Geometric SMEFT

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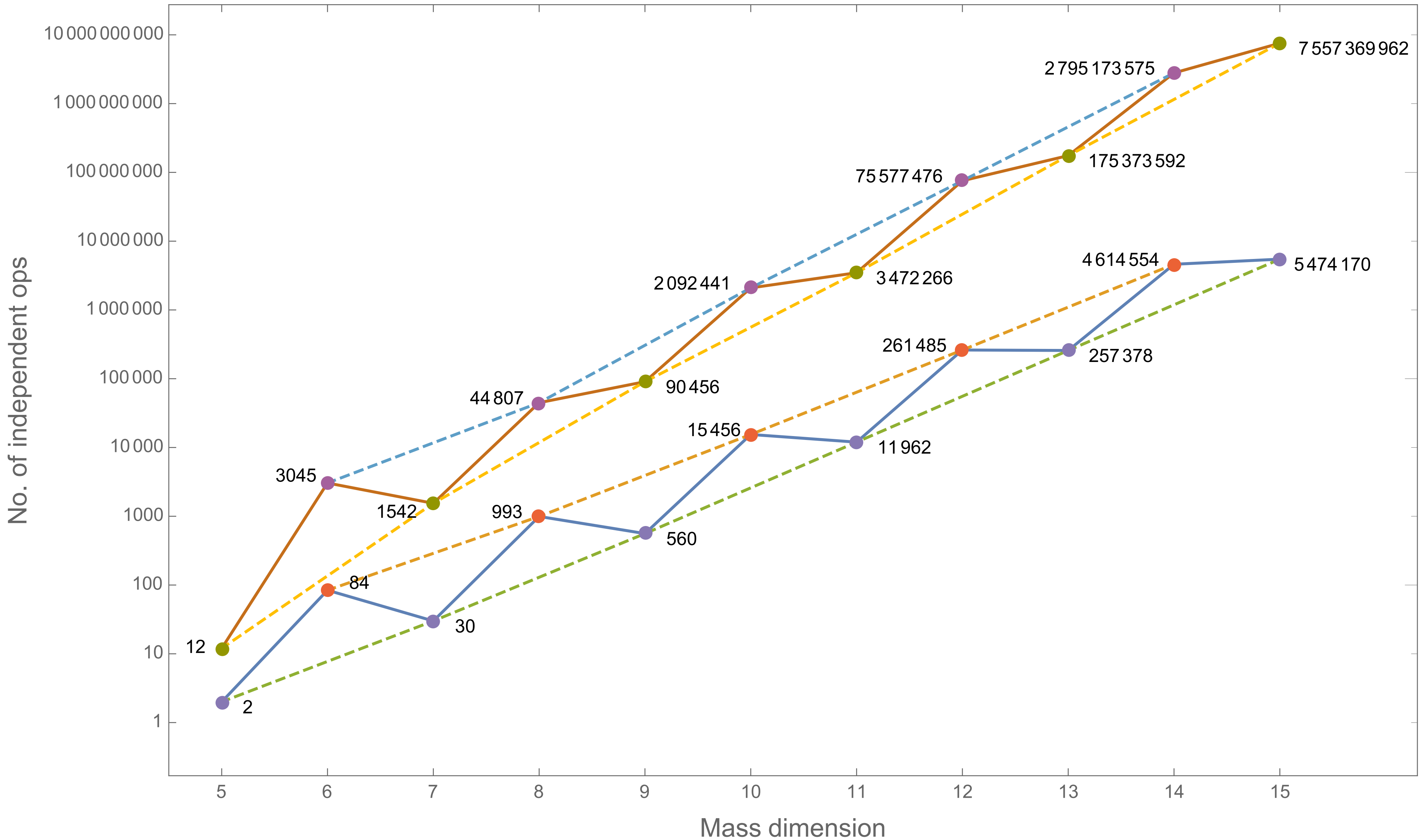
based on 2001.01453 with A. Helset and M. Trott (NBI) + work in progress w/ Hays, Helset, Trott

HEFT, April 15th 2020

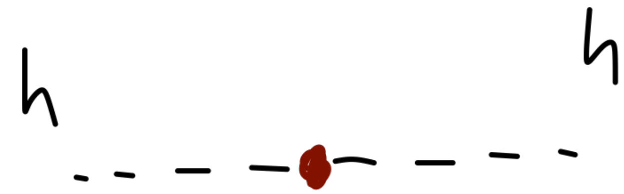
[Henning et al 1512.03433]



- # of SMEFT operators grows quickly with increasing mass dimension
- Looks overwhelming for phenomenology…

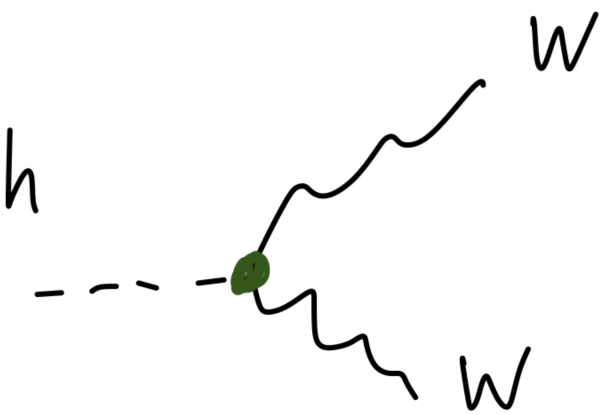
But what do these operators actually do?

Change field strength normalization/inputs



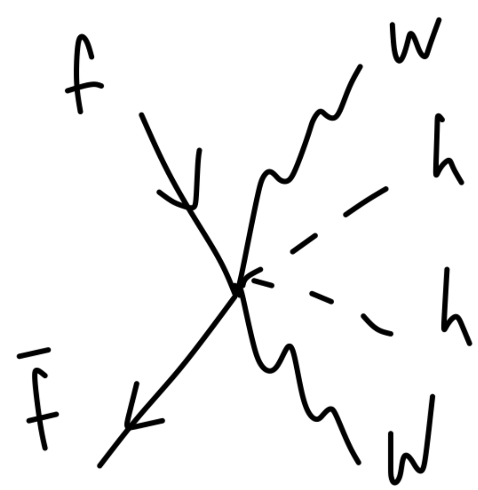
ex.) $(H^\dagger H)\Box(H^\dagger H)$

Modify existing vertices



ex.) $(H^\dagger H)\, W^a_{\mu\nu} W^{a,\mu\nu}$

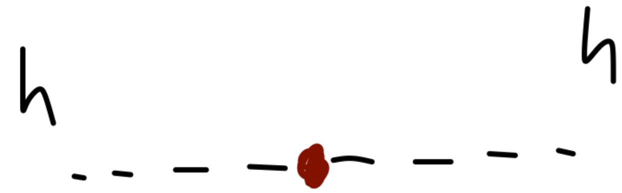
New multi-particle interactions



ex.) $(\bar{\psi}\psi)^2$

But what do these operators actually do?

Change field strength normalization/inputs



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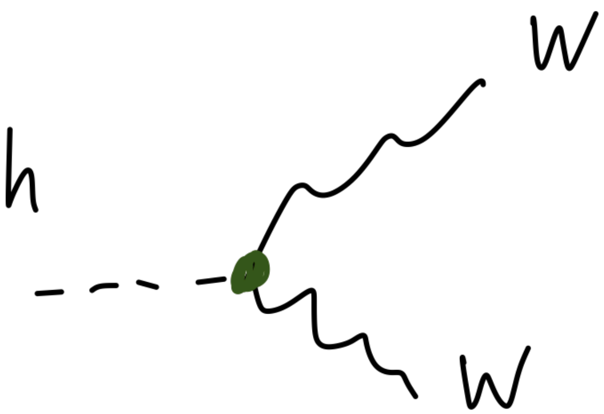

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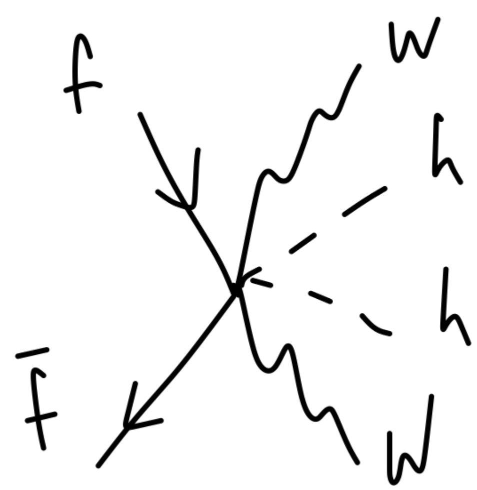
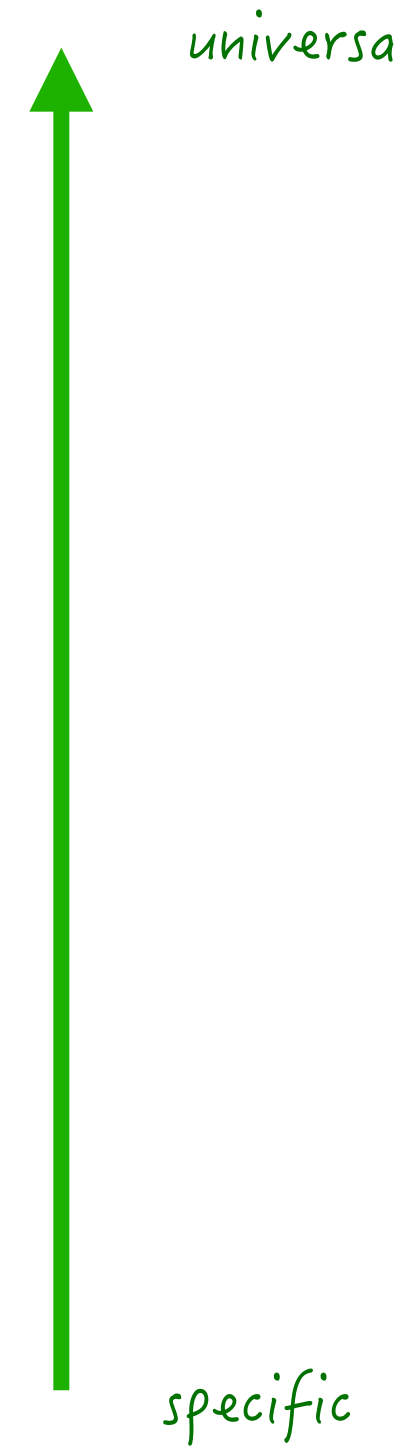
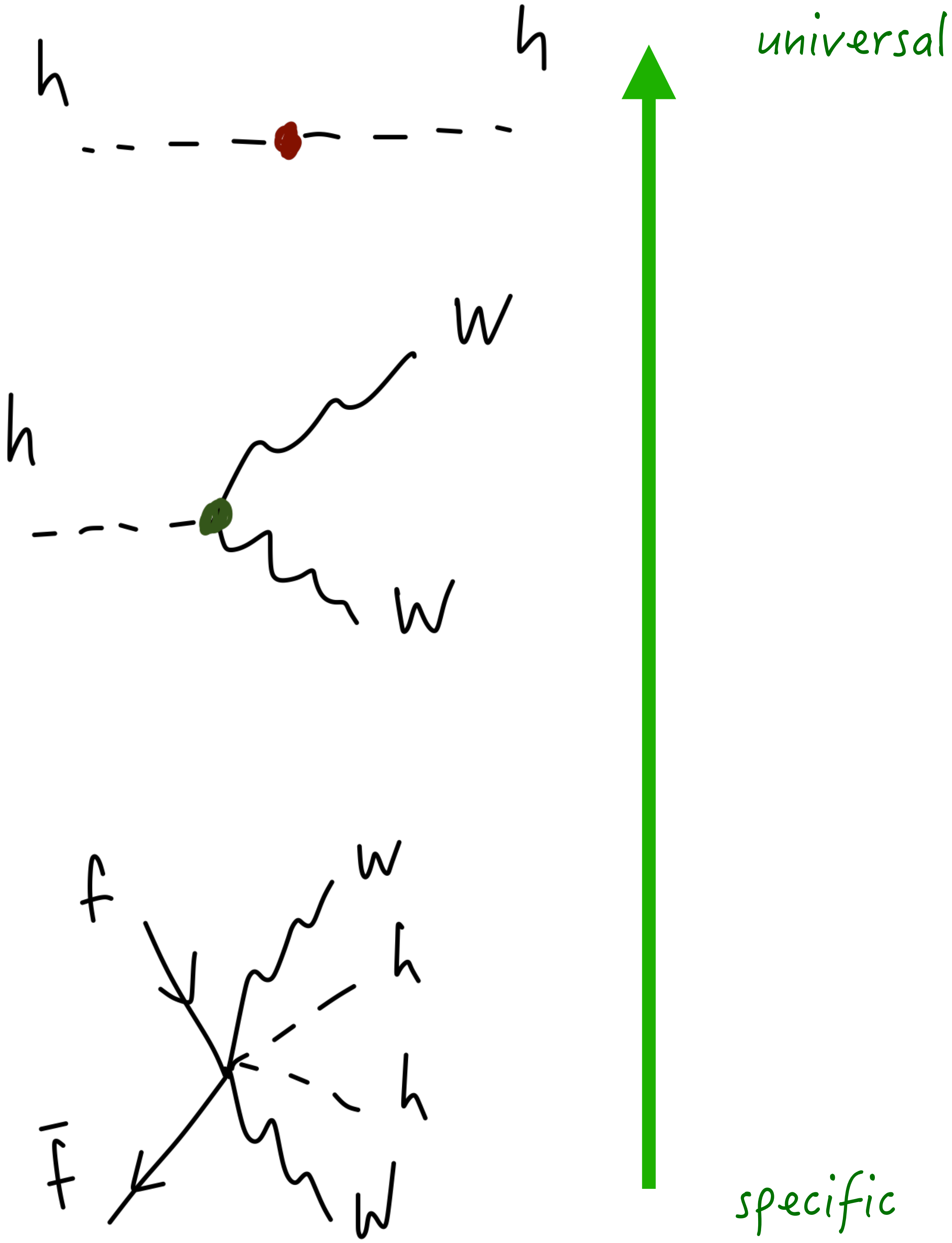




Punchline of this talk



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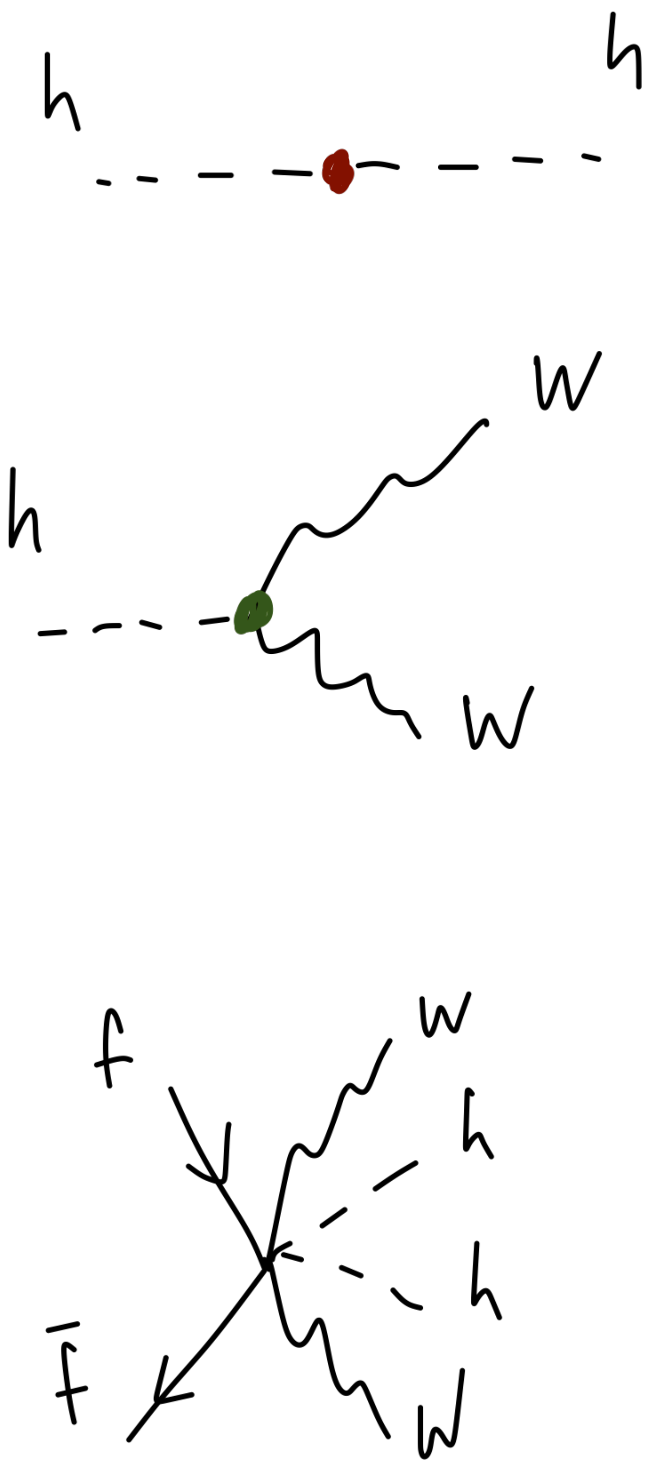
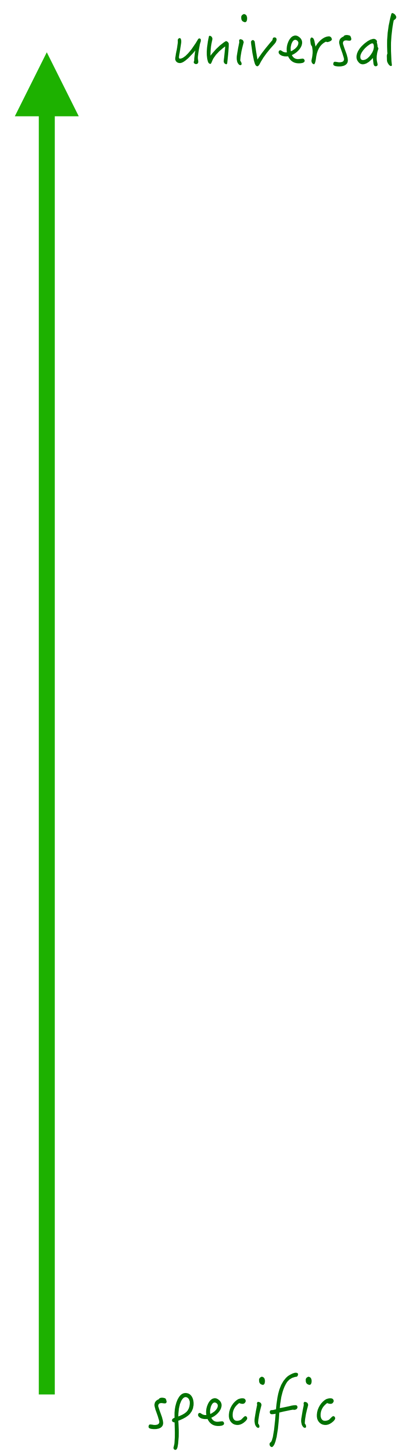




few operators

many operators

For **2- and 3-pt. interactions**, the # of contributing SMEFT operators is **small** and **constant with operator dimension**

Fixed dimension: can do lots of phenomenology with a relatively small set of operators

All dimensions: allows compact, all-orders expressions

First hint: Misiak et al 1812.11513

Only SMEFT operator types that contribute to bosonic 2-pt interactions are:

$$H^n\,,\ H^nX^2\,,\ D^2H^n$$

- Why not $(DH^\dagger)(DH)(DH^\dagger)(DH)$? — too many fields
- Why not $(D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H)$? — via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

Similar arguments can be made for operators with field strengths, more derivatives

Entire EW input set by two 'metrics': $h(H)(D_\mu H^\dagger D_\mu H)\,,\ g_{AB}(H)\mathscr{W}^A_{\mu\nu}\mathscr{W}^{B\mu\nu}$

$$\mathscr{W}^A = \left(W^1, W^2, W^3, B\right)$$

Even better:

Number of H^n , $H^n X^2$, $D^2 H^n$ type operators ~ doesn't change with mass dimension

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4

Just a consequence of group theory + Bose statistics (see backup for example). Verified with Hilbert series method

contributions to g_{AB}

$$
\begin{aligned}
Q_{HB}^{(6+2n)} &= (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu}, \\
Q_{HW}^{(6+2n)} &= (H^\dagger H)^{n+1} W_a^{\mu\nu} W^a_{\mu\nu}, \\
Q_{HWB}^{(6+2n)} &= (H^\dagger H)^n (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu}, \\
Q_{HW,2}^{(8+2n)} &= (H^\dagger H)^n (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu},
\end{aligned}
$$

Convenient to work with real fields:

$$H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

Using γ_A = generators in real representation and $\Gamma_A = \gamma_A \gamma_4$, translate

[Helset, Paraskevas, Trott 1803.08001]

$$H^\dagger \sigma_a H = -\frac{1}{2} \phi_I \Gamma^I_{a,J} \phi^J$$

$$H^\dagger \hat{D}^\mu H = -\phi_I \gamma^I_{4,J} \left(D^\mu \phi\right)^J = \left(D^\mu \phi\right)_I \gamma^I_{4,J} \phi^J$$

$$H^\dagger \hat{D} \hat{D}^\mu_a H = -\phi_I \gamma^I_{a,J} \left(D^\mu \phi\right)^J = \left(D^\mu \phi\right)_I \gamma^I_{a,J} \phi^J$$

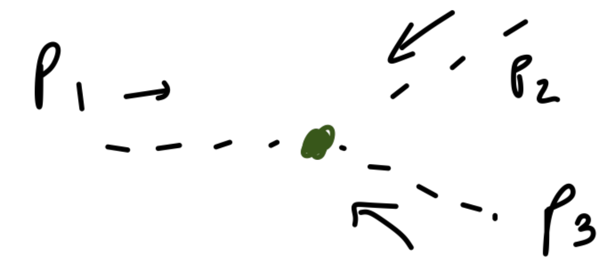
Can rewrite scalar quadratic form as a metric in field space $h_{IJ}(\phi)\left(D_\mu \phi\right)^I \left(D_\mu \phi\right)^J$

$$h_{IJ} = \left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+2} \left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right] \delta_{IJ} + \frac{\Gamma^I_{A,J} \phi_K \Gamma^K_{A,L} \phi^L}{2} \left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right)$$

SM, $g_{AB}, h_{IJ} = \mathbf{1}$. Including higher dimension operators, field space metrics become curved $\longrightarrow$ 'geometric' SMEFT

[Burgess, Lee, Trott '10, Alonso, Jenkins, Manohar '15, '16, Helset, Paraskevas, Trott 1803.08001]

What about 3-pt interactions? Similar story



- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

$D\psi, D\bar{\psi}, DX \rightarrow$ **2 fields** or **1 field + 1 momentum**

$DH \rightarrow$ **1 or 2 fields** or **1 field + 1 momentum**

- three-particle kinematics: $p_1 + p_2 + p_3 = 0 \longrightarrow$ all operators with derivative = momentum reduce via EOM/IBP

Limited options:

(thanks to X. Lu for this perspective)

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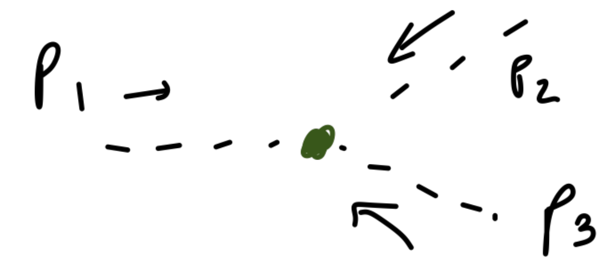

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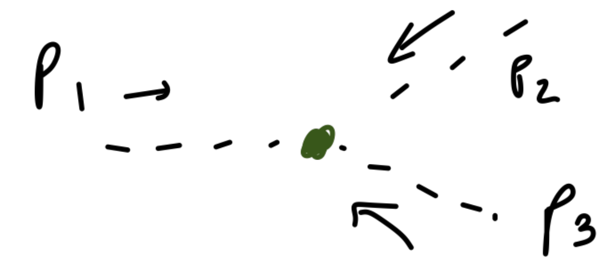

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Limited options:

- $DF_1\, DF_2\, DF_3\, DF_4$ ✗
- $(DX)^2\, H^2$ ✗
- $H^2\, X^3$ ✓
- $(D\bar{\psi})\, \psi\, (DH)\, H$ ✗
- $\bar{\psi}\psi\, (DH)\, H^3$ ✓

...

(thanks to X. Lu for this perspective)

Allowed 3-pt structures:

$$h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$

[+ versions with G^A]

$$Y(\phi)\bar{\psi}_1\psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar{\psi}_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$$

Higgs-dependent 'connections'

As before, # operators small and remains ~fixed for increasing mass dimension

Field space connection	Mass Dimension 6	8	10	12	14
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

Example: $L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I$

contributing operators

$$\mathcal{Q}^{1,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n H^\dagger\, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\psi_r,$$
$$\mathcal{Q}^{3,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n\, H^\dagger\, i\overleftrightarrow{D}^\mu_a H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$

higher dim. versions of class 7 operators

$$\mathcal{Q}^{2,(8+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n(H^\dagger\sigma_a H)\, H^\dagger\, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$
$$\mathcal{Q}^{\epsilon,(8+2n)}_{\substack{H\psi\\pr}} = \epsilon^a_{bc}\,(H^\dagger H)^n\,(H^\dagger\sigma_c H)\, H^\dagger\, i\overleftrightarrow{D}^\mu_b H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r.$$

new effects from $d \geq 8$

compact form for connection:

$$L^{\psi,pr}_{J,A} = -(\phi\,\gamma_4)_J\delta_{A4}\sum_{n=0}^\infty C^{1,(6+2n)}_{\substack{H\psi\\pr}}\left(\frac{\phi^2}{2}\right)^n - (\phi\,\gamma_A)_J(1-\delta_{A4})\sum_{n=0}^\infty C^{3,(6+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$
$$+\frac{1}{2}(\phi\,\gamma_4)_J(1-\delta_{A4})\left(\phi_K\Gamma^K_{A,L}\phi^L\right)\sum_{n=0}^\infty C^{2,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$
$$+\frac{\epsilon^A_{BC}}{2}(\phi\,\gamma_B)_J\left(\phi_K\Gamma^K_{C,L}\phi^L\right)\sum_{n=0}^\infty C^{\epsilon,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n$$

4-pt interactions: can we go 'full metric'?

Key part of 2- and 3-pt result is that special kinematics forbade

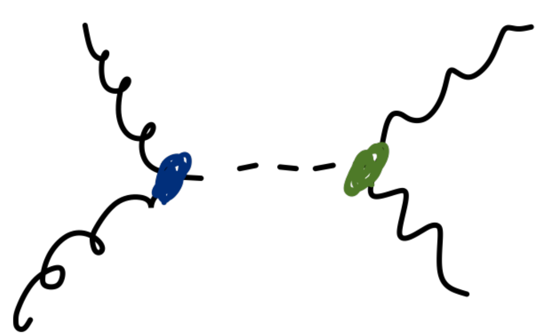
$$DF \sim \textbf{momentum}$$

No longer true at ≥ 4-pt interactions, operators can depend on

$$\mathcal{O} \sim s^n\, t^m$$

$\longrightarrow$ infinite set of higher derivative operators can contribute

- emphasizes the importance of on-resonance measurements for SMEFT



- still may be some surprising structure for $n \geq 4$ — worth thinking about

What can we do with this?

EW inputs are entirely in terms of g_{AB}, h_{IJ} metrics

$$D_\mu \psi = \left[\partial_\mu + i\bar{g}_3 \, \mathcal{G}^\mu_{\mathcal{A}} T^{\mathcal{A}} + i\frac{\bar{g}_2}{\sqrt{2}}\left(\mathcal{W}^+ T^+ + \mathcal{W}^- T^-\right) + i\bar{g}_Z \left(T_3 - s^2_{\theta_Z} Q_\psi\right) \mathcal{Z}^\mu + i\, Q_\psi \, \bar{e}\, \mathcal{A}^\mu\right] \psi.$$

$$\left.\begin{aligned}
\bar{g}_2 &= g_2 \sqrt{g}^{11} = g_2 \sqrt{g}^{22}, \\
\bar{g}_Z &= \frac{g_2}{c^2_{\theta_Z}}\left(c_{\bar{\theta}}\sqrt{g}^{33} - s_{\bar{\theta}}\sqrt{g}^{34}\right) = \frac{g_1}{s^2_{\theta_Z}}\left(s_{\bar{\theta}}\sqrt{g}^{44} - c_{\bar{\theta}}\sqrt{g}^{34}\right), \\
\bar{e} &= g_2 \left(s_{\bar{\theta}}\sqrt{g}^{33} + c_{\bar{\theta}}\sqrt{g}^{34}\right) = g_1 \left(c_{\bar{\theta}}\sqrt{g}^{44} + s_{\bar{\theta}}\sqrt{g}^{34}\right),
\end{aligned}\right\} \text{ couplings}$$

mixing angles

$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})}{g_2(\sqrt{g}^{33} c_{\bar{\theta}} - \sqrt{g}^{34} s_{\bar{\theta}}) + g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})},$$

$$s^2_{\bar{\theta}} = \frac{(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1 g_2 \sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

$$\left.\bar{m}^2_W = \frac{\bar{g}_2^2}{4}\sqrt{h_{11}}^2 \bar{v}^2_T, \qquad \bar{m}^2_Z = \frac{\bar{g}_Z^2}{4}\sqrt{h_{33}}^2 \bar{v}^2_T \qquad \bar{m}^2_A = 0. \right\} \text{ masses}$$

[Helset, Paraskevas, Trott 1803.08001, Misiak et al 1812.11513]

Can get 'all orders' expressions for $1 \rightarrow 2$ processes:

e.g) $h \rightarrow \gamma\gamma$

go to mass basis

$$+\langle h A^{\mu\nu} A_{\mu\nu}\rangle \mathcal{A}^{h\gamma\gamma}_{\rm SM} - \langle h A^{\mu\nu} A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\overline{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\overline{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\overline{e}^2}{g_1^2}\right]$$

H normalization

expand $g_{33}(\phi)\mathscr{W}^3_{\mu\nu}\mathscr{W}^{3\mu\nu}$ to get linear h piece

application: expanding to dimension-8, can check how well $1/\Lambda^4$ corrections are captured by (dim-6)^2

defining: $$\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} = \left[\frac{g_2^2\tilde{C}^{(6)}_{HB} + g_1^2\tilde{C}^{(6)}_{HW} - g_1 g_2\tilde{C}^{(6)}_{HWB}}{\left(g_1^2+g_2^2\right)\bar{v}_T}\right]$$

(dim-6)^2 estimate: $$\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \langle h|\gamma\gamma\rangle^2_{\mathscr{L}^{(6)}}$$

Can get 'all orders' expressions for $1 \to 2$ processes:

e.g) $h \to \gamma\gamma$

Using geoSMEFT setup, can easily find full $\mathcal{O}(1/\Lambda^4)$ result:

$$\left|\mathcal{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\left(1 + \left\langle\sqrt{h}^{44}\right\rangle_{\mathcal{L}^{(6)}}\right)\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}} + \left(1 + 4\bar{v}_T\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\right)\left(\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}}\right)^2$$

$$+2\,\mathrm{Re}\left(\mathcal{A}^{h\gamma\gamma}_{SM}\right)\left[\frac{g_2^2\tilde{C}^{(8)}_{HB} + g_1^2\left(\tilde{C}^{(8)}_{HW} - \tilde{C}^{(8)}_{HW,2}\right) - g_1 g_2 \tilde{C}^{(8)}_{HWB}}{\left(g_1^2 + g_2^2\right)\bar{v}_T}\right]$$

at, $1/\Lambda^4$ only involves $\mathcal{O}(10)$ operators

Significant differences between full and (dim6)^2 result!

Numerics + similar exercise for $h \to Z\gamma$ and $Z \to \bar{f}f$ to appear soon

[Hays, Helset, Martin, Trott]

[for other applications, see talk by Andreas]

Conclusions

Number of SMEFT operators that matter for 2-pt (masses, inputs) and 3-pt interactions is small and ~ constant with increasing operator dimension. Allowed structures phrased as "Geometric SMEFT"

Consequence of low # particles and special kinematics. Does not extend to ≥ 4 particle interactions

Applications:

- Fixed operator dimension (e.g. dim-8): a framework to study $1/\Lambda^4$ corrections
- Beyond fixed order: compact, all order expressions

Many interesting directions to explore!

THANK YOU

Backup

Example operator counting:

$(H^\dagger H)^n W_L^2$ ignore Lorentz, focus on $SU(2)_W$ reps.

$H = (1/2) \therefore H^n = (n/2)$ $\quad W_L^2 = (0 \oplus 2)$ enforced by Bose symm.

$H^\dagger = (1/2) \therefore (H^\dagger)^n = (n/2)$

$$(H^\dagger H)^n = (0 \oplus 1 \oplus 2 \oplus \ldots n) \quad \otimes \quad W_L^2 = (0 \oplus 2) \quad = \text{2 invariants}$$

[+1 for B_L^2 and +1 for $W_L B_L$ = 4]

To get $SU(2)_W$ 2, need $\geq$ 4 Higgses $\rightarrow$ operator dimension ≥ 8

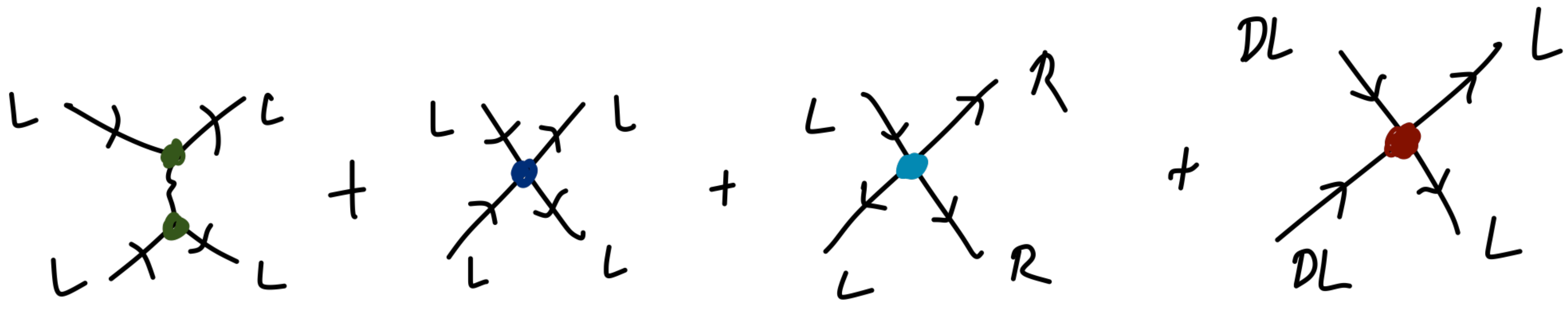
Operators in h_{IJ}:

$$Q_{HD}^{(8+2n)} = \left(H^\dagger H\right)^{n+2} \left(D_\mu H\right)^\dagger (D^\mu H)$$

$$Q_{H,D2}^{(8+2n)} = \left(H^\dagger H\right)^{n+1} \left(H^\dagger \sigma_a H\right) \left(D_\mu H\right)^\dagger \sigma^a (D^\mu H)$$

What about G_F?

G_F involves more than quadratic terms:



However, since G_F derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from L^4, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2} \left(H^\dagger H\right)^{1+n} \left(\bar{\ell}_2 \gamma^\mu \sigma^i \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_i \ell_1\right) \qquad i C^{(8+2n)}_{4\ell,5} \epsilon_{ijk} \left(H^\dagger H\right)^n \left(H^\dagger \sigma^i H\right) \left(\bar{\ell}_2 \gamma^\mu \sigma_j \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_k \ell_1\right)$$

All orders result is possible even for contact terms:

$$\mathscr{G}^{4pt}_F = \frac{1}{\bar{v}_T^2} \left(\tilde{C}^{(6)}_{\mu c c \mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n} \right)$$

[Hays, Helset, Martin, Trott to appear]

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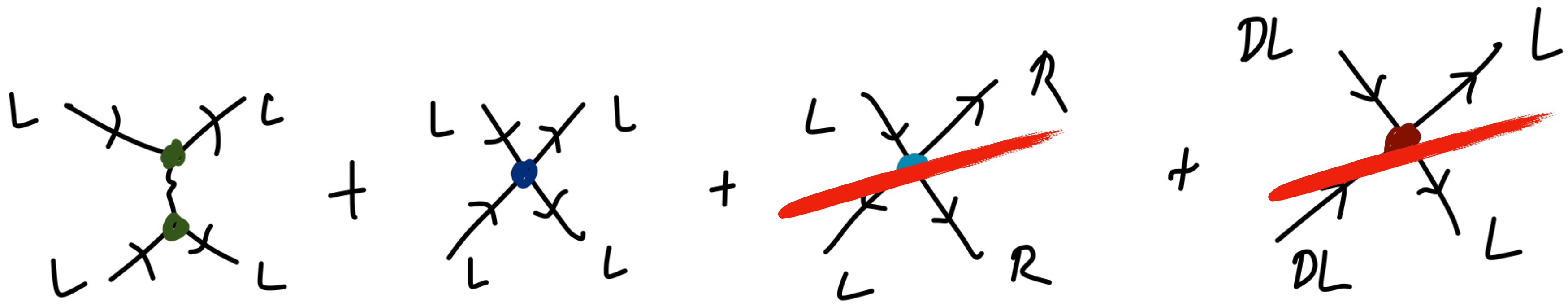

However, since G_F derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from L^4, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2} \left(H^\dagger H\right)^{1+n} \left(\bar{\ell}_2 \gamma^\mu \sigma^i \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_i \ell_1\right) \qquad iC^{(8+2n)}_{4\ell,5} \epsilon_{ijk} \left(H^\dagger H\right)^n \left(H^\dagger \sigma^i H\right) \left(\bar{\ell}_2 \gamma^\mu \sigma_j \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_k \ell_1\right)$$

All orders result is possible even for contact terms:

$$\mathcal{G}^{4pt}_F = \frac{1}{\bar{v}_T^2} \left(\tilde{C}^{(6)}_{\mu c c \mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n} \right)$$

[Hays, Helset, Martin, Trott to appear]