Low-energy EFT of spontaneously broken BSM theories:

SMEFT or HEFT?

related to work in progress
with
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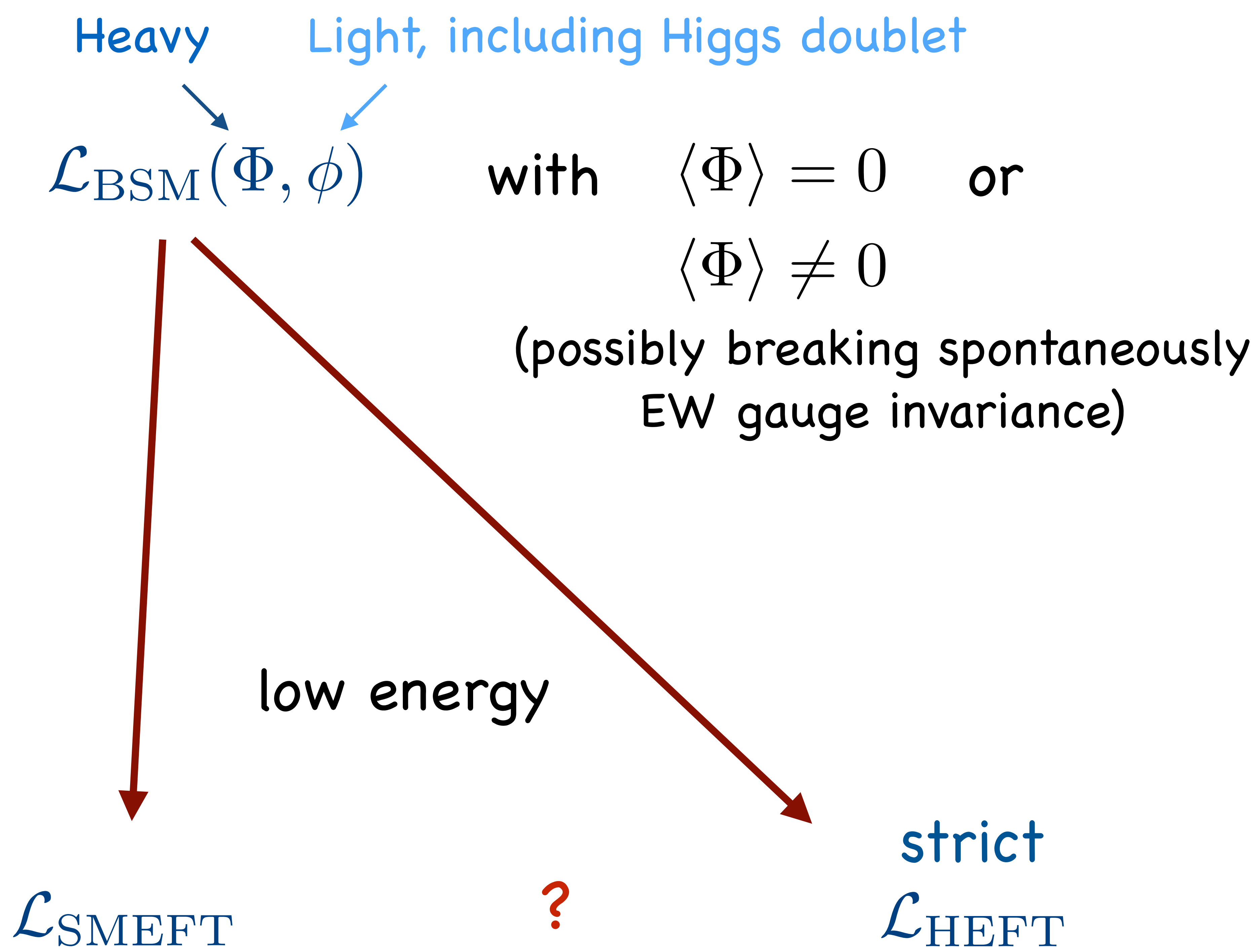
Heavy
Light, including Higgs doublet
$\mathcal{L}_{\rm BSM}(\Phi, \phi)$
with $\langle \Phi \rangle = 0$ or
$\langle \Phi \rangle \neq 0$
(possibly breaking spontaneously
EW gauge invariance)
low energy
$\mathcal{L}_{\rm SMEFT}$
?
strict
$\mathcal{L}_{\rm HEFT}$

Outline

I. Mixing ($\langle \Phi \rangle = 0$)

II. Heavy scalars with vev: $\langle \Phi \rangle \neq 0$

I. Mixing

Is mixing an issue?

Passarino, 2019

. SMEFT assumes a Higgs doublet, so any mixing among scalars (in general among heavy and light degrees of freedom) in the high-energy theory brings us to the HEFT/SMEFT dichotomy. Although there is a wide class of BSM models that support the linear SMEFT description, this realization does not always provide the appropriate framework.

The important point to stress is that the EFT extension of X, as defined in Eq.(1), requires absence of mixing between heavy and light degrees of freedom.

SMEFT at O $1/\Lambda^2$ reproduces the effect of scalar mixing on interactions involving one light Higgs scalar, but fails otherwise

To summarize, if X is the SM and we have in mind an X' without mixing, then the EFT extension of the SM is what we call SMEFT.

Φ Heavy scalar singlet

$$\kappa\,\Phi\phi^\dagger\phi \rightarrow \sqrt{2}\kappa v\,\Phi h$$

Mass eigenstates H, h′

Integrate H out ⟶ HEFT (h′ is not part of a doublet)

Integrate Φ out ⟶ SMEFT

Many other examples in SM extensions with new fermions or bosons

$$\lambda\bar{\Psi}_R\phi q_L \rightarrow \frac{1}{\sqrt{2}}\lambda v\bar{\Psi}_R q_L$$

Φ Heavy scalar singlet

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Mass eigenstates H, h′

Integrate H out ⟶ HEFT (h′ is not part of a doublet)

Integrate Φ out ⟶ SMEFT

Equivalent?

Many other examples in SM extensions with new fermions or bosons

$$\lambda \bar{\Psi}_R \phi q_L \to \frac{1}{\sqrt{2}} \lambda v \bar{\Psi}_R q_L$$

$$Z[J] = \int \mathcal{D}\phi \mathcal{D}\Phi \exp\{iS_{\rm UV}[\Phi, \phi] + J_\alpha \phi^\alpha\}$$
$$= \int \exp\{S_{\rm eff}[\phi] + J_\alpha \phi^\alpha\}$$

$$(\Phi, \phi) \rightarrow (\Phi', \phi') = F(\Phi, \phi)$$

ϕ' new low-energy interpolating field

$$Z'[J] = \int \mathcal{D}\phi' \mathcal{D}\Phi' \left(\frac{\delta F}{\delta(\phi', \Phi')} \right) \exp\{iS_{\rm UV}[F(\Phi', \phi')] + J_\alpha {\phi'}^\alpha\}$$
$$= \int \mathcal{D}\phi' \exp\{S'_{\rm eff}[\phi] + J_\alpha {\phi'}^\alpha\}$$

$Z' \sim Z$ ⟶ $S_{\rm eff}$ and $S'_{\rm eff}$ equivalent

In fact, G exists such that $S'_{\rm eff}[\phi] = S_{\rm eff}[G[\phi]]$ (J.M. Lizana)

Φ Heavy scalar singlet

$$\kappa\,\Phi\phi^\dagger\phi \rightarrow \sqrt{2}\kappa v\,\Phi h$$

Mass eigenstates H, h′

Integrate H out ⟶ HEFT (h′ is not part of a doublet)

Integrate Φ out ⟶ SMEFT

Equivalent

(Note that the power counting is different)

Brehmer, Freitas, López-Val, Plehn, 2016

Consider this action...

$$-\int d^dx\,\left[\frac{1}{2}\phi\,\Box\left(1+\frac{\Box}{m^2}\right)^2\phi+\frac{g}{m^2}\phi^3\Box\left(1+\frac{\Box}{m^2}\right)\phi+\frac{g^2}{2m^4}\phi^3\Box\phi^3\right]$$
$$-\int d^dx\,\left[\bar{c}(\Box+m^2)c+3g\phi^2\bar{c}c\right].$$

Consider this action...

$$-\int d^d x \left[\frac{1}{2}\phi\,\Box\left(1+\frac{\Box}{m^2}\right)^2\phi + \frac{g}{m^2}\phi^3\Box\left(1+\frac{\Box}{m^2}\right)\phi + \frac{g^2}{2m^4}\phi^3\Box\phi^3\right]$$
$$-\int d^d x \,\left[\bar{c}(\Box+m^2)c + 3g\phi^2\bar{c}c\right].$$

Actually, m is redundant: $\dfrac{\partial S_\phi}{\partial(1/m^2)} = \dfrac{\delta S_\phi}{\delta\phi^x}$ **(...)**

So, same on-shell amplitudes as

$$\int d^d x\, \tfrac{1}{2}\partial\phi\partial\phi$$

II. Heavy scalars with vev

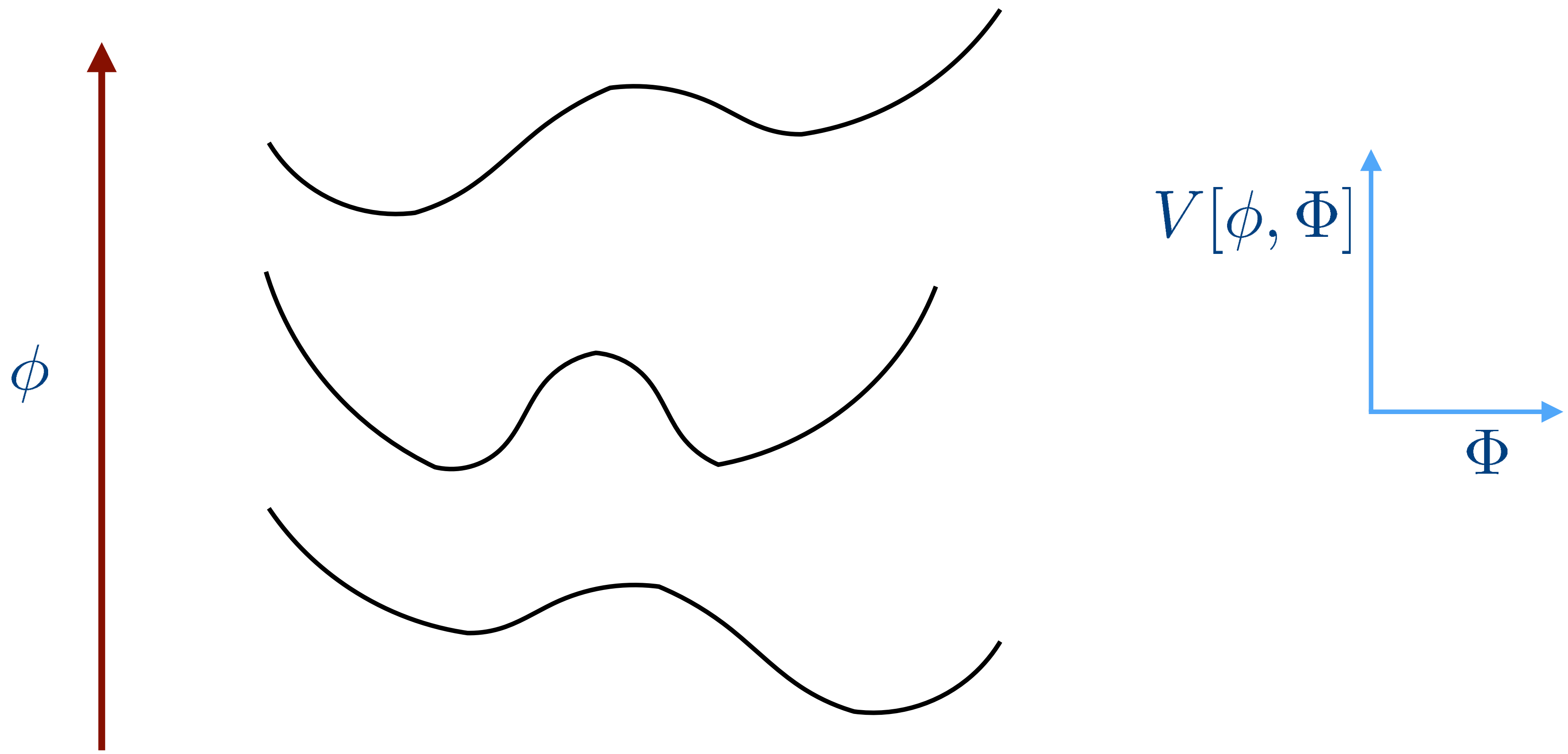
Coleman, Wess, Zumino, 1969
Alonso, Jenkins, Manohar, 2016
Falkowski, Rattazzi, 2019

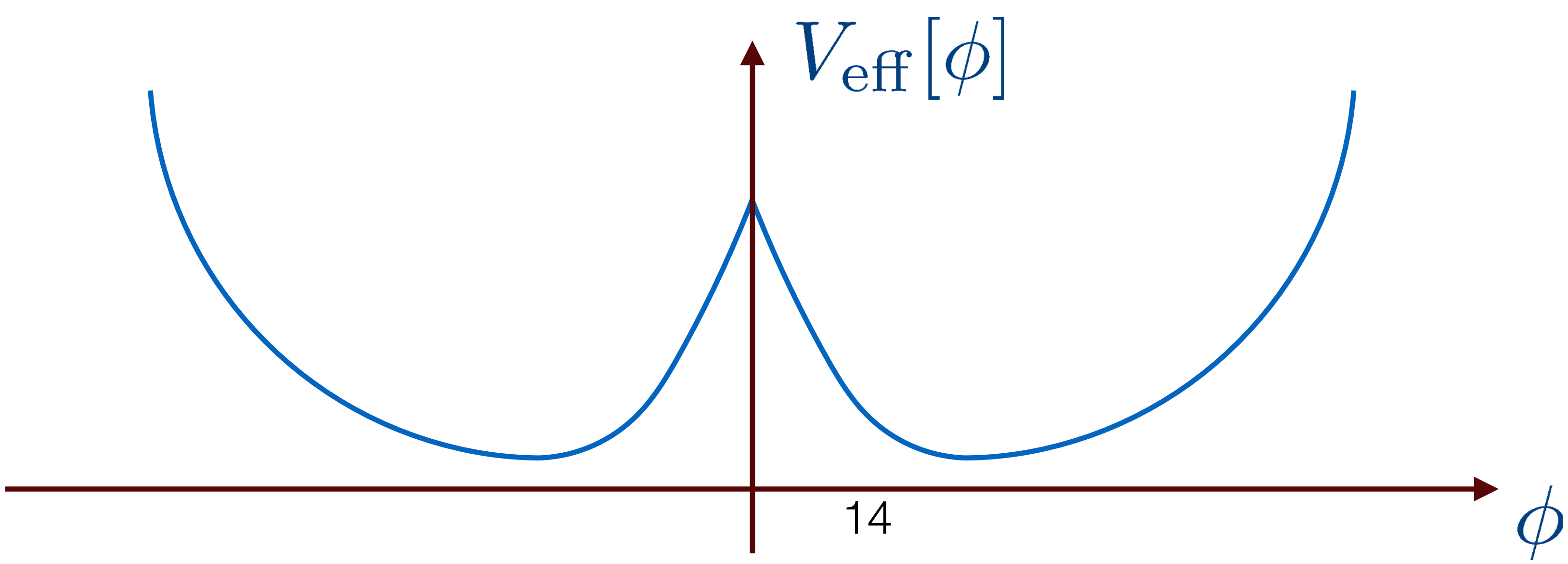
HEFT is SMEFT when there exists an SU(2)xU(1) invariant fixed point in the scalar manifold.

Then, there is a field redefinition giving rise to a Lagrangian **analytic** at $\phi = 0$ in a Higgs field ϕ transforming linearly as a doublet

A quantum phase transition?

$$S_{\text{eff}}[\phi] = \Gamma_\phi[\bar{\Phi}(\phi)] \quad \textbf{with} \quad 0 = \left. \frac{d\Gamma_\phi[\Phi_c]}{d\Phi_c} \right|_{\Phi_c = \bar{\Phi}(\phi)}$$





Singular
at
$\phi = 0$

Convex potential for all ϕ → SMEFT

it becomes non-convex for large ϕ → SMEFT

it becomes non-convex for $\phi < v$ → SMEFT but expansion in field not convergent for relevant values of ϕ !

Non-convex potential when $\phi = 0$ → HEFT

This occurs for instance in GUT $\rightarrow$SMEFT or in SMEFT $\rightarrow$ LEFT
and in limiting flat case: usual examples with fields taking mass from EWSB

Typically, we have a useful SMEFT if heavy scalar has large gauge-invariant mass $M^2 \gg v^2$

Example: SM extension with scalar triplet

$$\mathcal{L}_{\mathrm{UV}} = |D_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4 + \frac{1}{2}(D_\mu \Sigma)^2 - \frac{M^2}{2}\Sigma^2 - \kappa \phi^\dagger \Sigma \phi$$

For any sign of μ^2, convex in Σ for any ϕ

However,

hep-ph/0504286, hep-ph/0604102, hep-ph/0605302,

arXiv:0707.2323, arXiv:0712.0546, arXiv:0809.4185

Dawson et al, Chankowski et al, Chivukula et al find non-decoupling behaviour at one loop: $\rho \neq 1$ when $\kappa \sim M \to \infty$

Chankowski, Pokorski, Wagner, 2007

There are custodial-breaking contributions to Green functions that cannot be reproduced by SMEFT in the limit (the SM). They can be reproduced by HEFT at leading order

At odds with SMEFT expansion as checked by explicit matching in symmetric phase by Skiba in arXiv:1006.2142

(checked by Aguila, Kunszt, Santiago in 2016)

But SMEFT should be the correct low-energy theory even if the super-renormalizable coupling gets large

Let´s have a look:

$$\mathcal{L}_{\rm EFT} = |D_\mu\phi|^2 + c_\mu|\phi|^2 + c_\lambda|\phi|^4 + c_\phi|\phi|^6 + c_{\phi D}|\phi^\dagger D_\mu\phi|^2 + c_{\phi\Box}|\phi|^2\Box|\phi|^2.$$

$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} = \frac{\frac{v_{\rm EFT}^2 g^2}{4}}{\frac{v_{\rm EFT}^2 g^2}{4c_W^2}\left(1+\frac{c_{\phi D}v_{\rm EFT}^2}{2}\right)c_W^2} = \left(1-\frac{c_{\phi D}c_\mu}{4c_\lambda}\right)^{-1}$$

$$v_{\rm EFT}^2 = -c_\mu/(2c_\lambda)$$

$$c_\mu^{\rm tree} = \mu^2, \qquad c_\lambda^{\rm tree} = -\lambda + \frac{\kappa^2}{2M^2}\left(1-\frac{4\mu^2}{M^2}\right), \qquad c_\mu^{\rm loop} = \frac{\kappa^2}{16\pi^2}(1-\log\frac{M^2}{Q^2})$$

$$c_{\phi D}^{\rm tree} = -\frac{2\kappa^2}{M^4}, \qquad c_\phi^{\rm tree} = \frac{4\lambda\kappa^2}{M^4}, \qquad c_{\phi\Box}^{\rm tree} = \frac{\kappa^2}{2M^4}. \qquad c_{\phi D}^{\rm loop} = -\frac{3}{2}\frac{\kappa^2\lambda}{(4\pi)^2M^4} - \frac{\kappa^4}{(4\pi)^2M^6} + \frac{6\kappa^4}{(4\pi)^2M^6}$$

$$\rho \to 1 - \frac{\mu^2\kappa^2}{\lambda M^4} - \frac{\kappa^4}{16\pi^2\lambda M^4}(1-\log\frac{M^2}{Q^2}) + \ldots$$

SMEFT reproduces precisely the calculation in broken phase!

In fact, this result respects the decoupling theorem:

We must use low-energy observables
to determine the value of renormalized couplings

$v_{\rm EFT}$ determined by measured G_F

So, μ must be fine-tuned to keep M_W at EW scale:

$$\mu = \tilde{\mu} - c_\mu^{\rm loop}$$

$$\rho \to 1 - \frac{\tilde{\mu}^2 \kappa^2}{\lambda M^4} + \ldots$$

The same decoupling actually holds in broken phase calculation in an appropriate renormalization scheme

The scheme in Chankowski et al is actually meant to achieve this

Input parameters: $\alpha,\ G_F,\ M_Z,\ v_\Sigma$

But it fails because v_Σ must be correlated with v to minimize potential, so it is also fine tuned. (This is automatically incorporated in effective method)

If taken into account, ρ parameter goes to 0 when $\kappa \sim M \to \infty$

Conclusion

BSM theories with linearly-realized SM gauge symmetry and large positive square masses have SMEFT as low-energy theory. Operator coefficients can be computed in symmetric phase, in the standard way