Impact of the QCD dynamics on the determination of the high energy astrophysical neutrino flux

NuFact 20|21

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The number of UHE events at the neutrino detector:

$$
\begin{aligned}
N_{(l_\beta^\pm)} &= \epsilon t \int_{x_0}^{1} dx \int_{-1}^{1} dc\theta_\nu \int_{0}^{2\pi} d\phi_\nu \int_{E_{\nu,0}}^{E_{\nu,f}} dE_\nu \int_{E_{l,0}}^{E_{l,f}} dE_l \\
\times & \sum_{\alpha=e,\mu,\tau} \sum_{\bar{\nu},\nu} n_{\mathrm{t}}(E_\nu) \times \Phi_{\nu_\alpha}(E_\nu, c\theta_\nu, \phi_\nu) \times P_{\nu_\alpha \to \nu_\beta}(E_\nu, c\theta_\nu) \\
\times & \; S(E_\nu, c\theta_\nu, \sigma_{\nu N}) \times \frac{\partial \sigma_{\nu_\beta}(E_\nu, E_l, x)}{\partial E_\nu \partial E_l \partial x}
\end{aligned}
$$

$\sigma_{\nu N}$: main ingredient in the calculation of $N_{(l_\beta^\pm)}$

HESE: $\longrightarrow$ small x_{Bj} and large Q^2 at same time!

Neutrino DIS at NLO: $\rightarrow m_l \neq 0; \quad m_N \neq 0; \quad F_i \rightarrow F_i'$

S. Kretzer and M. Reno. Phys Rev **D 66** 113007 (2002)

$$
\begin{aligned}
\frac{\partial^2 \sigma^{\nu(\bar{\nu})}}{\partial x \partial y} &= \frac{G_F^2 m_N E_\nu}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \left\{ \left(xy^2 + \frac{m_l^2 y}{2E_\nu m_N} \right) F_1 \right. \\
&+ \left[\left(1 - \frac{m_l^2}{4E_\nu^2} \right) - \left(1 + \frac{m_N x}{2E_\nu} \right) y \right] F_2 \\
&\pm \left[xy \left(1 - \frac{y}{2} \right) - \frac{m_l^2 y}{4E_\nu^2 m_N} \right] F_3 \\
&+ \left. \frac{m_l^2 (m_l^2 + Q^2)}{4E_\nu^2 m_N^2 x} F_4 - \frac{m_l^2}{E_\nu m_N} F_5 \right\}
\end{aligned}
$$

Albright- Jarlskog relations @ LO:

- $F_1 = \frac{1}{2x} F_2 \quad , \quad F_5 = \frac{1}{2x} F_2 \quad$ and $\quad F_4 = 0$

The DGLAP equations give the evolution on Q^2 of the PDF's $\leftrightarrow$ only linear dynamics!

- The *initial conditions* are non-perturbative
- They must be infered from the experiments

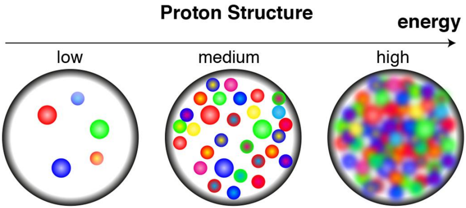
Several groups worldwide who calculate the PDF's

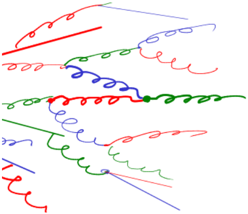
- ABM by S. Alekhin, J. Bluemlein, S. Moch
- CTEQ, from the CTEQ Collaboration
- GRV/GJR, from M. Gluck, P. Jimenez-Delgado, E. Reya, and A. Vogt
- HERA PDFs, by H1 and ZEUS collaborations from DESY
- MRST/MSTW, from A. D. Martin, R. G. Roberts, W. J. Stirling, R. S. Thorne, and G. Watt
- NNPDF, from the NNPDF Collaboration

PDF's and parton saturation

L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Rep. 100, 1(1983)

- At high energies (small x) the nucleon is a approximatelly a sea of gluons
- Both quarks and gluons emite gluons, which generates the growth of $xg(x, Q^2)$
- $xg(x, Q^2)$ so hight that the recombination $g + g \to g$ becomes important





L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Rep. 100, 1(1983)

DLLA approximation: $ln(1/x) >> 1$; $ln(Q^2) >> 1$

$$Q^2 \frac{\partial^2 xg(x, Q^2)}{\partial ln(1/x) \partial Q^2} = \frac{\alpha_s N_c}{\pi} xg(x, Q^2) - \frac{4\alpha_s^2 N_c}{3 C_F R^2} \frac{1}{Q^2} [xg(x, Q^2)]^2$$

The non-linear term is the responsible for the attenuation of the growth of $xg(x, Q^2)$

Few words from Prof. G. Altarelli @ INSS 2009

(at the coffee-break)

https://cds.cern.ch/record/41606?ln=en

Even worse, the theory does not predict the energy scale where such processes become important.

- There is no question about the existence of non-linear terms.
- Quark and gluon recombination are process contained in the SM.
- Also, it is expected that at some point the linear dynamics will need corrections.
- The problem is that the theory does not predict the importance of these corrections.
- We will have to extract it from the data!

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- We will have to extract it from the data.

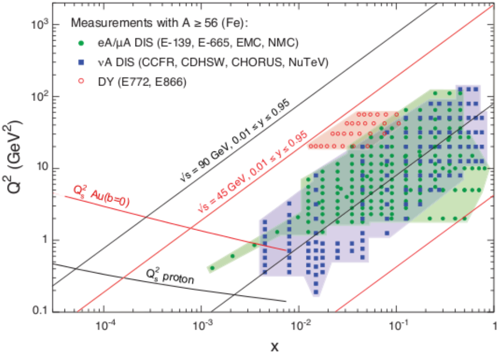
Until today there is no clear signature of saturation!

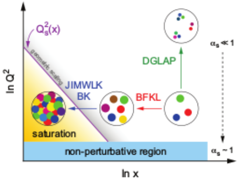
Prof. Yuri Kovchegov: Introduction to saturation physics

https://conference.ippp.dur.ac.uk/event/1005/?print=1

Can Saturation be Discovered at EIC?

EIC has an unprecedented small-x reach for DIS on large nuclear targets, allowing to seal the discovery of saturation physics and study of its properties:





BFKL (BGR18): V. Bertone, R. Gauld, J. Rojo, JHEP 01, 217 (2019).

$\longrightarrow$ Evolution is in $ln(1/x)$

- Estimation of the structure functions using the framework of collinear factorization at NNLO
- Take into account the small-x BFKL resummation up to next-to-leading logarithmic (NNLx)
- Basic motivation: to include the BFKL corrections due to $\alpha_s ln(1/x)$
- They are expected to contribute in the kinematical range probed in neutrino telescopes

Both approaches, DGLAP(CT14) and BFKL(BGR18), predict the increasing of $\sigma_{\nu N}$ with E_ν

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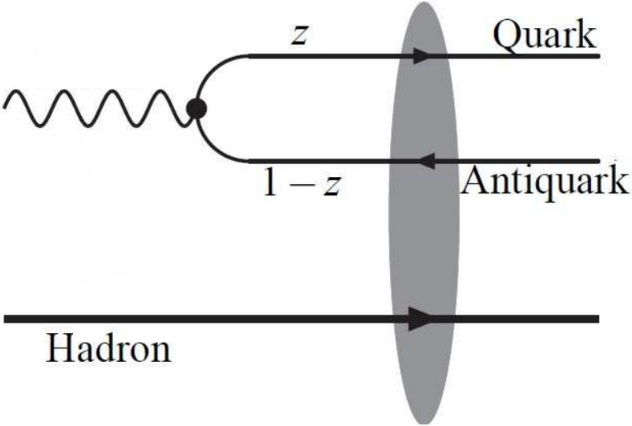
Both approaches, DGLAP(CT14) and BFKL(BGR18), predict the increasing of $\sigma_{\nu N}$ with E_ν

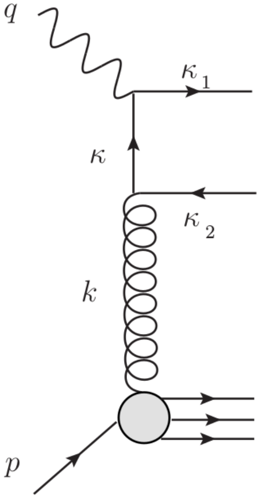
- This is expected since they are based on linear evolution Eqs.
- Only consider parton emissions ($g \to gg$)
- Disregard recombination effects ($gg \to g$) at high partonic density

Including Non-Linear Effects with Dipole Formalism



Including Non-Linear Effects within Dipole Formalism

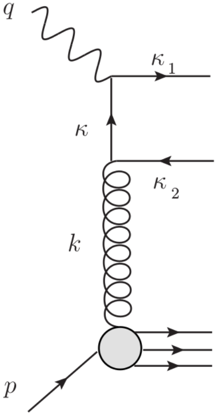
- In this picture the boson, $W^{\pm}(q)$ or $Z^0(q)$ splits into a pair $q\bar{q}$
- Color dipole then interacts with the target
- r denotes the transverse size of the dipole
- z is the longitudinal momentum fraction carried by a quark
- $F_2^{CC} = F_T^{CC} + F_L^{CC}$



Including Non-Linear Effects within Dipole Formalism

$$F_{T,L}^{CC}(x, Q^2) = \frac{Q^2}{4\pi^2} \int_0^1 dz \int d^2\mathbf{r} |\Psi_{T,L}^W(\mathbf{r}, z, Q^2)|^2 \sigma^{dh}(\mathbf{r}, x)$$

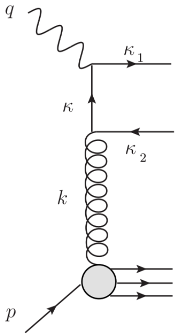
- $|\Psi_{T,L}^W(\mathbf{r}, z, Q^2)|^2$ are proportional to the wave functions
- $\sigma^{dh}(\mathbf{r}, x)$ describes the interaction of the color dipole with the target
- $\sigma^{dh}(\mathbf{r}, x) \rightarrow$ strong interaction
- $\sigma^{dh}(\mathbf{r}, x) \rightarrow$ Model dependent!



Including Non-Linear Effects within Dipole Formalism

$$\sigma^{dh}(x, \mathbf{r}) = 2 \int d^2\mathbf{b}\, \mathcal{N}^h(x, \mathbf{r}, \mathbf{b}) \ ,$$

- $\mathcal{N}^h$ is the forward dipole-target scattering amplitude
- $\mathbf{b}$ is the paramenter of impact
- $\mathcal{N}(x, \mathbf{r}, \mathbf{b}) = \mathcal{N}(x, \mathbf{r}) S(\mathbf{b})$
- $\sigma^{dp}(x, \mathbf{r}) = \sigma_0 \mathcal{N}(x, \mathbf{r})$
- $\mathcal{N}(x, \mathbf{r})$ energy evolution can be given by the running coupling Balitsky - Kovchegov equation



[I. Balitsky, Nucl. Phys. B 463, 99 (1996); Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999); J. L. Albacete. et. al. Phys.Rev. D71 (2005) 014003]

Color Glass Condensate formalism: CGC (IIMS)

Iancu, E., Itakura, K.; e Munier, S.Physics Letters B, v. 590, p. 199208, 2004. ⊕

Rezaeian, A. H.; Schmidt, I.: Phys. Rev. D88, n. 7, p. 074016, 2013

Assymtoptic solutions of BK equation $\longrightarrow$ Phenomelogical model:

$$\mathcal{N}(r,Y) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{ln(2/rQ_s)}{k\lambda Y}\right)} & \rightarrow \quad rQ_s \leq 2 \\ 1 - e^{-A ln^2(BrQ_s)} & \rightarrow \quad rQ_s > 2 \end{cases}$$

- $rQ_s = 2$ is the saturation condition
- A and B must be determined from the continuity condition at the saturation condition

$$A = \frac{\mathcal{N}_0^2 \gamma_s^2}{(1-\mathcal{N}_0^2) ln(1-\mathcal{N}_0^2)} \quad ; \quad B = \frac{1}{2}(1-\mathcal{N}_0)^{-\frac{1-\mathcal{N}_0}{\mathcal{N}_0^2\gamma_s^2}}$$

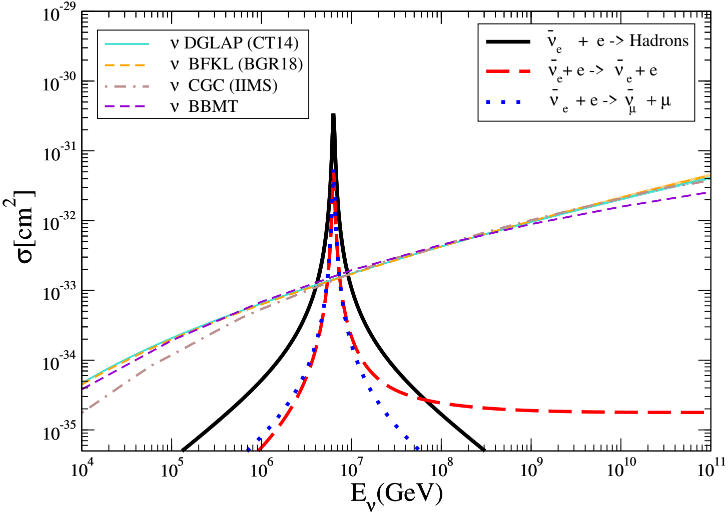
Froissart bound (BBMT) M.M. Block, L. Durand, P. Ha, D.W. McKay, Phys. Rev. D 88(1) (2013)

- Alternative way to include the unitarity (saturation) effects at all orders
- Motivation: Successful descriptions of hadron - hadron and γ-hadron total cross sections:
- $\sigma_{ip} \propto ln^2(S) \quad ; \quad i = \gamma, p$
- Main assumption for $\sigma_{\nu N}$: the growth on the proton structure function is limited by the Froissart bound at high energies
- Bound on F_2 when $x \to 0$.
- Bound on neutrino-hadron cross-section:

$$\sigma_{\nu N} \propto ln^3(E_\nu)$$

Our results:

V.P. Gonçalves *et. al.* Eur. Phys. Journ. C (2021) 81:496

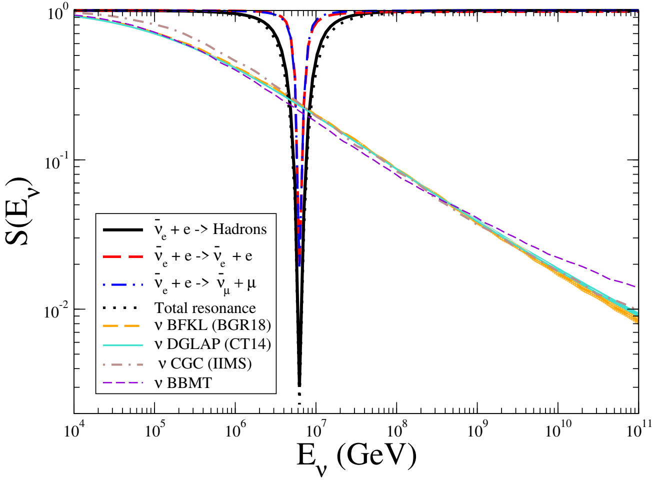


CT14 and BGR18 error bands are included and are small!

The ν absorption by the Earth

V.P. Gonçalves *et. al.* Eur. Phys. Journ. C (2021) 81:496

$$S(E_\nu) = \int_0^1 dcos(\theta_{\rm zenith}) exp\left\{-k_j \sigma_{\nu j}(E_\nu) \int_0^{r(\theta_{\rm zenith})} \rho(r) dr\right\}$$



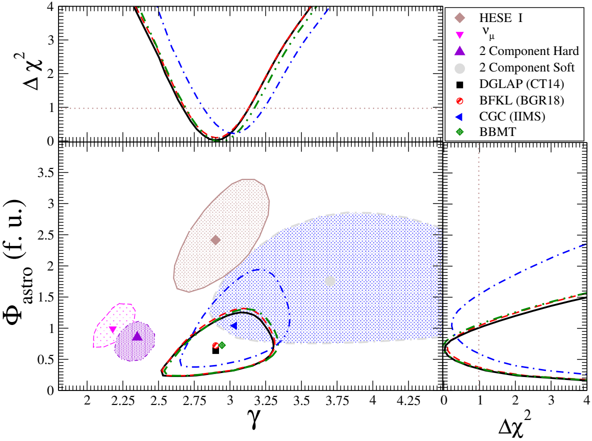
$k_N = N_A$ and $k_e = < Z/A > N_A$

Errata!!!

- After the publication of this results, we found an error in our code in the definition of variables associated with the astrophysical neutrino flux normalization
- As consequence, the values of the astrophysical neutrino flux normalization we report in Table 1 must be multiplied by a factor of 3
- It also implies modifications in Figs 3 and 4 of the paper
- Fortunately, this error does not affect any other aspect of our analysis nor the conclusions of the paper
- Indeed, this correction implied a significant improvement in the agreement of our results and the value reported by the IceCube Collaboration in what concerns the astrophysical neutrino flux normalization
- In what follows, we correct the Table and figures above mentioned

Errata: Fig 3 of V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496

$$\Phi_\nu(E_\nu) = \Phi_{astro} \times (E_\nu/10^5\,GeV)^{-\gamma}\ ;\quad \Phi_{astro} = 3\phi_0 \approx 2\ [f.u.]$$



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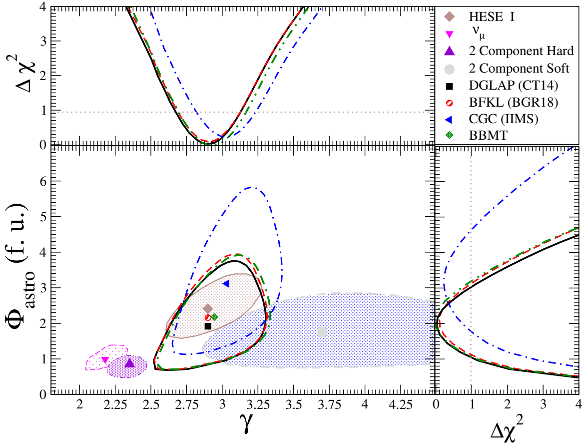


Better agreement with results from IceCube 6Y!

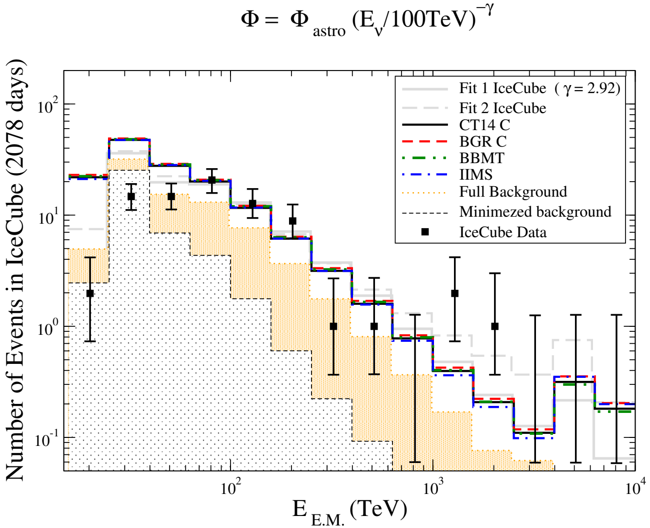
Errata: Table 1 of V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496

$\Phi_\nu(E_\nu) = \Phi_{astro} \times (E_\nu/10^5\,GeV)^{-\gamma}$; $\Phi_{astro} = 3\phi_0 \approx 2$ [$f.u.$]

$\gamma \pm \delta\gamma$	$\Phi_{astro} \pm \delta\Phi_{astro}$(f.u.)	$\mathcal{X}^2_{min}$
	DGLAP (CT14)	
$2.90^{+0.23}_{-0.22}$	$0.64^{+0.33}_{-0.28} \rightarrow 1.92^{+0.33}_{-0.28}$	10.81
	BFKL (BGR18)	
$2.94^{+0.19}_{-0.26}$	$0.72^{+0.44}_{-0.20} \rightarrow 2.16^{+0.44}_{-0.20}$	10.90
	CGC (IIMS)	
$3.03^{+0.20}_{-0.18}$	$1.04^{+0.48}_{-0.47} \rightarrow 3.12^{+0.48}_{-0.47}$	11.02
	BBMT	
$2.94^{+0.26}_{-0.25}$	$0.72^{+0.33}_{-0.40} \rightarrow 2.16^{+0.33}_{-0.40}$	10.76

Best fit values for the extra-galactic neutrino flux derived assuming different approaches for the QCD dynamics.

V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496



IceCube Gen2:

- We also Consider the future extension of IceCube to the IceCube Gen2 configuration
- Increasing of 40 times the actual exposure
- We assume the predictions from CT14 as the signal
- Then we study the sensitivity of IceCube Gen2 to the models

Errata: V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496

$$\Phi_\nu(E_\nu) = \Phi_{astro} \times (E_\nu/10^5\,GeV)^{-\gamma}\ ; \quad \Phi_{astro} = 3\phi_0 \approx 2\ [f.u.]$$

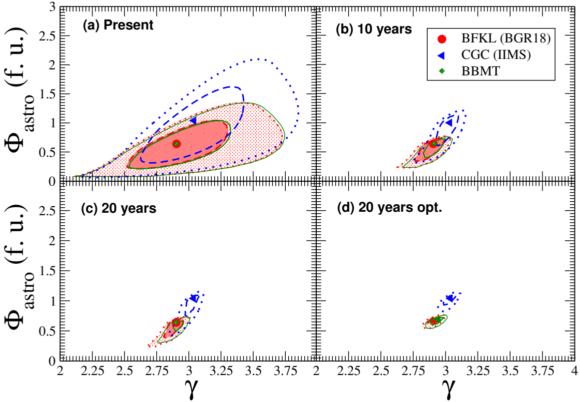


Figure: **The values of Φ_{astro} we report must be multiplied by 3**

Errata: V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496

$$\Phi_\nu(E_\nu) = \Phi_{astro} \times (E_\nu/10^5\,GeV)^{-\gamma}\ ;\quad \Phi_{astro} = 3\phi_0 \approx 2\ [f.u.]$$

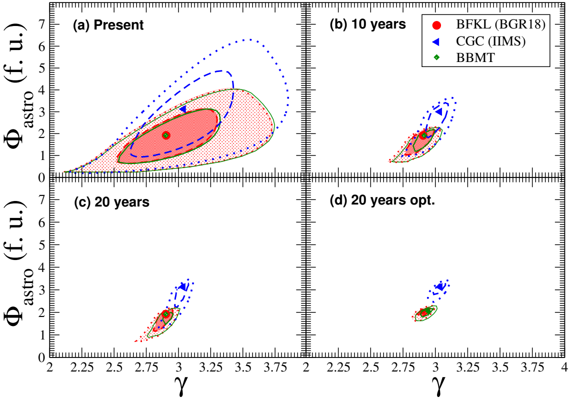


Figure: Effects in the allowed region of parameters due to increments in the IceCube exposition. In all the cases, we assume the DGLAP (CT14) prediction as the observed number of events.

Errata: V.P. Gonçalves *et. al.* Eur. Phys. J. C (2021) 81:496

- I would like to make it clear that it was MY MISTAKE !!
- The other authors, and in special the Ph. D. student, had nothing to do with it ;)

Conclusions from 6Y

- $E_i(E_{vis})$ at IceCube including DGLAP, BFKL, CGC, BBMT.
- We find the best estimates for Φ_{ASTRO} and γ using a maximum likelihood fit .
- The modifications in the noralization and energy dependence of $\sigma_\nu(E_\nu)$ can be compensated by different values for the Φ_{ASTRO} and γ.
- All the models can describe de data successfully and cannot be disregard even at 68% of C.L.

Conclusions from IceCube-Gen2 sensitivities

- The higher number of events implies the reduction of the overlap of the allowed areas for the neutrino flux parameters.
- The opt. case, where $\sigma_{\nu,pt} \rightarrow 0.15$ let to completelly different allowed regions for the flux parameters.
- The increase of the detection exposure is not enough to allow us to fully discriminate between the models studied.
- Next step: Angular distribution of events!

Thank you!!

BACKUP SLIDES

1-D Approach To compute the N_ν @ the IceCube detector

Taking into account only the energy deppendence:

$$\begin{aligned} dN_{events} &= T \sum_{\nu+\bar{\nu}} N_{eff,\alpha}(E_\nu) \otimes \Phi\nu_\alpha(E_\nu) \otimes \sigma_{\nu_\alpha}(E_\nu) \\ &\times \frac{1}{2}(1 + S(E_\nu)) \times d(E_{vis}), \end{aligned}$$

- Must sum the contributions from $\bar{\nu}_\alpha$ and ν_α
- T is the time of data taken, 2028 days
- $S(E_\nu)$ is the absorption function inside the Earth

Systematics: *Pull Method* $\longrightarrow \Phi_{\mu_{atm}}, \Phi_{\nu_{atm}}, \Phi_{\mu_{prompt}}, \delta E_{E.M.}$

$$\begin{aligned}
\mathcal{X}^2 &\equiv -2\ ln(\lambda_{Poi}) + \text{Systematics} \\
&= 2\sum_{i=1}^{n}\left\{(E_i - O_i) + O_i\ ln\left(\frac{O_i}{E_i}\right)\right\} \\
&+ \sum_{j=1}^{m}\left(\frac{\theta_j - \theta_j^*}{\sigma_j}\right)^2
\end{aligned}$$

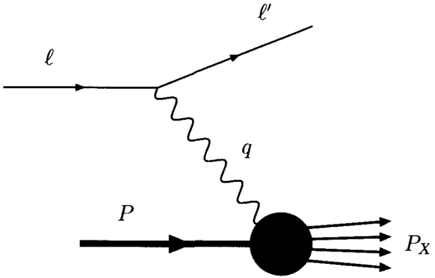
- $E_i = Signal + BKGD = E_{astro,i} + E_{\mu_{atm},i} + E_{\nu_{atm},i} + E_{\nu_{charm},i}$
- $E_{astro,i} = \int dN_{\text{events}}(\Phi_{\text{astro}}, \gamma, \sigma_\nu, \delta E_{E.M.})$
- $E_{\mu_{atm},i} = \Phi_{\mu_{atm}} E'_{\mu_{atm},i}$; $E_{\nu_{atm},i} = \Phi_{\nu_{atm}} E'_{\nu_{atm},i}$
- $E_{\mu_{prompt},i} = \Phi_{\mu_{prompt}} E'_{\mu_{prompt},i}$

The priors we adopt: $\theta^* \pm \sigma_j$

- Atmospheric muon flux normalization, $\Phi_\mu \longrightarrow (1.00 \pm 0.5)$
- Conventional atmospheric neutrino flux normalization, $\Phi_\nu \longrightarrow (1.00 \pm 0.30)$
- Atmospheric *prompt neutrinos* normalization, $\Phi_{\nu_{pt}} \longrightarrow (0.00 \pm 0.65)$
- Energy resolution, $\delta E \longrightarrow (1.00 \pm 0.15)$

M. G. Aartsen et al. [IceCube Collaboration], Astrophys.J. 809, no. 1, 98 (2015)

Neutrino Interaction at UHE: Deep Inelastic Scattering (from Barone and Predazzi book)



$$W^2 = (P+q)^2$$

$$\nu \equiv \frac{P.q}{m_N} = \frac{W^2 + Q^2 - m_N^2}{2m_N}$$

$$x \equiv \frac{Q^2}{2P.q} = \frac{Q^2}{2m_N \nu}$$

$$x = \frac{Q^2}{W^2 + Q^2 - m_N^2}$$

$$y \equiv \frac{P.q}{P.l} = \frac{W^2 + Q^2 - m_N^2}{s - m_N}$$

Deep Inelastic Scattering Formalism

V. Barone and E. Predazzi book:

$$d\sigma = \frac{1}{4(l.P)} \frac{1}{2} \sum_{s_l s_{l'}} \frac{1}{2} \sum_{S} \sum_{X} \int \frac{dP_X}{(2\pi)^3 2P_X^0} \frac{d^3 l'}{(2\pi)^3 2E'} \times (2\pi)^4 \delta^4(P + l - P_X - l') |\mathcal{M}|^2$$

- Average over initial lepton spin and nucleon spin
- Sum over spin of final state
- $|\mathcal{M}|^2$ contains all the dynamic of the interaction

$|\mathcal{M}|^2$ contains all the dynamic of the interaction

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{q^2}[\bar{u}_{\nu_l}\gamma^a(1-\gamma_5)u_l][\bar{u}_{\nu_l}\gamma^c(1-\gamma_5)u_l]^* \\
&\times \langle X|J^a(0)|P,S\rangle^*\langle X|J^c(0)|P,S\rangle
\end{aligned}
$$

The leptonic tensor:

$$
\begin{aligned}
L^{ac}(\nu) &\equiv \frac{1}{2}\sum_{s_l s_{l'}}[- - - - - -]^*[- - - - - -] \\
&= 8(l^a l'^c + l^c l'^a + g^{ac} l.l' - i l'_\alpha l_\beta \epsilon^{a\alpha c\beta})
\end{aligned}
$$

The hadronic tensor

$$W^{ac} \equiv \frac{1}{2\pi}\frac{1}{2}\sum_S \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} \delta^4(P + q - P_X) \times \langle P, S|J^a(0)|\rangle \langle X|J^c(0)|P, S\rangle$$

$$W^{ac} = \frac{1}{2\pi} \int d^4 z e^{iq.z} \langle N|J^a(z) J^c(0)|N\rangle$$

The hadronic tensor can be parameterized in terms of the structure functions $F_i(x, Q^2)$

$$\frac{1}{2m_N} W^{ac}(l^{\pm}) \equiv 2\left(-g_{ac} + \frac{q_a q_c}{q^2}\right) F_1(x, Q^2)$$
$$+ \frac{2}{(P.q)}\left[\left(P_a - \frac{P.q}{q^2}q_a\right)\left(P_c - \frac{P.q}{q^2}q_c\right)\right] F_2(x, Q^2)$$

Where

$$\begin{aligned} F_1(x, Q^2) &\equiv m_N W_1(\nu, Q^2) \\ F_2(x, Q^2) &\equiv \nu W_2(\nu, Q^2) \\ \nu &= \frac{P.q}{m_N} = \frac{W^2 + Q^2 - m_N}{2m_N} \end{aligned}$$

The Factorization Theorem relates the Hadronic W^{ac} and the partonic $\hat{\omega}^{ac}$ forward matrix element

- μ is the factorization scale
- ζ is the fraction of momentum of the nucleon carried by the parton
- $f(\zeta, \mu^2)$ is the parton distribution function
- $P_N^+ = (P_N^0 + P_N^z)/\sqrt{2}$

$$W_N^{ac} = \int \frac{d\zeta}{\zeta} \, f(\zeta, \mu^2) \, \hat{\omega}_p^{ac}|_{p^+ = \zeta P_N^+}$$

Neutrino DIS: LO $\times$ NLO

Albright- Jarlskog relations @ LO

- $F_1 = \frac{1}{2x} F_2$
- $F_4 = 0$
- $F_5 = \frac{1}{2x} F_2$

S. Kretzer and M. Reno. Phys Rev **D 66** 113007 (2002)

$\longrightarrow$ Target mass corrections (charm mass)

Neutrino DIS at NLO: $\rightarrow m_l \neq 0; \quad m_N \neq 0; \quad F_i \rightarrow F_i'$

Kinematic corrections @ the integration limits:

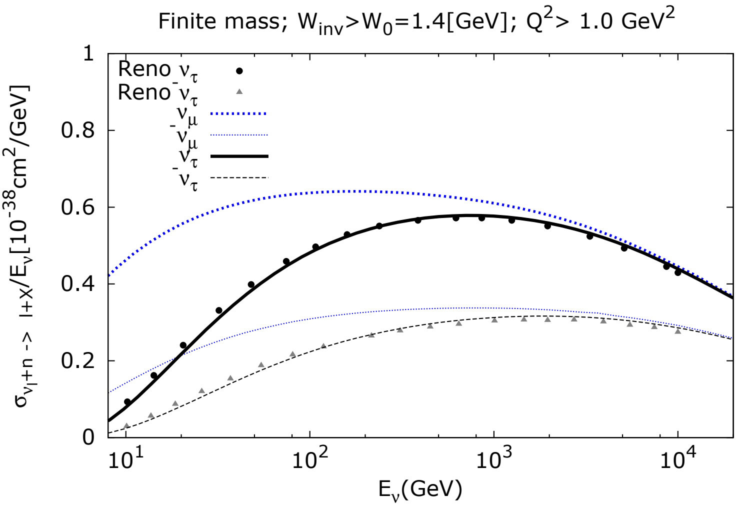
$$
\begin{aligned}
\frac{m_l^2}{2m_N(E_\nu - m_l)} \leq & \quad x \quad \leq 1 \\
a - b \leq & \quad y \quad \leq a + b
\end{aligned}
$$

where:

$$
\begin{aligned}
a/d &= 1 - m_l^2 \left(\frac{1}{2m_N E_\nu x} + \frac{1}{2E_\nu^2} \right) \\
b/d &= \sqrt{\left(1 - \frac{m_l^2}{2m_N E_\nu x}\right)^2 - \frac{m_l^2}{E_\nu^2}} \\
d &= 2\left(1 + \frac{m_N x}{2E_\nu}\right)
\end{aligned}
$$

Neutrino DIS at NLO: $\rightarrow$ $m_l \neq 0$; $m_N \neq 0$; $F_i \rightarrow F_i'$

We use CTEQ14



The DGLAP equations give the evolution on Q^2 of the PDF's $\leftrightarrow$ only linear dynamics!

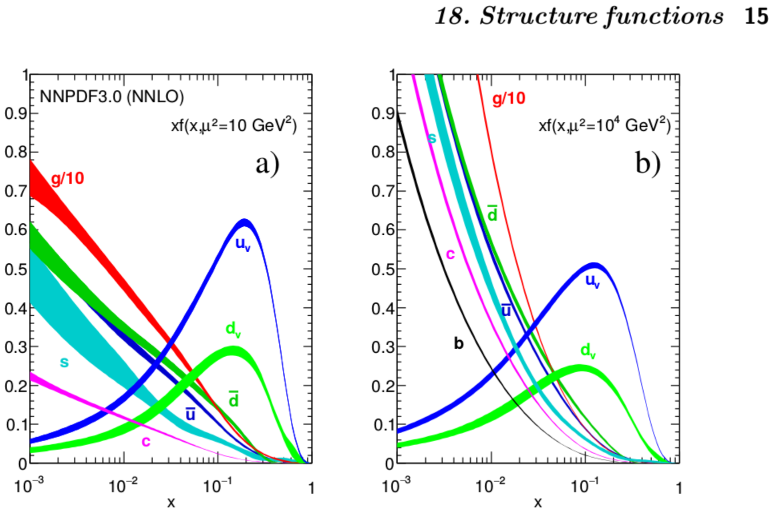
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)

$$\frac{\partial q_i(x,Q^2)}{\partial ln(Q^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dx_1}{x_1}\left(P_{qq}(\frac{x}{x_1})qi(x_1,Q^2) + P_{qg}(\frac{x}{x_1})g(x_1,Q^2)\right)$$

$$\frac{\partial g(x,Q^2)}{\partial ln(Q^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dx_1}{x_1}\left(P_{gq}(\frac{x}{x_1})q_i(x_1,Q^2) + P_{gg}(\frac{x}{x_1})g(x_1,Q^2)\right)$$

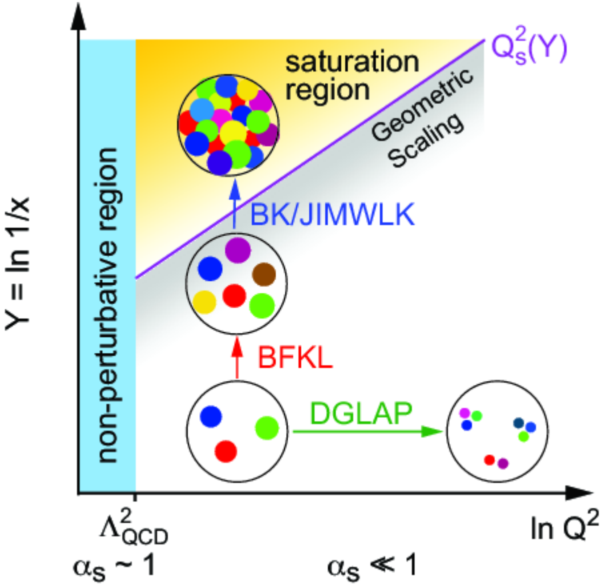
Where P_{ij} are the splitting functions related with the probability of a parton j emits a parton i

PDF's and parton saturation



Particle Data Book 2018: Notice the gluon distribution at $x \to 0$

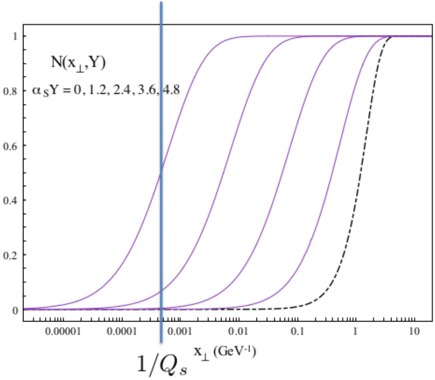
The saturation scale $Q^2 \equiv Q_s^2 = \frac{4\pi\alpha_s}{3C_F R^2} xg(x, Q^2)$



Prof. Yuri Kovchegov: Introduction to saturation physics

https://conference.ippp.dur.ac.uk/event/1005/?print=1

Solution of BK equation



numerical solution by J. Albacete '03 (earlier solutions were found numerically by Golec-Biernat, Motyka, Stasto, by Braun, and by Lublinsky et al in '01)

BK solution preserves the black disk limit, N<1 always (unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

Froissart Bound (general definition)

A limit on the rate at which the cross section of a completely absorptive collision between hadrons can increase with energy, so that the interaction radius cannot increase more rapidly than the logarithm of the energy.

The nuclear strong interactions are typically mediated by bosons, the lightest of which is the pion. In classical mechanics, it is impossible for a particle to exchange a pion with another particle, because this would violate energy conservation at some time. In quantum mechanics, however, this is possible, during very small time intervals, because of Heisenberg's uncertainty principle. But this possibility, also called *tunnel effect* decreases exponentially with the distance.

Its maximum is thus reached when the incident particle is closest to the target particle, at a distance – measured perpendicular to the incident line-of-flight – b which is called impact parameter.

http://nsmn1.uh.edu/rbellwied/classes/PHY7397-fall2013/7397-lec6.pdf

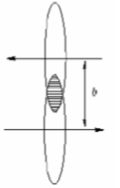
Froissart Bound = unitarity condition

Intuitive derivation of the Froissart bound (by Heisenberg)
Reaction can occur only if the energy density in the overlap region is high enough to create at least a pion pair:

$$\underbrace{\kappa\, e^{-m_\pi b}}_{\text{overlap of pion cloud}} \sqrt{s} \geq k_0$$

k_0 : average energy of two pions

$\sqrt{s}$: Total energy



$$\Rightarrow \quad b_{\max} = \frac{1}{m_\pi} \ln \frac{\sqrt{s}\kappa}{k_0}$$

edge of the reaction zone

Rough estimate of the cross section reads

$$\sigma \simeq \pi b_{\max}^2 = \frac{\pi}{m_\pi^2} \ln^2 \frac{\sqrt{s}\kappa}{k_0} \quad \propto \quad \ln^2 s \quad (s \to \infty)$$

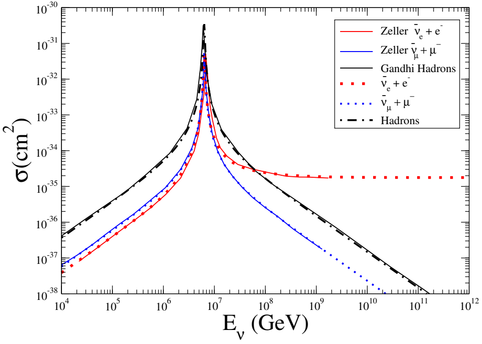
Saturation is implicit

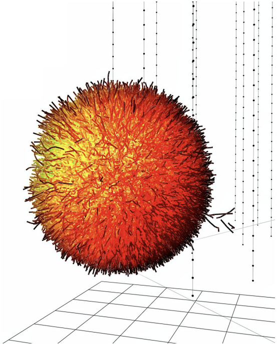
BFKL solution violates the unitarity bound.

$$\sigma \sim s^{\omega \bar{\alpha}_s} \qquad \omega = 4 \ln 2$$

The Glashow Resonance: $\bar{\nu}_e + e^- \longrightarrow W^\pm$ (real)

Glashow, Sheldon L. "Resonant Scattering of Antineutrinos". Phys. Rev. 118 (1): 316317 (1960) $\rightarrow$ 1^{st} probe of SMPC



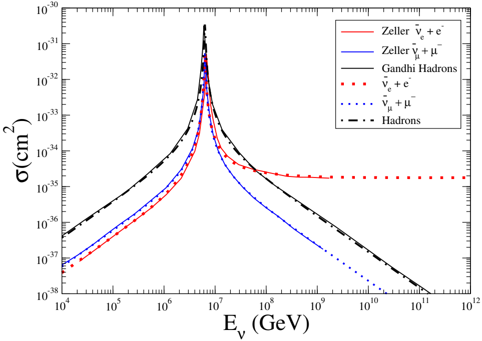


https://icecube.wisc.edu/gallery/icecube-sees-a-glashow-resonance-event/

IceCube Collaboration:Nature vol 591, pages 220-224 (2021)

The Glashow Resonance: $\bar{\nu}_e + e^- \longrightarrow W^\pm$ (real)

1^a probe of SMPC



$$\frac{d\sigma}{dy}(\bar{\nu}_e e \to \bar{\nu}_\mu \mu) = \frac{G_F^2 m_e E_\nu}{2\pi} \frac{4(1-y)^2(1-(m_\mu^2 - m_e^2)/(2m_e E_\nu))^2}{(1-2m_e E_\nu/M_W^2)^2 + \Gamma_W^2/M_W^2}$$

The higher the σ_ν the higher the count of ν-induced events?

R. Gandhi, C. Quigg, M. H. Reno and I. Sarcevic. Astropart. Phys. 5 (1996) 81-110

$$\mathcal{L}^N = \frac{1}{\sigma_{\nu N,\bar{\nu} N}(E_\nu) N_A}$$
$$\mathcal{L}^e = \frac{1}{\sigma_{\nu e,\bar{\nu} e}(E_\nu)\langle Z/A\rangle N_A}$$