The CKM parameters in the SMEFT

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arXiv:1812.08163 [hep-ph] w/ Descotes-Genon, Falkowski, Gonzalez-Alonso, Fedele

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Pheno analyses in the SMEFT

$$\mathcal{L}_{\rm SMEFT} = g_i\, \mathcal{O}_i^{D\leq 4} + \mathcal{C}_i\, \mathcal{O}_i^{D>4}$$

1. **Bottom-Up**
   - Write low-energy observables as $O(g_i, \mathcal{C}_i)$
   - Plug in determinations of "SM" couplings g_i (**)
   - Use EXP determination of observables to determine or bound $\mathcal{C}_i$
2. **Top-down**
   - Start with your UV-complete model with couplings λ_i
   - Do a matching to the SMEFT to find $g_i(\lambda_i)$ and $\mathcal{C}_i(\lambda_i)$

Meeting point is the matching scale, where Low-E pheno meets the UV.

(**) This is the subject of this talk

Simple example: Leptonic Tau decay

Consider using $\Gamma(\tau \to e\nu\bar{\nu})$ to constrain a Lepton-Universal BSM scenario:

$$\mathcal{L}_{BSM} = \sum_{i,j\in\{e,\mu,\tau\}} \left[C_{H\ell}^{(3)}\right](\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_i \sigma^I \gamma^\mu \ell_i) + \left[C_{\ell\ell}\right](\bar{\ell}_i \gamma^\mu \ell_j)(\bar{\ell}_j \gamma_\mu \ell_i)$$

Then (at tree level): $\mathcal{A}(\tau \to e\nu\bar{\nu}) \propto \frac{1}{v^2} + 2\, C_{H\ell}^{(3)} - C_{\ell\ell} \quad (= SM + NP)$

Going to the PDG, $v = 246.21965(6)\text{GeV}$

A measurement of $\Gamma(\tau \to e\nu\bar{\nu})$ then constrains $[2\, C_{H\ell}^{(3)} - C_{\ell\ell}]$

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WRONG : The PDG value of v comes from the measurement of τ_μ, which in this scenario is strictly speaking a determination of exactly $\tilde{v}$, with

$$\frac{1}{\tilde{v}^2} \equiv \frac{1}{v^2} + 2\, C_{H\ell}^{(3)} - C_{\ell\ell}$$

Simple example: Leptonic Tau decay

How to do it properly? (in general)

1. Reinterpret the PDG value: $\tilde{v} = 246.21965(6)\text{GeV}$

with $$\frac{1}{\tilde{v}^2} \equiv \frac{1}{v^2} + \big[C_{H\ell}^{(3)}\big]_{\mu\mu} + \big[C_{H\ell}^{(3)}\big]_{ee} - \big[C_{\ell\ell}\big]_{\mu ee\mu}$$

2. Rewrite the $\tau \to e\nu\bar{\nu}$ amplitude:

$$\begin{aligned}\mathcal{A}(\tau \to e\nu\bar{\nu}) \quad &\propto \quad \frac{1}{v^2} + \big[C_{H\ell}^{(3)}\big]_{\tau\tau} + \big[C_{H\ell}^{(3)}\big]_{ee} - \big[C_{\ell\ell}\big]_{\tau ee\tau} \\ &= \quad \frac{1}{\tilde{v}^2} + \big[C_{H\ell}^{(3)}\big]_{\tau\tau} - \big[C_{H\ell}^{(3)}\big]_{\mu\mu} - \big[C_{\ell\ell}\big]_{\tau ee\tau} + \big[C_{\ell\ell}\big]_{\mu ee\mu}\end{aligned}$$

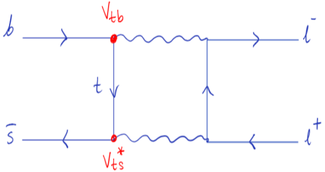
3. Substitute $\tilde{v} = 246.21965(6)\text{GeV}$ and use $\Gamma(\tau \to e\nu\bar{\nu})$ to constrain C_i.

Fixing CKM couplings in the SMEFT

The procedure to fix the "SM" couplings in NP scenarios is well known and has been discussed extensively.

But... what about Quark Flavor transitions?

Imagine you want to produce a SM prediction for $\mathcal{B}(B_s \to \mu^+\mu^-)$:

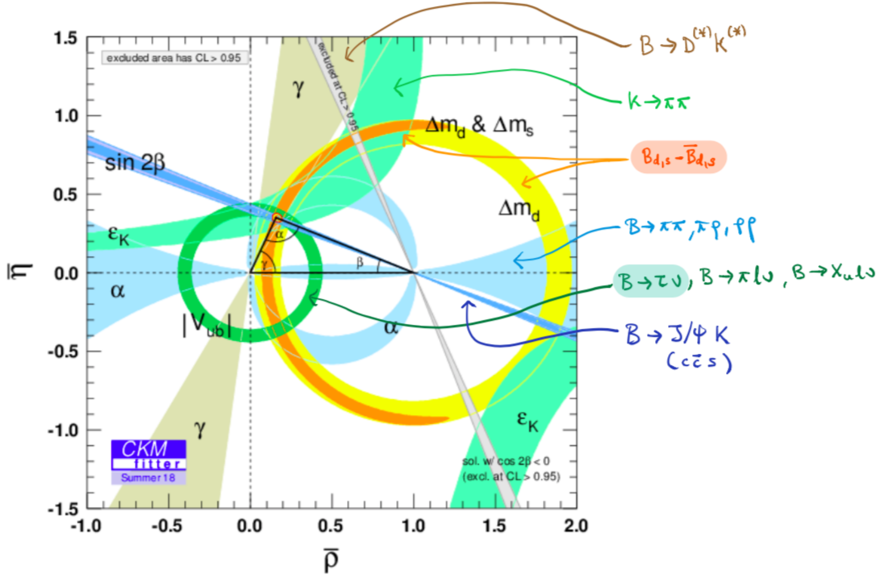


$$\mathcal{B}(B_s \to \mu^+\mu^-) \propto |V_{ts} V_{tb}|^2 \, f_{B_s} \times F\,(C_i\,)$$

In a general case with contributions to many SMEFT operators.

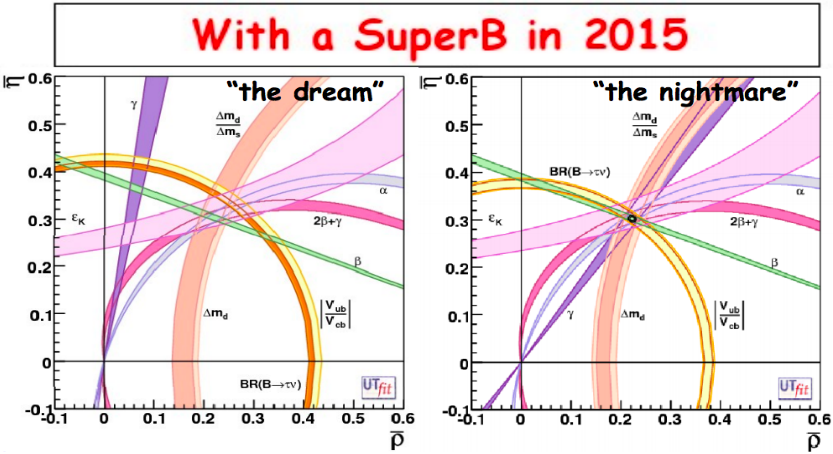
What should you use for V_{ij} ?

The SM CKM fit



The SM CKM fit

L.Silvestrini, Lattice'2008



How to translate this into BSM constraints / a BSM pattern ?

The CKM Matrix, Unitarity and the Wolfenstein Parameters

- $\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \mathcal{L}_{D>4} = \mathcal{L}_{\rm SM} + \sum_i C_i\, Q_i^{(6)} + \cdots\ ,$
- In the broken phase, $\mathcal{L}_{m_\psi} = -\sum_{\psi=u,d,e} \overline{\psi}_{R,i}\, [M_\psi]_{ij}\, \psi_{L,j} + \text{h.c.}$
- $\exists$ a weak basis s.t. $M_e = \text{diag},\ M_u = \text{diag},\ M_d = \text{diag} \cdot V^\dagger$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \text{CKM Matrix}$$

$$= \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(1+\frac{1}{2}\lambda^2)(\bar\rho-i\bar\eta) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\bar\rho-i\bar\eta) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\bar\rho-i\bar\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)\,.$$

- Wolfenstein Parameters : $W_i = \{\lambda, A, \bar\rho, \bar\eta\}$

Beyond The SM: Two paths

Path 1. Global fit to NP including the CKM parameters as free parameters.

Path 2. Take the model-independent determination of some "effective CKM parameters" from precise measurements (independent on the ones you use in your analysis) and use them as inputs.

Path 2 has some advantages (if you are looking at individual observables, or your analysis is very different from flavor and at a different scale [i.e. collider, top, higgs, ...]) and works very well under certain conditions (hierarchy of precisions, etc).

I advocate for and discuss Path 2.

The Strategy

We do:

1. Choose 4 "optimal" observables that depend on 4 orthogonal combinations of Wolfenstein parameters.
2. Absorb NP contributions into "effective" Wolfenstein parameters $\widetilde{W}_j$.
3. Extract numerical values for $\widetilde{W}_j$, and quote $W_j = \widetilde{W}_j - \delta W_j(C_k^{D=6})$.

$$O_i^{\rm exp} \stackrel{!}{=} O_i^{\rm th}(W_j) = \underbrace{O_i^{\rm SM}(W_j)}_{\sim 1} + \underbrace{O_i^{\rm NP}(W_j)}_{\sim 1/\Lambda^2} \equiv O_i^{\rm SM}(\widetilde{W}_j) \quad \Rightarrow \quad \widetilde{W}_j = \#_j$$

You do:

4. To calculate your observables $P_i(W_j, C_k^{D=6})$, you substitute $W_j \to \widetilde{W}_j - \delta W_j(C_k^{D=6})$, and re-expand in $1/\Lambda$.

Example: $\widetilde{V}_{us}$ from $K_{\mu 2}$

From now on it is convenient (not necessary) to define $\widetilde{V} \equiv V(\tilde{\lambda}, \tilde{A}, \tilde{\rho}, \tilde{\eta})$

$$\Gamma(K^- \to \mu^- \bar{\nu}_\ell) = \underbrace{|V_{us}|^2 (1 + \Delta_{K\mu 2})}_{|\widetilde{V}_{us}|^2} \frac{f_K^2 \, m_P m_\mu^2}{16\pi \tilde{v}^4} \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 (1 + \delta_{K\mu})$$

$$\Delta_{K\mu 2} = 2\,\mathrm{Re}(\epsilon_A^{\mu us}) - \frac{2\, m_P^2}{(m_u + m_q) m_\mu} \mathrm{Re}(\epsilon_P^{\mu us}) + 4\,\frac{\delta v}{v} + \mathcal{O}(\Lambda^{-4})$$

$$\epsilon_A^{\mu us} \equiv -1 - \frac{v^2}{2V_{us}} \left(\left[L_{\nu edu}^{V,LL}(\mu_s)\right]^*_{\mu\mu su} - \left[L_{\nu edu}^{V,LR}(\mu_s)\right]^*_{\mu\mu su} \right) ,$$

$$\epsilon_P^{\mu us} \equiv -\frac{v^2}{2V_{us}} \left(\left[L_{\nu edu}^{S,RR}(\mu_s)\right]^*_{\mu\mu su} - \left[L_{\nu edu}^{S,RL}(\mu_s)\right]^*_{\mu\mu su} \right) ,$$

A number for $|\widetilde{V}_{us}|$ can be used by trading $|V_{us}| \to |\widetilde{V}_{us}|(1 - \Delta_{K\mu 2}/2 + \cdots)$ in the observable of interest.

Choice of Observables

X **CP Asymmetries in Non-leptonic Decays :** (Matrix Elements)

$B \to \pi\pi, \rho\pi, \rho\rho$ (for α) $\quad B \to J/\psi K^{(*)}, (c\bar{c})K$ (for β)

$B \to D^{(*)}K^{(*)}$ (for γ) $\quad B_s \to J/\psi\phi, \psi(2S)\phi$ (for β_s)

X $b \to c\ell\nu$ **transitions :** (inclusive vs. exclusive)

X **Semi-leptonic Decays :** (momentum dependence)

$$K \to \pi\ell\nu\ (V_{us}),\ D \to K\ell\nu\ (V_{cs}),\ B \to \pi\ell\nu\ (V_{ub}), \ldots$$

✓ **Leptonic Decays :**

→ For λ, $K_{\ell 2}$ better than $D_{\ell 2}$ – (precision).

→ $K_{\ell 2}/\pi_{\ell 2}$ (f_K/f_π) better than $K_{\ell 2}$ (f_K) – (precision, lattice scale).

→ $B_{\ell 2}$ necessary.

✓ $\Delta M_{d,s}$: All Matrix Elements known from Lattice

Choice of Observables

Our subjective and time-dependent choice is :

$$\frac{\Gamma(K \to \mu\nu_\mu)}{\Gamma(\pi \to \mu\nu_\mu)} \qquad \Gamma(B \to \tau\nu_\tau)$$

$$\Delta M_d \qquad \Delta M_s$$

This choice is based on some criteria which are not universal, and on some circumstances that are local in time (e.g. precision, tensions, theory, . . .)

These observables fix the combinations

$$|\widetilde{V}_{us}/\widetilde{V}_{ud}| \qquad |\widetilde{V}_{ub}|$$

$$|\widetilde{V}_{tb}\widetilde{V}_{td}| \qquad |\widetilde{V}_{tb}\widetilde{V}_{ts}|$$

Extraction of the Wolfenstein Parameters

$$
\begin{aligned}
|\widetilde{V}_{us}/\widetilde{V}_{ud}| &= 0.23131 \pm 0.00050 &&= \tilde{\lambda} + \frac{1}{2}\tilde{\lambda}^3 + \frac{3}{8}\tilde{\lambda}^5 + \mathcal{O}(\lambda^7)\,, \\
|\widetilde{V}_{ub}| &= 0.00425 \pm 0.00049 &&= \tilde{A}\sqrt{\tilde{\rho}^2 + \tilde{\eta}^2}\left[\tilde{\lambda}^3 + \frac{1}{2}\tilde{\lambda}^5 + \mathcal{O}(\lambda^7)\right] \\
|\widetilde{V}_{tb}\widetilde{V}_{td}| &= 0.00851 \pm 0.00025 &&= \tilde{\lambda}^3\tilde{A}\sqrt{(1-\tilde{\rho})^2 + \tilde{\eta}^2} + \mathcal{O}(\lambda^7)\,, \\
|\widetilde{V}_{tb}\widetilde{V}_{ts}| &= 0.0414 \pm 0.0010 &&= \tilde{\lambda}^2\tilde{A} - \frac{1}{2}\tilde{\lambda}^4\tilde{A}(1-2\tilde{\rho}) + \mathcal{O}(\lambda^6)\,.
\end{aligned}
$$

$$
\begin{pmatrix} \tilde{\lambda} = \lambda + \delta\lambda \\ \tilde{A} = A + \delta A \\ \tilde{\rho} = \bar{\rho} + \delta\bar{\rho} \\ \tilde{\eta} = \bar{\eta} + \delta\bar{\eta} \end{pmatrix} = \begin{pmatrix} 0.22537 \pm 0.00046 \\ 0.828 \pm 0.021 \\ 0.194 \pm 0.024 \\ 0.391 \pm 0.048 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 & -0.16 & 0.05 & -0.03 \\ \cdot & 1 & -0.25 & -0.24 \\ \cdot & \cdot & 1 & 0.83 \\ \cdot & \cdot & \cdot & 1 \end{pmatrix}
$$

Comparison to SM fits

Comparing apples and oranges:

CKMfitter (SM) [1]	UTfit (SM) [2]	This work (SMEFT)
$\lambda = 0.224747^{+0.000254}_{-0.000059}$	$\lambda = 0.2250 \pm 0.0005$	$\tilde{\lambda} = 0.22537 \pm 0.00046$
$A = 0.8403^{+0.0056}_{-0.0201}$	$A = 0.826 \pm 0.012$	$\tilde{A} = 0.828 \pm 0.021$
$\bar{\rho} = 0.1577^{+0.0096}_{-0.0074}$	$\bar{\rho} = 0.148 \pm 0.013$	$\tilde{\rho} = 0.194 \pm 0.024$
$\bar{\eta} = 0.3493^{+0.0095}_{-0.0071}$	$\bar{\eta} = 0.348 \pm 0.010$	$\tilde{\eta} = 0.391 \pm 0.048$

[1] CKMfittler collaboration, http://ckmfitter.in2p3.fr/www/html/ckm results.html

[2] UTfit collaboration, http://www.utfit.org/UTfit/Results

NP contributions to Wolfenstein Parameters

The NP contributions to the Wolfenstein Parameters are:

$$\begin{pmatrix} \delta\lambda \\ \delta A \\ \delta\bar{\rho} \\ \delta\bar{\eta} \end{pmatrix} = M(\tilde{\lambda}, \tilde{A}, \tilde{\rho}, \tilde{\eta}) \begin{pmatrix} \Delta_{K/\pi} \\ \Delta_{B\tau 2} \\ \Delta_{\Delta M_d} \\ \Delta_{\Delta M_s} \end{pmatrix} .$$

Numerically,

$$M(\tilde{\lambda}, \tilde{A}, \tilde{\rho}, \tilde{\eta}) = \begin{pmatrix} 0.1070(2) & 0 & 0 & 0 \\ -0.786(20) & -0.0040(9) & 0.0167(6) & 0.402(10) \\ 0.286(24) & 0.094(22) & -0.390(10) & 0.296(23) \\ -0.385(18) & 0.200(19) & 0.184(10) & -0.384(19) \end{pmatrix}$$

where the correlations can & should be included (see paper).

Applications I : Leptonic Decays

Pion decay :

$$\Gamma(\pi \to \mu\nu) = \left|1 - \frac{\tilde{\lambda}^2}{2} - \frac{\tilde{\lambda}^4}{8}\right|^2 \frac{f_{\pi^\pm}^2 m_{\pi^\pm} m_\mu^2}{16\pi \tilde{v}^4} \left(1 - \frac{m_\mu^2}{m_{\pi^\pm}^2}\right)^2 (1 + \delta_{\pi\mu}) \left[1 + \widetilde{\Delta}_{\pi\mu 2}\right]$$

$$\widetilde{\Delta}_{\pi\mu 2} = 2\,\mathrm{Re}(\epsilon_A^{\mu ud}) - \frac{2 m_{\pi^\pm}^2}{(m_u + m_d) m_\mu} \mathrm{Re}(\epsilon_P^{\mu ud}) + 4\frac{\delta v}{v} + 2\tilde{\lambda}(1 + \tilde{\lambda}^2)\delta\lambda + \mathcal{O}(\Lambda^{-4}, \tilde{\lambda}^6)$$

$$\left.\begin{array}{l} \mathcal{B}(\pi \to \mu\nu) = 0.9998770(4) \\ \tau_\pi = 2.6033(5) \cdot 10^{-8} s \end{array}\right\} \quad \Rightarrow \quad \widetilde{\Delta}_{\pi\mu 2} = 0.004 \pm 0.013$$

D meson decay :

$$\Gamma(D \to \ell\nu) = |\tilde{\lambda}|^2 \frac{f_{D^\pm}^2\, m_{D^\pm}\, m_\ell^2}{16\pi\tilde{v}^4} \left(1 - \frac{m_\ell^2}{m_{D^\pm}^2}\right)^2 (1 + \delta_{D\ell}) \left[1 + \widetilde{\Delta}_{D\ell 2}\right]$$

$$\widetilde{\Delta}_{D\ell 2} = 2\,\mathrm{Re}(\epsilon_A^{\ell cd}) - \frac{2\, m_{D^\pm}^2}{(m_c + m_d)m_\ell}\, \mathrm{Re}(\epsilon_P^{\ell cd}) + 4\frac{\delta v}{v} - 2\frac{\delta\lambda}{\tilde{\lambda}} + \mathcal{O}(\Lambda^{-4}, \tilde{\lambda}^4)$$

$$\left.\begin{array}{l} \mathcal{B}(D \to \mu\nu) = 3.74\,(17) \cdot 10^{-4} \\ \tau_{D^\pm} = 1.040(7) \cdot 10^{-12} s \end{array}\right\} \quad \Rightarrow \quad \widetilde{\Delta}_{D\mu 2} = -0.089 \pm 0.043$$

Applications II : Exclusive Hadronic W Decays

Corrections to W couplings in the SMEFT:

$$\mathcal{L}_{\rm SMEFT} \supset \frac{\tilde{g}_L}{\sqrt{2}} W^{\mu+} \bar{u}_{Lj}\gamma_\mu \left(V_{jk} + \left[\delta g_L^{Wq}\right]_{jk} \right) d_{Lk} + \text{h.c.}$$

with

$$\begin{aligned} \left[\delta g_L^{Wq}\right]_{jk} &= [C_{Hq}^{(3)}]_{jl} V_{lk} + \frac{\tilde{g}_L^2 \tilde{v}^2}{\tilde{g}_L^2 - \tilde{g}_Y^2} \Bigg[-\frac{\tilde{g}_Y}{\tilde{g}_L} C_{HWB} - \frac{1}{4} C_{HD} + \frac{1}{4} [C_{\ell\ell}]_{e\mu\mu e} \\ &\quad + \frac{1}{4}[C_{\ell\ell}]_{\mu e e\mu} - \frac{1}{2}[C_{H\ell}^{(3)}]_{ee} - \frac{1}{2}[C_{H\ell}^{(3)}]_{\mu\mu} \Bigg] V_{jk} + \mathcal{O}(\Lambda^{-4}) \end{aligned}$$

Taking $V_{jk} \to \tilde{V}_{jk} - \delta V_{jk}$, we have

$$\frac{\Gamma(W \to u_j d_k)}{\Gamma(W \to u_j d_k)_{\rm SM}} = 1 + 2\,\text{Re}\left(\frac{\left[\delta g_L^{Wq}\right]_{jk} - \delta V_{jk}}{\widetilde{V}_{jk}} \right),$$

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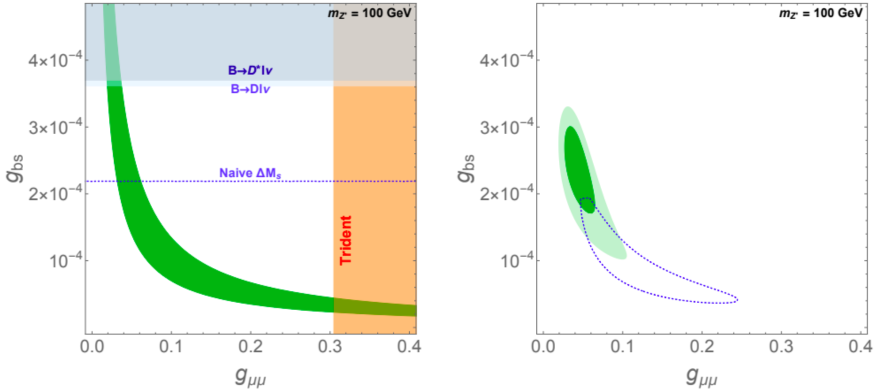
Where the result of our analysis gives:

$$\begin{aligned}
\delta V_{ud} &= \delta V_{cs} = -\tilde{\lambda}\,\delta\lambda + \mathcal{O}(\tilde{\lambda}^4)\ , \\
\delta V_{us} &= -\delta V_{cd} = \delta\lambda + \mathcal{O}(\tilde{\lambda}^5)\ , \\
\delta V_{ub} &= 3\tilde{A}\tilde{\lambda}^2(\tilde{\rho} - i\tilde{\eta})\,\delta\lambda + \tilde{\lambda}^3(\tilde{\rho} - i\tilde{\eta})\,\delta A + \tilde{A}\tilde{\lambda}^3(\delta\rho - i\delta\eta) + \mathcal{O}(\tilde{\lambda}^5)\ , \\
\delta V_{cb} &= 2\tilde{A}\,\tilde{\lambda}\,\delta\lambda + \tilde{\lambda}^2\,\delta A + \mathcal{O}(\tilde{\lambda}^6)\ .
\end{aligned}$$

Applications III : $b \to s\ell\ell$ Anomalies

$$\mathcal{L}_{BSM} = g_{bs} Z'_\rho \left(\bar{q}_2 \gamma^\rho q_3 + \text{h.c.} \right) - g_{\mu\mu} Z'_\rho \bar{\ell}_2 \gamma^\rho \ell_2$$



Imagine having NP contributions to $b \to c\ell\nu$ transitions also.

Summary

☐ Determination of CKM parameters affected by $D = 6$ ops in the SMEFT

☐ Cannot use SM fit. We set up a consistent strategy.

☐ We identify a set of 4 good observables to extract the "tilde" Wolf Pars:

$\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu)\ , \quad \Gamma(B \to \tau\nu_\tau)\ , \quad \Delta M_d\ , \quad \Delta M_s$

☐ Our results are : (with a given correlation matrix)

$\tilde{\lambda} = 0.22537(46)\ , \quad \tilde{A} = 0.828(21)\ , \quad \tilde{\rho} = 0.194(24)\ , \quad \tilde{\eta} = 0.391(48)\ .$

☐ Also necessary are the NP contributions $\delta W(C_i) = \{\delta\lambda, \delta A, \delta\rho, \delta\eta\}$ depending on the SMEFT Wilson coefficients C_i at the matching scale Λ.

Any observable can now be written as $Obs = Obs(W, C_i) = Obs(\widetilde{W}, C_i)$

☐ **Important for automation of EFT Analyses**