

Parameterising CCQE interactions in the Spectral Function model for neutrino oscillation analyses

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1. Introduction

2. Neutrino-nucleus interactions

3. Systematic uncertainties in CCQE interactions

4. Fits to published data

5. Summary and prospects

1. Introduction

- First hints of CP violation
- Currently: NOvA, T2K (w/ ND upgrade)
- Future: DUNE, Hyper-Kamiokande
- Still limited by statistics, but not for long!



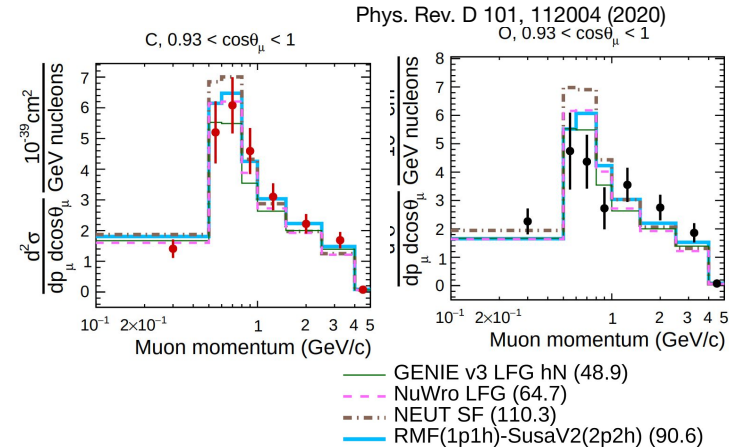
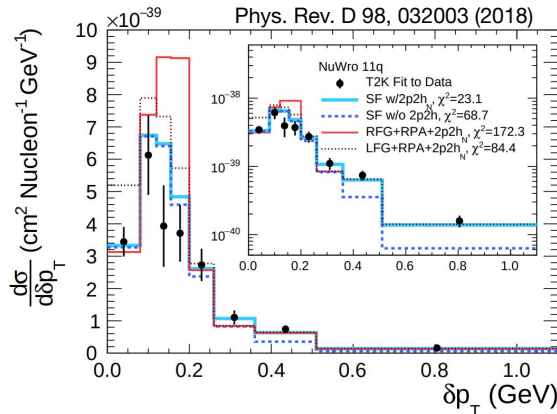
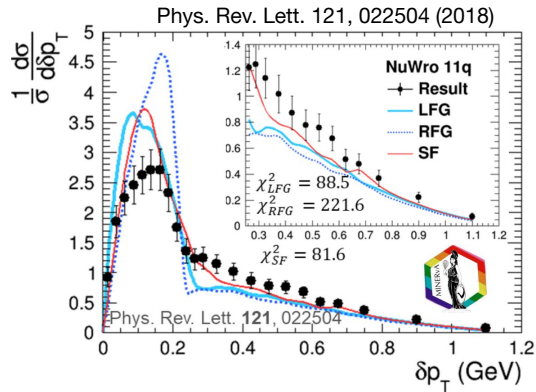
Current systematic uncertainties in T2K

Nature 580, 339-344

| Source | $N(\nu_e)$ |
|----------------|------------|
| Binding energy | 7.1% |
| Total | 8.8% |

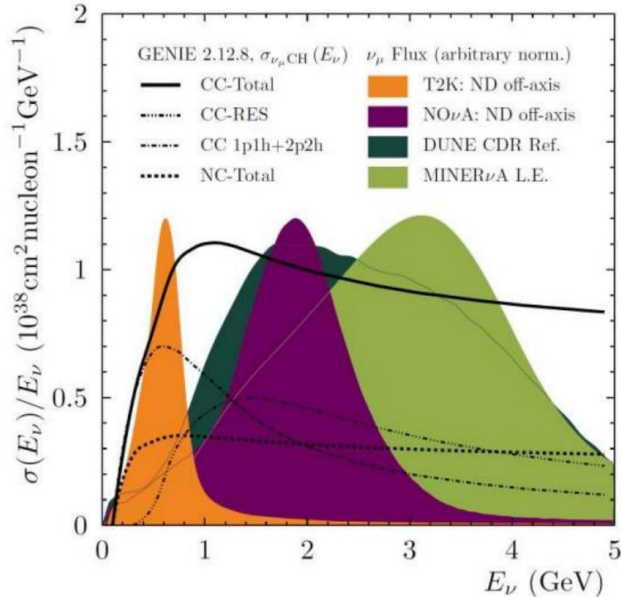
Neutrino interaction uncertainties must be reduced!

- Cross section measurements allow to test our neutrino interaction models
- Current models struggle to describe (semi-)inclusive measurements
- In this talk we introduce new theory-driven freedoms to robustly improve agreement

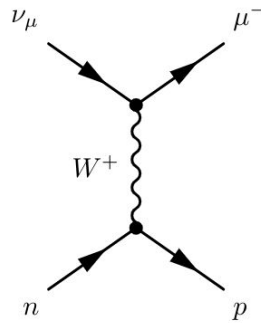


The background of the slide is a grid of 15 rectangular panels, arranged in 5 rows and 3 columns. Each panel contains a different physics diagram. The top row shows particle tracks or data points. The second row shows particle tracks with various interaction points and loops. The third row shows particle tracks with a central vertex and branching paths. The fourth row shows particle tracks with a central vertex and branching paths, similar to the third row. The bottom row shows empty panels. The text '2. Neutrino-nucleus interaction models' is overlaid on the second and third rows.

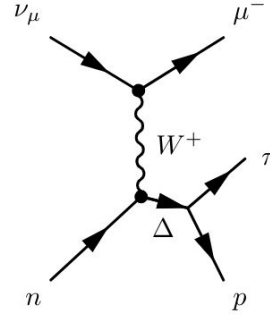
2. Neutrino-nucleus interaction models



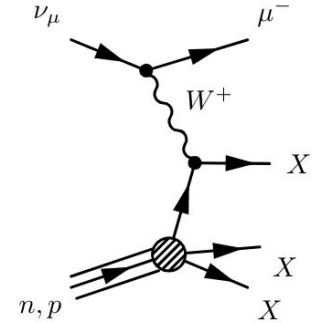
Quasi-elastic (CCQE)



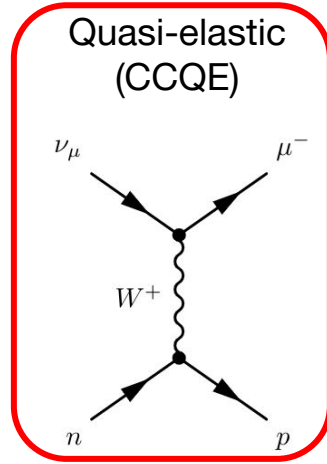
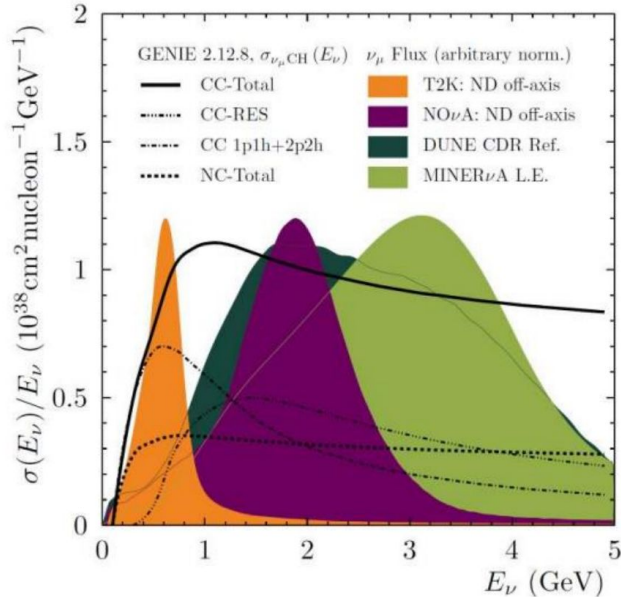
Resonant (CCRES)



Deep Inelastic Scattering (CCDIS)

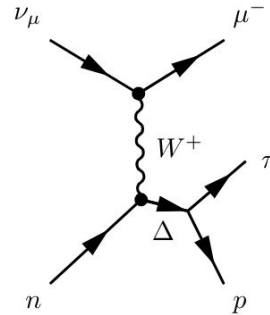


- In order to estimate neutrino energy, a good understanding of neutrino-nucleus interactions is necessary
- CCQE is the dominant interaction in T2K/HK, and is a significant mode in NOvA, MINERvA and DUNE

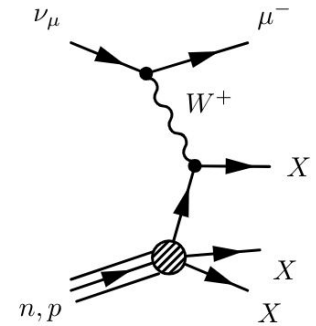


This talk

Resonant (CCRES)

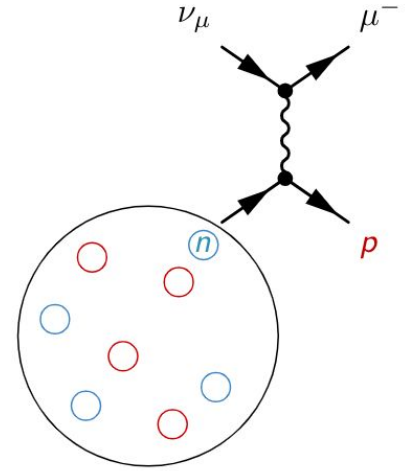


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- Neutrinos can interact with nucleons bound within nuclei (Carbon, Oxygen, Argon...)
- Initial state nucleons are non-static: Fermi motion
- How to model this?



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Fermi gas

Relativistic Fermi Gas (RFG)

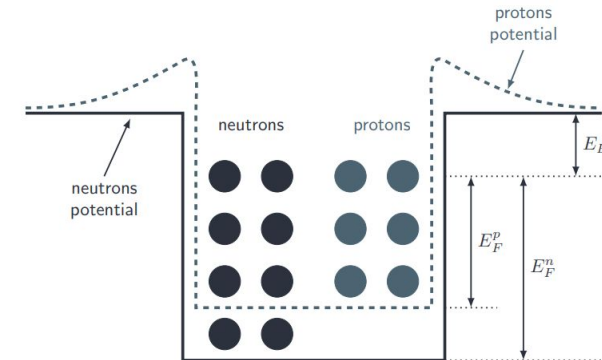
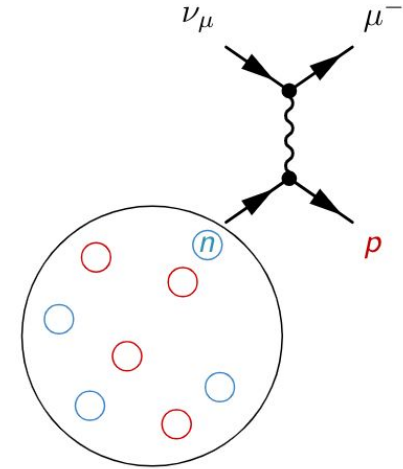
Nucleons move freely in a constant binding energy within the nuclear volume

$$p_F = \left(3\pi^2 \rho \frac{Z}{A}\right)^{1/3}$$

Local Fermi Gas (LFG)

The nucleus is described with the local density approximation

$$p_F(r) = \left(3\pi^2 \rho(r) \frac{Z}{A}\right)^{1/3}$$



T. Golan

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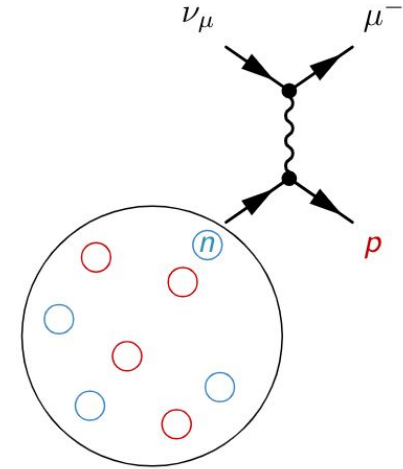
Spectral Function (SF)

The probability of removing of a nucleon with momentum p_m and leaving residual nucleus with excitation energy E_m

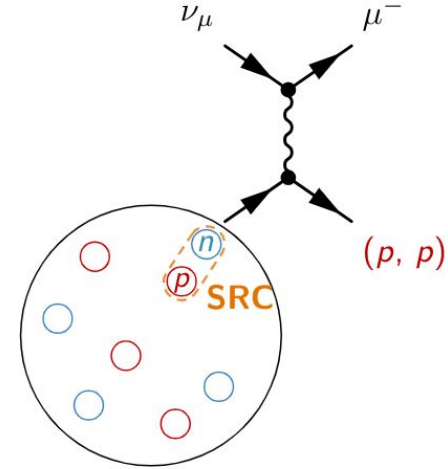
$$P(p_m, E_m) = \boxed{P_{MF}(p_m, E_m)} + P_{corr}(p_m, E_m)$$

Independent nucleons, moving in a mean-field potential within the shell-model picture → built from (e,e'p) data (~80%)

=> **One outgoing nucleon is produced**



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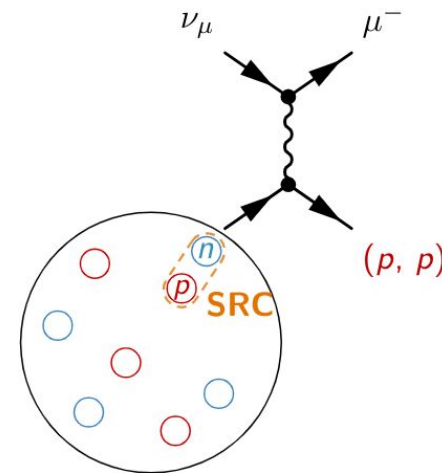
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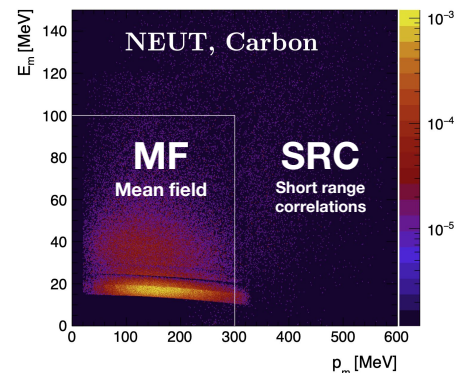
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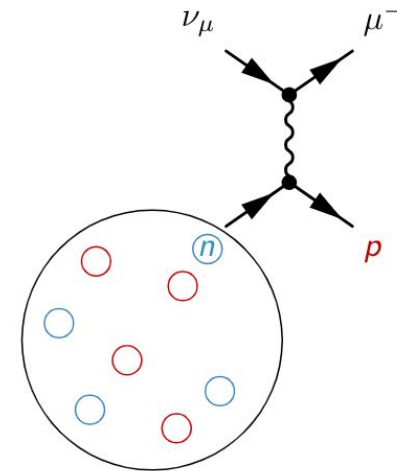
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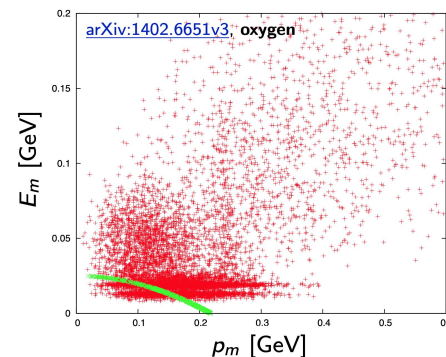
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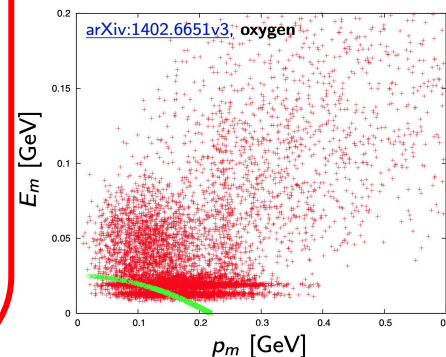
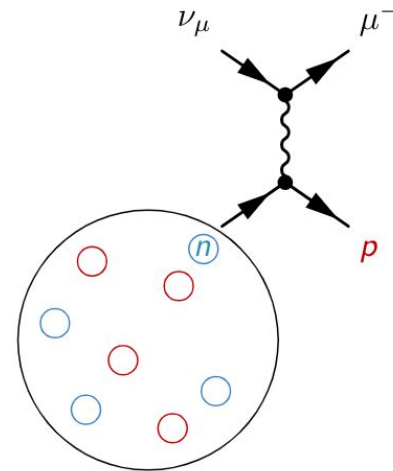
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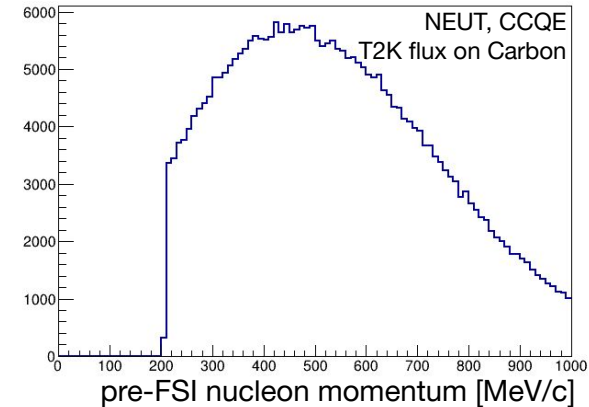
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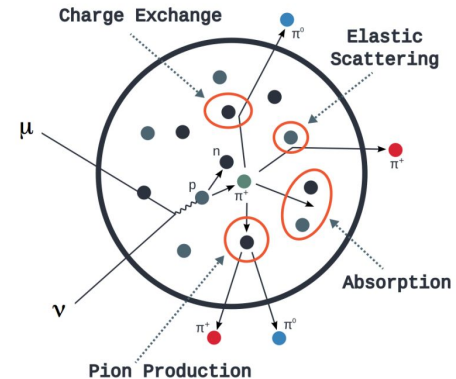
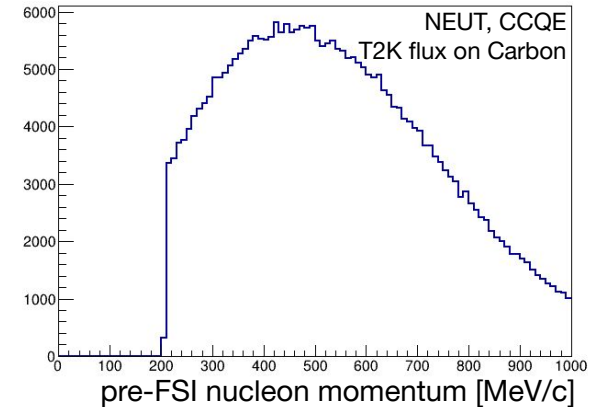


SF
RFG

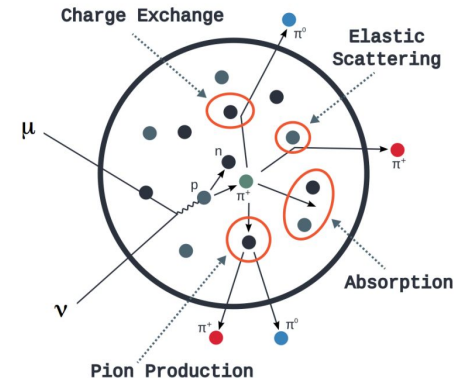
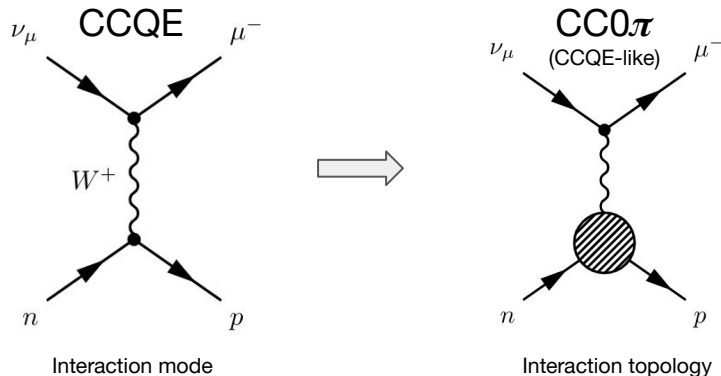
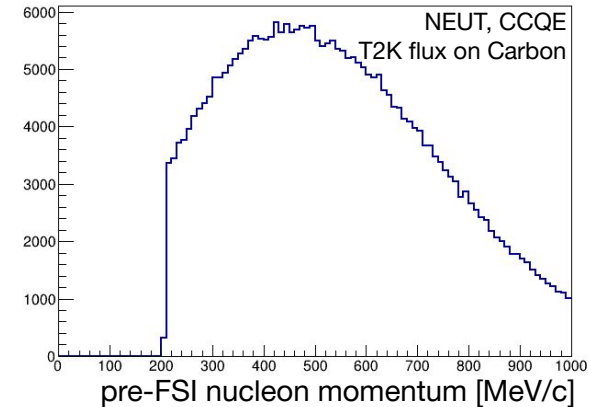
- Pauli blocking (PB):
 - By Pauli principle, an interaction cannot occur if it leads to the creation of a nucleon in a state that is already occupied
 - Simple model: reject events with outgoing nucleon momentum below Fermi level



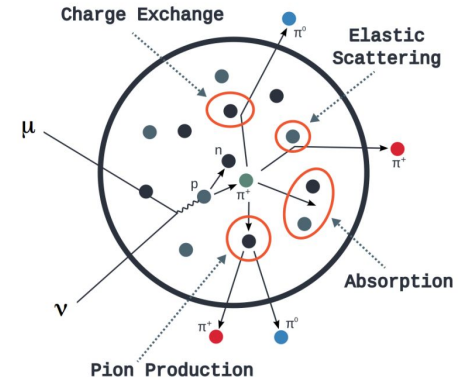
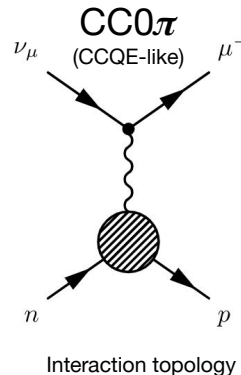
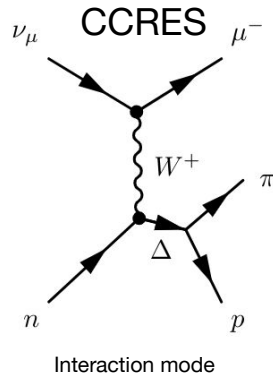
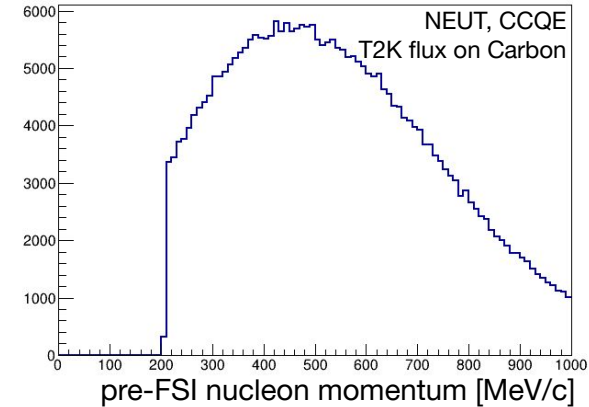
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 - Outgoing particles may re-interact with the nuclear matter



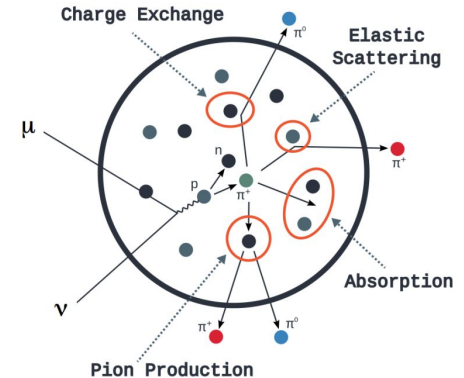
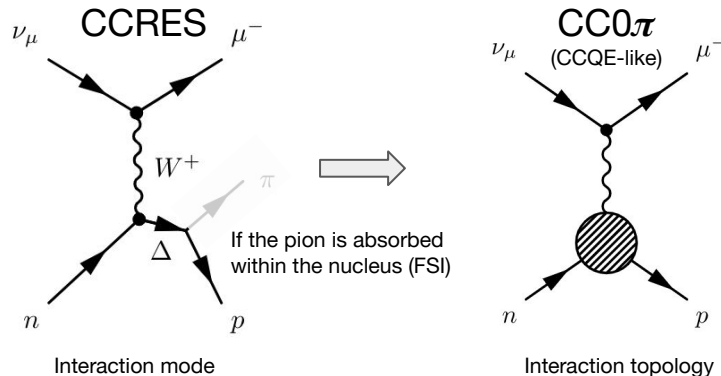
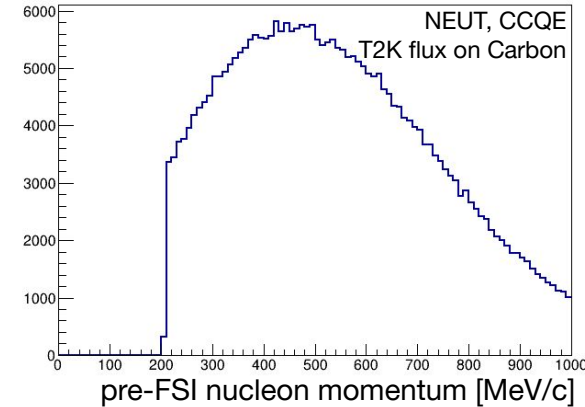
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 - Can cause different interactions to have the same final state

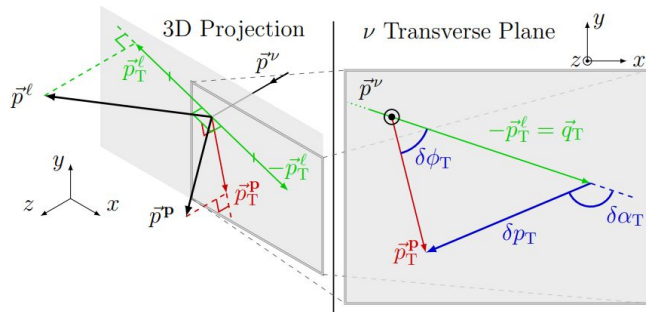


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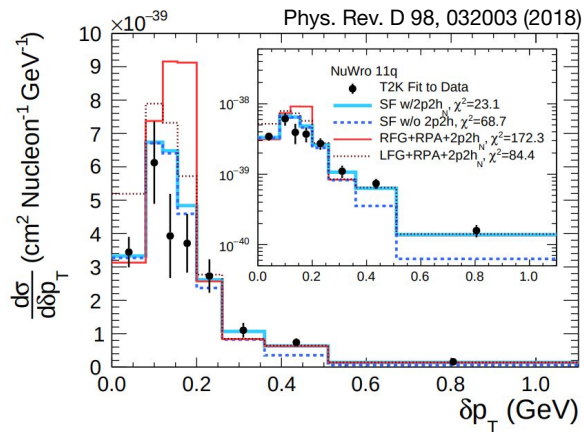
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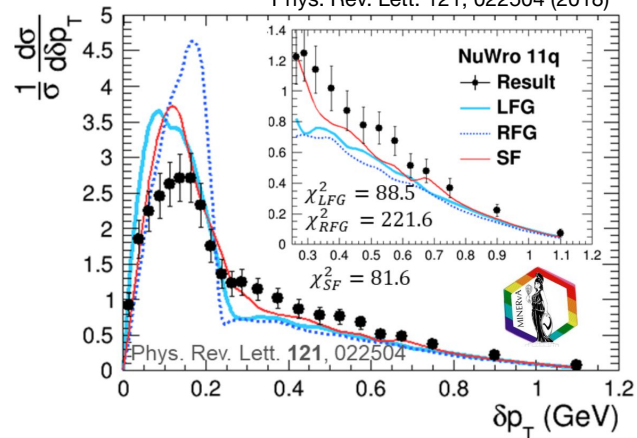
T2K

Phys. Rev. D 98, 032003 (2018)

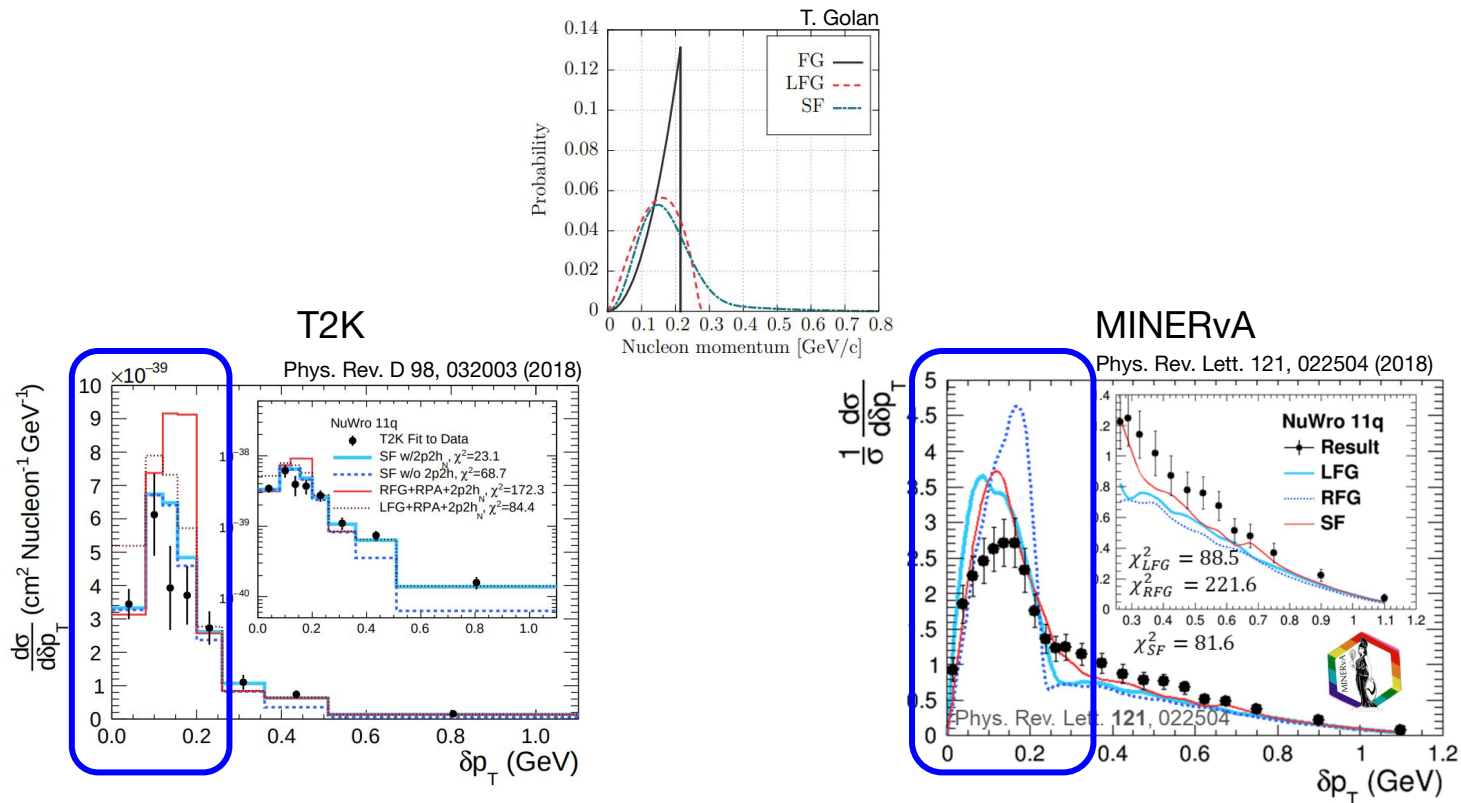


MINERvA

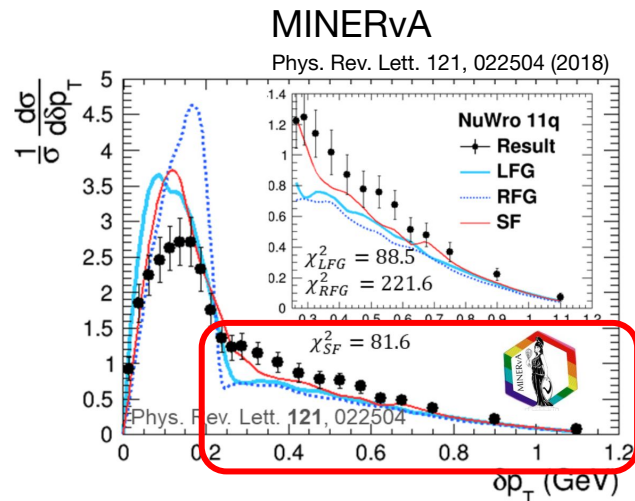
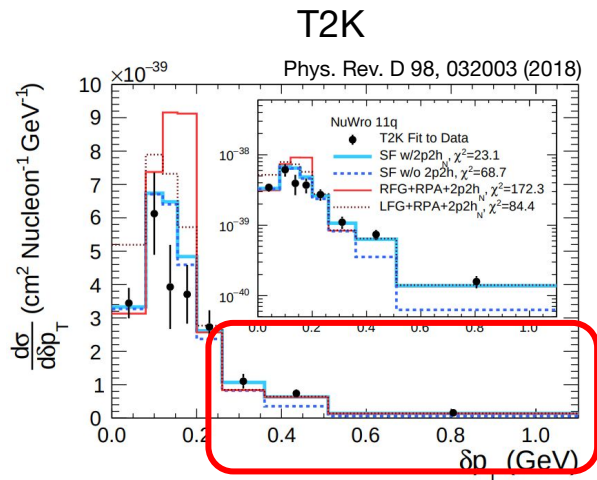
Phys. Rev. Lett. 121, 022504 (2018)



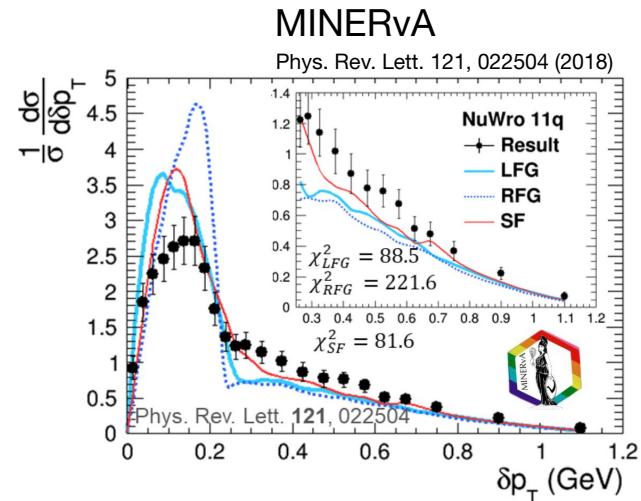
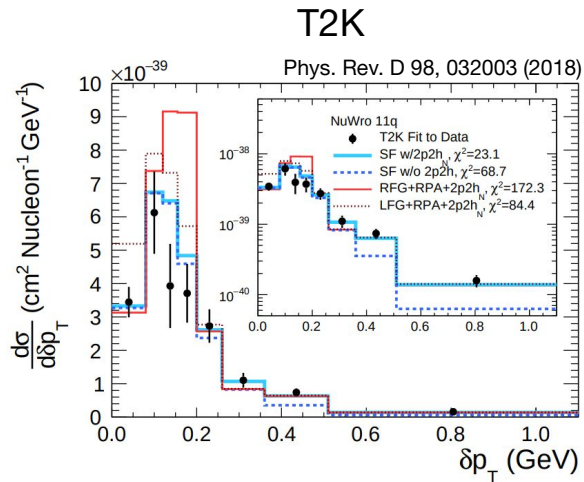
- The bulk of the distribution is sensitive to the initial state nucleon momentum



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- The tail of the distribution is sensitive to FSI, SRC, 2p2h



- The bulk of the distribution is sensitive to the initial state nucleon momentum
- The tail of the distribution is sensitive to FSI, SRC, 2p2h
- None of the models describe well the data...





3. Systematic uncertainties in CCQE interactions

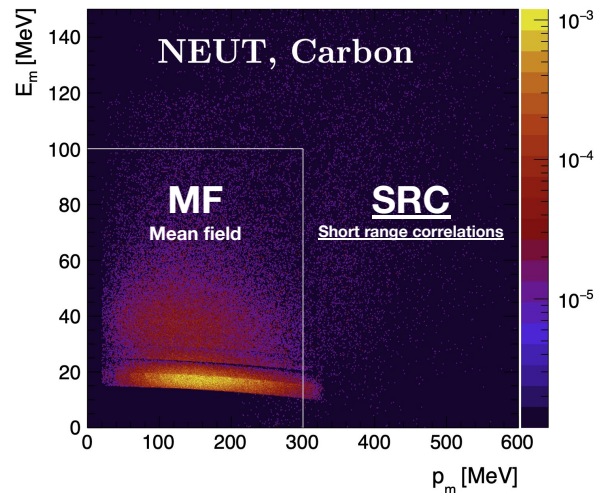
- The models do not describe well semi-inclusive data
- A new parameterisation of the CCQE SF model is implemented in [NUISANCE](#) to take into account systematic uncertainties
- Applied on NEUT 5.4.0 model ([arxiv:2106.15809](https://arxiv.org/abs/2106.15809))



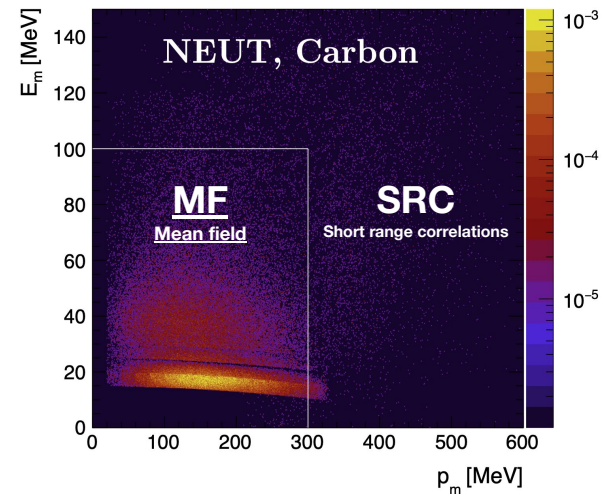
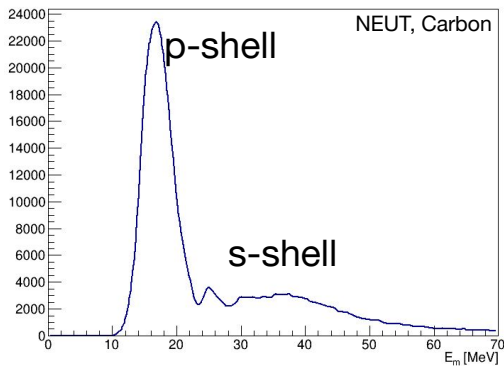
- SF model:
$$P(p_m, E_m) = P_{MF}(p_m, E_m) + P_{corr}(p_m, E_m)$$
 - SRC contribution : 5% of CCQE in NEUT vs. 15% in NuWro **pairs of strongly-correlated nucleons (~20%)**

➤ **Systematic:** SRC normalization

⇒ **Two outgoing nucleons are produced** (before FSI)



- SF model: $P(p_m, E_m) = P_{MF}(p_m, E_m) + P_{corr}(p_m, E_m)$



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$$P(p_m, E_m) = P_{MF}(p_m, E_m) + P_{corr}(p_m, E_m)$$

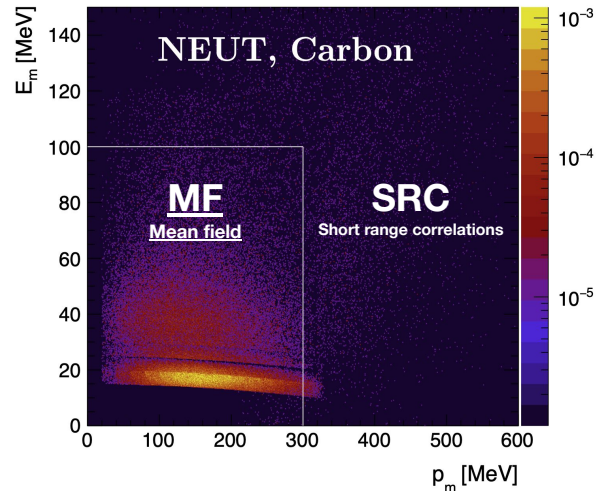
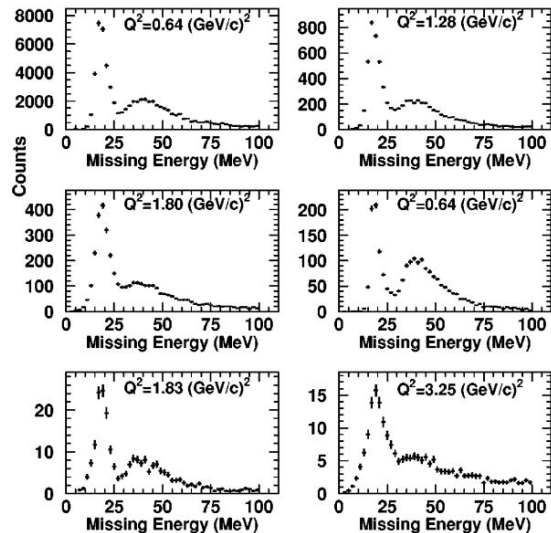
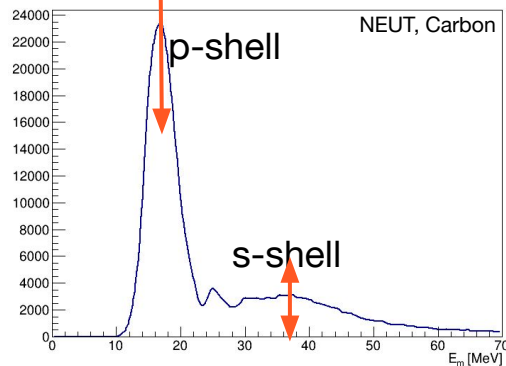
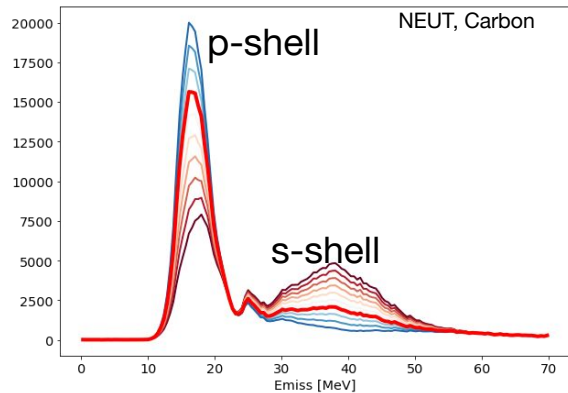


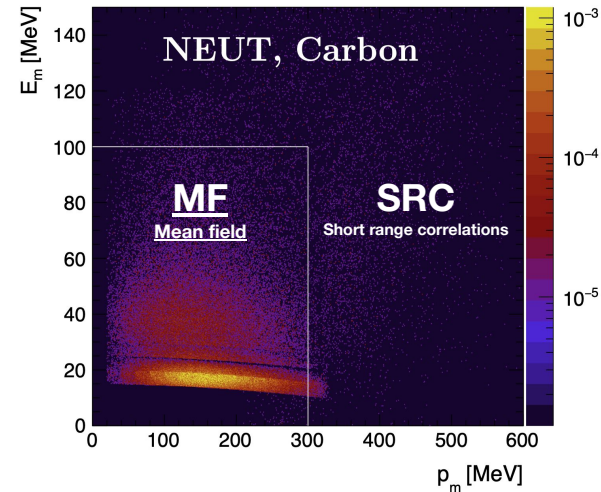
FIG. 3. Measured missing energy spectra for carbon at the different Q^2 . The top left and middle right panels refer to the forward and backward electron angle kinematics, respectively, at $Q^2=0.64$ (GeV/c) 2 and the middle left and bottom left panels refer to the forward and backward electron angle kinematics, respectively, at $Q^2=1.80$ (GeV/c) 2 . These are the L - T separation kinematics.

Phys Rev C 68, 064603 (2003)

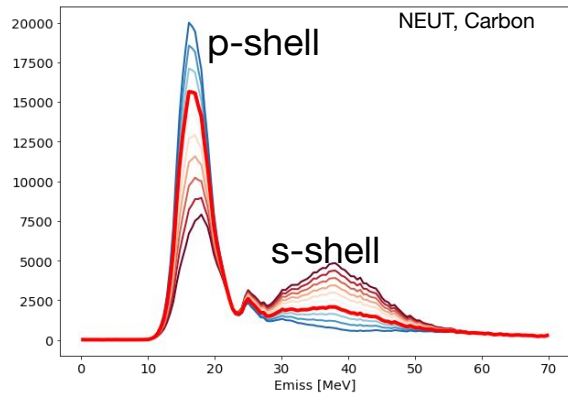
- SF model: $P(p_m, E_m) = P_{MF}(p_m, E_m) + P_{corr}(p_m, E_m)$



➤ **Systematics:** shell occupancies

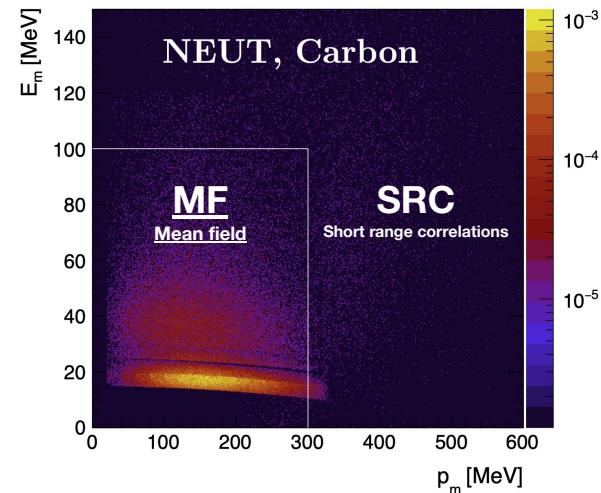
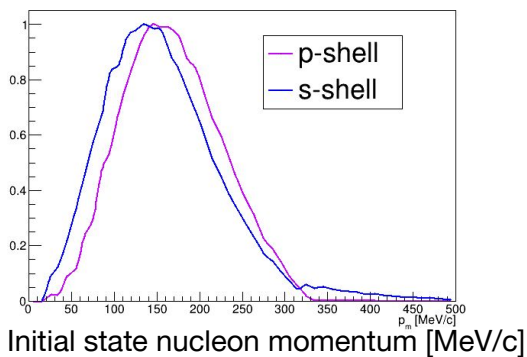
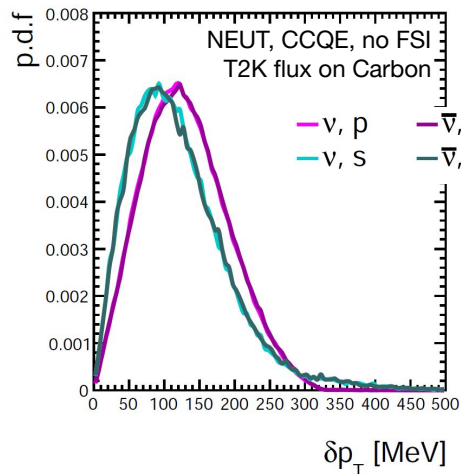


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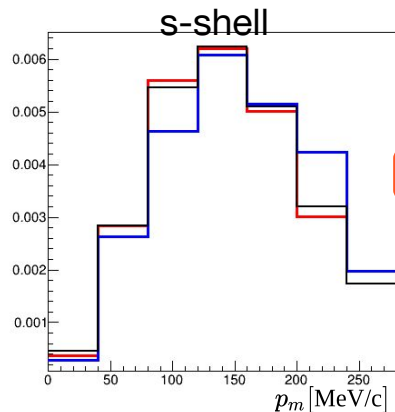
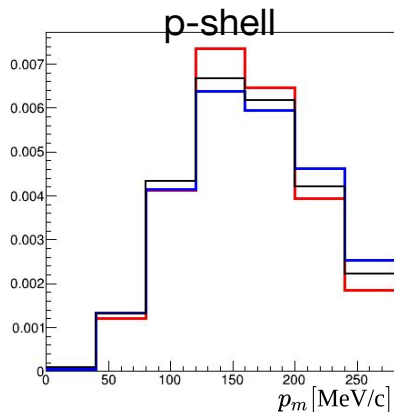
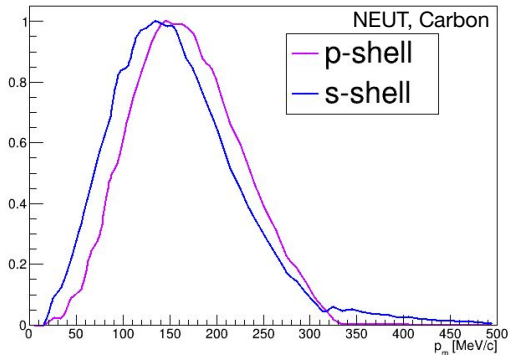
➤ **Systematics:** shell occupancies

- Impacts the outgoing nucleon kinematics

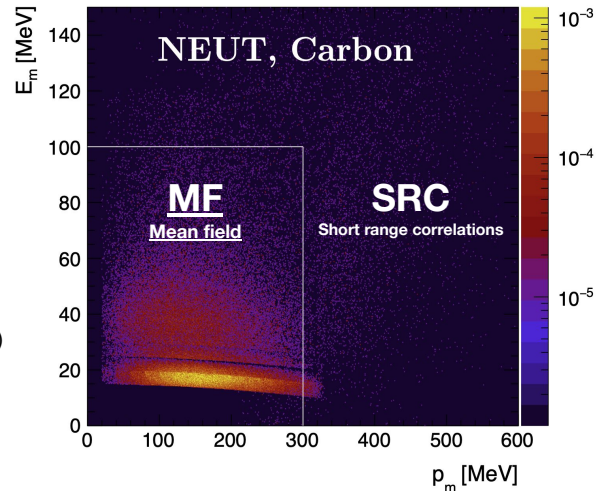


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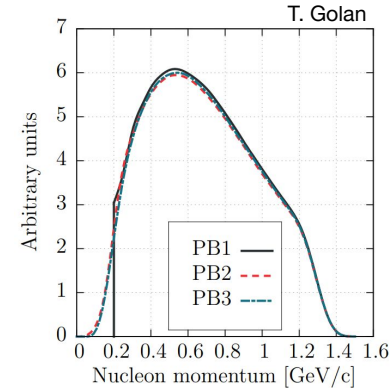
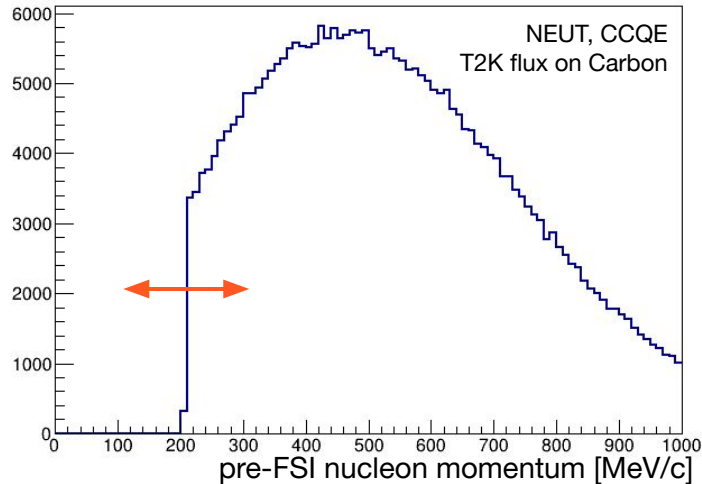
➤ **Systematics:** shape variations of the initial state momentum in each shell



— NEUT
— From (e,e') data
 Phys Rev C 49 955 (1994)
 Phys Rev C 68, 064603 (2003)



- By Pauli principle, an interaction cannot occur if it leads to the creation of a nucleon in a state that is already occupied

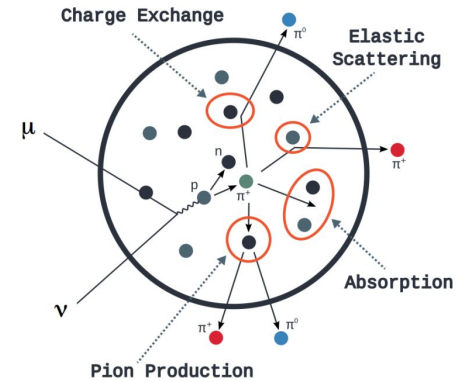


➤ **Systematic:** Pauli Blocking threshold

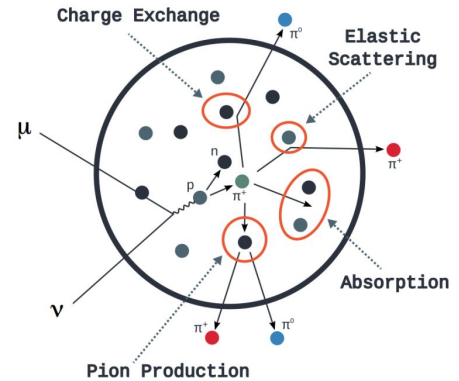
- Since this affects very low momentum protons, this parameter may not be well constrained by samples like CC0pi+Np (proton momentum threshold in detectors)

- Outgoing nucleon:

➤ **Systematic:** Nucleon FSI: simple parameter to change the amount of events undergoing FSI w.r.t. no FSI



- Outgoing nucleon:
 - **Systematic:** Nucleon FSI: simple parameter to change the amount of events undergoing FSI w.r.t. no FSI
 - Outgoing lepton:
 - The SF model does not include the impact of FSI on the differential cross section in lepton kinematics
 - This can be accounted by altering the lepton kinematics with an optical nuclear potential (Phys. Rev. D 91, 033005 (2015))
 - Available in NuWro (used to generate reweight templates for NEUT)
- **Systematic:** strength of the optical potential correction





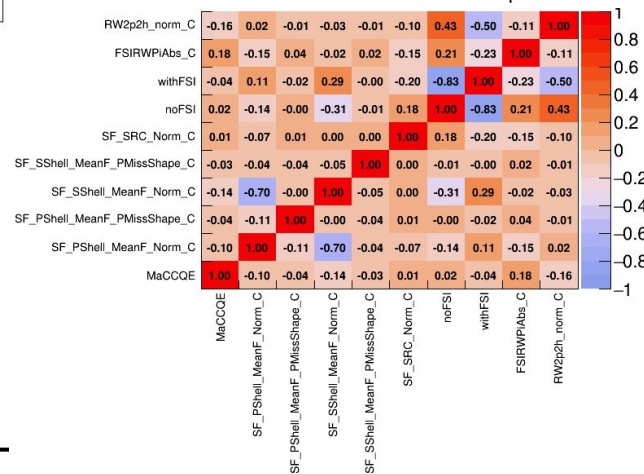
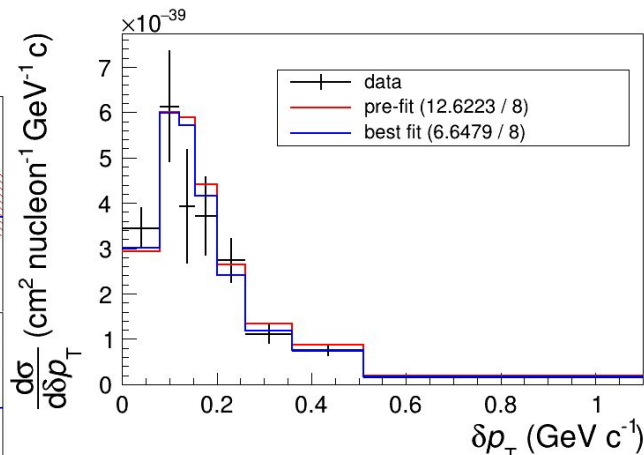
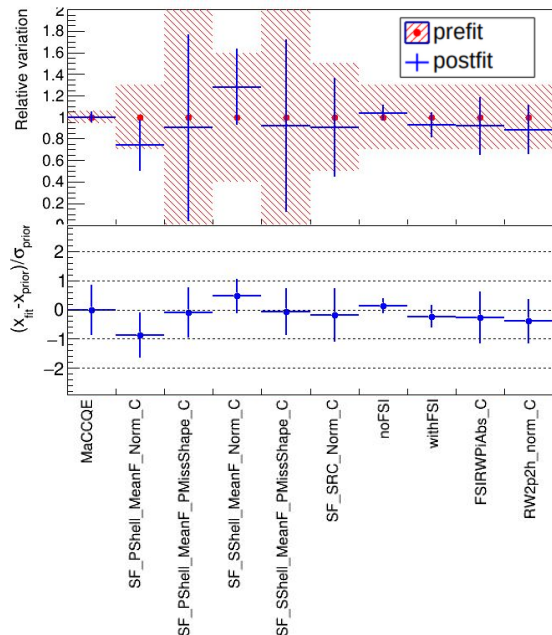
4. Fits to published data

- Ingredients:
 - Model: NEUT with SF model (for Oxygen and Carbon)
 - Parameters: SF model parameters (+ normalisation parameters for other interactions)
 - Data: cross section measurements from T2K and MINERvA
- Chi-square(*): $\chi_{\text{data}}^2 = \sum_{1 \leq i, j \leq n} (B_i - B_i^{MC}) (M^{-1})_{ij} (B_j - B_j^{MC})$
- How is this parameterisation able to match the data?

(*) Peelle's Pertinent Puzzle was avoided using a different decomposition of the data histogram and covariance matrix, see back-up

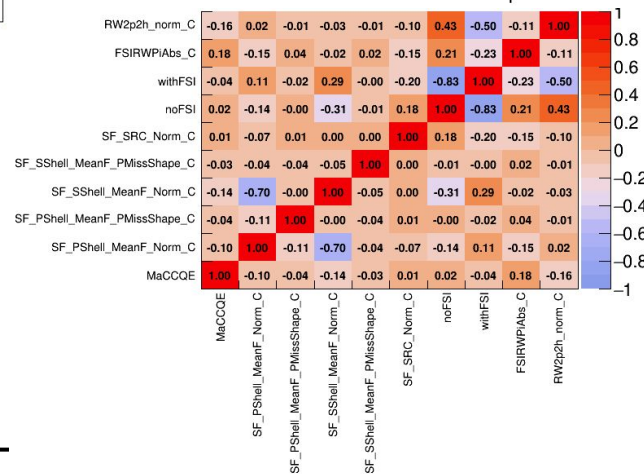
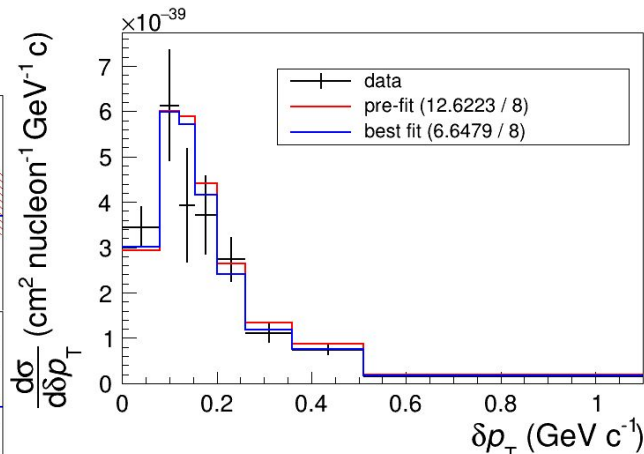
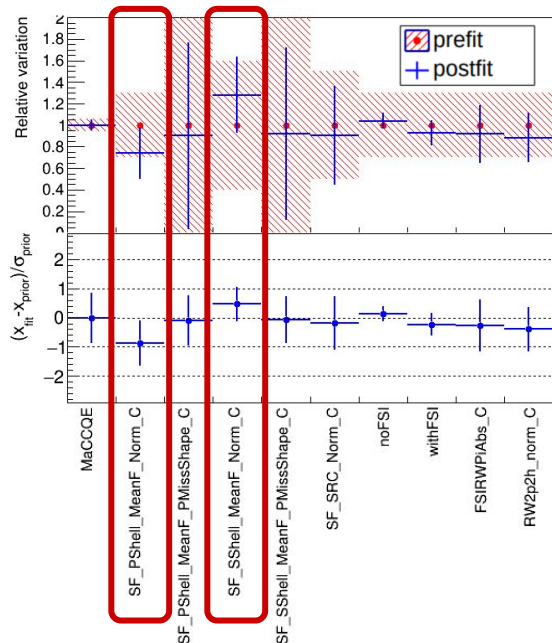
Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- No FSI / with FSI (correlated)
- Pion absorption normalization
- 2p2h normalization



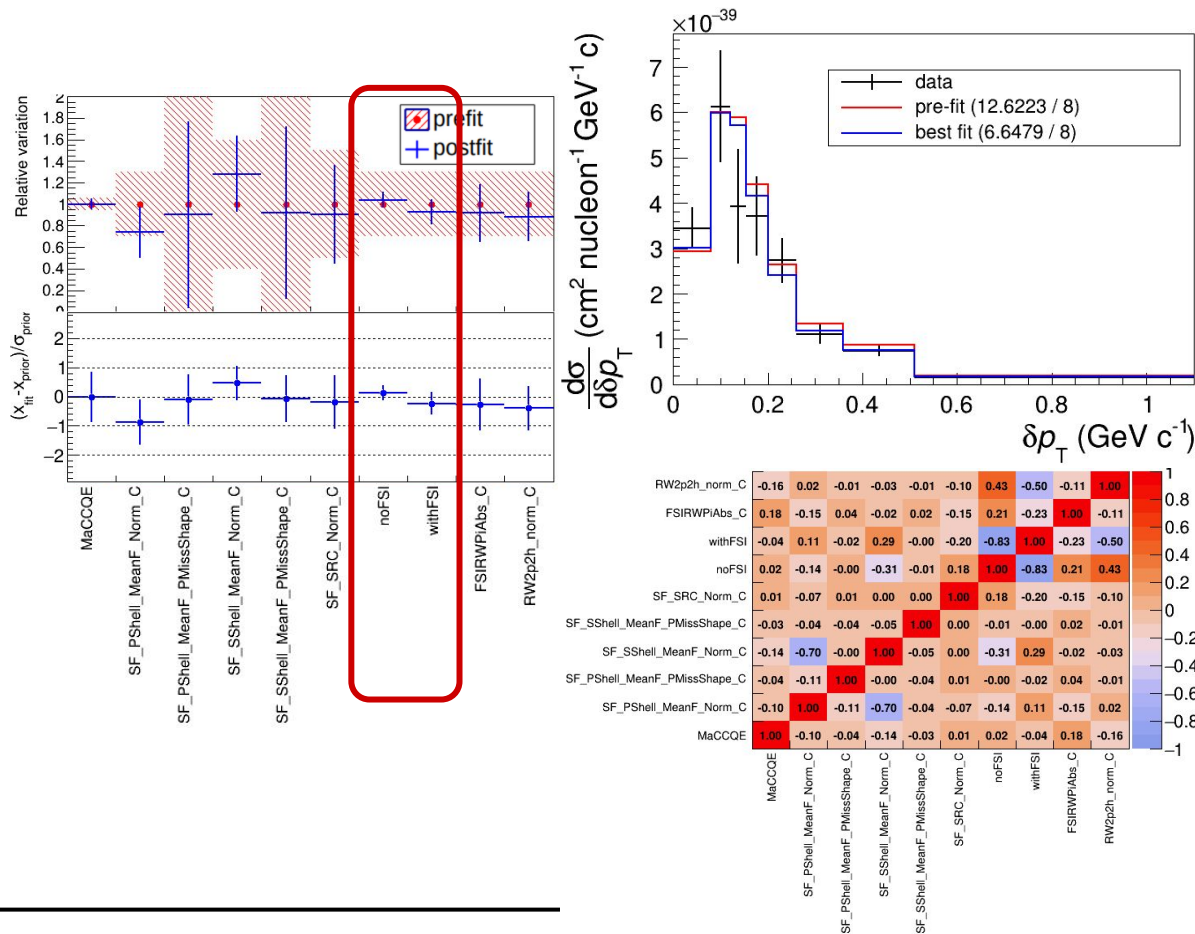
Parameters used in this fit:

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- **Shell occupancies**
- p_m shape
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- 2p2h normalization



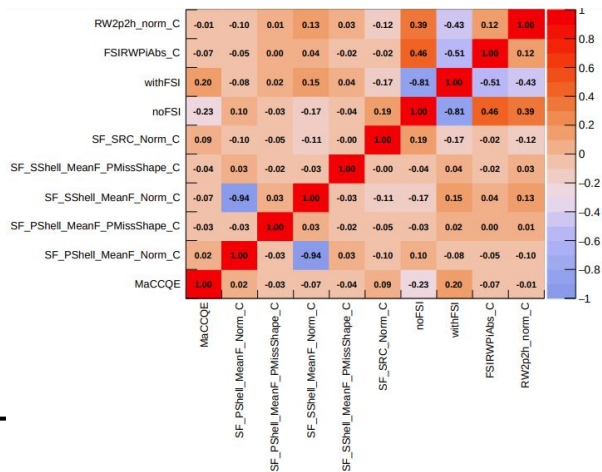
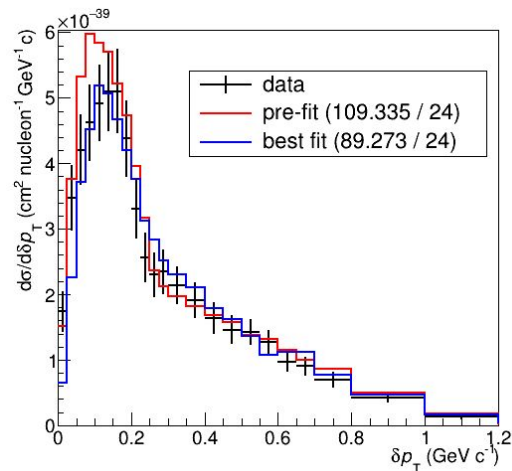
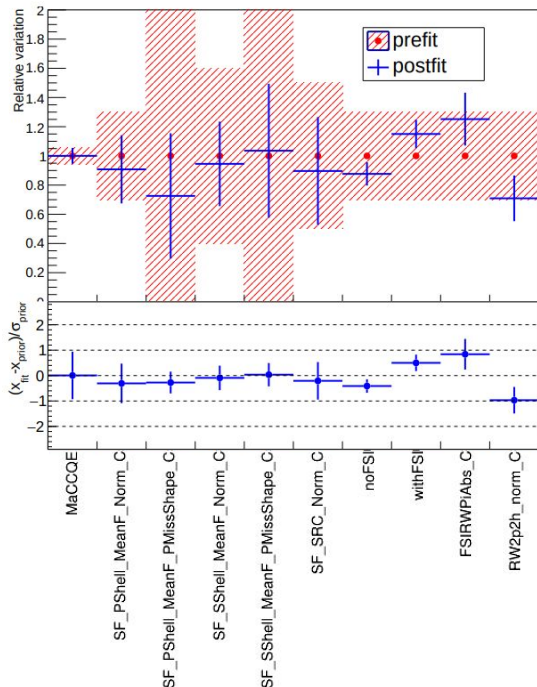
Parameters used in this fit:

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- SRC norm
- **No FSI / with FSI** (correlated)
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- 2p2h normalization



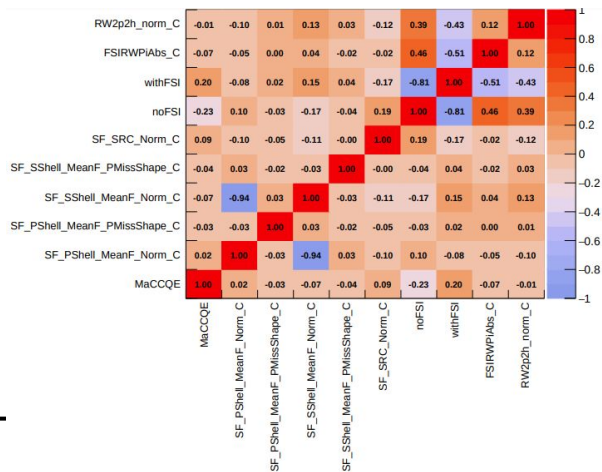
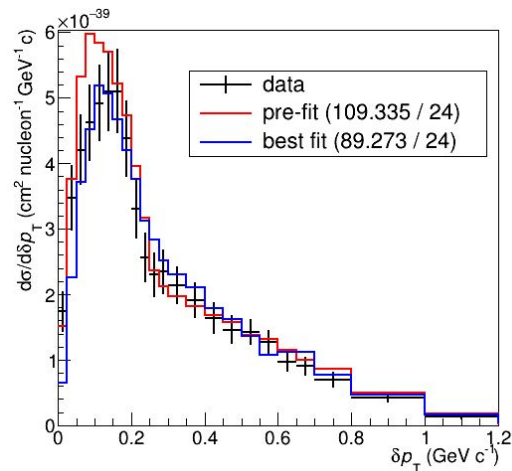
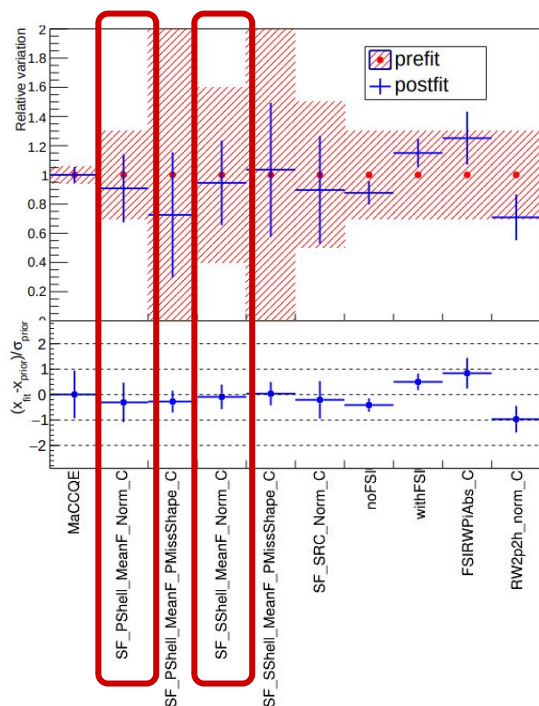
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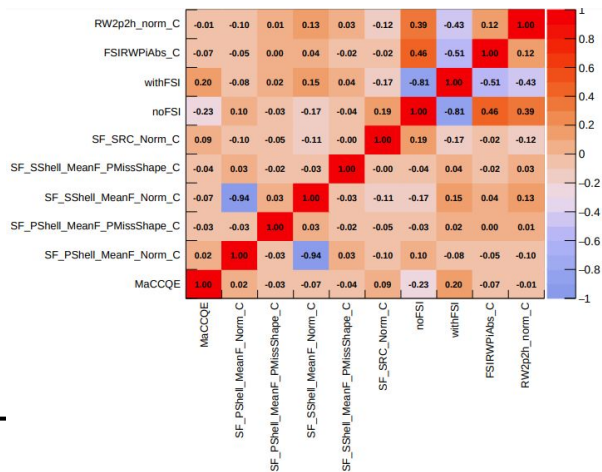
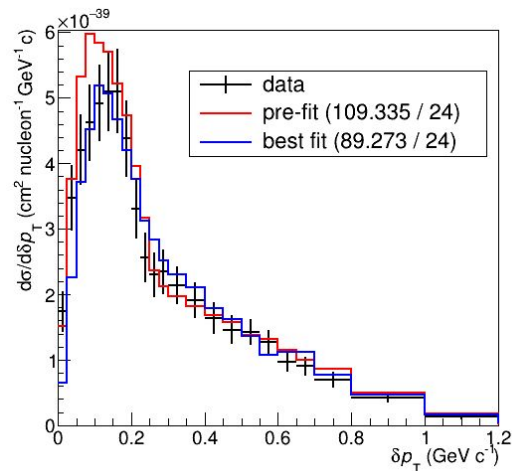
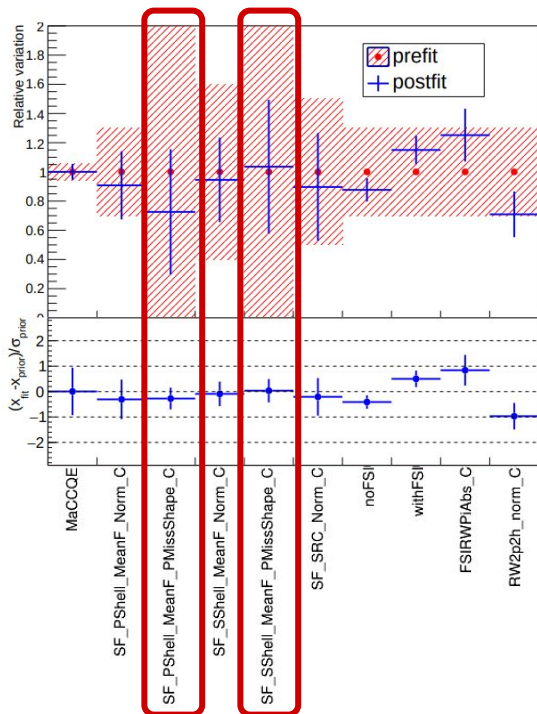
Parameters used in this fit:

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- **Shell occupancies**
- p_m shape
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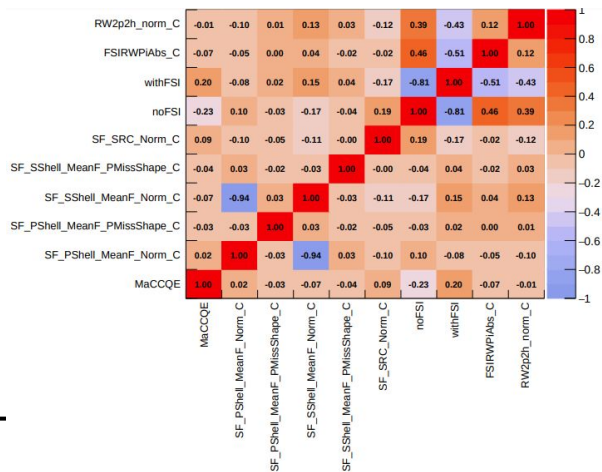
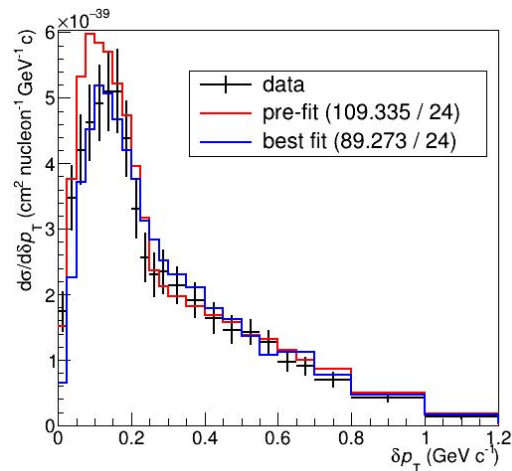
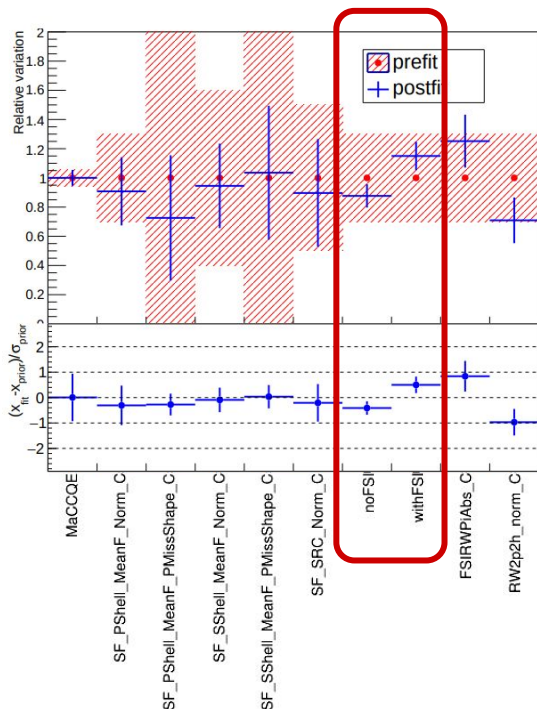
Parameters used in this fit:

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- Shell occupancies
- p_m shape
- SRC norm
- No FSI / with FSI (correlated)
- Pion absorption normalization
- 2p2h normalization



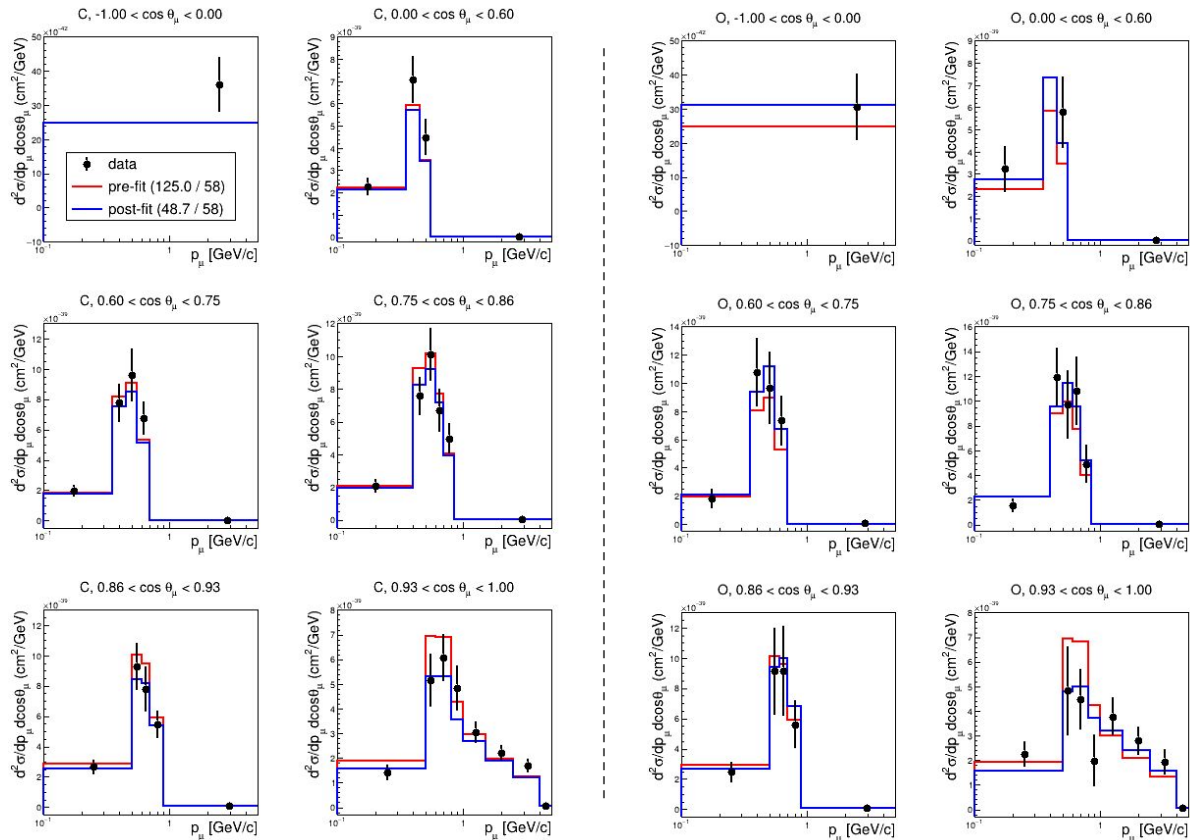
Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- **No FSI / with FSI** (correlated)
- Pion absorption normalization
- 2p2h normalization



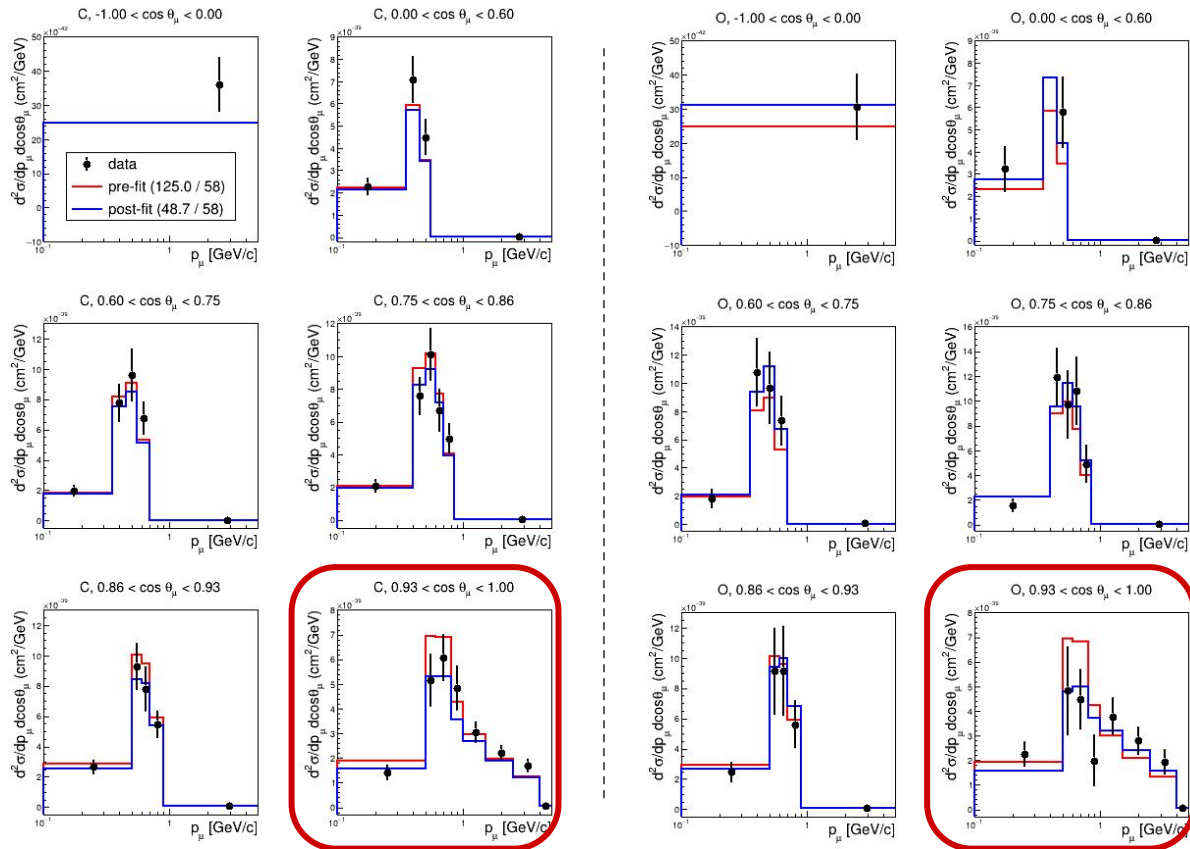
Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- Pauli blocking
- Optical potential correction
- Pion absorption normalization
- 2p2h normalization



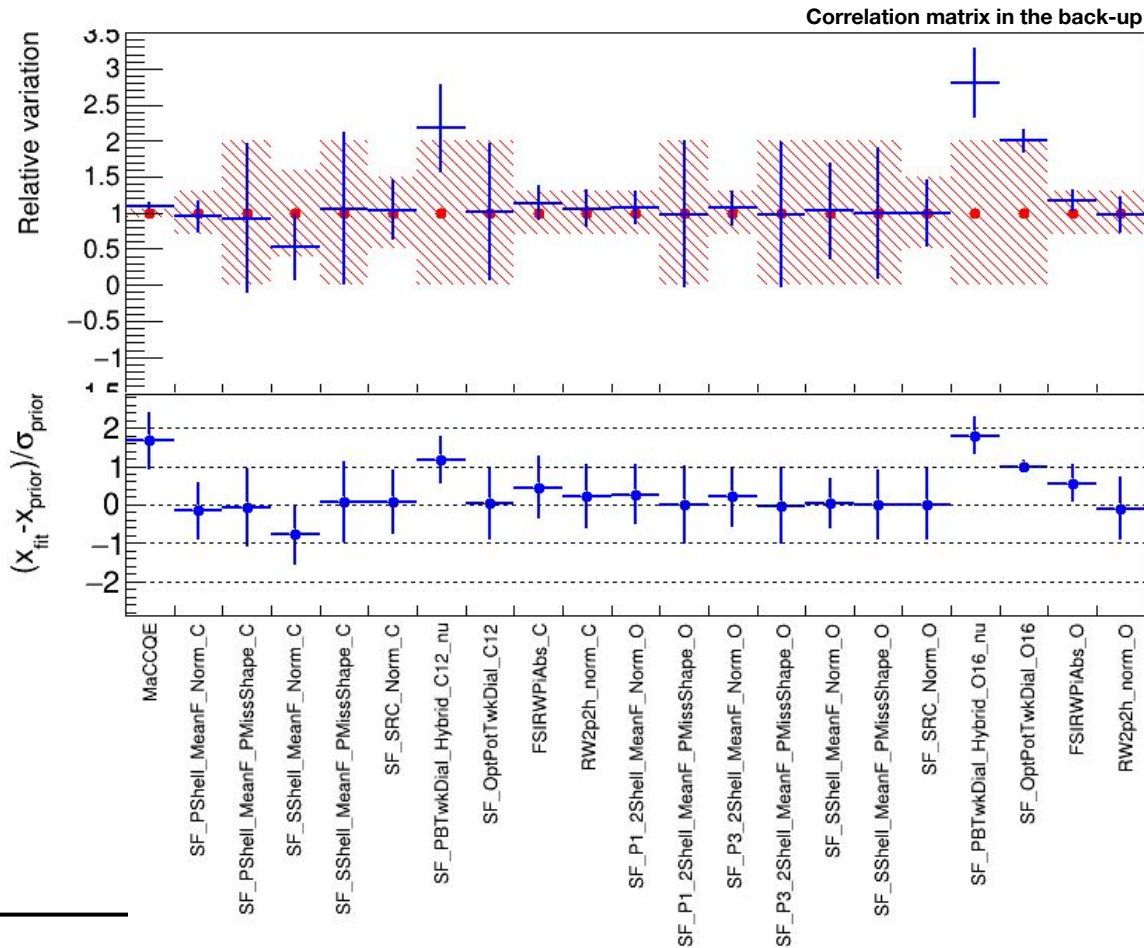
Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- **Pauli blocking**
- **Optical potential correction**
- Pion absorption normalization
- 2p2h normalization



Parameters used in this fit:

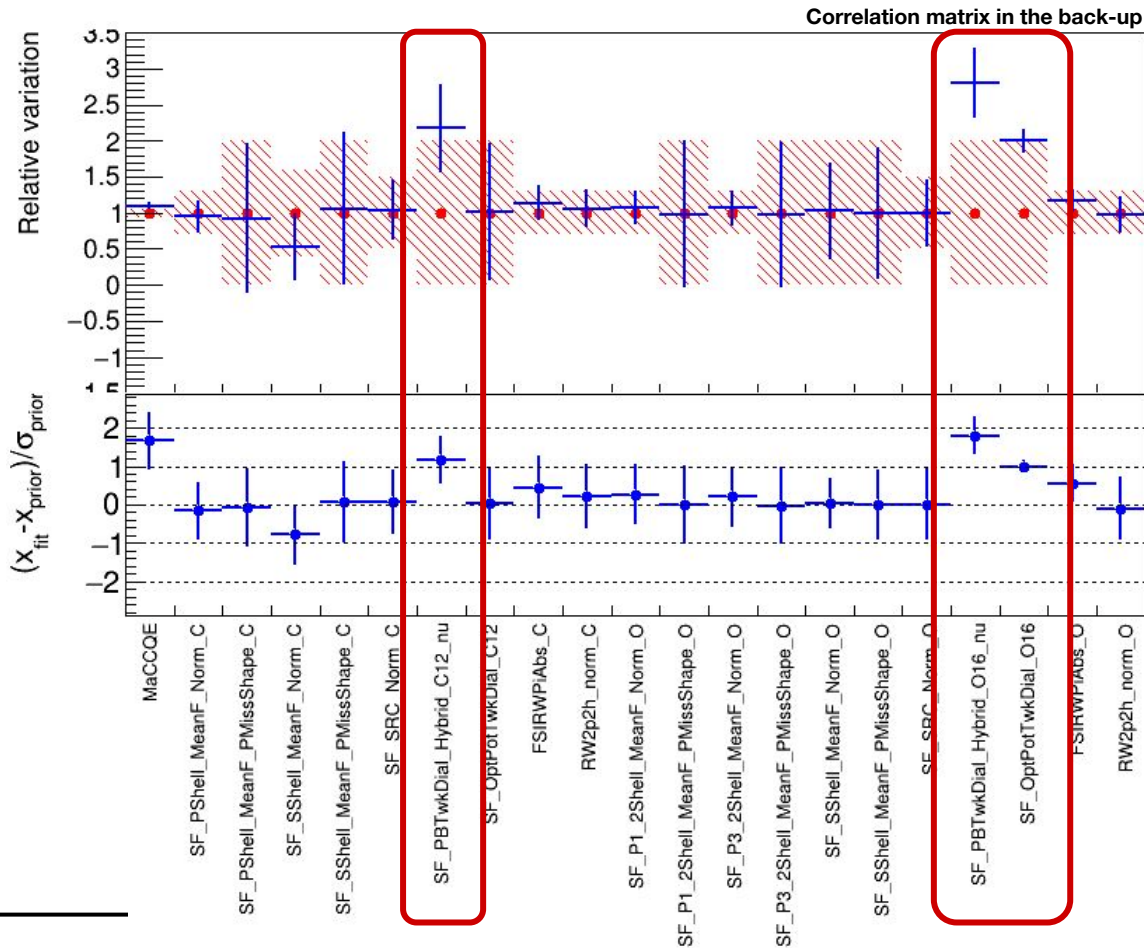
- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- Pauli blocking
- Optical potential correction
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Parameters used in this fit:

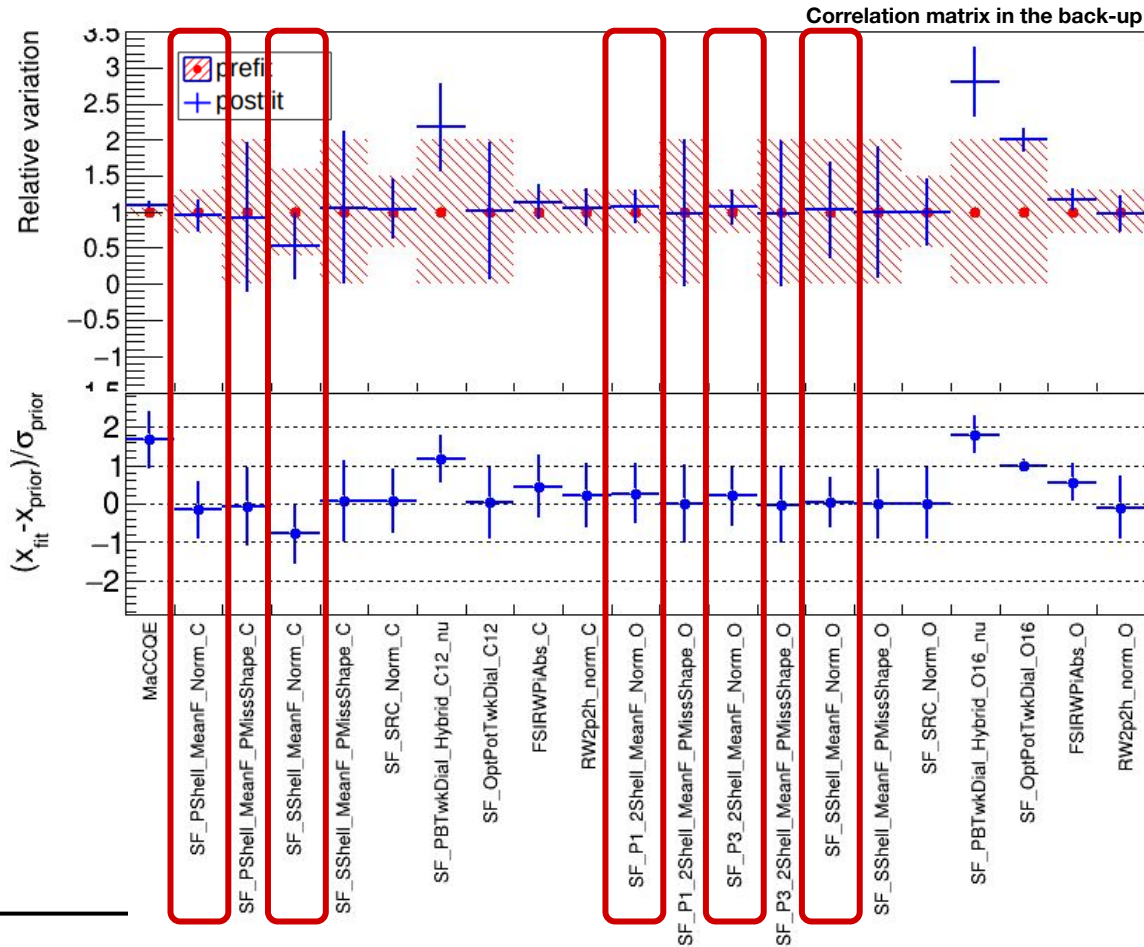
- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- **Pauli blocking**
- **Optical potential correction**
- Pion absorption normalization
- 2p2h normalization

Nominal PB threshold \rightarrow 209 MeV/c
 1σ shift in PB \rightarrow 30 MeV/c



Parameters used in this fit:

- MaCCQE
- **Shell occupancies**
- p_m shape
- SRC norm
- Pauli blocking
- Optical potential correction
- Pion absorption normalization
- 2p2h normalization

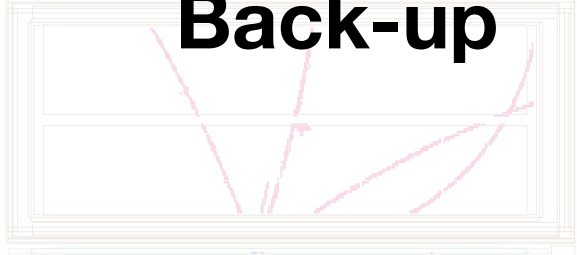
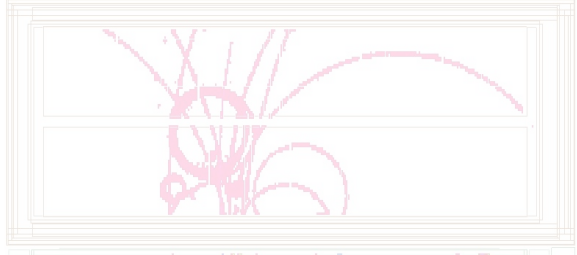
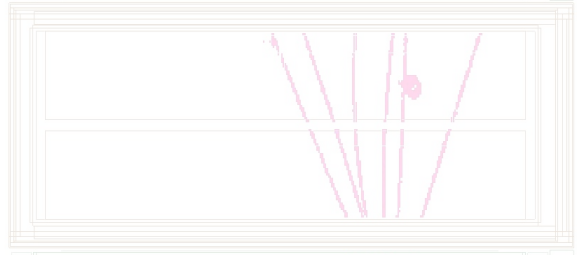
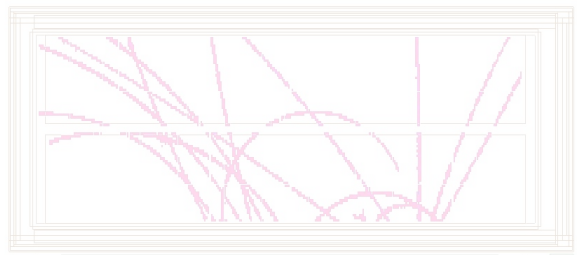




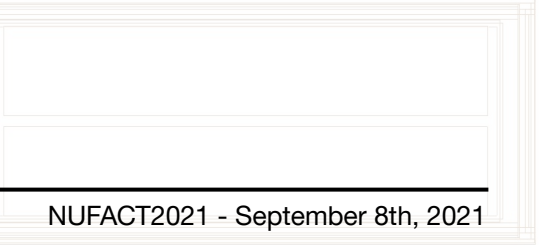
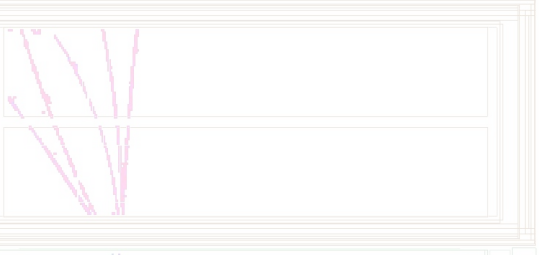
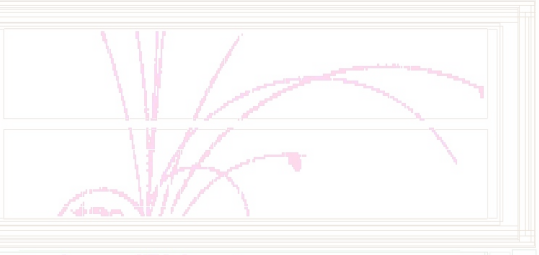
5. Summary and prospects

- A new set of physically-motivated uncertainties on the (semi-)inclusive predictions of the CCQE Spectral Function model were introduced
- Fitting to external data shows great improvement for T2K
- MINERvA data is sensitive to this parameterisation but shows little improvement in the data/MC agreement

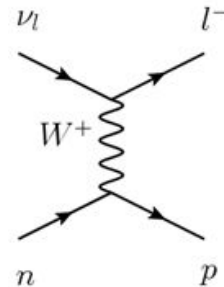
- This parameterisation may still need further improvement especially for FSI and low energy transfer effects (e.g. see Jordan's talk next)
- New data and an appropriate choice of observables may help to better constrain and improve the model
- Some of this work has been used in a recent sensitivity study of T2K's ND upgrade (see [arXiv:2108.11779](https://arxiv.org/abs/2108.11779))
- A complete set of uncertainties on semi-inclusive models will be crucial for upcoming oscillation measurements



Back-up



- By Pauli principle, an interaction cannot occur if it leads to the creation of a nucleon in a state that is already occupied
- Different possible models for SF:



This talk

$$\Theta(p_F - p_{final})P(p_m, E_m)$$

(PB1)
RFG-like

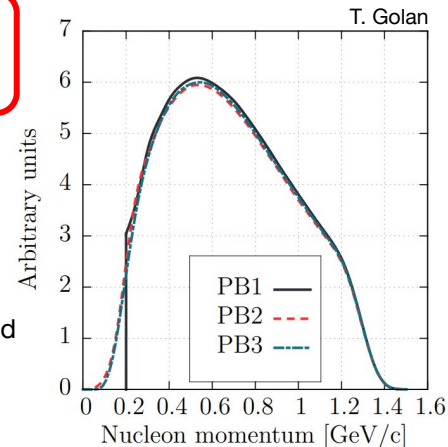
$$\Theta(p_F(r) - p_{final})P(p_m, E_m)$$

(PB2)
LFG-like

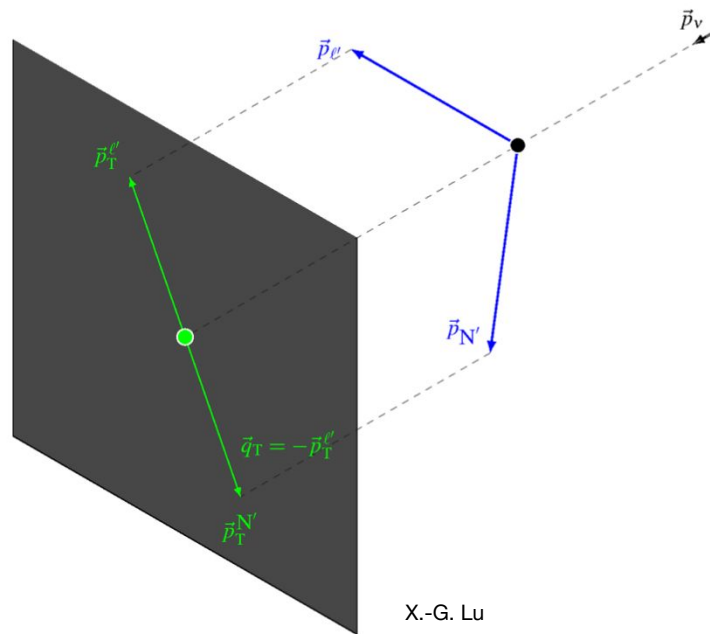
$$\Theta(n(p_{final}) - \text{rand}[0, 1])P(p_m, E_m)$$

(PB3)
MC method

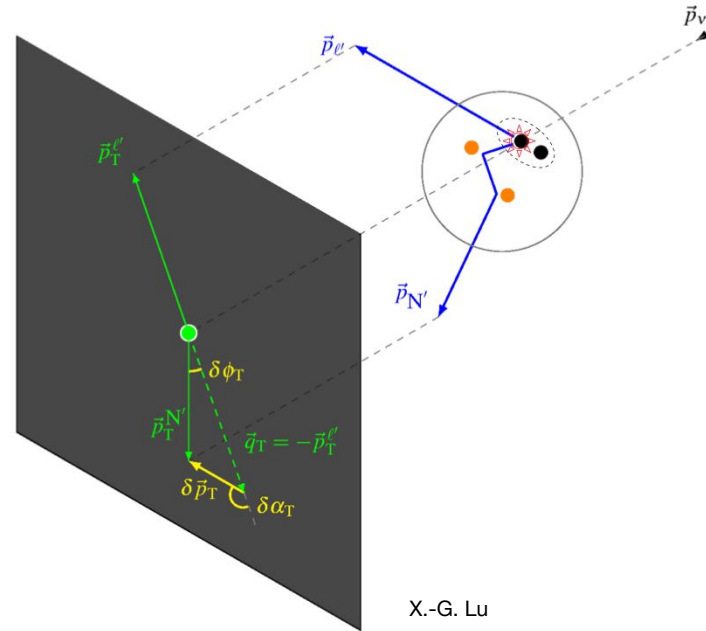
$P(p_m, E_m)$



Static nucleon target



Nucleon bound within nuclear target



- Need the reconstruction of both muons and protons
- Probe nuclear effects (Fermi motion, FSI, ...)

- Attempts to fit neutrino cross-section data with a parameterisation of the interaction model gives us seemingly unphysically low normalisations PPP
- The reason this happens is subtle, but is related to the strong correlations in published covariance matrices and the corresponding “type” of Gaussian errors approximation
- In our standard approach we assume the absolute uncertainty on the cross section is independent of its normalisation
 - i.e. our uncertainties state that a 10 fb / GeV uncertainty on some bin remains at 10 fb / GeV even if we had underestimated our flux by 10% (and so the cross section is lower than measured)
 - This implies the relative uncertainty is larger if fitting to models that predict lower normalisations. This is what give us PPP
- We could alternatively suggest that it should be the relative uncertainty that is independent of its normalisation (D’Agostini does)
 - i.e. our uncertainties would state that a 10% uncertainty on some bin remains at 10% even if we had underestimated our flux by 10%
 - This implies the absolute uncertainty is larger if fitting to models that predict lower normalisations

- We can construct a covariance matrix that keeps the relative uncertainties constant when the normalization changes: a “Norm-Shape” covariance where one row contains the normalization of the data and the rest contains the shape

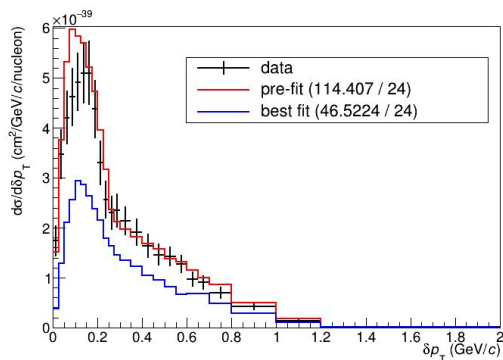
$$H_1 : \{B_1, \dots, B_n\} \rightarrow H_2 : \{C_1, \dots, C_n\} \quad C_i = \begin{cases} \frac{B_i}{\sum_k B_k} & , i < n \\ \sum_k B_k & , i = n \end{cases}$$

- We can obtain $\text{Cov}[\{C_i\}]$ (norm-shape covariance) directly from the data covariance given by experiments $\text{Cov}[\{B_i\}]$
- Perform the fit in this new basis using $N = \text{Cov}[\{C_i\}]$

$$\chi_{\text{NS}}^2 = \sum_{1 \leq i, j \leq n} (C_i - C_i^{\text{MC}}) (N^{-1})_{ij} (C_j - C_j^{\text{MC}})$$

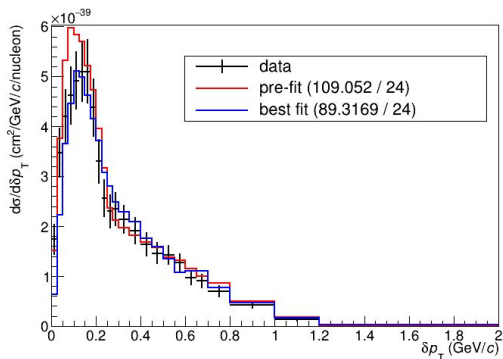
- Example of MINERvA dpt fit with the same parameterisation of slide 41:

Fit using minimizing the usual chi2

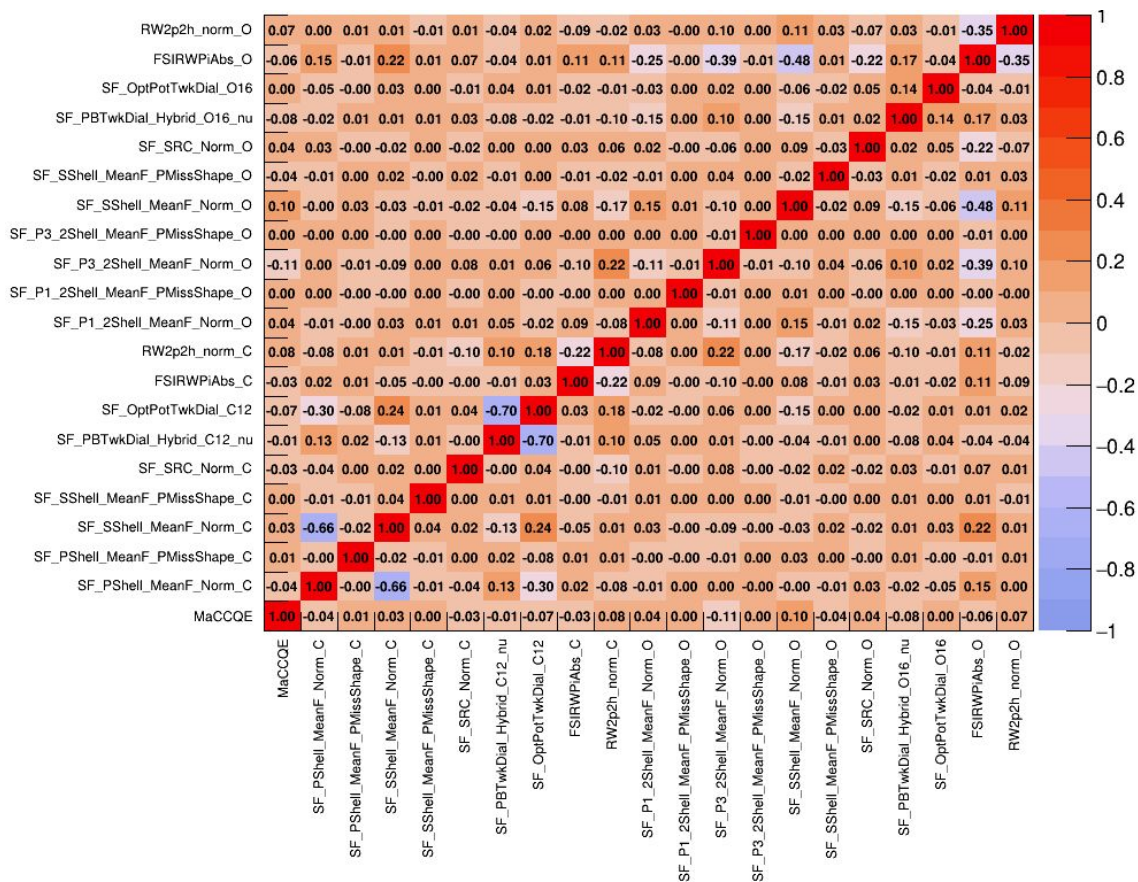


| | Normal chi2 | NS chi2 |
|----------------|----------------|---------|
| prefit | 114.407 | 109.052 |
| postfit | 46.5224 | 108.201 |

Fit using minimizing the new chi2



| | Normal chi2 | NS chi2 |
|----------------|-------------|----------------|
| prefit | 114.407 | 109.052 |
| postfit | 85.2858 | 89.3169 |



Parameters used in this fit:

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- Shell occupancies
- p_m shape
- SRC norm
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- Pion absorption normalization
- 2p2h normalization

