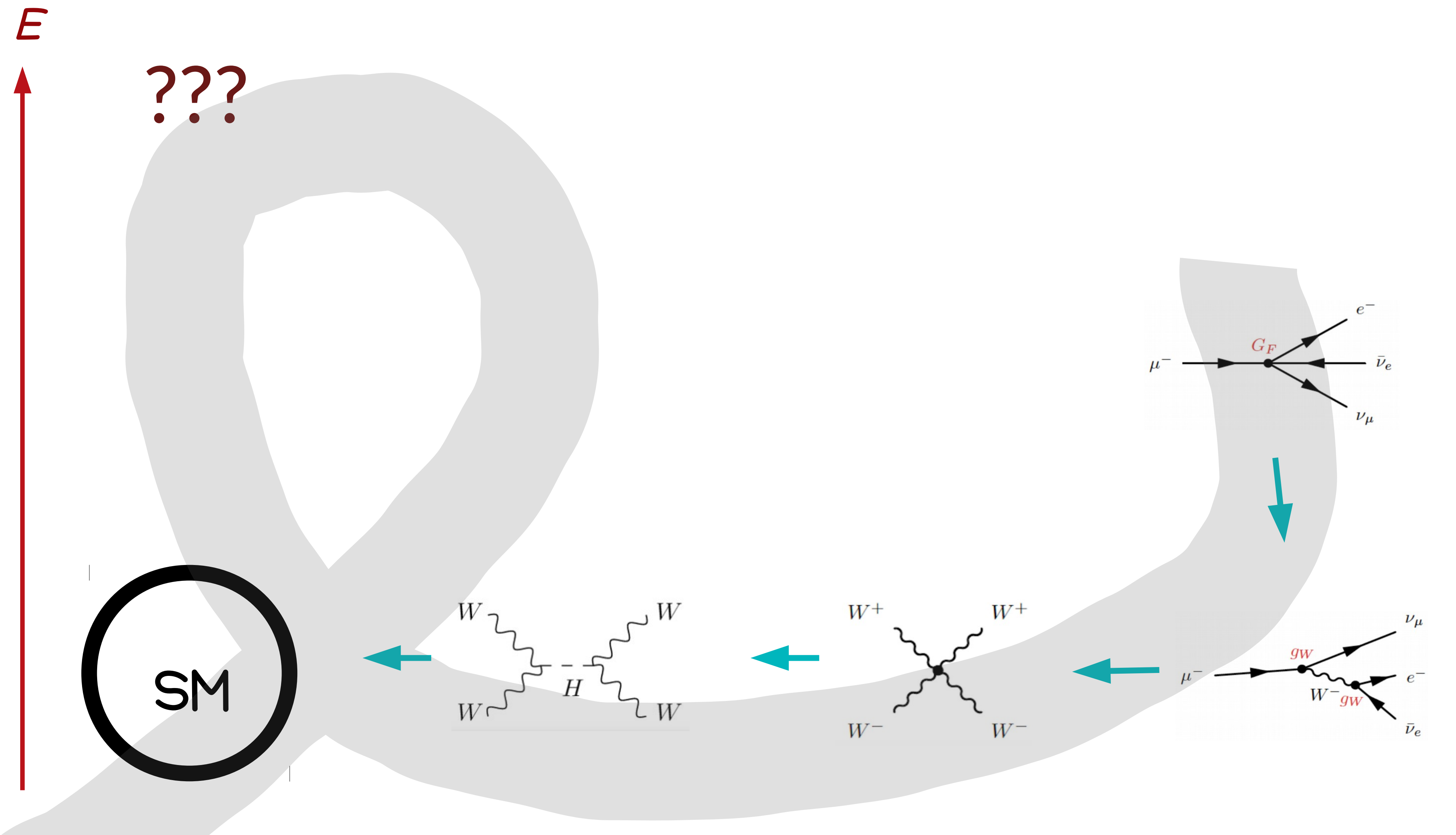
Work in progress, with **Joan Elias Miró** and **James Ingoldby**

XXth Century particle physics from a XXIst Century perspective:



-DM
-GUT
-Connection with String Theory
-Explain why EW symm. Broken
-CPV, Baryogenesis
-...
-Why not?

Starting from the discovery of muon decay, path towards Higgs & beyond is straightforward:


E
???
SM
W
W
H
W
W
W^+
W^+
W^-
W^-
μ^-
g_W
g_W
W^-
ν_μ
e^-
$\bar{\nu}_e$
G_F
μ^-
e^-
$\bar{\nu}_e$
ν_μ

$$\mathcal{L}\,?$$

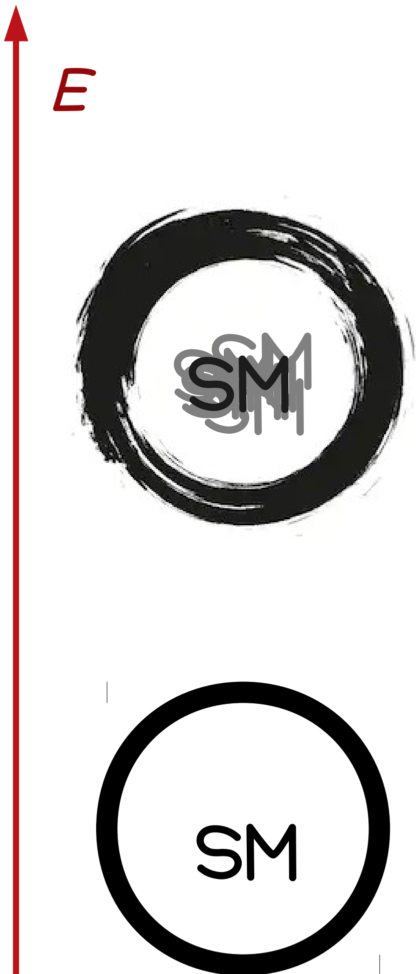
E

Particle Physics is back to the origin, is again the exploration of the unknown.

SM

$$\mathcal{L} = \mathcal{L}_{\rm SM}$$

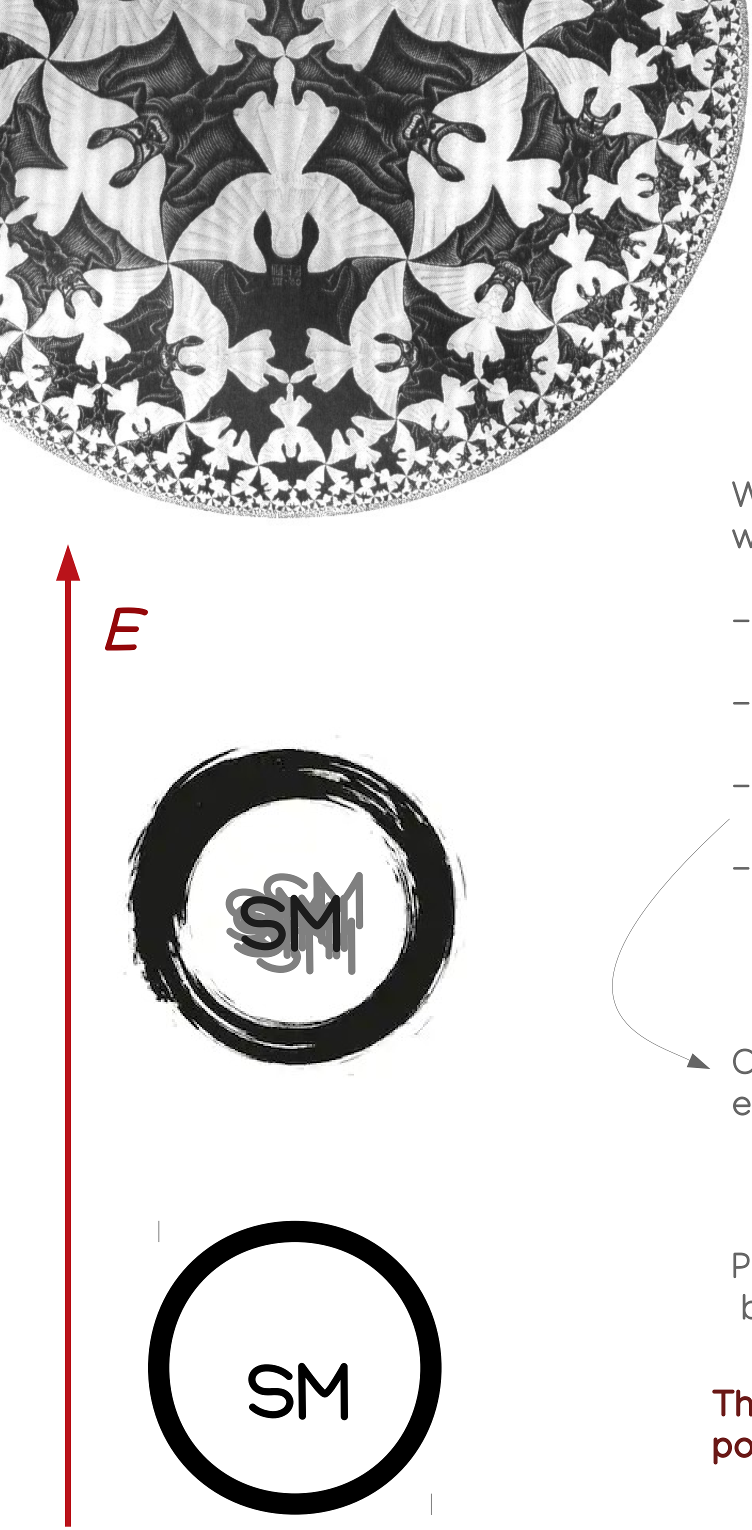
$$\mathcal{L}\,?$$



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda}\mathcal{O}_i$$

EFT operators encode information
about the heavy dynamics,
and tells us in which way
the SM is deformed.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$



$\mathcal{L}$?

While waiting for the next collider,
we might get some hints from precision experiments.

– Flavour

– mu3e (4 orders of mag. improvement)

– EDMs (1 or 2 orders of mag. Improvement in next 10 years)

– ...

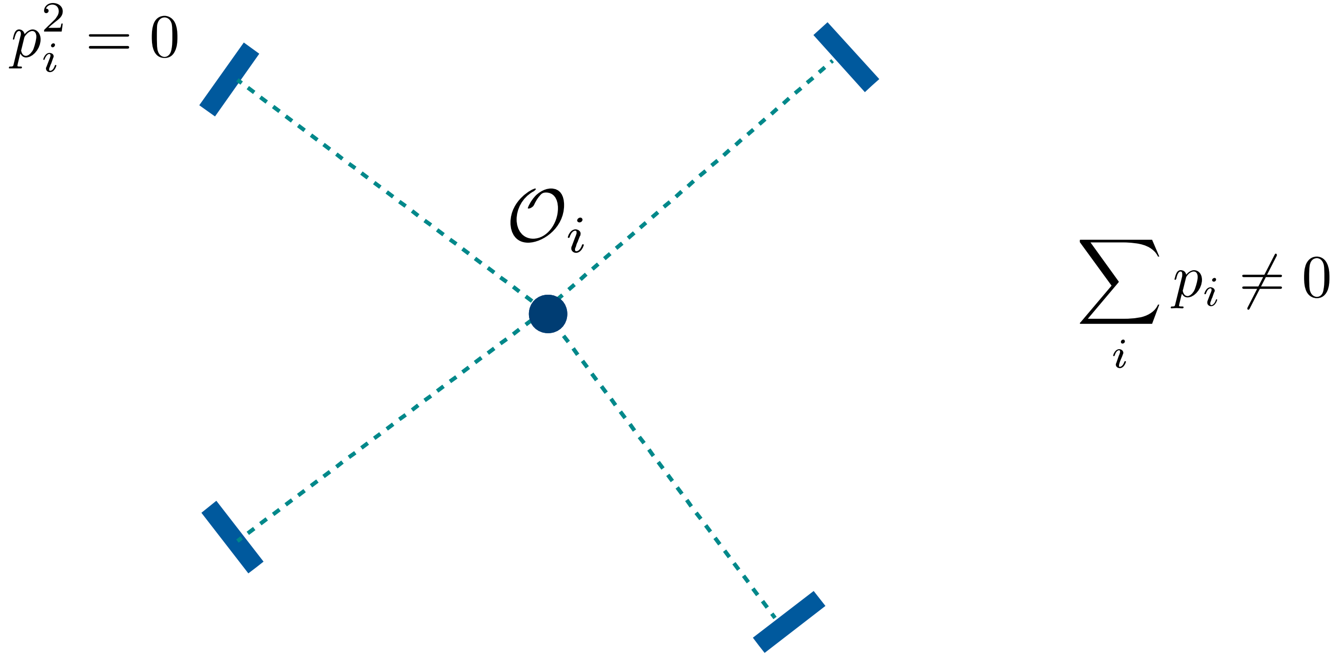
Current bounds already testing dynamics at several TeV
even if only affect the dipole at two loops [Pomarol, Panico, MR]

Precision experiments might explore dynamics at two loops...
but how to get this precision?

This talk is about a new way to compute anomalous dimensions, potentially reaching higher loops

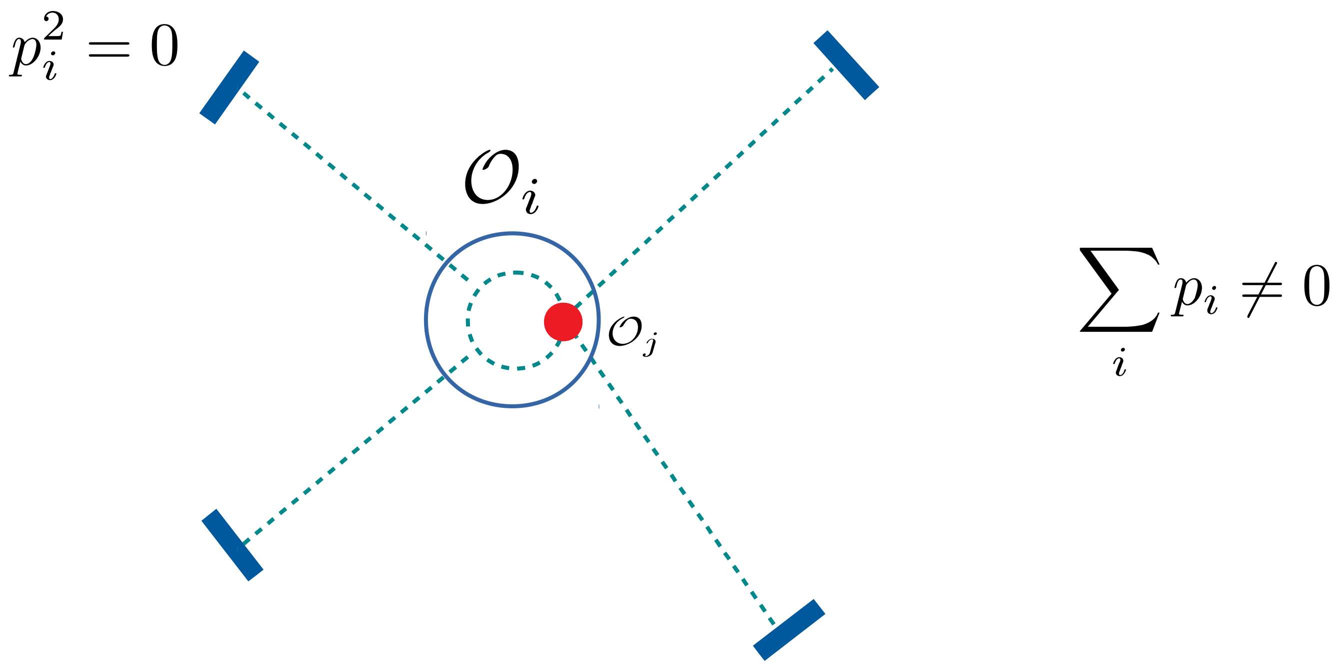
Form Factors of specific operators give the response of a given state once we insert that operator

$$F_{\mathcal{O}_i} = \langle p_1, p_2, p_3, p_4 | \mathcal{O}_i | 0 \rangle \overset{\text{e.g.}}{=} \delta_{ab}(s+t)$$



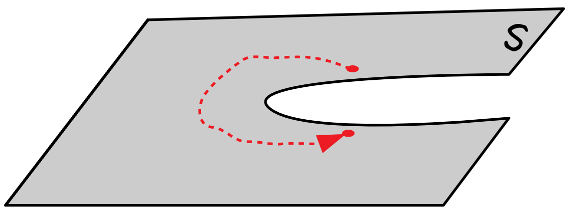
They receive, of course, loop corrections.

$$F_{\mathcal{O}_i} = \langle p_1, p_2, p_3, p_4 | \mathcal{O}_i | 0 \rangle \overset{\text{e.g.}}{=} \delta_{ab}(s+t)\left(1 + c_j \frac{g^2}{16\pi^2} \log \frac{s}{\Lambda}\right)$$



In perturbation theory (and perhaps beyond), a Form Factor is related across both sides of the cut by the **reality condition**

$$F(s_{ij}+i\epsilon) = F^*(s_{ij}-i\epsilon)$$



This is generated by a complex rotation of the momenta,

$$F = e^{-i\pi\sum_i p_i \frac{\partial}{\partial p_i}} F^* = e^{-i\pi D} F^*$$

On the other hand, **unitarity** implies

CPT

$$F = {}_{out}\langle\alpha|\mathcal{O}|0\rangle = \sum_\beta {}_{out}\langle\alpha|\beta\rangle_{in} \boxed{{}_{in}\langle\beta|\mathcal{O}|0\rangle} = S\boxed{F^*}$$

So,

CPT
Unitarity
Analyticity

$$\boxed{e^{-i\pi D}F^* = SF^*}$$

(This is a generalization of the Watson equation)

The dilatation operator is proportional to the phase of the S-matrix

see [Caron-Huot, Wilhelm '16] for alternative derivation

[Caron-Huot, Wilhelm '16]

$$e^{-i\pi D}F = SF$$

Dilatation operator related to anomalous dimensions by RG equation:

$$DF \sim \mu\frac{\partial}{\partial\mu}F \sim (\gamma_{UV} - \gamma_{IR} + \beta(g^2)\frac{\partial}{\partial g})F$$

Convolution of FF with S-matrix:

$$S = 1 + i\mathcal{M}$$

At LO, dependence on beta ignored,

$$(\gamma_{UV} - \gamma_{IR})\langle\alpha|\mathcal{O}|0\rangle = -\frac{1}{\pi}\langle\alpha|\mathcal{M}\otimes\mathcal{O}|0\rangle$$

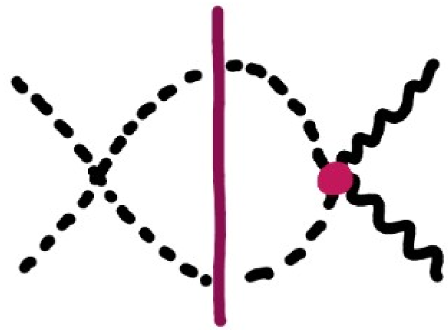
For this talk, we focus on elements with no IR divergences

$$\gamma_{i\leftarrow j} = -\frac{1}{\pi}\frac{\langle\alpha|\mathcal{M}\otimes\mathcal{O}_j|0\rangle}{\langle\alpha|\mathcal{O}_i|0\rangle}$$

I'm going to use this equation over and over in a series of explicit examples, so it is a good point to **tune in again**!

Example 1: Self renormalization of $\mathcal{O}_{FF} = |H|^2 F_{\mu\nu}^2$

$$\langle \alpha | \mathcal{M} \otimes \mathcal{O}_{FF} | 0 \rangle =$$



$$\langle 1_i 2_j^* | \mathcal{M} | 3_k 4_l^* \rangle = 2\lambda(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})$$

$$\langle 1_{\phi_i} 2_{\phi_j^*} 3^- 4^- | H^\dagger H F_{\mu\nu} F_{\mu\nu} | 0 \rangle = 2\delta_{ij} \langle 34 \rangle^2$$

$$= \int [\mathrm{d}p_{1'}][\mathrm{d}p_{2'}] 4\lambda \langle 34 \rangle^2 (\delta_{ij}\delta_{i'j'} + \delta_{ii'}\delta_{jj'})\delta_{i'j'}$$

$$= (2\delta_{ij}\langle 34 \rangle^2)\, 4\lambda(n_s + 1) \int [\mathrm{d}p_{1'}][\mathrm{d}p_{2'}]$$

$$= (2\delta_{ij}\langle 34 \rangle^2)\, 4\lambda(n_s + 1) \frac{1}{16\pi}$$

next slide I'll tell you.
Not important now.

So, $\gamma_{FF \leftarrow FF} = \dfrac{\lambda}{16\pi^2} 4(n_s + 1)$

Spinor notation for massless momenta:

See [Elvang-Huang] for a review

$$p_\mu \sigma^\mu_{a\dot{b}} = p_{a\dot{b}} = -|p]_a \langle p|_{\dot{b}} \qquad\qquad |p\rangle^{\dot{a}} = \sqrt{2E}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\phi} \end{pmatrix}$$

The important point is that angle and square brackets carry opposite little group weight, or helicity.
Example:

$$\bar{f}\gamma_\mu f \phi \overleftrightarrow{D}_\mu \phi \;\rightarrow\; \bar{u}_\pm(p_1)(\not{p}_3 - \not{p}_4) v_\mp(p_2) \begin{array}{l} \xrightarrow{+-} [13]\langle 32\rangle + [14]\langle 42\rangle \\ \xrightarrow[-+]{} \langle 13\rangle[32] + \langle 14\rangle[42] \end{array}$$

Phase space integral:

The previous integral was trivial, but in general it is not.
It will be useful to write it as

$$\int [\mathrm{d}p][\mathrm{d}q] = \frac{1}{16\pi}\int_0^{\pi/2} 2\cos\theta\sin\theta \mathrm{d}\theta \int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}$$

The angles parametrize the rotation to base spinors,

$$\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta e^{i\phi} \\ s_\theta e^{-i\phi} & c_\theta \end{pmatrix}\begin{pmatrix} |p\rangle \\ |q\rangle \end{pmatrix}$$

Example 2: 4–fermions

– There are two fundamentally different types of 4–fermions, with and without net helicity

$$(\bar\ell\gamma_\mu\ell)(\bar\ell\gamma_\mu\ell) = \langle 1|\gamma_\mu|2]\langle 3|\gamma_\mu|4] = 2\cdot\langle 13\rangle[24]$$

$$(\bar\ell u)(\bar q e) = \langle 12\rangle\langle 34\rangle$$

– We'll compute anomalous dimensions of the second type due to a U(1):

$$\mathcal{A}(f^- f^+ f^- f^+) = g^2\langle 13\rangle[24]\left(\frac{1}{s}+\frac{1}{u}\right)$$

– Only two flavour structures will be independent due to Schouten identity:

$$0 = \langle \ell u\rangle\langle qe\rangle + \langle \ell q\rangle\langle eu\rangle + \langle \ell e\rangle\langle uq\rangle$$

$$f_1 f_2 \in \{\ell u, eq, \ell e, eu, \ell q, qu\}$$

$$\gamma = \sum_{f_1 f_2}\langle f_1 f_2|\mathcal{M}|xy\rangle\langle f_3 f_4 xy|\mathcal{O}_{\ell equ}|0\rangle$$

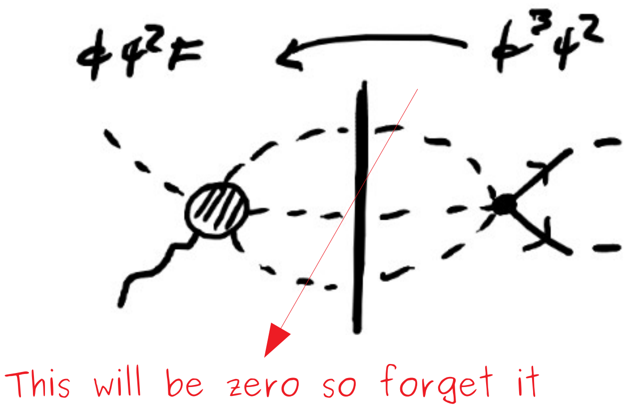
$$= \int \mathrm{d}\Omega g^2\left(Y_\ell Y_u\frac{\langle \ell u\rangle[xy]\langle xe\rangle\langle qy\rangle}{s_{\ell x}} + Y_e Y_q\frac{\langle eq\rangle[xy]\langle \ell x\rangle\langle yu\rangle}{s_{ex}} + \dots\right)$$

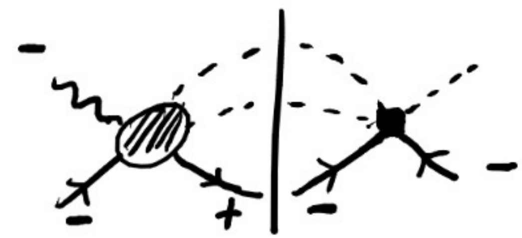
$$= \int \mathrm{d}\Omega\, g^2\,(Y_\ell Y_u + Y_\ell Y_q + Y_e Y_u + Y_e Y_q)\langle \ell u\rangle\langle qe\rangle + \langle \ell equ\rangle\dots$$

$$= \frac{g^2}{16\pi^2}(Y_\ell + Y_e)(Y_q + Y_u)\langle \ell u\rangle\langle qe\rangle$$

Example 3: A 2-loop example, Yukawa to dipole

Two types of diagrams, involving 3-particle cut and 5-point amplitudes:





The entire difficulty of this calculation is to write down the 5-point amplitude in a simple way so that the integral is easily doable. Let's focus on the pure gauge part:

$$\mathcal{A}(f^- f^+ \phi\phi\gamma^-) = g^3 \left(Q_f Q_\phi^2 \frac{[23][24]}{[12][35][45]} - Q_f^2 Q_\phi \frac{[23][24]}{[15][25][34]} \right)$$

Example 3: A 2-loop example, Yukawa to dipole

$$\mathcal{A}(f^- f^+ \phi\phi\gamma^-) = g^3 \left(Q_f Q_\phi^2 \frac{[23][24]}{[12][35][45]} - \cancelto{0}{Q_f^2 Q_\phi \frac{[23][24]}{[15][25][34]}} \right)$$

Since integral symmetric under 3, 4 exchange, and this term is odd

$$\text{integral} = s_{14} \int d\mu \frac{[xy][xz]}{[1x][y4][z4]} \langle x2 \rangle = \langle 14 \rangle \langle 42 \rangle \cdot N$$

dipole FF

with

$$N = \int_0^{\pi/2} 2 s_{\theta_1} c_{\theta_1} \mathrm{d}\theta_1 \int_0^{\pi/2} 4 s_{\theta_2}^3 c_{\theta_2} \mathrm{d}\theta_2 \int_0^{\pi/2} 2 s_{\theta_3} c_{\theta_3} \mathrm{d}\theta_3 \, \frac{c_{\theta_1}^2}{s_{\theta_2}^2} = 1$$

Adding the flavour structure, one gets the result in the literature,

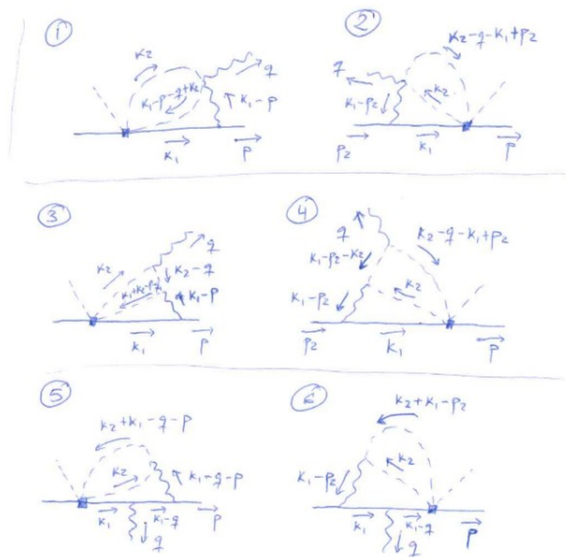
$$\frac{d}{d\ln\mu} \begin{pmatrix} C_{eB} \\ C_{eW} \end{pmatrix} = \frac{g^3}{(16\pi^2)^2} \frac{3}{4} \begin{pmatrix} t_{\theta_W} Y_H + 4 t_{\theta_W}^3 Y_H^2 (Y_L + Y_e) \\ \frac{1}{2} + \frac{2}{3} t_{\theta_W}^2 Y_H (Y_L + Y_e) \end{pmatrix} C_{y_e}$$

from [Pomarol, Panico, MR]

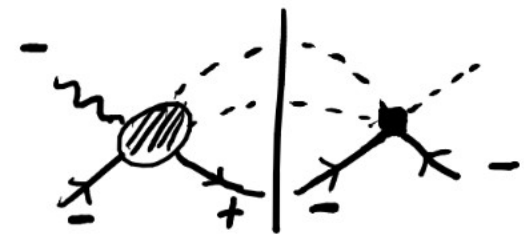
Example 3:

My brain is not a standard candle, but to get an idea...

The Verdict



VS



One month+ of struggle,
of sign-chasing, of looking
for factors of two and comparing
with collaborators...

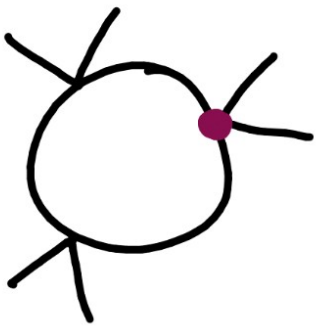
One day to get the amplitude
in a nice form.
30min of writing the rotation and
do the integrals in Mathematica.

So, is this method better for everything?

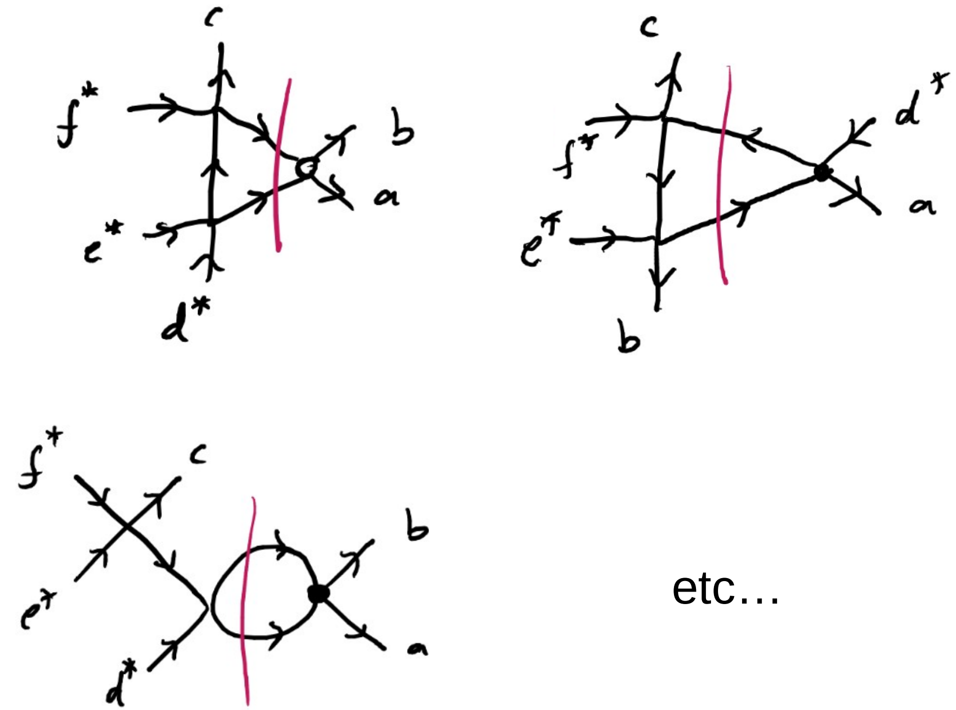
NO, IT IS NOT

Example 4:

OH to O6



VS



etc…

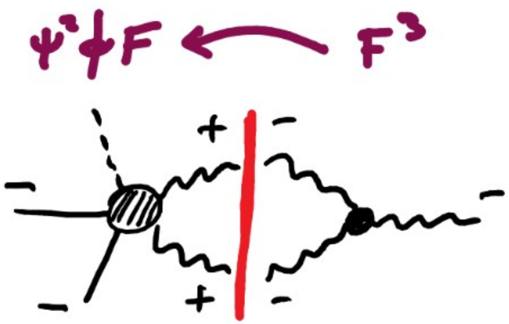
Write down the amplitude
Check Peskin's page 807
Done

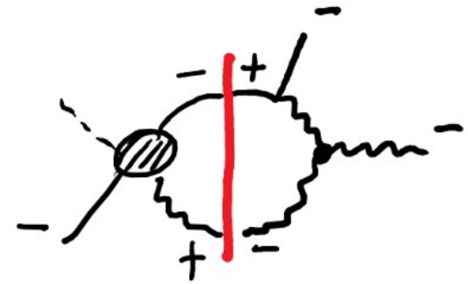
Write down diagrams
Don't forget the nonminimal FF
Compute 6-point amplitudes
Do the integrals, get logs
Try to cancel different logs
Try to understand 6-particle kinematics
Give up and spend you life in something else

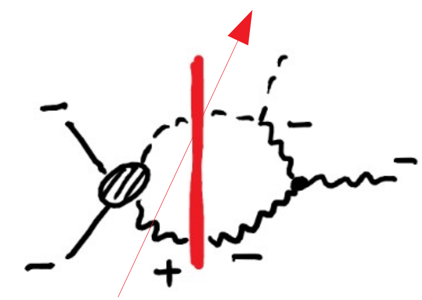
Example 5: F^3 to dipole

Let's see in a simpler example what is going on:

assume scalar singlet under SU(N)







This 5-point amplitude is actually easy since you can use BCFW

This is the key part, some contributions contain nonminimal form factors

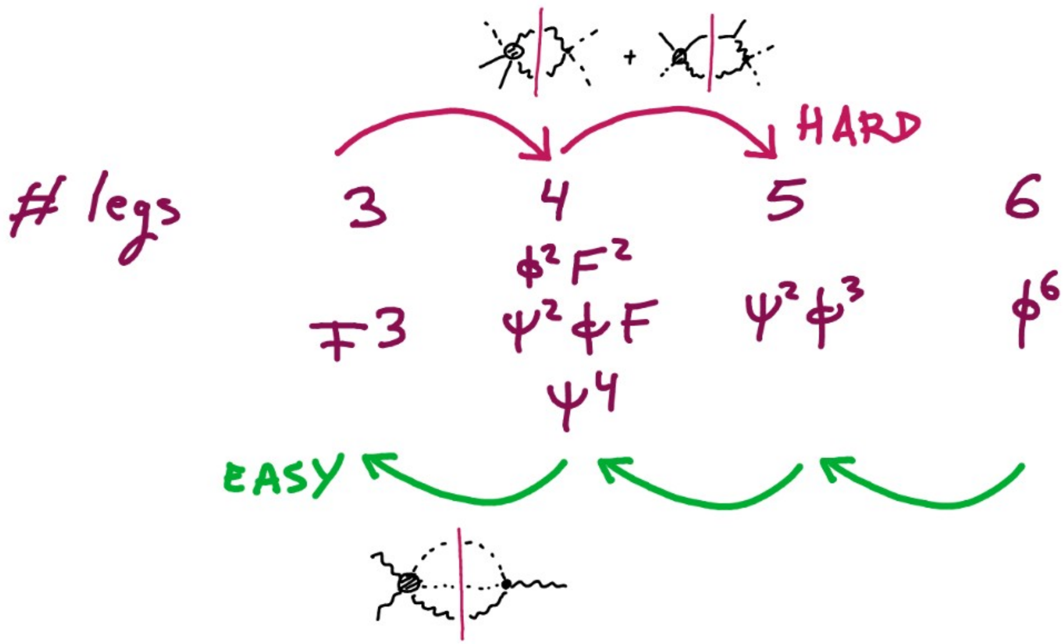
Polynomial + logs + Polynomial + logs

Polynomial = (dipole FF)*(number)

The logs cancel among the diagrams giving just a polynomial in lambdas.
The cancellation of logs is really nontrivial, and involved when more that 4 external particles.

The difficulty of the method DOES NOT scale with the number of loops,
it scales with the number of legs you need to remove/add from the initial operator.

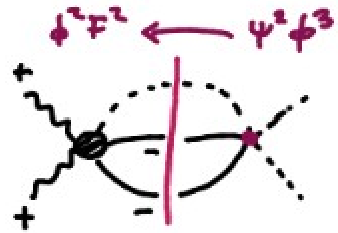
To decrease # of legs, send particles through the phase space → easy
To increase # of legs, need high point amps, nonminimal FF and cancel logs → hard

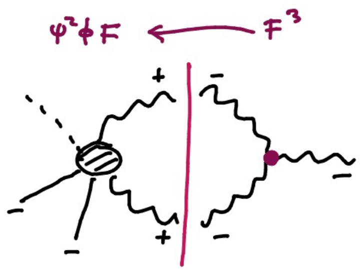


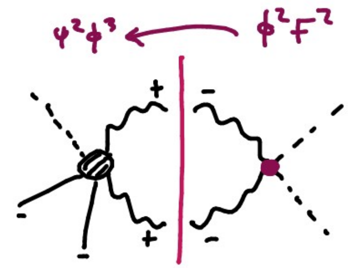
This is VERY convenient, since the 'easy' contributions of this method are precisely the transitions forbidden at one loop.

So it is possible to get all the relevant 2 loop anomalous dimensions.

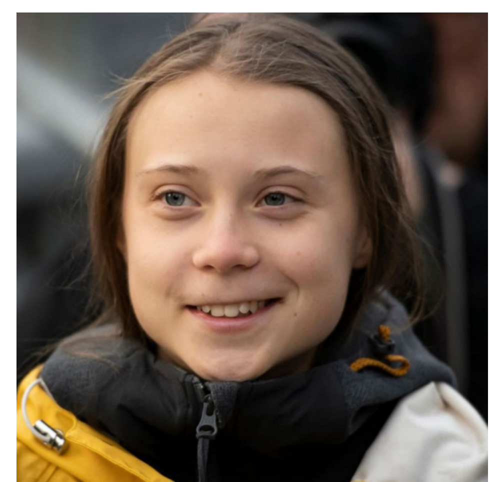
Last but not least, an obvious but important aspect of the method is that it is GREEN







Same amplitude appears in several computations, in this case $\mathcal{A}(f^- f^- \phi\, g^+ g^+)$



Conclusions...

- Anomalous dimensions of the SMEFT can be computed from on-shell data

- The method is not a magic wand, some computations are easier, some harder

- The ‘easy’ 2-loop are actually the relevant ones since are the forbidden contributions at one loop

… and outlook

- Similar manipulations let us use to optical theorem to get

$$(e^{-i\pi D}-1)M_{2\to n}(s_{ij}+i\epsilon)=\langle 2|\,M^\dagger\,|i\rangle\otimes\langle i|\,M\,|n\rangle$$

It works in many cases but IR divs. under current investigation

- Better undestranding of the nonminimal form factors in needed.
Preliminary exploration shows that writing the phase space integral as the Grassmanian might help