

The Higgs EFT is a black hole
based on an upcoming paper with T. Cohen, N. Craig and X.
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Study the (simplified) EFT of the SM scalar sector

On first principles, what is the difference between scalar sectors of:

- ▶ **SMEFT**: built out of fields $\vec{\phi}$ that linearly realise electroweak symmetry, and
- ▶ **HEFT**: built out of fields $h, \vec{\pi}$ that don't?

With the simplifications of

- ▶ *ignoring fermions & vectors;*
- ▶ *(global) custodial symmetry;*
- ▶ *no lagrangian terms with more than two derivatives.*

We draw **heavily** on recent work in the literature

- ▶ (Alonso, Jenkins, and Manohar 2016)
- ▶ (Chang and Luty 2019)
- ▶ (Falkowski and Rattazzi 2019)

to understand the importance of analyticity and convergence in a geometric setting.

SMEFT (Standard Model Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016) for details

Comprising four equivalent real scalars

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \quad \vec{\phi} \rightarrow O\vec{\phi}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$. Electroweak symmetry is *linearly realised* on the $\vec{\phi}$.

Then the terms in the Lagrangian are

$$\mathcal{L}_{\text{SM}} = \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\mathcal{L}_{\text{SMEFT}} = \frac{1}{2}A(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}B(\vec{\phi} \cdot \vec{\phi})(\vec{\phi} \cdot \partial\vec{\phi})^2 - V(\vec{\phi} \cdot \vec{\phi}) + \mathcal{O}(\partial^4),$$

[Canonically $A(0) = 1$]

HEFT (Higgs Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016) for details

Built out of a singlet h and a unit vector \vec{n} comprising three Goldstone fields π^i

$$h, \quad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v_0 \\ n_2 = \pi_2/v_0 \\ n_3 = \pi_3/v_0 \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix},$$

upon which the electroweak symmetry is *non-linearly realised*

$$h \rightarrow h \quad , \quad \vec{n} \rightarrow O\vec{n}, \quad O \in O(4).$$

The lagrangian is

$$\mathcal{L}_{\text{SM}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} (v_0 + h)^2 (\partial \vec{n})^2 - \frac{1}{4} \lambda (h^2 + 2v_0 h)^2$$
$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} [v_0 F(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4).$$

[Canonically $F(0) = 1$, $V'(0) = 0$]

Do SMEFT and HEFT describe the same physics?

In other words, can we write SMEFT as HEFT, and HEFT as SMEFT?

Ultimately SMEFT \leftrightarrow HEFT is just a field redefinition

$$\vec{\phi} = f(h) \vec{n}(\pi);$$

(E.g.

$$\vec{\phi} = (v_0 + h) \vec{n}(\pi);$$

for SM \leftrightarrow H(EFT).)

Since when did physics care about a field redefinition?

Geometric picture of scalar effective field theory

(Coleman, Wess, and Zumino 1969)

Field redefinitions that leave the S -matrix unchanged are real analytic and invertible¹

$$\phi = \phi[\eta] = \sum \frac{\delta^n \phi}{\delta \eta^n} \eta^n \sim \eta + \eta^2 + \eta^3 + \dots$$

so each field choice is a choice of coordinates on a real analytic manifold.

On this real analytic manifold, lagrangian defines a metric and a potential function

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial \phi^i \partial \phi^j - V(\phi) = \frac{1}{2} \left(g_{ij}(\eta) \frac{\delta \phi^i}{\delta \eta^m} \frac{\delta \phi^j}{\delta \eta^n} \right) \partial \eta^m \partial \eta^n - V(\eta)$$

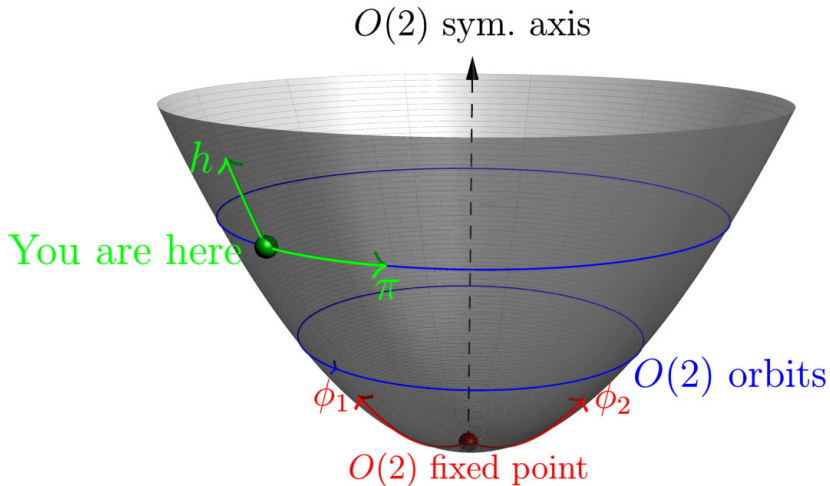
We want the lagrangian to be analytic, so we can expand in a series of local EFT operators

$$\mathcal{L} = \frac{1}{2} \left(\sum \frac{1}{n!} g_{ij, k_1 \dots k_n}(0) \phi^{k_1} \dots \phi^{k_n} \right) \partial \phi^i \partial \phi^j - \left(\sum \frac{1}{n!} V_{, k_1 \dots k_n}(0) \phi^{k_1} \dots \phi^{k_n} \right)$$

¹NOTE: No derivatives as working to fixed derivative order in \mathcal{L}

Geometric picture of SMEFT & HEFT

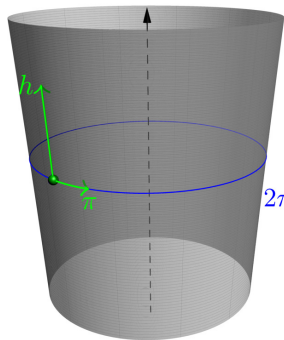
Cartesian vs. polar coordinates, see (Alonso, Jenkins, and Manohar 2016)



When is a HEFT not a SMEFT?

1) When it's a funnel (Alonso, Jenkins, and Manohar 2016)

$O(2)$ sym. axis



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[v_0 F(h)]^2 (\partial \vec{n})^2 - V(h)$$

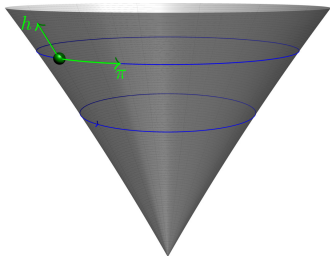
Require geodesic distance of closed $O(2)$ orbits to be non-zero everywhere

$$F(h) \neq 0$$

then there's no fixed point about which to expand in SMEFT coordinates.

When is a HEFT not a SMEFT?

2) When it's a cone



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[v_0 F(h)]^2(\partial \vec{n})^2 - V(h)$$

(To avoid a singularity, it doesn't suffice for $F(h)$, $V(h)$ to be analytic in h , because the chart is degenerate at the singularity. Cf. r in polar coordinates. Diagnose singularity through curvature invariants.)

In coordinate invariant language, at the putative conical singularity, $R \rightarrow \infty$ and/or $\nabla^2 V \rightarrow \infty$.^a

^aFor any non-analyticity, $\exists n$, $(\nabla^2)^n R \rightarrow \infty$ and/or $(\nabla^2)^{n+1} V \rightarrow \infty$

In HEFT coordinates, if $F(-v) = 0$ for some v , then $F''(-v) \neq 0$ and/or $F'(-v) \neq \frac{1}{v_0}$ and/or $V'(-v) \neq 0$. E.g.

$$\left[\nabla^2 V = \frac{\partial_h(F^{N-1} \partial_h V)}{F^{N-1}} = V'' + \frac{NF'}{F} V' \rightarrow \infty \Leftrightarrow V'(-v) \neq 0 \right]$$

Non-analyticities from integrating out massless particles

Look for discontinuities in the partition function

$$\exp(iS_{\text{eff}}[\phi_{\text{SM}}]) = \int \mathcal{D}\phi_{\text{BSM}} \exp(iS[\phi_{\text{SM}}, \phi_{\text{BSM}}])$$

when varying background of SM fields ϕ_{SM} .

Example: 'heavy' vector A , **funnel**

$$\mathcal{L}_{\text{UV}} = -A_{\mu}^i \partial_{\mu} \phi^i - \frac{1}{2} (M^2 + \lambda \vec{\phi} \cdot \vec{\phi}) A_{\mu}^i A^{i,\mu}$$
$$\Delta \mathcal{L}_{\text{eff}} = \frac{1}{2} \frac{1}{M^2 + \lambda \vec{\phi} \cdot \vec{\phi}} \partial_{\mu} \vec{\phi} \cdot \partial^{\mu} \vec{\phi}$$

Example: 'heavy' scalar Φ , **cone**

$$\mathcal{L}_{\text{UV}} = -\frac{1}{2} \Phi (\partial^2 + M^2 + \lambda \vec{\phi} \cdot \vec{\phi}) \Phi$$
$$16\pi^2 \Delta \mathcal{L}_{\text{eff}} = -\frac{1}{4} (M^2 + \lambda \vec{\phi} \cdot \vec{\phi})^2 \ln \left(\frac{M^2 + \lambda \vec{\phi} \cdot \vec{\phi}}{\mu^2} - \frac{3}{2} \right) + \frac{1}{24} \frac{[\partial_{\mu} (M^2 + \lambda \vec{\phi} \cdot \vec{\phi})]^2}{M^2 + \lambda \vec{\phi} \cdot \vec{\phi}}$$

Amplitudes are covariant quantities

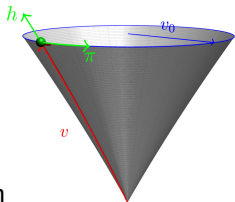
$$\mathcal{L} = \frac{1}{2} \left(\sum \frac{1}{n!} g_{ij,k_1 \dots k_n} \phi^{k_1} \dots \phi^{k_n} \right) \partial \phi^i \partial \phi^j - \left(\sum \frac{1}{n!} V_{,k_1 \dots k_n} \phi^{k_1} \dots \phi^{k_n} \right)$$

The partial derivatives $g_{ij,(k_1 \dots k_n)}$ and $V_{,k_1 \dots k_n}$ must combine in amplitudes form coordinate invariant objects: **covariant derivatives** of the Riemann curvature tensor and potential

$$M(\pi_i \pi_j \rightarrow h^n) = \begin{array}{c} \pi_i \\ \pi_j \end{array} \begin{array}{c} h \\ h \\ \dots \\ h \end{array} = sR_{\pi_i h h \pi_j; h \dots h} + V_{;(\pi_i \pi_j h \dots h)}$$

Singularity causes factorial growth of amplitudes

If R diverges at fixed point, so does Taylor expansion about vacuum



$$\sum_n R_{\underbrace{hh\dots h}_n} \frac{(-v)^n}{n!} \rightarrow \infty \implies |R_{\underbrace{hh\dots h}_n}| \rightarrow \frac{n!}{v^n}$$

As $R = -2g^{hh}g^{\pi_i\pi_j}R_{\pi_i hh\pi_j} + g^{\pi_i\pi_j}g^{\pi_k\pi_l}R_{\pi_i\pi_k\pi_j\pi_l}$, then

$$|R_{\pi_i hh\pi_j; h\dots h}| \rightarrow \frac{n!}{v^n} g^{\pi_i\pi_j}, \text{ and/or}$$

$$|R_{\pi_i\pi_k\pi_j\pi_l; h\dots h}| \rightarrow \frac{n!}{v^n} (g^{\pi_i\pi_l}g^{\pi_k\pi_j} - g^{\pi_i\pi_j}g^{\pi_k\pi_l})$$

at our vacuum in the large n limit.

(Similarly if $\nabla^2 V = g^{hh}V_{;hh} + g^{\pi_i\pi_j}V_{;\pi_i\pi_j}$ diverges at fixed point

$$|V_{;\pi_i\pi_j h\dots h}| \rightarrow \frac{n!}{v^n} g^{\pi_i\pi_j}$$

in the large n limit.)

... and an exponential growth in the inelastic cross section

(Chang and Luty 2019), (Falkowski and Rattazzi 2019)

$$M(\pi_i \pi_j \rightarrow h^n) = s R_{\pi_i h h \pi_j; h \dots h} + V_{i(\pi_i \pi_j h \dots h)}$$

$$|R_{\pi_i h h \pi_j; h \dots h}|, |V_{i(\pi_i \pi_j h \dots h)}| \rightarrow \frac{n!}{v^n} g_{\pi_i \pi_j}$$

Reprising (very schematically) the argument of (Falkowski and Rattazzi 2019)

$$\begin{aligned} 1 &\gtrsim \sum_X |M(\pi_i \pi_j \rightarrow X)|^2 \gtrsim \sum_n \int \frac{d\Pi_n}{n!} |M(\pi_i \pi_j \rightarrow h^n)| \\ &\sim \sum_n \frac{s^n}{(4\pi)^{2n} (n!)^3} \theta(s - nm_h^2) |s R_{\dots} + V_{\dots}|^2 \\ &\sim \sum_n \frac{1}{n!} \left(\frac{s}{(4\pi v)^2} \right)^n \theta(s - nm_h^2) \sim \exp\left(\frac{s}{(4\pi v)^2} \right) \end{aligned}$$

leading to perturbative unitarity violation at $s \sim (4\pi v)^2$.

(Ultimately unitarised by the particle whose mass comes entirely from EWSB. E.g.

$$m^2 = \lambda |H|^2 \lesssim (4\pi v)^2.)$$

Summary (1)

A geometric picture makes manifest the connection between scalar field theory and its amplitudes, and the physical significance of non-analyticities.

A custodially symmetric HEFT can be thought of as having a singularity at the point on the scalar field manifold where electroweak symmetry is restored, caused by massless BSM fields.

This begets

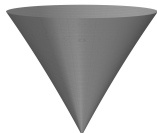
- ▶ $\frac{n!}{v^n}$ growth in amplitudes with n Higgses (v is geodesic distance to singularity from our vacuum);
- ▶ perturbative unitarity violations $s \lesssim (4\pi v)^2$.

Summary (2)



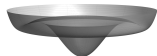
~~X~~SMEFT

Singularity infinite distance away. From integrating out massless particles.



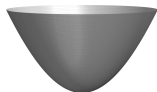
~~X~~SMEFT

Singularity finite distance away, HEFT power counting and exponential growth of amplitudes. From integrating out massless particles.



\sim SMEFT

Valid SMEFT expansion about fixed point, does not converge at our vacuum. Impractical.



✓SMEFT

Valid SMEFT expansion converges at fixed point and our vacuum. All BSM particles get minority of mass from EWSB.

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