The One-Loop Renormalization Group Equations of the Higgs-Electroweak Chiral Lagrangian

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In collaboration with: Gerhard Buchalla, Oscar Catà, Alejandro Celis, and Marc Knecht,
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arXiv:2004.soon,

With the data of the first LHC runs, EFTs became more popular.

- With no direct signal for new physics seen at LHC, there is now an increased interest in (bottom-up) EFTs.

⇒ This is the 8^{th} HEFT workshop!

- In the past 10 years, we have seen the development of two different EFTs for Higgs physics in and beyond the Standard Model:
  The SMEFT and the EWChL (sometimes called HEFT)

- A lot of work has been done towards understanding these EFTs at the 1-loop level.
  For the EWChL: Delgado/Dobado/Llanes-Estrada [1311.5993,JHEP],
  Gavela/Kanshin/Machado/Saa [1409.1571,JHEP],
  Guo/Ruiz-Femena/Sanz-Cillero [1506.04204,PRD],
  Alonso/Jenkins/Manohar [1511.00724,PLB], Alonso/Kanshin/Saa [1710.06848,PRD], and...

Previously . . .

- . . . we derived a Master Formula to find all UV-divergences of a given Lagrangian.
- . . . we applied it to the bosonic SMEFT operators at dimension 6 and confirmed the results of Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].
- . . . we applied it to the EWChL to find all divergences and . . .
- . . . we commented on the consistency of the operator basis and the power counting.

So now. . .

- . . . we perform the full projection to the operator basis in detail.
- . . . we work out all the RGEs for LO and NLO operators.
- . . . we perform some non-trivial cross-checks of our results.

The One-Loop Renormalization Group Equations of the Higgs-Electroweak Chiral Lagrangian

Part I: The leading-order Lagrangian

$$\mathcal{L}_2 = \frac{v^2}{4}\langle L_\mu L^\mu \rangle F(h/v) + \dots$$

$$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$$

Part II: The Master Formula for 1-loop divergences [1710.06412]

Part III: The result [1710.06412, 2004.abcde]

$$16\pi^2 \frac{d}{d\ln\mu} F_i(h) = -\Gamma_i(h)$$

$\frac{v^2}{4}\langle L_\mu L^\mu \rangle F(h/v) + \ldots$

I: The EFT is defined by 3 Ingredients.

Ingredients:

- Particles: all SM particles, but we do not assume a relation between the GB and the Higgs
- Symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$, B, L
- Power counting: in terms of chiral dimensions

Buchalla/Catà/CK [1312.5624,PLB]

$$2L + 2 = [\text{couplings}]_\chi + [\text{derivatives}]_\chi + [\text{fields}]_\chi$$

$$[\text{bosons}]_\chi = 0,$$
$$[\text{fermion bilinears}]_\chi = [\text{derivatives}]_\chi = [\text{weak couplings}]_\chi = 1$$

$\frac{v^2}{4}\langle L_\mu L^\mu \rangle F(h/v) + \ldots$

I: The EWChL studies the Higgs particle.

$$\mathcal{L}_2 = \frac{v^2}{4}\langle L_\mu L^\mu \rangle \; F(h/v) \; + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) \; - \mathcal{V}(h)$$
$$+ i\bar{\psi}\not{D}\psi \; - (v\,\bar{\psi} U \, \mathcal{M}(h/v)\psi + \text{h.c.})$$
$$- \frac{1}{2}\langle G_{\mu\nu} G^{\mu\nu}\rangle \; - \frac{1}{2}\langle W_{\mu\nu} W^{\mu\nu}\rangle \; - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

with $L_\mu = iUD_\mu U^\dagger, \qquad U = \exp(i\varphi_a \sigma_a / v), \qquad \psi = (u_i, d_i, \nu_i, e_i)^T$

Feruglio[hep-ph/9301281], Bagger *et al.*[hep-ph/9306256], Chivukula *et al.*[hep-ph/9312317], Wang/Wang[hep-ph/0605104], Grinstein/Trott[0704.1505], Contino[1005.4269], Alonso *et al.*[1212.3305], Buchalla/Catà/CK [1307.5017], Buchalla/Catà/Celis/CK [1603.03062], . . . & talk by JuanJo Sanz-Cillero

Properties:

- It has generalized Higgs-couplings compared to the SM.
  $\Rightarrow$ related to the κ-formalism at LO.
  Buchalla/Catà/Celis/CK [1504.01707,EPJC]
  de Blas/Eberhardt/CK [1803.00939,JHEP]
- It is not just a field-redefinition of the SMEFT, the expansion is different.
- It captures the low-energy effects of strongly-coupled new physics.
- It is non-renormalizable at LO $\Rightarrow$ NLO operators are needed at 1 loop!

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Part II: The Master Formula for 1-loop divergences [1710.06412]

Part III: The result [1710.06412, 2004.abcde]

$$16\pi^2 \frac{d}{d\ln\mu} F_i(h) = -\Gamma_i(h)$$

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ II: We use the Background-Field Method

Starting from the generating functional:

$$Z[j,\rho,\bar\rho] = e^{iW[j,\rho,\bar\rho]} = \int [d\phi d\psi d\bar\psi] \quad e^{i(\mathcal{S}[\phi,\psi,\bar\psi]+j\phi+\bar\psi\rho+\bar\rho\psi)},$$

$$\phi \to \hat\phi + \phi, \qquad\qquad \psi \to \hat\psi + \psi,$$

$$\Rightarrow e^{iW_{L=1}} = \int [d\phi d\psi d\bar\psi] \quad e^{i\mathcal{S}^{(2)}[\hat\phi,\hat\psi,\hat{\bar\psi};\phi,\psi,\bar\psi]}$$

Abbott '81

Quantum gauge fixing:

$$\mathcal{L}_{\text{g-f}} = -\frac{1}{2}\left(\left(\hat D^\mu_G G^A_\mu\right)^2 - \left(\partial_\mu B^\mu + \tfrac{g'}{2} v\varphi_3\right)^2 - 2\langle\left(\hat D^\mu_W W_\mu - \tfrac{g}{2} v \hat U \varphi \hat U^\dagger\right)^2\rangle\right)$$

- The $\hat D^\mu_X$ are background covariant derivatives.
- The terms proportional to φ will cancel gauge-GB mixing terms in subsequent steps.

see also Andreas Helset's talk

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ II: and the Super-Heat-Kernel Expansion

- In Superspace, we can write the "Gaussian" integral as: $W_{L=1} = \frac{i}{2}\,\mathrm{Str}\ln\Delta$, with $\Delta = \Delta[\mathcal{S}]$
- The logarithm is then expanded in Seeley-DeWitt coefficients.
- Only a single term of the expansion gives all the UV-divergences of $W_{L=1}$.

$\Rightarrow$ Finding the one-loop divergencies of $\mathcal{S}^{(2)}$ is now an algebraic problem.

Donoghue/Golowich/Holstein '92, Neufeld/Gasser/Ecker [hep-ph/9806436]

We get

$$W_{L=1,div} = \frac{1}{32\pi^2\epsilon}\int d^4x\,\mathrm{str}\left[\frac{1}{12}\Lambda_{\mu\nu}\Lambda^{\mu\nu}+\frac{1}{2}\Sigma\Sigma\right],$$

with $\Delta = (\partial_\mu+\Lambda_\mu)(\partial^\mu+\Lambda^\mu)+\Sigma$ and $\Lambda_{\mu\nu} = \partial_\mu\Lambda_\nu-\partial_\nu\Lambda_\mu+[\Lambda_\mu,\Lambda_\nu]$.

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ II: ...to find the Master Formula.

In the EWChL (and the SM), we have

$$\mathcal{L}_2 = -\frac{1}{2}\phi^i A_i^j \phi_j + \bar{\psi}\left(i\partial\!\!\!/ - G\right)\psi + \bar{\psi}\Gamma^i\phi_i + \phi^i\bar{\Gamma}_i\psi,$$

with $A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y$ and $G \equiv (r + \rho_\mu\gamma^\mu)P_R + (l + \lambda_\mu\gamma^\mu)P_L$.

This gives

$$\mathcal{L}_{\text{div}} = \frac{1}{32\pi^2\varepsilon}\left(\text{tr}\left[\frac{1}{12}N^{\mu\nu}N_{\mu\nu} + \frac{1}{2}Y^2 - \frac{1}{3}\left(\lambda^{\mu\nu}\lambda_{\mu\nu} + \rho^{\mu\nu}\rho_{\mu\nu}\right)\right]\right.$$
$$\left. + \text{tr}\left[2D^\mu l D_\mu r - 2lrlr\right] + \bar{\Gamma}\left(i\partial\!\!\!/ + iN\!\!\!/ + \frac{1}{2}\gamma^\mu G\gamma_\mu\right)\Gamma\right)$$

with

$$N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu]\,,$$
$$\lambda_{\mu\nu} \equiv \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu + i[\lambda_\mu, \lambda_\nu]\,, \qquad \rho_{\mu\nu} \equiv \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + i[\rho_\mu, \rho_\nu]\,,$$
$$D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il\lambda_\mu\,, \qquad D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir\rho_\mu\,.$$

'tHooft '73,NPB; Buchalla/Català/Celis/Knecht/CK [1710.06412,NPB]

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III: Projection to Operator Basis

The Master Formula gives us $\mathcal{L}_{\rm div}$: Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

$$\mathcal{L}_{\rm div} \equiv \underbrace{\Delta\mathcal{L}_2}_{\rm LO} + \underbrace{\Delta\mathcal{L}_{\beta_1} + \Delta\mathcal{L}_{UhD^4} + \Delta\mathcal{L}_{\psi^2 UhD} + \Delta\mathcal{L}_{\psi^2 UhD^2} + \Delta\mathcal{L}_{\psi^4 Uh}}_{\rm NLO}$$

$\Delta\mathcal{L}_2$ is not in canonical form, yet.

- Use the e.o.m to eliminate $(\partial_\mu h)(\partial^\mu h)\ A_h(h)$ and $\bar{\psi} i \not{D} \psi\ A_\psi(h)$.
  (more convenient than non-linear field redefinition $\tilde{h} = \int_0^h \sqrt{A_h(s)/2}\, ds$)
  Buchalla/Catà/CK [1307.5017,NPB]
- Shift h, such that $V'(0) = 0$.
  (this shift has a divergent and a finite contribution)
- Now $\Delta\bar{\mathcal{L}}_2$ has the canonical form of $\Delta\mathcal{L}_2$ and we can determine the RGEs.

$$16\pi^2 \frac{d}{d\ln\mu} F_i(h)$$

III: Results: Observations at LO

Gauge couplings: The β-functions are as in the SM

$$\beta_{g_s} = -7g_s^3, \quad \beta_g = -\left(\frac{22}{3} - \frac{N_c+1}{3} N_g - \frac{1}{6}\right) g^3, \quad \beta_{g'} = \left(\left(\frac{11N_c}{27} + 1\right) N_g + \frac{1}{6}\right) g'^3$$

This can also be seen in SMEFT: $\beta_{\phi X} \sim c_{\phi X}$ $(\sim c_{\phi WB})$ and the SM

Since $\frac{v^2}{4}\langle L_\mu L^\mu\rangle = \frac{1}{2}\partial_\mu\varphi^a\partial^\mu\varphi^a + \frac{v^2}{8}(gW_\mu^a - g'B_\mu\delta^{a3})^2 + \dots$, we see that $Z_\varphi = Z_v$
$\Rightarrow$ $U(\varphi)$ is not renormalized.

$F(h/v)$, $V(h)$, and $\mathcal{M}(h/v)$ contribute all to each other's running, e.g.:

$$\begin{aligned}\beta_{F_1} &= \frac{3}{64V_2} F_1(F_1^2 - 4F_2)(3g^4 + 2g^2g'^2 + g'^4) - \frac{g^2}{12} F_1\left[\frac{37}{4}(F_1^2 - 4F_2) + 17(F_2 - 1)\right] - \frac{3}{16} g'^2 F_1(F_1^2 - 4)\\ &+ V_2\left[F_1\left(\frac{5}{2}(F_1^2 - 4F_2) + 4(F_2 - 1)\right) + 12F_3\right] + \frac{2F_1}{v^2}\langle\langle\mathcal{M}_0^\dagger\mathcal{M}_0 + \mathcal{M}_1^\dagger\mathcal{M}_1\rangle\rangle\\ &- \frac{4}{v^2}\langle\langle\mathcal{M}_1^\dagger\mathcal{M}_0 + \mathcal{M}_0^\dagger\mathcal{M}_1\rangle\rangle - \frac{F_1^2 - 4F_2}{v^4V_2}\langle\langle\mathcal{M}_0^\dagger\mathcal{M}_0(\mathcal{M}_0^\dagger\mathcal{M}_1 + \mathcal{M}_1^\dagger\mathcal{M}_0)\rangle\rangle\end{aligned}$$

$$16\pi^2 \frac{d}{d\ln\mu} F_i(h)$$

III: Results: Observations at NLO

- We project to the basis of Buchalla/Catà/CK [1307.5017,NPB].
- Some operators require to split off the h-independent part to avoid further redundancies. Buchalla/Catà [1203.6510,JHEP]

The result can be written as

$$\mathcal{L}_4 = \sum_i \mathcal{O}_i \left(F_i + \frac{\Gamma_i}{16\pi^2} \frac{1}{d-4} \right) \mu^{d-4} \quad \xrightarrow{\text{yielding}} \quad 16\pi^2 \frac{d}{d\ln\mu} F_i(h) = -\Gamma_i(h)$$

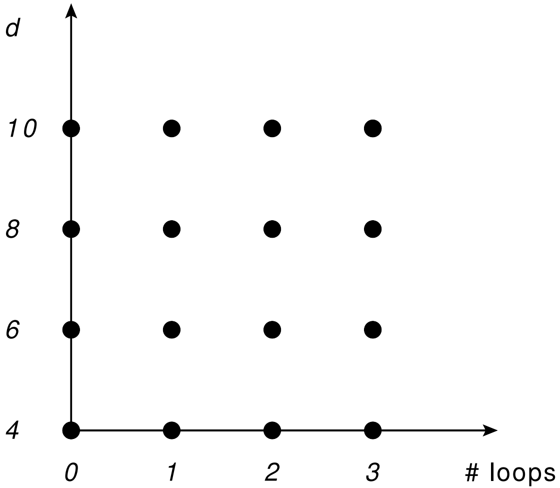
- The NLO RGEs do not depend on the NLO, only on LO coefficients.
- The sum above includes:

  the operator $\mathcal{O}_{\beta_1}$ 5 ops. of UhD^4 all 13 ops. of $\psi^2 UhD$

  12 ops. (+h.c.) of $\psi^2 UhD^2$ 22 ops. (+h.c.) of $\psi^4 Uh$

$16\pi^2 \frac{d}{d\ln\mu} F_i(h)$ III: Non-trivial cross-checks with SMEFT

Buchalla/Catà/CK [1412.6356,NPB]



$$16\pi^2 \frac{d}{d\ln\mu} F_i(h)$$

III: Non-trivial cross-checks with SMEFT

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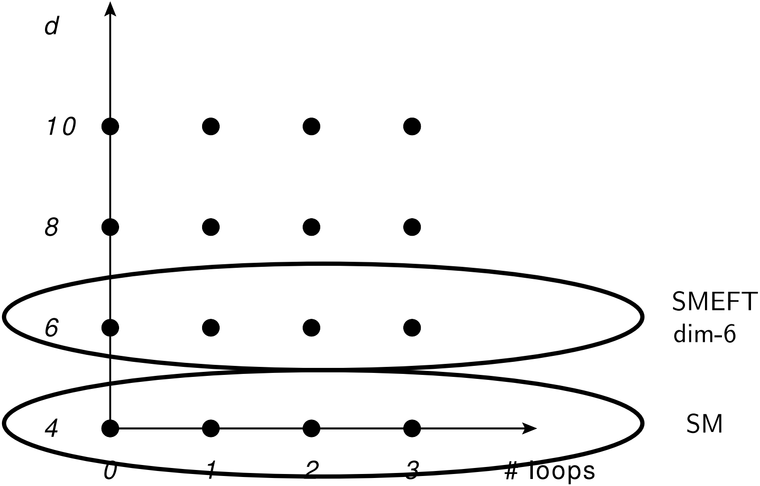

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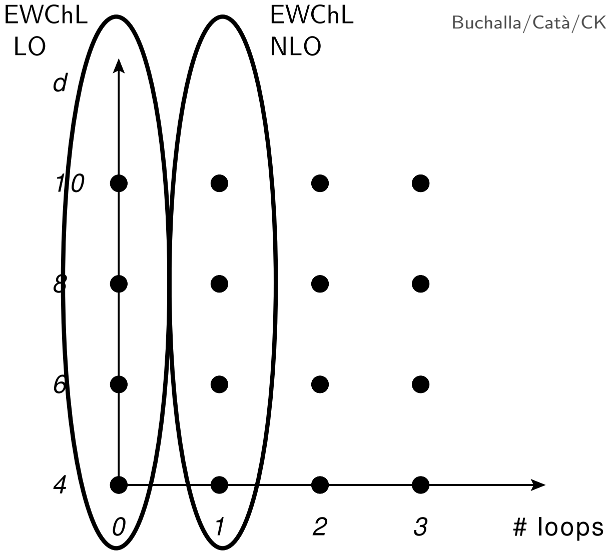

$16\pi^2 \frac{d}{d \ln \mu} F_i(h)$

III: Non-trivial cross-checks with SMEFT

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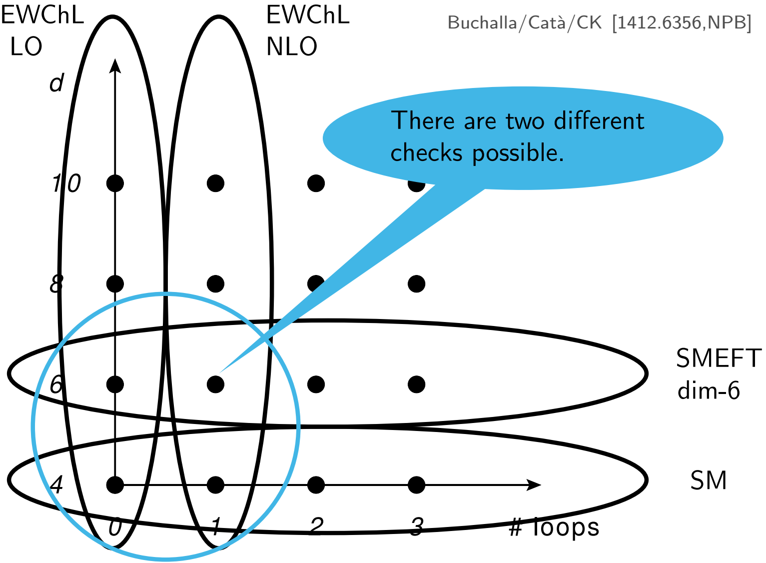

$$16\pi^2 \frac{d}{d\ln\mu} F_i(h)$$

III: Non-trivial cross-checks with SMEFT

1) If $\mathcal{L}_2$ is restricted to the $d \leq 4$ $\mathcal{L}_{SM}$, we find:

- $\Gamma_{NLO} = \beta_{NLO} = 0$.
- The remaining divergent structures map back to $\mathcal{L}_{SM}$.
- We recover all β_{SM}. Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

2) If $\mathcal{L}_2$ is restricted to $d \leq 6$ as $\mathcal{L}_{SM} + \frac{1}{\Lambda^2}\left(c_{\phi\Box}\mathcal{O}_{\phi\Box} + c_\phi\mathcal{O}_\phi + c^{rs}_{\psi\phi}\mathcal{O}^{rs}_{\psi\phi}\right)$:

- We need to expand the result to first order in c_i ...
- ...and consider the field renormalization in c_i, then ...
- ...we recover all contributions of $\mathcal{O}_i$ as in Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].

The One-Loop Renormalization Group Equations of the Higgs-Electroweak Chiral Lagrangian — Summary —

- Previously, we had derived a master formula for the $1/\epsilon$-poles based on the super-heat-kernel expansion.

$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

- We applied it to different theories, like the SM, the bosonic sector of the SMEFT, and the EWChL.

- We projected all divergences of the EWChL to our basis at LO and NLO.

$$\boxed{16\pi^2 \frac{d}{d\ln\mu} F_i(h) = -\Gamma_i(h)}$$

- We derived the RGEs for all couplings at LO and NLO.

- There is a small overlap with the SMEFT renormalization that we used for cross-checks. Different runnings could help distinguish the EFTs.