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Ward Identities for the Standard Model Effective Field Theory

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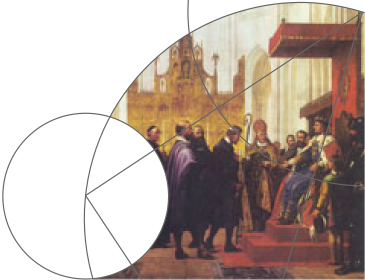

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Outline

1. Curved field space

2. Gauge fixing

3. Ward Identities

Standard Model Effective Field Theory

SMEFT is an *effective field theory* based on SM field content and symmetries:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots \tag{1}$$

$$\mathcal{L}^{(d)} = \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}, \qquad d > 4 \tag{2}$$

$C_i^{(d)}$: Wilson coefficient
$Q_i^{(d)}$: Effective operator

Ward Identities

The Background Field Method generating functional of the SMEFT is

$$Z[\hat{F}, J] = \int \mathcal{D}F \det\left[\frac{\Delta G^A}{\Delta \alpha^B}\right] e^{i\left(S[F+\hat{F}]+\mathcal{L}_{\rm GF}+\text{source terms}\right)} \tag{3}$$

Ward Identities

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We want the generating functional to be background field gauge invariant

$$\frac{\delta Z[\hat{F}, 0]}{\delta \hat{\alpha}^A} \stackrel{?}{=} 0 \tag{4}$$

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Objective: background field gauge invariant gauge fixing term

Curved field space

Geometry of scalar field space

The bilinear field interactions can be thought of in terms of connections on the field space manifold

$$\mathcal{L}_{\mathrm{scalar,kin}} = (D_\mu H)^\dagger (D^\mu H) + \frac{C_{H\Box}}{\Lambda^2} \left(H^\dagger H\right) \Box \left(H^\dagger H\right) + \frac{C_{HD}}{\Lambda^2} \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D^\mu H\right) \tag{5}$$

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$$
\begin{aligned}
\mathcal{L}_{\text{scalar,kin}} =& (D_\mu H)^\dagger (D^\mu H) + \frac{C_{H\Box}}{\Lambda^2} \left(H^\dagger H\right) \Box \left(H^\dagger H\right) \\
&+ \frac{C_{HD}}{\Lambda^2} \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D^\mu H\right) \\
=& \frac{1}{2} h_{IJ}(\phi) (D_\mu \phi)^I (D^\mu \phi)^J
\end{aligned} \tag{5}
$$

where $I, J \in \{1, \ldots, 4\}$ and

$$
H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}. \tag{6}
$$

Metric of the scalar field manifold

The metric is non-trivial

$$h_{IJ}(\phi) = \delta_{IJ} - 2\frac{C_{H\Box}}{\Lambda^2}\phi_I\phi_J + \frac{1}{2}\frac{C_{HD}}{\Lambda^2} f_{IJ}(\phi), \tag{7}$$

where

$$f_{IJ}(\phi) = \begin{bmatrix} a & 0 & d & c \\ 0 & a & c & -d \\ d & c & b & 0 \\ c & -d & 0 & b \end{bmatrix}, \quad \begin{aligned} a &= \phi_1^2 + \phi_2^2 \\ b &= \phi_3^2 + \phi_4^2 \\ c &= \phi_1\phi_4 + \phi_2\phi_3, \\ d &= \phi_1\phi_3 - \phi_2\phi_4. \end{aligned} \tag{8}$$

- The Riemann curvature tensor calculated from the scalar field metric is non-vanishing. The scalar manifold is curved due to the power counting expansion.

Geometry of gauge field space

Analogously, we can describe the kinetic part of the gauge fields in terms of connections on the field space manifold

$$\begin{aligned}\mathcal{L}_{\text{gauge,kin}} = &-\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} \\ &+ \frac{C_{HW}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}\end{aligned} \tag{9}$$

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where

$$\begin{aligned} g_{ab} &= \left(1 - 4\frac{C_{HW}}{\Lambda^2}H^\dagger H\right)\delta_{ab}, \qquad g_{44} = 1 - 4\frac{C_{HB}}{\Lambda^2}H^\dagger H, \\ g_{a4} &= g_{4a} = -2\frac{C_{HWB}}{\Lambda^2}H^\dagger \sigma_a H, \qquad a = 1,2,3. \end{aligned} \tag{10}$$

The gauge field manifold is curved.

Real representation of the scalar field

We use the real representation

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}. \tag{11}$$

We have that

$$\begin{aligned} [\gamma_a, \gamma_b] &= 2\epsilon^c_{ab}\gamma_c, \qquad \Gamma^I_{A,K} = \gamma^I_{A,J}\gamma^J_{4,K} \\ [\gamma_a, \gamma_4] &= 0, \end{aligned} \tag{12}$$

Gauge fixing

Background field method

- The background field method splits fields into background and quantum fields $F \rightarrow \hat{F} + F$.
  $\hat{F}$: background field
  F: quantum field

- The background field method provides technical simplifications due to the background field gauge invariance being preserved and the resulting Ward identities.

- The Standard Model was formulated using the background field method [Denner, Dittmaier, Weiglein]

Gauge fixing the Standard Model Effective Field Theory

A gauge fixing choice which preserves the geometric structure of the theory is

$$\mathcal{L}_{\mathrm{GF}} = -\frac{\hat{g}_{AB}}{2\xi}\mathcal{G}^A\mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu W^{X,\mu} - \tilde{\epsilon}^X_{CD}\hat{W}^C_\mu W^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\hat{h}_{IK}\tilde{\gamma}^K_{C,J}\hat{\phi}^J. \quad (13)$$

Background field gauge invariance is preserved.

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Background field gauge invariance is preserved.

- We can gauge fix the Standard Model Effective Field Theory preserving the background field gauge invariance.

- All-orders in the power counting expansion.

The key point

We gauge fix the fields on the curved field space due to the power counting expansion.

Ward Identities

Ward Identities

The Background Field Method generating functional of the SMEFT is

$$Z[\hat{F}, J] = \int \mathcal{D}F \det\left[\frac{\Delta G^A}{\Delta \alpha^B}\right] e^{i\left(S[F+\hat{F}]+\mathcal{L}_{\mathrm{GF}}+\text{source terms}\right)} \quad (14)$$

Due to the gauge-fixing term, the generating functional is background field gauge invariant

$$\frac{\delta Z[\hat{F}, 0]}{\delta \hat{\alpha}^A} \overset{!}{=} 0 \quad (15)$$

Ward Identities

In position space, the identities are

$$0 = \left(\partial^\mu \delta^A_B - \tilde{\epsilon}^A_{BC} \hat{\mathcal{W}}^{C,\mu}\right) \frac{\delta \Gamma}{\delta \hat{\mathcal{W}}^\mu_A} - \frac{\tilde{\gamma}^I_{B,J}}{2} \hat{\phi}^J \frac{\delta \Gamma}{\delta \hat{\phi}^I} + \sum_j \left(\bar{f}_j \bar{\Lambda}^J_{B,i} \frac{\delta \Gamma}{\delta \bar{f}_i} - \frac{\delta \Gamma}{\delta f_i} \Gamma^i_{B,j} f_j \right). \tag{16}$$

$\Gamma[\hat{F}]$: effective action

Mass eigenstates

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

$\mathcal{W}^{A,\nu}$: weak eigenstate
$\mathcal{A}^{A,\nu}$: mass eigenstate

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix}, \qquad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Mass eigenstate generators

$$\gamma^{I}_{C,J} = \frac{1}{2}\tilde{\gamma}^{I}_{A,J}\sqrt{g}^{AB}U_{BC} \tag{17}$$

$$\gamma^{I}_{1,J} = \frac{\overline{g}_2}{2\sqrt{2}}\begin{bmatrix} 0 & 0 & i & -1 \\ 0 & 0 & -1 & -i \\ -i & 1 & 0 & 0 \\ 1 & i & 0 & 0 \end{bmatrix}, \qquad \gamma^{I}_{2,J} = \frac{\overline{g}_2}{2\sqrt{2}}\begin{bmatrix} 0 & 0 & -i & -1 \\ 0 & 0 & -1 & i \\ i & 1 & 0 & 0 \\ 1 & -i & 0 & 0 \end{bmatrix},$$

$$\gamma^{I}_{3,J} = \frac{\overline{g}_Z}{2}\begin{bmatrix} 0 & -(c^2_{\theta_Z} - s^2_{\theta_Z}) & 0 & 0 \\ (c^2_{\theta_Z} - s^2_{\theta_Z}) & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \gamma^{I}_{4,J} = \overline{e}\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Geometric definition of couplings

The geometric definition of the canonically normalized mass eigenstate gauge couplings are

$$\bar{g}_2 = g_2 \sqrt{g}^{11} = g_2 \sqrt{g}^{22}, \tag{18}$$

$$\bar{g}_Z = \frac{g_2}{c^2_{\theta_Z}} \left(c_{\bar{\theta}} \sqrt{g}^{33} - s_{\bar{\theta}} \sqrt{g}^{34} \right) = \frac{g_1}{s^2_{\theta_Z}} \left(s_{\bar{\theta}} \sqrt{g}^{44} - c_{\bar{\theta}} \sqrt{g}^{34} \right), \tag{19}$$

$$\bar{e} = g_2 \left(s_{\bar{\theta}} \sqrt{g}^{33} + c_{\bar{\theta}} \sqrt{g}^{34} \right) = g_1 \left(c_{\bar{\theta}} \sqrt{g}^{44} + s_{\bar{\theta}} \sqrt{g}^{34} \right), \tag{20}$$

Geometric mixing angles and masses

Similarly, consistency also dictates the field space geometric definitions of the mixing angles

$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})}{g_2(\sqrt{g}^{33} c_{\bar{\theta}} - \sqrt{g}^{34} s_{\bar{\theta}}) + g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})}, \tag{21}$$

$$s^2_{\bar{\theta}} = \frac{(g_1 \sqrt{g}^{44} - g_2 \sqrt{g}^{34})^2}{g_1^2[g^{34} + g^{44}] + g_2^2[g^{33} + g^{34}] - 2 g_1 g_2 \sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}. \tag{22}$$

and masses

$$\bar{m}^2_W = \frac{\bar{g}_2^2}{4} \sqrt{h_{11}}^2 \bar{v}_T^2, \qquad \bar{m}^2_Z = \frac{\bar{g}_Z^2}{4} \sqrt{h_{33}}^2 \bar{v}_T^2 \qquad \bar{m}^2_A = 0. \tag{23}$$

Ward Identities – Photon/Z boson

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^\mu \delta \hat{\mathcal{A}}^{Y\nu}}, \qquad 0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^\mu \delta \hat{\Phi}^I}. \tag{24}$$

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^\nu \delta \hat{\mathcal{Z}}^\mu}. \tag{25}$$

SM-like Ward Identities

Ward Identities – Z boson

$$0 = \partial^{\mu} \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{Z}}^{\mu} \delta \hat{\mathcal{A}}^{Y\nu}} - \bar{M}_Z \frac{\delta^2 \Gamma}{\delta \hat{\Phi}^3 \delta \hat{\mathcal{A}}^{Y\nu}}, \quad (26)$$

$$0 = \partial^{\mu} \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{Z}}^{\mu} \delta \hat{\Phi}^I} - \bar{M}_Z \frac{\delta^2 \Gamma}{\delta \hat{\Phi}^3 \delta \hat{\Phi}^I} + \frac{\bar{g}_Z}{2} \frac{\delta \Gamma}{\delta \hat{\Phi}^4} \left(\sqrt{h}_{[4,4]} \sqrt{h}^{[3,3]} \right) \delta^3_I \quad (27)$$

Note the appearence of $\overline{M}_Z$ and $\overline{g}_Z$.

Conclusion

Higher-dimensional operators with Higgs field define curved field space:

- Geometric definition of couplings, masses, and mixing angle
- Gauge fix with background field method
- Background field Ward Identities

Conclusion

Higher-dimensional operators with Higgs field define curved field space:

- Geometric definition of couplings, masses, and mixing angle
- Gauge fix with background field method
- Background field Ward Identities
  Valid to all orders in both perturbative and v/Λ expansions

Thank you!

Backup

Background field gauge transformations

It is useful to note the following background field gauge transformations ($\delta\hat{F}$), with infinitesimal local gauge parameters $\delta\hat{\alpha}_A(x)$ when verifying the explicitly the background field gauge invariance of this expression

$$
\begin{aligned}
\delta\,\hat{\phi}^I &= -\delta\hat{\alpha}^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}\hat{\phi}^J, \\
\delta\,(D^\mu\hat{\phi})^I &= -\delta\hat{\alpha}^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}(D^\mu\hat{\phi})^J, \\
\delta\hat{\mathcal{W}}^{A,\mu} &= -\partial^\mu(\delta\hat{\alpha}^A) - \tilde{\epsilon}^A{}_{BC}\,\delta\hat{\alpha}^B\,\hat{\mathcal{W}}^{C,\mu}, \\
\delta\hat{h}_{IJ} &= \hat{h}_{KJ}\,\frac{\delta\hat{\alpha}^A\,\tilde{\gamma}^K_{A,I}}{2} + \hat{h}_{IK}\,\frac{\delta\hat{\alpha}^A\,\tilde{\gamma}^K_{A,J}}{2}, \\
\delta\hat{\mathcal{W}}^A_{\mu\nu} &= -\tilde{\epsilon}^A{}_{BC}\,\delta\hat{\alpha}^B\,\hat{\mathcal{W}}^C_{\mu\nu}, \\
\delta\hat{g}_{AB} &= \hat{g}_{CB}\,\tilde{\epsilon}^C{}_{DA}\,\delta\hat{\alpha}^D + \hat{g}_{AC}\,\tilde{\epsilon}^C{}_{DB}\,\delta\hat{\alpha}^D.
\end{aligned}
\tag{28}
$$

The background field gauge invariance is established by using these transformations in conjuction with a linear change of variables on the quantum fields

$$\mathcal{W}^{A,\mu} \to \mathcal{W}^{A,\mu} - \tilde{\epsilon}^{A}_{\ BC}\, \delta\hat{\alpha}^{B}\, \mathcal{W}^{C,\mu},$$
$$\phi^{I} \to \phi^{I} - \frac{\delta\hat{\alpha}^{B}\, \tilde{\gamma}^{I}_{B,K}}{2}\phi^{K}. \tag{29}$$

The transformation of the gauge fixing term is

$$\delta\mathcal{G}^{X} = -\tilde{\epsilon}^{X}_{\ AB}\, \delta\hat{\alpha}^{A}\mathcal{G}^{B}. \tag{30}$$

With these transformations, the background field gauge invariance of the gauge fixing term is directly established.

Ghost term

The quantum fields gauge transformations are

$$\Delta\mathcal{W}^A_\mu = -\partial_\mu \Delta\alpha^A - \tilde{\epsilon}^A{}_{BC}\,\Delta\alpha^B\,(\mathcal{W}^C_\mu + \hat{\mathcal{W}}^C_\mu),$$
$$\Delta\phi^I = -\Delta\alpha^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}\,(\phi^J + \hat{\phi}^J). \qquad (31)$$

As the hatted field metrics depend only on the background fields and do not transform under quantum field gauge transformations, the Faddeev-Popov ghost term still follows directly; we find

$$\begin{aligned}\mathcal{L}_{\rm FP} = & -\hat{g}_{AB}\bar{u}^B\Big[-\partial^2\delta^A_C - \overleftarrow{\partial}_\mu\tilde{\epsilon}^A{}_{DC}(\mathcal{W}^{D,\mu} + \hat{\mathcal{W}}^{D,\mu}) \\ & + \tilde{\epsilon}^A{}_{DC}\hat{\mathcal{W}}^D_\mu\overrightarrow{\partial}^\mu - \tilde{\epsilon}^A{}_{DE}\tilde{\epsilon}^E{}_{FC}\hat{\mathcal{W}}^D_\mu(\mathcal{W}^{F,\mu} + \hat{\mathcal{W}}^{F,\mu}) \\ & - \frac{\xi}{4}\hat{g}^{AD}(\phi^J + \hat{\phi}^J)\tilde{\gamma}^I_{C,J}\,\hat{h}_{IK}\,\tilde{\gamma}^K_{D,L}\,\hat{\phi}^L)\Big]\,u^C. \end{aligned} \qquad (32)$$

Ward Identities – Two-point functions

The Ward Identities for two-point functions are

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\mathcal{W}}^{B,\mu}} - \frac{\tilde{\gamma}^I_{B,J}}{2} \langle \hat{\phi}^J \rangle \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\phi}^I} \tag{33}$$

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\phi}^K \delta \hat{\mathcal{W}}^{B,\mu}} - \frac{\tilde{\gamma}^I_{B,J}}{2} \left(\langle \hat{\phi}^J \rangle \frac{\delta^2 \Gamma}{\delta \hat{\phi}^K \delta \hat{\phi}^I} + \delta^I_K \frac{\delta \Gamma}{\delta \hat{\phi}^I} \right) \tag{34}$$

Ward Identities – Three-point functions

$$0 = \partial^\mu \frac{\delta^3 \Gamma}{\delta \hat{W}^{A,\nu} \delta \hat{W}^{B,\mu} \delta \hat{W}^{C,\rho}} - \tilde{\epsilon}^D_{\ BC} \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{W}}^{D,\rho} \delta \hat{\mathcal{W}}^{A,\nu}} - \frac{\tilde{\gamma}^I_{B,J}}{2} \langle \hat{\phi}^J \rangle \frac{\delta^3 \Gamma}{\delta \hat{\phi}^I \delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\mathcal{W}}^{C,\rho}}, \tag{35}$$

$$0 = \partial^\mu \frac{\delta^3 \Gamma}{\delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\mathcal{W}}^{B,\mu} \delta \hat{\phi}^K} - \tilde{\epsilon}^D_{\ BA} \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{W}}^{D,\nu} \delta \hat{\phi}^K} - \frac{\tilde{\gamma}^I_{B,J}}{2} \left(\langle \hat{\phi}^J \rangle \frac{\delta^3 \Gamma}{\delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\phi}^I \delta \hat{\phi}^K} + \delta^J_K \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{W}}^{A,\nu} \delta \hat{\phi}^I} \right), \tag{36}$$

$$0 = \partial^\mu \frac{\delta^3 \Gamma}{\delta \hat{\mathcal{W}}^{B,\mu} \delta \hat{\phi}^K \delta \hat{\phi}^L} - \frac{\tilde{\gamma}^I_{B,J}}{2} \langle \hat{\phi}^J \rangle \frac{\delta^3 \Gamma}{\delta \hat{\phi}^I \delta \hat{\phi}^K \delta \hat{\phi}^L} - \frac{\tilde{\gamma}^I_{B,J}}{2} \left(\delta^J_K \frac{\delta^2 \Gamma}{\delta \hat{\phi}^I \delta \hat{\phi}^L} + \delta^J_L \frac{\delta^2 \Gamma}{\delta \hat{\phi}^I \delta \hat{\phi}^K} \right). \tag{37}$$