EFT of Gravity to All Orders

HIGGS AND EFFECTIVE FIELD THEORY 2020

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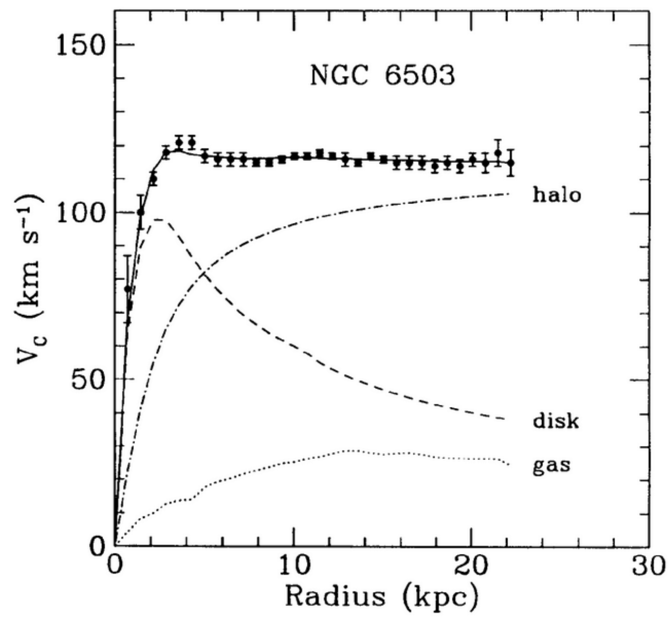
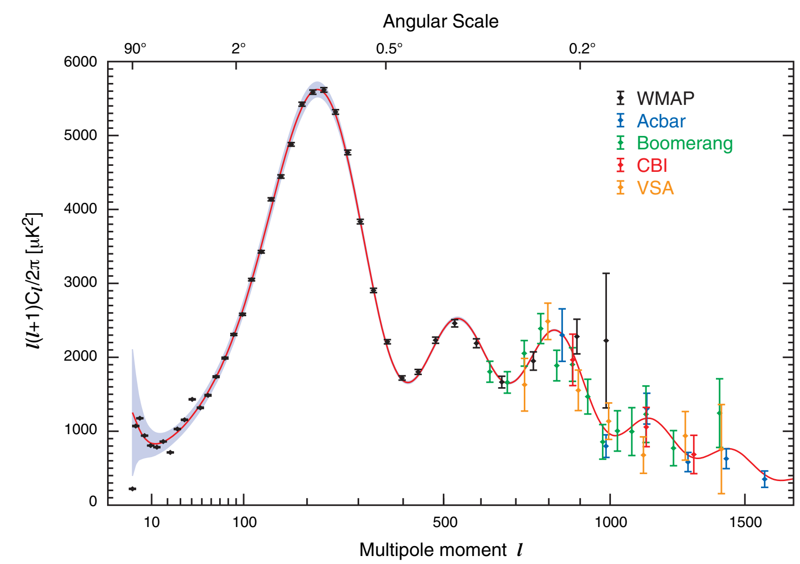
Motivations

Gravity is related to many of the deepest puzzles in fundamental physics today.

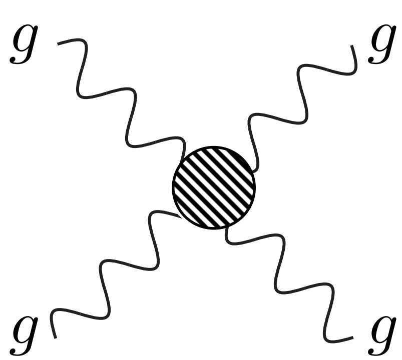
dark matter



inflation



strong quantum gravity

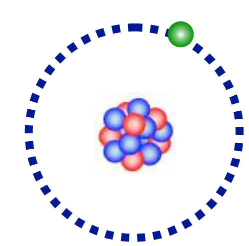


$$\mathcal{M} \sim \left(\frac{E}{M_{\rm Pl}}\right)^2$$

cosmological constant

$$\frac{\Lambda_{\rm CC}}{M_{\rm Pl}^4} \sim 10^{-120}$$

electroweak scale



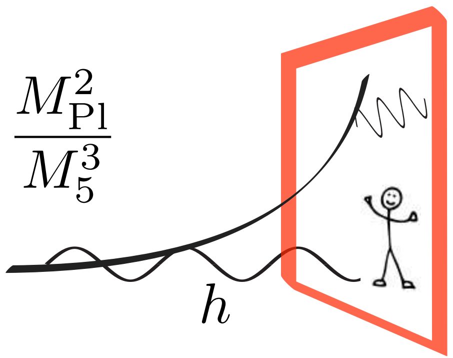
$$\frac{v^2}{M_{\rm Pl}^2} \sim 10^{-30}$$

Tests of Gravity

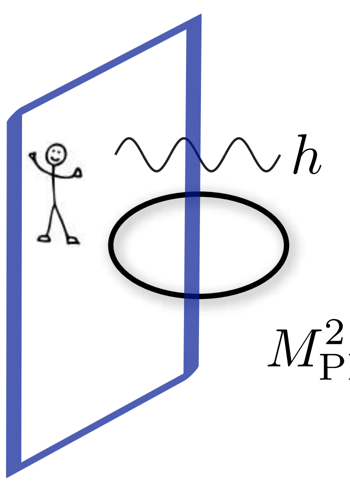
experimental

DGP model

$$\ell_{\rm DGP} = \frac{M_{\rm Pl}^2}{M_5^3}$$



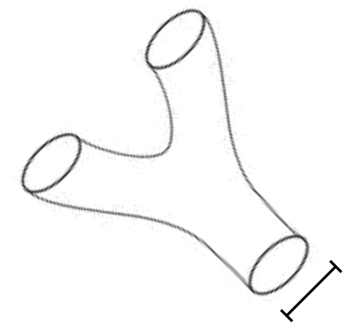
large extra-dimensions (ADD)



$$M_{\rm Pl}^2 = M_*^{2+n} R^n$$

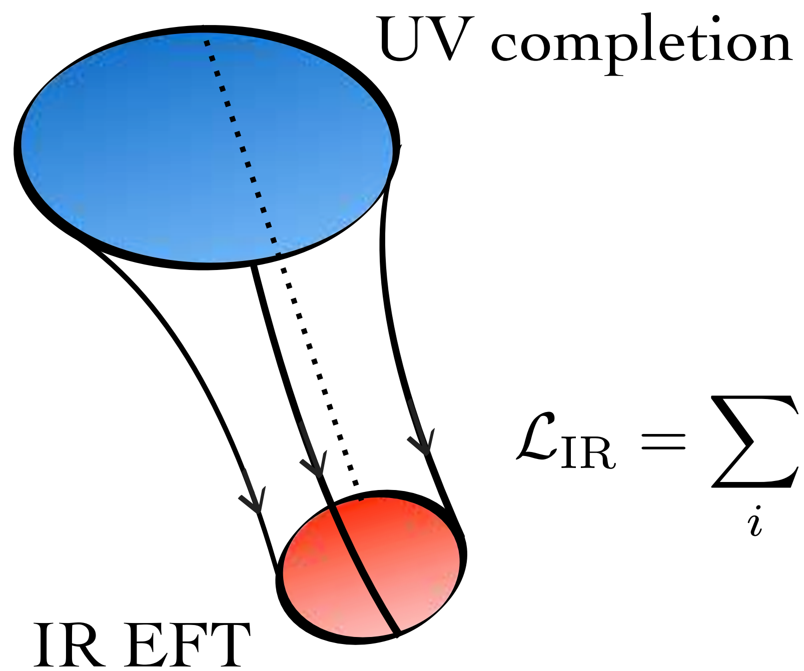
theoretical

string theory



$$\ell_{\rm Pl} \sim \frac{\hbar}{c} \frac{1}{M_{\rm Pl}}$$

Effective Field Theory Approach



$$\mathcal{L}_{\rm IR} = \sum_i \frac{\mathcal{O}_i(x)}{\Lambda^{\Delta_i - 4}}$$

model ↓ *independent*

- Novel ways to test heavy gravitational physics beyond the Standard Models.

  e.g. GRSMEFT, most general set of deformations of leading dynamics.

- Different and robust understanding of the properties of UV completions of gravity.

  e.g. positivity constraints on Wilson coefficients.

Method

Non-redundant Operator Basis

First main goal: identify most general set of IR deformations, w/o redundancies.

- Group relations:

  a.k.a. algebraic identities

$$F_{\mu\nu} = -F_{\nu\mu}$$

- Equations of motion (EOM):

  i.e. field redefinitions

$$\Box\phi = -m^2\phi + \cdots$$

- Integration by parts (IBP):

  i.e. momentum conservation

$$\mathcal{O}_1 = \mathcal{O}_2 + \partial_\mu \mathcal{O}_3^\mu$$

Important recent progress on these issues by Henning, Lu, Melia, Murayama '17.

Hilbert Series

Counting independent operators, as invariants of group

$$G = \text{Lorentz} \times \text{gauge} \times \text{global}$$

Euclidean

$SO(4)$

if desired

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

built out of (extended) representations, single particle module, e.g. $R_\phi = \begin{pmatrix} \phi \\ \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$

(Henning, Lu, Melia, Murayama '17)

$$\mathcal{H}(\mathcal{D}, \{\phi_i\}) = \int d\mu_{\text{Lorentz}}(x) \int d\mu_{\text{gauge}}(y) \frac{1}{P(\mathcal{D}, x)} \prod_i \text{PE}\left[\frac{\phi_i}{\mathcal{D}^{\Delta_i}} \chi_{R_{\phi_i}}\right] + \Delta\mathcal{H}$$

Hilbert Series

Counting independent operators, as invariants of group

$$G = \text{Lorentz} \times \text{gauge} \times \text{global}$$

Euclidean → $SO(4)$

gauge → $SU(3)_C \times SU(2)_L \times U(1)_Y$

global: if desired

built out of (extended) representations, single particle module, e.g. $R_\phi = \begin{pmatrix} \phi \\ \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$

(Henning, Lu, Melia, Murayama '17)

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(spurions)

derivative: $\mathcal{D}$; fields: $\{\phi_i\}$

group integration: $\int d\mu_{\text{Lorentz}}(x) \int d\mu_{\text{gauge}}(y)$

momentum generating function: $\frac{1}{P(\mathcal{D}, x)}$

plethystic exponential: PE

character: $\chi_{R_{\phi_i}}$

Hilbert Series

Counting independent operators, as invariants of group

$$G = \text{Lorentz} \times \text{gauge} \times \text{global}$$

Euclidean

$SO(4)$

if desired

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

built out of (extended) representations, single particle module, e.g. $R_\phi = \begin{pmatrix} \phi \\ \partial_\mu \phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$

$$\mathcal{H}(\mathcal{D}, \{\phi_i\}) = \sum_{r_1,\ldots,r_n,k} c_{r_1,\ldots,r_n,k}\, \phi_1^{r_1} \cdots \phi_n^{r_n} \mathcal{D}^k$$

number of operators of each class

Group Characters

Characterize the representation $\mathbf{r}$ of a group G.

$$\chi_{\mathbf{r}}(g) = \mathrm{Tr}_{\mathbf{r}}(g) \quad g \in G \quad \longrightarrow \quad \chi_{\mathbf{r}}(z)\,, \quad z = \{z_1, \ldots, z_{\mathrm{rank}(G)}\}$$

Plethystic exponential performs symmetric tensor products at the character level.

$$\mathbf{r} \otimes \mathbf{r} \otimes \cdots \otimes \mathbf{r} \quad \longrightarrow \quad \mathrm{PE}[\phi_{\mathbf{r}} \chi_{\mathbf{r}}(z)] = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} \phi_{\mathbf{r}}^n \, \chi_{\mathbf{r}}(z^n)\right]$$

Group integral projects outs invariants.

$$\mathcal{H}(\phi_{\mathbf{r}}, \ldots, \varphi_{\mathbf{r}'}) = \int d\mu_G(z)\, \mathrm{PE}[\phi_{\mathbf{r}} \chi_{\mathbf{r}}(z)] \cdots \mathrm{PE}[\varphi_{\mathbf{r}'} \chi_{\mathbf{r}'}(z)]$$

- $SU(2)$ fundamental scalar:

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_A \oplus \mathbf{3}_S$$

$$\mathrm{PE}[\phi_{\mathbf{2}} \chi_{\mathbf{2}}(y)] = 1 + \chi_{\mathbf{2}}(y)\, \phi_{\mathbf{2}} + \chi_{\mathbf{3}}(y) \phi_{\mathbf{2}}^2 + \mathcal{O}(\phi^3)$$

$$\mathcal{H}(\phi_{\mathbf{2}}, \phi_{\mathbf{2}}^\dagger) = 1 + \phi_{\mathbf{2}} \phi_{\mathbf{2}}^\dagger + \mathcal{O}(\phi_{\mathbf{2}}, \phi_{\mathbf{2}}^\dagger)^3$$

Single Particle Modules

When adding derivatives, we need to take care, besides Lorentz, of EOM and IBP.

- Scalar field EOM: $\Box\phi = -m^2\phi + \cdots$

$$R_\phi = \begin{pmatrix} \phi \\ \partial_\mu\phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\phi \\ \vdots \end{pmatrix}$$ contains only symmetrized, traceless derivatives.

Momentum generating functional generates tower of symmetrized derivatives,

$$\chi_{R_\phi}(\mathcal{D}, x) = \mathcal{D}\, P(\mathcal{D}, x)(1 - \mathcal{D}^2)$$

Lorentz variables

and accounts for IBP redundancy.

$$\mathcal{H}(\mathcal{D}, \{\phi_i\}) = \int d\mu_{\text{Lorentz}}(x) \int d\mu_{\text{gauge}}(y) \frac{1}{P(\mathcal{D}, x)} \prod_i \text{PE}\left[\frac{\phi_i}{\mathcal{D}^{\Delta_i}} \chi_{R_{\phi_i}}\right] + \Delta\mathcal{H}$$

Gravity

Weyl Tensor

Riemann tensor contains gravitational degrees of freedom — Lorentz reducible.

$$R_{\mu\nu\rho\sigma} \sim (\mathbf{0},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{0}) \oplus (\mathbf{0},\mathbf{2})$$

R

$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu}$

$C^{\mu\nu\rho\sigma}_{L/R} = \frac{1}{2}(C^{\mu\nu\rho\sigma} \pm i\,\tilde{C}^{\mu\nu\rho\sigma})$

$$C_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - \left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) + \frac{1}{3} g_{\mu[\rho}g_{\sigma]\nu}R$$

Only the Weyl tensor, C, is non-vanishing on-shell, i.e. on the (vacuum) EOM.

$$\mathcal{S}_{\rm EH} = \frac{1}{2}M_{\rm Pl}^2 \int d^4x \sqrt{-g}\, R \quad \longrightarrow \quad R_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}\right)$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

Operators w/ Ricci tensor and Ricci scalar can be redefined as pure matter operators.

Gravity's Single Particle Module and Character

Bianchi identities imply Weyl tensor EOM and redundancy of Laplacian — in vacuum:

$$\nabla^\mu C_{\mu\nu\rho\sigma} = 0$$

$$\nabla^2 C_{\mu\nu\rho\sigma} = -2C^\lambda{}_{\mu\rho\alpha} C_{\lambda\nu\sigma}{}^\alpha - 2C^\lambda{}_{\nu\rho\alpha} C_{\mu\lambda\sigma}{}^\alpha - C^\lambda{}_{\alpha\rho\sigma} C_{\mu\nu\lambda}{}^\alpha$$

↓ antisymmetric covariant derivatives

$$[\nabla_\mu, \nabla_\nu] V^\rho = R_{\mu\nu\sigma}{}^\rho V^\sigma$$

$$R_C = \begin{pmatrix} C_{\mu\nu\rho\sigma} \\ \nabla_{\{\mu_1} C_{\mu\}\nu\rho\sigma} \\ \nabla_{\{\mu_1} \nabla_{\mu_2} C_{\mu\}\nu\rho\sigma} \\ \vdots \end{pmatrix}$$

$$\chi_{R_{C_L}}(\mathcal{D}, x) = \mathcal{D}^2 \, P(\mathcal{D}, x) \big(\chi_{(2,0)}(x) - \chi_{(3/2,1/2)}(x) \, \mathcal{D} + \chi_{(1,0)}(x) \, \mathcal{D}^2 \big)$$

Lorentz $SO(4)$ group characters

Applications

Pure Gravity

$$\mathcal{L}_6 + \mathcal{L}_8 = \frac{c_1}{\Lambda^2}\mathcal{I} + \frac{c_2}{\Lambda^2}\tilde{\mathcal{I}} + \frac{d_1}{\Lambda^4}\mathcal{C}^2 + \frac{d_2}{\Lambda^4}\mathcal{C}\tilde{\mathcal{C}} + \frac{d_3}{\Lambda^4}\tilde{\mathcal{C}}^2$$

(Endlich, Gorbenko, Huang, Senatore '17)

$$\mathcal{L}_{10} = \frac{e_1}{\Lambda^6}\mathcal{I}\mathcal{C} + \frac{e_2}{\Lambda^6}\tilde{\mathcal{I}}\mathcal{C} + \frac{e_3}{\Lambda^6}\mathcal{I}\tilde{\mathcal{C}} + \frac{e_4}{\Lambda^6}\tilde{\mathcal{I}}\tilde{\mathcal{C}} + \frac{e_5}{\Lambda^6}\mathcal{F}\mathcal{C} + \frac{e_6}{\Lambda^6}\mathcal{F}\tilde{\mathcal{C}} + \frac{e_7}{\Lambda^6}\tilde{\mathcal{F}}\tilde{\mathcal{C}}$$

(Ruhdorfer, JS, Weiler '19)

$$\mathcal{I} = C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma}\,, \quad \tilde{\mathcal{I}} = C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}\tilde{C}_{\alpha\beta\rho\sigma}\,, \quad \mathcal{C} = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\,, \quad \tilde{\mathcal{C}} = C_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$$

$$\mathcal{F} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha C^{\mu\nu\rho\sigma})\,, \quad \tilde{\mathcal{F}} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha \tilde{C}^{\mu\nu\rho\sigma})$$

Comments

$\mathcal{I}$, $\tilde{\mathcal{I}}$, $\mathcal{C}$, $\tilde{\mathcal{C}}$ completely fix spacetime curvature in d = 4 and operator basis w/o derivatives.

$$\mathcal{H}(C_L, C_R) = \frac{1}{(1-C_L^2)(1-C_R^2)(1-C_L^3)(1-C_R^3)}$$

Pure Gravity

$$\mathcal{L}_6 + \mathcal{L}_8 = \frac{c_1}{\Lambda^2}\mathcal{I} + \frac{c_2}{\Lambda^2}\tilde{\mathcal{I}} + \frac{d_1}{\Lambda^4}\mathcal{C}^2 + \frac{d_2}{\Lambda^4}\mathcal{C}\tilde{\mathcal{C}} + \frac{d_3}{\Lambda^4}\tilde{\mathcal{C}}^2$$

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$$\mathcal{F} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha C^{\mu\nu\rho\sigma})\,, \quad \tilde{\mathcal{F}} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha \tilde{C}^{\mu\nu\rho\sigma})$$

Comments

Operator basis + Naive Dimensional Analysis implies pure gravity is 1-loop finite.

Renormalization group evolution at 2 loops:

$$\mu \frac{\partial c_1}{\partial \mu} = \frac{1}{120} \frac{\Lambda^2}{M_{\rm Pl}^2} \frac{1}{(4\pi)^4}$$

(Goroff, Sagnotti '86)

Pure Gravity

$$\mathcal{L}_6 + \mathcal{L}_8 = \frac{c_1}{\Lambda^2}\mathcal{I} + \frac{c_2}{\Lambda^2}\tilde{\mathcal{I}} + \frac{d_1}{\Lambda^4}\mathcal{C}^2 + \frac{d_2}{\Lambda^4}\mathcal{C}\tilde{\mathcal{C}} + \frac{d_3}{\Lambda^4}\tilde{\mathcal{C}}^2$$

(Endlich, Gorbenko, Huang, Senatore '17)

$$\mathcal{L}_{10} = \frac{e_1}{\Lambda^6}\mathcal{I}\mathcal{C} + \frac{e_2}{\Lambda^6}\tilde{\mathcal{I}}\mathcal{C} + \frac{e_3}{\Lambda^6}\mathcal{I}\tilde{\mathcal{C}} + \frac{e_4}{\Lambda^6}\tilde{\mathcal{I}}\tilde{\mathcal{C}} + \frac{e_5}{\Lambda^6}\mathcal{F}\mathcal{C} + \frac{e_6}{\Lambda^6}\mathcal{F}\tilde{\mathcal{C}} + \frac{e_7}{\Lambda^6}\tilde{\mathcal{F}}\tilde{\mathcal{C}}$$

(Ruhdorfer, JS, Weiler '19)

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$$\mathcal{F} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha C^{\mu\nu\rho\sigma})\,, \quad \tilde{\mathcal{F}} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha \tilde{C}^{\mu\nu\rho\sigma})$$

Comments

Operators are subject to theoretical constraints from unitarity, locality, causality.

$$d_1, d_3 > 0\,, \quad d_2 \lesssim d_1 d_3\,, \quad c_{1,2} \lesssim 1$$

(Gruzinov, Kleban '07
Bellazzini et al. '16
Endlich et al. '17
Camanho et al. '14)

(Ruhdorfer, JS, Weiler '19)

U(1) Nambu-Goldstone Boson

First-time basis (i.e. non-redundant operators) at dim-6:

$$\mathcal{L}_6^{\mathrm{CP-even}} = \frac{c_1}{\Lambda^2}\left[(\nabla_\mu\phi)^2\right]^3 + \frac{c_2}{\Lambda^2}(\nabla_\mu\phi)^2(\nabla_\rho\nabla_\sigma\phi)^2 + \frac{c_3}{\Lambda^2}C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma} + \frac{c_4}{\Lambda^2}(C_{\alpha\beta\rho\sigma})^2(\nabla_\mu\phi)^2 + \frac{c_5}{\Lambda^2}C_{\mu\nu\rho\sigma}(\nabla^\mu\phi)(\nabla^\rho\phi)(\nabla^\nu\nabla^\sigma\phi)$$

Hilbert series (i.e. operator counting) at dim-8:

$$\mathcal{H}_8 = C_L^4 + C_L^2C_R^2 + C_R^4 + d\phi^8 + 2d\phi^6\mathcal{D}^2 + d\phi^5\mathcal{D}^3 + d\phi^4\mathcal{D}^4 + d\phi^5\mathcal{D}C_L + d\phi^5\mathcal{D}C_R + d\phi^4C_LC_R + d\phi^4\mathcal{D}^2C_L + d\phi^4C_L^2 + d\phi^4\mathcal{D}^2C_R + d\phi^4C_R^2 + 2d\phi^3\mathcal{D}C_L^2 + 2d\phi^3\mathcal{D}C_R^2 + d\phi^2\mathcal{D}^2C_LC_R + 2d\phi^2\mathcal{D}^2C_L^2 + d\phi^2C_L^3 + 2d\phi^2\mathcal{D}^2C_R^2 + d\phi^2C_R^3$$

Comments

(shift-symmetric) Topological terms are missed by the Hilbert series.

(Henning, Lu, Melia, Murayama '17)

$$\phi C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\,, \quad \phi C_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$$

(Ruhdorfer, JS, Weiler '19)

GRSMEFT

The most general non-redundant set of deformations of the leading GR + SM dynamics.

$$\begin{aligned}\mathcal{L}_6 = &\frac{c_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma} + \frac{\tilde{c}_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} \tilde{C}_{\alpha\beta\rho\sigma} \\ &+ \frac{c_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\tilde{c}_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \\ &+ \frac{c_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \\ &+ \frac{c_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \,.\end{aligned}$$

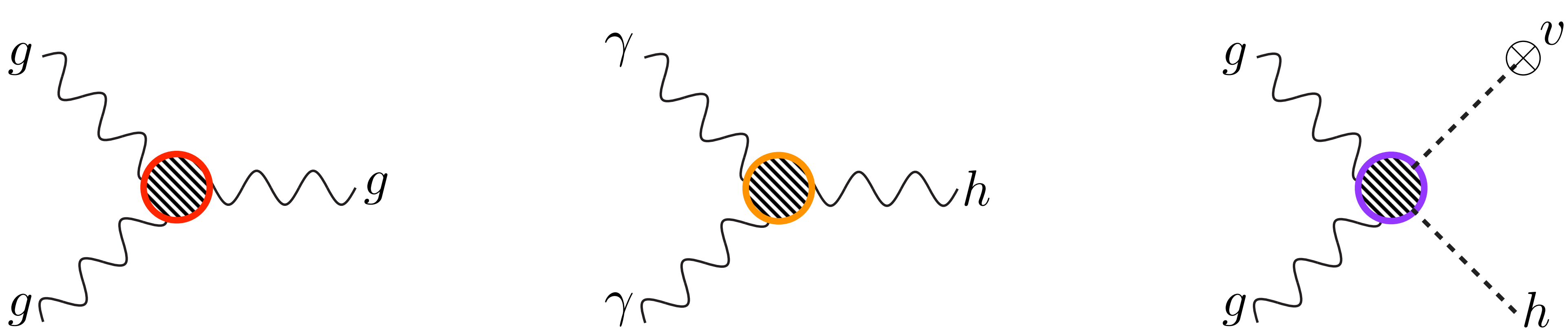
GRSMEFT

The most general non-redundant set of deformations of the leading GR + SM dynamics.

$$
\begin{aligned}
\mathcal{L}_6 = & \frac{c_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma} + \frac{\tilde{c}_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} \tilde{C}_{\alpha\beta\rho\sigma} \\
& + \frac{c_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\tilde{c}_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \\
& + \frac{c_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \\
& + \frac{c_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \, .
\end{aligned}
$$

Comments

All these operators give rise to anomalous 3-point vertices.



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$$\mathcal{L}_6 = \frac{c_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma} + \frac{\tilde{c}_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} \tilde{C}_{\alpha\beta\rho\sigma}$$
$$+ \frac{c_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\tilde{c}_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma}$$
$$+ \frac{c_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma}$$
$$+ \frac{c_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \, .$$

Comments

Similar operators relevant for the weak gravity conjecture,
subject to constraints from unitarity, locality, causality.

(e.g. Arkani-Hamed et al. '06
Endlich et al. '17
Bellazzini, Lewandowski, JS '19)

Some results are known on (non-)renormalization group evolution.

(Deser et al. '75, '81)

GRSMEFT

Basis of dim-8 operators also counted and constructed.

Structure	N_f	$N_f=1$	$N_f=3$	Representative Operator
C^4	3	3	3	$(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})^2$
C^3H^2	2	2	2	$H^\dagger H(C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma})$
C^2H^4	2	2	2	$(H^\dagger H)^2(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
C^2X^2	18	18	18	$B_{\mu\nu}B^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma}$
CH^2X^2	8	8	8	$H^\dagger H(C^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\sigma})$
CX^3	4	4	4	$C^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\alpha}B^{\alpha}{}_{\sigma}$
$C^2H\psi^2$	$12N_f^2$	12	108	$\bar{Q}_L H d_R(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
$CHX\psi^2$	$16N_f^2$	16	144	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\sigma^{\mu\nu}d_R)\tau^a H\, W^{a,\,\rho\sigma}$
$C\psi^4$	$\frac{N_f^2}{3}(17N_f^2+3N_f-2)$	6	480	$\epsilon_{jk}C_{\mu\nu\rho\sigma}(\bar{Q}^j_L\sigma^{\mu\nu}u_R)(\bar{L}^k_L\sigma^{\rho\sigma}e_R)$
$CX\psi^2\mathcal{D}$	$20N_f^2$	20	180	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\gamma^\mu\tau^a\nabla^\nu Q_L)W^{a,\,\rho\sigma}$
$C^2H^2\mathcal{D}^2$	2	2	2	$(\nabla_\mu H)^\dagger(\nabla^\mu H)(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
$CH^2X\mathcal{D}^2$	4	4	4	$C_{\mu\nu\rho\sigma}(\nabla^\mu H)^\dagger\tau^a(\nabla^\nu H)W^{a,\,\rho\sigma}$
$CH\psi^2\mathcal{D}^2$	$6N_f^2$	6	54	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\sigma^{\mu\nu}\nabla^\rho d_R)\nabla^\sigma H$
Total	$43+\frac{N_f^2}{3}(17N_f^2+3N_f+160)$	103	1009	

GRSMEFT

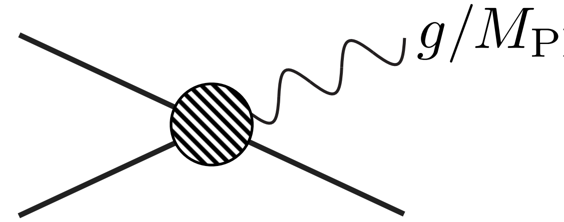
Basis of dim-8 operators also counted and constructed.

Structure	N_f	$N_f = 1$	$N_f = 3$	Representative Operator
C^4	3	3	3	$(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})^2$
C^3H^2	2	2	2	$H^\dagger H(C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma})$
C^2H^4	2	2	2	$(H^\dagger H)^2(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
C^2X^2	18	18	18	$B_{\mu\nu}B^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma}$
CH^2X^2	8	8	8	$H^\dagger H(C^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\sigma})$
CX^3	4	4	4	$C^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\alpha}B^\alpha{}_\sigma$
$C^2H\psi^2$	$12N_f^2$	12	108	$\bar{Q}_L H d_R(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
$CHX\psi^2$	$16N_f^2$	16	144	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\sigma^{\mu\nu}d_R)\tau^a H\, W^{a,\,\rho\sigma}$
$C\psi^4$	$\frac{N_f^2}{3}(17N_f^2+3N_f-2)$	6	480	$\epsilon_{jk}C_{\mu\nu\rho\sigma}(\bar{Q}^j_L\sigma^{\mu\nu}u_R)(\bar{L}^k_L\sigma^{\rho\sigma}e_R)$
$CX\psi^2\mathcal{D}$	$20N_f^2$	20	180	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\gamma^\mu\tau^a\nabla^\nu Q_L)W^{a,\,\rho\sigma}$
$C^2H^2\mathcal{D}^2$	2	2	2	$(\nabla_\mu H)^\dagger(\nabla^\mu H)(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$
$CH^2X\mathcal{D}^2$	4	4	4	$C_{\mu\nu\rho\sigma}(\nabla^\mu H)^\dagger\tau^a(\nabla^\nu H)W^{a,\,\rho\sigma}$
$CH\psi^2\mathcal{D}^2$	$6N_f^2$	6	54	$C_{\mu\nu\rho\sigma}(\bar{Q}_L\sigma^{\mu\nu}\nabla^\rho d_R)\nabla^\sigma H$
Total	$43+\frac{N_f^2}{3}(17N_f^2+3N_f+160)$	103	1009	

(e.g. Falkowski '19)

GRSMEFT Probes

Need to fight the $M_{\rm Pl}$ suppression associated with (on-shell) graviton emission.

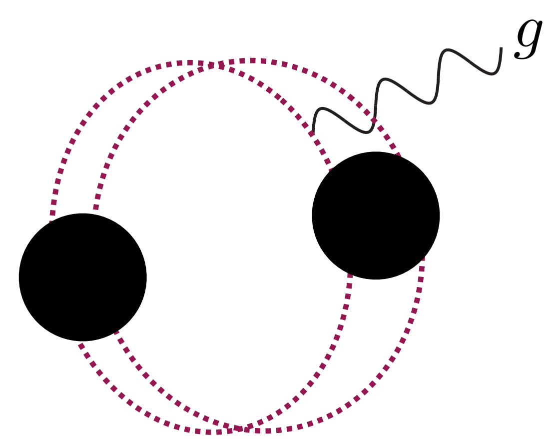


Generically, experiments will be sensitive to low Λ scales (for high energy standards).

- Gravitational waves: from BH mergers

  (Endlich, Gorbenko, Huang, Senatore '17)

$$\frac{1}{\Lambda^4}(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})^2$$



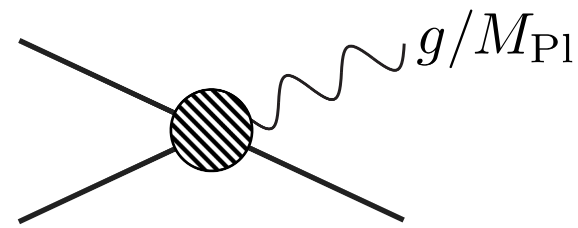
$$\Lambda \gtrsim \frac{1}{R_{\rm Sch}} \sim \frac{1}{100\,{\rm km}}$$

↓

Need systems with large curvatures.

GRSMEFT Probes

Need to fight the $M_{\rm Pl}$ suppression associated with (on-shell) graviton emission.

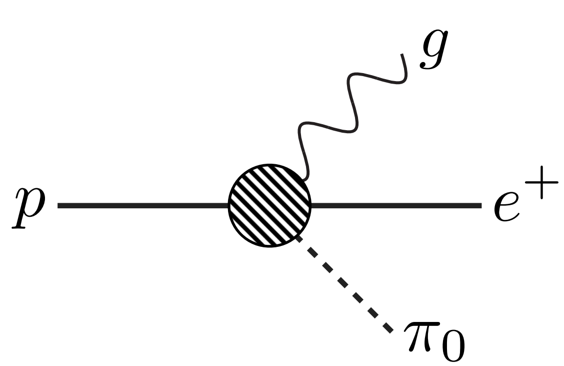


Generically, experiments will be sensitive to low Λ scales (for high energy standards).

- Proton decay:

  (preliminary)

$$\frac{c_{\not B}}{\Lambda^4}(d_R C\sigma_{\mu\nu}u_R)(u_R C\sigma_{\rho\sigma}e_R)C^{\mu\nu\rho\sigma}$$



$$\Gamma_{p\to g e^+ \pi_0} \sim \frac{1}{4\pi}\frac{m_p^6}{\Lambda^4 M_{\rm Pl}^2}\left(\frac{m_p}{\Lambda}\right)^4 m_p$$

$\tau \gtrsim 10^{34}\,\text{yr}$

$$\Lambda/c_{\not B}^{1/4} \gtrsim 1\,\text{TeV}$$

Conclusions/Outlook

To test gravity is a very important goal.

Tied to deep mysteries of nature.

Basis of operators for EFTs with gravity can be systematically counted — next constructed.

Weyl tensor as basic building block of Hilbert series.

- pure gravity.
- U(1) NGB.
- GRSMEFT.

Method can be applied to higher orders and spacetime dimensions (thanks to B. Henning).

Results consistent with amplitudes approach to EFTs (Falkowski; Durieux, Machado '19).

Much can still be explored within the EFT approach to gravity.

- identify potentially interesting probes.
- constraints from unitarity, locality, causality.
- matching to UV completions.
- renormalization group evolution.

Thank you.