

HIGGS DECAY TO FERMIONS AT NLO IN SMEFT

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$$\mathcal{L}^{\text{SMEFT}} = \mathcal{L}^{\text{SM}} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + (\text{dim-8 and higher})$$

- Q_i are 59 dimension-6 operators (2499 parameters with flavour indices)
- $C_i \sim 1/\Lambda_{\text{NP}}^2$ are Wilson coefficients. $C_i \neq 0 = \text{BSM}$
- well defined QFT: renormalizable order by order in $1/\Lambda_{\text{NP}}^2$

MOTIVATION FOR $h \rightarrow b\bar{b}$ AT NLO IN SMEFT

1) Phenomenology:

- $h \rightarrow b\bar{b}$ largest Higgs branching fraction
- can measure Higgs SM Yukawa at (sub)percent level at Higgs factory
- \Rightarrow NLO SMEFT calculation sets long-term baseline for analysis in EFT

2) SMEFT development:

- reveals many non-trivial features of SMEFT at NLO in (relatively) simple setting
- analytic results useful for benchmarking automated codes for NLO SMEFT

Some things we dealt with in full NLO calculation:

[Jonathan Cullen, B.P., Darren Scott: [arXiv:1904.06358](https://arxiv.org/abs/1904.06358)]

- renormalize e, M_W, M_Z, m_b, C_i , plus external b -quark, h -boson fields
(45 C_i appear at NLO, checks 100s of entries in 1-loop anom. dims. [[Alonso, Jenkins, Manohar, Trott](#)])
- gauge invariance: tadpoles and gauge fixing in SMEFT
- hybrid renormalization schemes and decoupling relations

OUTLINE OF NLO CALCULATION

Basic outline:

- specify input parameters and renormalization scheme
- write down LO and UV counterterm amplitudes for $h \rightarrow bb$
- calculate one-loop $h \rightarrow bb$ and 2-point functions for counterterms
- calculate real emissions of photons, gluons, add together with UV-renormalized virtual corrections to get IR finite answer

In general, every piece of calculation gets dim-4 (SM) and dim-6 (SMEFT) contributions, dim-8 terms are dropped

We use mass basis Lagrangian parameters:

$$\alpha_s, \alpha, m_f, m_H, M_W, M_Z, V_{ij}, C_i$$

- C_i are renormalized in the $\overline{\text{MS}}$ scheme
- M_W, M_Z renormalized in on-shell scheme
- renormalization scheme for e and $m_b \neq 0$ kept flexible
- M_H, m_t, m_τ kept non-zero but not renormalized at NLO. all other $m_f = 0$.
- we approximate $V_{ij} = \delta_{ij}$

THE LO AMPLITUDE

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \rightarrow b\bar{b}) = -i\bar{u}(p_b) \left(\mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R \right) v(p_{\bar{b}})$$

- split into dim-4 and dim-6 contributions

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

- Explicit results ($\hat{v}_T \equiv 2M_w \hat{s}_w / e$, $\hat{c}_w \equiv \cos \theta_W = M_W / M_Z$)

$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T},$$

$$\mathcal{M}_L^{(6,0)} = m_b \hat{v}_T \left[C_{H\Box} - \frac{C_{HD}}{4} \left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{C_{bH}^*}{\sqrt{2}} \right]$$

- $Q_{bH} \sim (H^\dagger H)(\bar{b}_L b_R H) + \text{h.c.}$ contributes to LO Feynman diagram
- other Q_i , where $Q_i \xrightarrow{\text{SSB}} v_T^2 Q_i^{(4)}$ appear from rotation to mass basis

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad Q_{HWB} = H^\dagger \sigma^I H W_{\mu\nu}^I B^{\mu\nu}, \quad Q_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$$

THE COUNTERTERM AMPLITUDE

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{\hat{v}_T} \left(\frac{\delta m_b^{(4)}}{m_b} - \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*} \right)$$

dimension-6 counterterm is

$$\begin{aligned} \delta\mathcal{M}_L^{(6)} = & \frac{m_b}{\hat{v}_T} \left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*} \right) \\ & + \mathcal{M}_L^{(6,0)} \left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*} \right) \\ & - \frac{\hat{v}_T^2}{\sqrt{2}} C_{bH}^* \left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b} \right) + m_b \hat{v}_T \left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w} C_{HD} \right] \delta \left(\frac{\hat{c}_w}{\hat{s}_w} \right)^{(4)} \\ & + m_b \hat{v}_T \left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4} \left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} \delta C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{\delta C_{bH}^*}{\sqrt{2}} \right) \end{aligned}$$

where

$$\frac{\delta\hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta\hat{s}_w}{\hat{s}_w} = -\frac{\hat{c}_w^2}{\hat{s}_w^2} \left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z} \right), \quad \delta \left(\frac{\hat{c}_w}{\hat{s}_w} \right)^{(4)} = -\frac{1}{\hat{c}_w \hat{s}_w} \left(\frac{\delta\hat{s}_w^{(4)}}{\hat{s}_w} \right)$$

CALCULATIONAL PROCEDURE

One-loop $h \rightarrow bb$ matrix elements and two-point functions for counterterms involve many Feynman diagrams and dim-6 operators

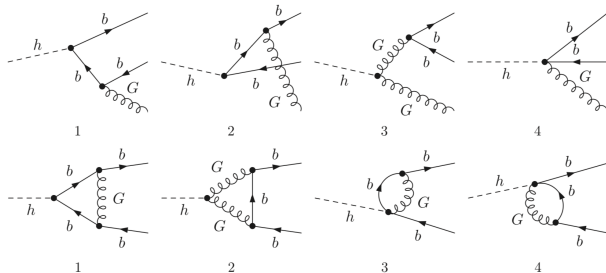
- **automation:** Feynrules (in-house model file, including gauge fixing and ghosts), Feynarts, FormCalc, Package X
- loop integrals obtained analytically in terms of Passarino-Veltmann integrals and standard functions

Decay rate also requires real emission corrections $h \rightarrow bb(g, \gamma)$

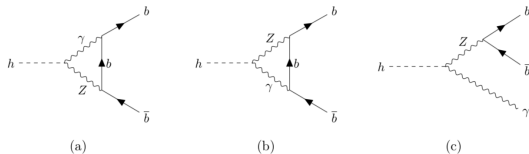
- squared matrix elements generated with automated tools
- 3-body phase space integrals done by hand

EXAMPLE: QCD-QED CORRECTIONS

- QCD corrections by far simplest to calculate [Gauld, B.P., Scott '16]
- UV-renormalized one-loop amplitudes have IR divergences canceled by real emissions



- most corrections involving photons can be obtained analogously, exception is graphs involving $h\gamma Z$ vertex



ANALYTIC STRUCTURE OF $h\gamma Z$ CORRECTIONS

$$\Gamma_{h\gamma Z} \propto v_b \left[2(C_{HB} - C_{HW})\hat{c}_w\hat{s}_w + C_{HWB}(\hat{c}_w^2 - \hat{s}_w^2) \right] F_{h\gamma Z} \left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2} \right)$$

$$\begin{aligned} F_{h\gamma Z}(z, \hat{\mu}^2, b) = & \frac{3}{4}\beta(8z - 5) - \beta^3 \left(\frac{39}{4} + \frac{z}{b} \right) - \frac{4}{3}\beta^2\pi^2\bar{z} + \frac{4}{3}\pi^2 z\bar{z} + 6\beta \left(\beta^2 - \frac{2}{3}z \right. \\ & \left. + \frac{(2b - \beta^2)z^2}{12b^2} \right) \ln(b) + 2(\beta^2 - z)\bar{z} \ln(x_z)^2 - 4\beta_z z\bar{z} \ln(x_{\beta z}) \\ & + \ln(x) \left(-\frac{1}{8} (15 + 7\beta^4 + 8z(4z - 7) + \beta^2(2 + 8z)) + 2(z - \beta^2)\bar{z} \ln(x_z) \right. \\ & \left. + 4(\beta^2 - z)\bar{z} \ln(1 - xx_z) + 2(\beta^2 - z)\bar{z} \ln(x_{\beta z}) \right) \\ & + \ln(x_z) \left(\frac{\beta\beta_z z (\beta^2(2b + z) - 2bz)}{2b^2} + 2(z - \beta^2)\bar{z} \ln(x_{\beta z}) \right) \\ & + 4\beta_z z\bar{z} \ln(\bar{z}) + \frac{\beta^3(\beta^2 + 2b)z^2 \ln(z)}{2b^2} - 6\beta^3 \ln(\hat{\mu}^2) \\ & + 4(\beta^2 - z)\bar{z} \left(\text{Li}_2 \left(\frac{x}{x_z} \right) + \text{Li}_2(xx_z) \right) \end{aligned}$$

where

$$\beta = \sqrt{1 - 4b}, \quad \beta_z = \sqrt{1 - \frac{4b}{z}}, \quad x = \frac{1 - \beta}{1 + \beta}, \quad x_z = \frac{1 - \beta_z}{1 + \beta_z}, \quad x_{\beta z} = \frac{\beta - \beta_z}{\beta + \beta_z}, \quad \bar{z} = 1 - z$$

CROSS-CHECKS AND FEATURES

Results involve 45 different Wilson coefficients (generally complex). Cross-checks:

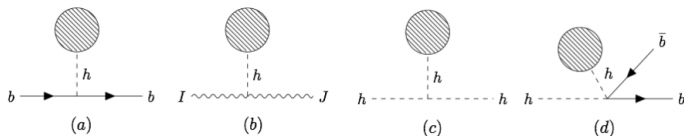
- all UV and IR poles cancel (and μ -dependence consistent with RG eqns)
- SM results reproduced from dim. 4 terms
- all results calculated in unitary and Feynman gauge with full agreement

Interesting features:

- structure of wave-function renormalization of b -quark field
- Higgs- Z mixing
- Ward identities and electric charge renormalization
- structure of tadpole contributions
- decoupling relations and hybrid renormalization schemes (mix of $\overline{\text{MS}}$ and on-shell)

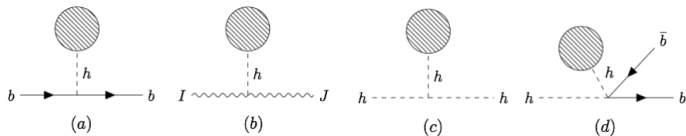
TADPOLES I

- we used FJ tadpole scheme [Fleischer, Jegerlehner '80]
- discussion in [Denner, Jenniches, Lang, Sturm '16] shows FJ scheme implemented by simply calculating all tadpole contributions to n -point functions
- tadpoles needed for $h \rightarrow bb$ in SMEFT



- we calculated tadpoles in Feynman and unitary gauge and verified:
 - (1) tadpoles cancel in on-shell scheme
 - (2) mass and electric charge counterterms, and matrix elements +wavefunction renormalization separately gauge invariant after adding tadpoles
- structure of tadpoles contributions in SMEFT richer than in SM

TADPOLES II



$$IJ \in \{\gamma\gamma, \gamma Z, WW, ZZ\}$$

- (a) contributes to δm_b in SM and SMEFT, but also to δZ_b^L in SMEFT

$$\delta Z_{b,\text{tad.}}^L = -\frac{i\sqrt{2}\hat{v}_T^2}{m_H^2 m_b} \text{Im}(C_{bH}) T^{(4)}$$

- (b) contributes to $\delta M_W, \delta M_Z$ in SM and SMEFT, also to δe in SMEFT ($IJ = \gamma\gamma$)

$$\frac{\delta e^{\text{cl.4,(6)}}}{e} = \frac{1}{16\pi^2} \left[c_{h\gamma\gamma} A_0(m_H^2) + 4\hat{c}_w \hat{s}_w C_{HWB} (4M_W^2 - 3A_0(M_W^2)) \right] - 2c_{h\gamma\gamma} \frac{\hat{v}_T}{m_H^2} T_{\text{un.}}^{(4)}$$

- (c) contributes to δZ_h in SMEFT (through C_{HD} and $C_{H\Box}$), but not in SM
- (d) contributes to $h \rightarrow b\bar{b}$ matrix element in SMEFT, but not in SM

ENHANCED NLO CORRECTIONS I: QCD CORRECTIONS

- QCD/QED corrections generate $\ln m_b/m_H$ terms when $\mu = m_H$:

$$\frac{\Gamma_{g,\gamma}^{(1)}}{\Gamma(4,0)} \approx \ln^2 \left(\frac{m_b^2}{m_H^2} \right) \frac{\hat{v}_T^2}{\pi} (C_F \alpha_s C_{HG} + Q_b^2 \alpha c_{h\gamma\gamma})$$

$$+ c_{m_b} \ln \left(\frac{m_b^2}{m_H^2} \right) \frac{3}{2} \left(\frac{C_F \alpha_s + Q_b^2 \alpha}{\pi} \right) \left[1 + 2\hat{v}_T^2 \left(C_{H\Box} - \frac{C_{HD}}{4} \left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2} \right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b} \frac{C_{bH}}{2\sqrt{2}} \right) \right]$$

- double logs of IR origin remain and are largest NLO correction
 - $Q_{HG} = H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$ gives hgg vertex strongly constrained from Higgs production
- $c_{m_b} = 1$ in on-shell scheme, $c_{m_b} = 0$ ($\overline{\text{MS}}$ scheme) for m_b .
- \Rightarrow QCD/QED prefers $\overline{\text{MS}}$ scheme for m_b (running mass resums single UV logs)

ENHANCED NLO CORRECTIONS II: TADPOLES

- QCD and QED corrections prefer $\overline{\text{MS}}$ scheme for m_b and e
- however, in $\overline{\text{MS}}$ scheme **tadpoles** contribute m_t^4 enhanced corrections.
Example: SM in $m_t \rightarrow \infty$ limit

$$\overline{\text{MS}} \text{ scheme: } \frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2} \frac{m_t^4}{\hat{v}_T^2 m_H^2} \approx -15\%$$

$$\text{on-shell scheme: } \frac{[\Gamma_t]^{\text{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2 \hat{v}_T^2} \left(-6 + N_c \frac{7 - 10\hat{c}_w^2}{3\hat{s}_w^2} \right) \approx -3\%$$

- similar behaviour in SMEFT contributions to decay rate
- \Rightarrow **EW corrections prefer on-shell scheme for m_b and e large corrections from top loops (and heavy EW bosons)**

Combining EW and QCD corrections is a non-trivial problem. Would like to calculate QCD in $\overline{\text{MS}}$ scheme, but EW in on-shell scheme...

DECOUPLING RELATIONS I

- decoupling relations connect \overline{MS} parameters in SM, with those in low-energy theory where top and heavy bosons integrated out:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z) \overline{m}_b^{(\ell)}(\mu)$$

- decoupling constants ζ_i contain contributions from heavy particles.
- ζ_b calculated by relating on-shell with \overline{MS} mass in SM and low-energy theory:

$$m_b = z_b^{-1}(\mu, m_b, m_t, m_H, M_W, M_Z) \overline{m}_b(\mu) = \left[z_b^{(\ell)}(\mu, m_b) \right]^{-1} \overline{m}_b^{(\ell)}(\mu)$$

$$\Rightarrow \zeta_b(\mu, m_t, m_H, M_W, M_Z) = \frac{z_b(\mu, m_b, m_t, m_H, M_W, M_Z)}{z_b^{(\ell)}(\mu, m_b)} \Big|_{m_b \rightarrow 0}$$

- works analogously for electric charge. The connection between low energy parameters and experiment are:

$$\text{from } B\text{-physics: } \overline{m}_b^{(\ell)}(\overline{m}_b^{(\ell)}) \approx 4.2 \text{ GeV}$$

$$\text{from LEP: } \overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z) \left(1 + \frac{100\alpha}{27\pi} \right), \quad \alpha(M_Z) \approx 1/129, \alpha \approx 1/137$$

DECOUPLING RELATIONS II

- dim.4 contributions to ζ_i well known, we calculated dim.6 corrs. Example:

$$\zeta_e^{(4,1)} = \frac{\alpha}{\pi} \left[-\frac{1}{12} - \frac{7}{8} \ln \left(\frac{\mu^2}{M_W^2} \right) + \frac{N_c}{6} Q_t^2 \ln \left(\frac{\mu^2}{m_t^2} \right) \right]$$

$$\zeta_e^{(6,1)} = \frac{\alpha}{\pi} \left[\sqrt{2} \hat{v}_T m_t N_c Q_t \left(\hat{c}_w \frac{\text{Re}(C_{tB})}{e} + \hat{s}_w \frac{\text{Re}(C_{tW})}{e} \right) \ln \left(\frac{\mu^2}{m_t^2} \right) + 9 \frac{C_W}{e} \hat{s}_w M_W^2 \ln \left(\frac{\mu^2}{M_W^2} \right) \right] \\ + \frac{\delta e^{\text{cl.4(6)}}}{e} \Big|_{\text{fin.}, m_b \rightarrow 0}$$

- relation between NLO decay rate using low-energy parameters vs. SM params:

$$\bar{\Gamma}_\ell^{(4,1)} = \bar{\Gamma}^{(4,1)} + 2\bar{\Gamma}^{(4,0)} \left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)} \right),$$

$$\bar{\Gamma}_\ell^{(6,1)} = \bar{\Gamma}^{(6,1)} + 2\bar{\Gamma}^{(4,0)} \left(\zeta_b^{(6,1)} + \zeta_e^{(6,1)} \right) + 2\bar{\Gamma}^{(6,0)} \zeta_b^{(4,1)} + \sqrt{2} C_{bH} \frac{(\bar{v}^{(\ell)})^3}{\bar{m}_b} \bar{\Gamma}^{(4,0)} \left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)} \right)$$

- illustrative results: QCD-QED corrections and EW corrections in $m_t \rightarrow \infty$ limit:

$$\bar{\Gamma}_{\ell,g,\gamma} = \bar{\Gamma}_{g,\gamma}, \quad \bar{\Gamma}_{\ell,t} = [\Gamma_t]^{\text{O.S.}}$$

- interpretation: QCD-QED corrections in $\overline{\text{MS}}$ scheme (UV logs resummed), heavy-particle EW corrections in on-shell (tadpoles cancel)

NUMERICAL RESULTS

- independent renormalization scales for Wilson coefficients (μ_C) and $\overline{\text{MS}}$ parameters (μ_R)
- varying $\mu_C, \mu_R = m_H$ by factors of 2 and adding in quadrature:

$$\Delta^{\text{LO}}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ \begin{aligned} &(3.74 \pm 0.36) \tilde{C}_{HWB} + (2.00 \pm 0.21) \tilde{C}_{H\Box} - (1.41 \pm 0.07) \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + (1.24 \pm 0.14) \tilde{C}_{HD} \\ &\pm 0.35 \tilde{C}_{HG} \pm 0.19 \tilde{C}_{Hq}^{(1)} \pm 0.18 \tilde{C}_{Ht} \pm 0.11 \tilde{C}_{Hq}^{(3)} + \dots \end{aligned} \right\}$$

$$\Delta^{\text{NLO}}(m_H, m_H) = 1.13_{-0.04}^{+0.01} + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ \begin{aligned} &(4.16_{-0.14}^{+0.05}) \tilde{C}_{HWB} + (2.40_{-0.09}^{+0.04}) \tilde{C}_{H\Box} \\ &+ (-1.73_{-0.03}^{+0.04}) \frac{\bar{v}^{(\ell)}}{\bar{m}_b^{(\ell)}} \tilde{C}_{bH} + (1.33_{-0.04}^{+0.01}) \tilde{C}_{HD} + (2.75_{-0.48}^{+0.49}) \tilde{C}_{HG} \\ &+ (-0.12_{-0.01}^{+0.04}) \tilde{C}_{Hq}^{(3)} + (-0.08_{-0.01}^{+0.05}) \tilde{C}_{Ht} + (0.06_{-0.05}^{+0.00}) \tilde{C}_{Hq}^{(1)} + (0.00_{-0.04}^{+0.07}) \frac{\tilde{C}_{tG}}{g_s} + \dots \end{aligned} \right\}$$

- in general, **scale uncertainties in LO result overlap with NLO one, and scale uncertainties decrease between LO and NLO**
- exception is C_{HG} , which gives large corrections unrelated to RG eqns.
- scale variation of C_{HG} gives rise to C_{tG} , which is a significant contribution although it is two-loop order

CORRECTIONS TO LO RESULTS

	SM	\tilde{C}_{HWB}	$\tilde{C}_{H\Box}$	\tilde{C}_{bH}	\tilde{C}_{HD}
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO large- m_t	-3.1%	-4.6%	3.2%	3.5%	-9.0%
NLO remainder	-2.2%	-1.9%	-1.2 %	0.6%	-2.0%
NLO correction	12.9%	11.3%	20.2%	22.3%	7.1%

TABLE: Size of NLO corrections to different terms in LO decay rate, split into QCD-QED, large m_t , and remaining components.

- applying SM K -factor to dim.6 operators bad approximation for EW corrections

$h \rightarrow \ell^+ \ell^-$ AND $h \rightarrow c \bar{c}$

- can use same techniques to calculate $\Gamma(h \rightarrow f \bar{f})$ [Cullen, BP, to appear]
- taking ratios of decay rates particularly useful in SMEFT, because C_i related to renormalization of v_τ drop out
- simple example: ratio of $\Gamma(h \rightarrow \mu^+ \mu^-) / \Gamma(h \rightarrow \tau^+ \tau^-)$

Denoting $\Delta(h \rightarrow f \bar{f}) = \Delta_f$

$$\frac{\Delta_\mu^{\text{LO}}(m_H)}{\Delta_\tau^{\text{LO}}(m_H)} = 1 + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ -1.41 \left[\frac{\bar{v}^{(\ell)}}{\bar{m}_\mu^{(\ell)}} \tilde{C}_{\mu H} - \frac{\bar{v}^{(\ell)}}{\bar{m}_\tau^{(\ell)}} \tilde{C}_{\tau H} \right] \right\} \quad (1.1)$$

$$\begin{aligned} \frac{\Delta_\mu^{\text{NLO}}(m_H)}{\Delta_\tau^{\text{NLO}}(m_H)} &= 1 + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\text{NP}}^2} \left\{ -1.50 \left[\frac{\bar{v}^{(\ell)}}{\bar{m}_\mu^{(\ell)}} \tilde{C}_{\mu H} - \frac{\bar{v}^{(\ell)}}{\bar{m}_\tau^{(\ell)}} \tilde{C}_{\tau H} \right] + 0.33 c_{h\gamma\gamma} \right. \\ &+ \left(2.7 \left[\frac{\bar{v}^{(\ell)}}{\bar{m}_\mu^{(\ell)}} \tilde{C}_{2233}^{(1) \text{lequ}} - \frac{\bar{v}^{(\ell)}}{\bar{m}_\tau^{(\ell)}} \tilde{C}_{3333}^{(1) \text{lequ}} \right] + 1.8 \left[\tilde{C}_{22}^{(3) \text{HI}} - \tilde{C}_{33}^{(3) \text{HI}} \right] \right. \\ &- 1.0 \left[\tilde{C}_{2222}^{\text{le}} - \tilde{C}_{3333}^{\text{le}} \right] \times 10^{-2} + \left(4 \left[\tilde{C}_{22}^{(1) \text{HI}} - \tilde{C}_{33}^{(1) \text{HI}} \right] - 4 \left[\tilde{C}_{H\mu} - \tilde{C}_{H\tau} \right] \right. \\ &\left. \left. \left[\frac{\bar{v}^{(\ell)}}{\bar{m}_\mu^{(\ell)} \bar{e}^{(\ell)}} \tilde{C}_{\mu W} - \frac{\bar{v}^{(\ell)}}{\bar{m}_\tau^{(\ell)} \bar{e}^{(\ell)}} \tilde{C}_{\tau W} \right] \right) \times 10^{-3} - (4 \times 10^{-4}) \left[\frac{\bar{v}^{(\ell)}}{\bar{m}_\mu^{(\ell)} \bar{e}^{(\ell)}} \tilde{C}_{\mu B} - \frac{\bar{v}^{(\ell)}}{\bar{m}_\tau^{(\ell)} \bar{e}^{(\ell)}} \tilde{C}_{\tau B} \right] \right\} \end{aligned}$$

- calculated full NLO corrections to $h \rightarrow bb$ in SMEFT
- forms basis for serious phenomenological analysis of $h \rightarrow bb$ in SMEFT
- calculation involved many non-trivial conceptual issues in SMEFT, and can be used as a template for future calculations combining EW and QCD corrections

Backup Slides

INPUT PARAMETERS II

- many non-trivial SMEFT effects are involved in rotation to mass basis
- example 1: Higgs vev $\langle H^\dagger H \rangle \equiv v_T^2/2$

$$\frac{1}{v_T} = \frac{1}{\hat{v}_T} \left(1 + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w} \left[C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD} \right] \right); \quad \hat{v}_T = 2M_W \hat{s}_w / e; \quad \hat{c}_w = \frac{M_W}{M_Z}$$

- example 2: covariant derivative in mass basis

$$\begin{aligned} D_\mu = & \partial_\mu - i \frac{e}{\hat{s}_w} \left[1 + \frac{\hat{c}_w^2 \hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB} \right] (\mathcal{W}_\mu^+ \tau^+ + \mathcal{W}_\mu^- \tau^-) \\ & - i \left[\frac{e}{\hat{c}_w \hat{s}_w} \left(1 + \frac{(2\hat{c}_w^2 - 1) \hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB} \right) (\tau^3 - \hat{s}_w^2 Q) \right. \\ & \left. + e \left(\frac{\hat{c}_w \hat{v}_T^2}{2\hat{s}_w} C_{HD} + \hat{v}_T^2 C_{HWB} \right) Q \right] \mathcal{Z}_\mu - ieQ \mathcal{A}_\mu, \end{aligned}$$

- gauge fixing is also much more involved than SM

FERMION W.F. RENORMALIZATION IN SMEFT

- Decompose two-point function of fermion f as

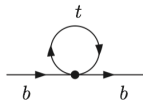
$$\Gamma^f(p) = i(\not{p} - m_f) + i \left[\not{p} \left(P_L \Sigma_f^L(p^2) + P_R \Sigma_f^R(p^2) \right) + m_f \left(\Sigma_f^S(p^2) P_L + \Sigma_f^{S*}(p^2) P_R \right) \right]$$

- In on-shell renormalization scheme

$$\delta Z_f^L = -\widetilde{\text{Re}} \Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) - m_f^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2=m_f^2},$$

$$\delta Z_f^R = -\widetilde{\text{Re}} \Sigma^{f,R}(m_f^2) - m_f^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2) \right] \Big|_{p^2=m_f^2}$$

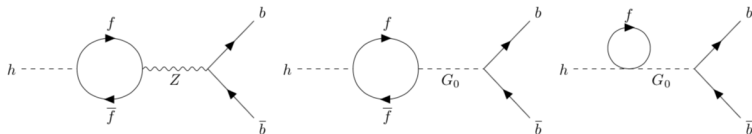
- $\Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2)$ vanishes in SM, but is proportional $\text{Im}(C_i)$ in SMEFT.
- appears in many places in renormalization of amplitude – example:



$$Z_b^L = \frac{1}{\epsilon} \left[-\frac{m_t^3}{m_b} \left((2N_c + 1) \left(C_{qtqb}^{(1)} - C_{qtqb}^{(1)*} \right) + c_{F,3} \left(C_{qtqb}^{(8)} - C_{qtqb}^{(8)*} \right) \right) \right] + \text{finite}$$

HIGGS-Z (GOLDSTONE) MIXING

- unlike SM, in SMEFT Higgs can mix into Z and neutral Goldstone boson G_0



- h - G_0 mixing is that between real and imaginary parts of doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}i\phi^+(x) \\ [1 + C_{H,\text{kin.}}] h(x) + i \left[1 - \frac{\hat{v}_T^2}{4} C_{HD} \right] \phi^0(x) + v_T \end{pmatrix}$$

- mixing is therefore proportional to imaginary parts of Wilson coefficients and reads

$$\eta_5 = \frac{\sqrt{2}}{\hat{v}_T} \text{Im} [N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H} + \dots]$$

- this term exactly cancels one appearing in renormalization of Q_{bH} (i.e. that in \dot{C}_{bH} calculated in [\[Jenkins, Manohar, Trott '13\]](#))

ELECTRIC CHARGE RENORMALIZATION

SM: $ff\gamma$ vertex related to two-point fcn through Ward identities:

- result

$$\frac{\delta e^{(4)}}{e} = \frac{1}{2} \frac{\partial \Sigma_T^{AA(4)}(k^2)}{\partial k^2} \Big|_{k^2=0} - \frac{(v_f^{(4)} - a_f^{(4)}) \Sigma_T^{AZ(4)}(0)}{Q_f M_Z^2}$$

- $v_f^{(4)} - a_f^{(4)} = -Q_f \hat{s}_w / \hat{c}_w \Rightarrow \delta e^{(4)}$ independent of fermion charge

SMEFT: we find by renormalizing $ff\gamma$ vertex directly (not using Ward identities)

- result

$$\frac{\delta e^{(6)}}{e} = \frac{1}{2} \frac{\partial \Sigma_T^{AA(6)}(k^2)}{\partial k^2} \Big|_{k^2=0} + \frac{1}{M_Z^2} \left(\frac{\hat{s}_w}{\hat{c}_w} \Sigma_T^{AZ(6)}(0) - \frac{\hat{v}_T^2}{4\hat{c}_w \hat{s}_w} C_{HD} \Sigma_T^{AZ(4)}(0) \right)$$

- $v_f^{(6)} - a_f^{(6)} = 2C_{Hf} \hat{v}_T^2 / 4\hat{c}_w \hat{s}_w \Rightarrow$ Naive application of SM Ward identity is wrong, due to operators of form $Q_{Hf} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{f} \gamma^\mu f)$