The anomalous magnetic moment of the muon in the Standard Model

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Magnetic moment

- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio
- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM

Electron vs. muon magnetic moments

- influence of heavier virtual particles of mass M scales as

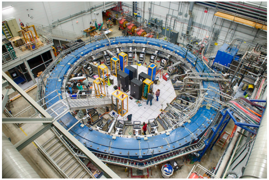
$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

- $(m_\mu/m_e)^2 \approx 4 \times 10^4 \Rightarrow$ muon is much more sensitive to **new physics**, but also to **EW and hadronic contributions**
- a_τ experimentally not yet known precisely enough

Muon anomalous magnetic moment $(g-2)_\mu$

recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**
- theory still needs to reduce **SM uncertainty**!



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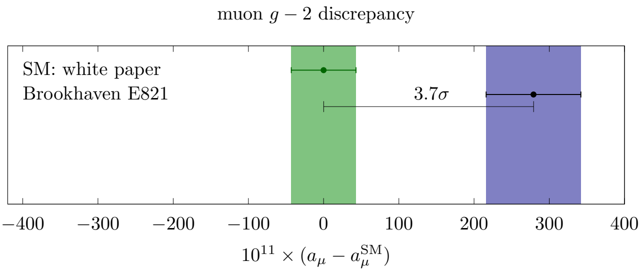

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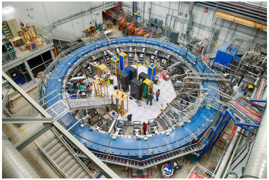

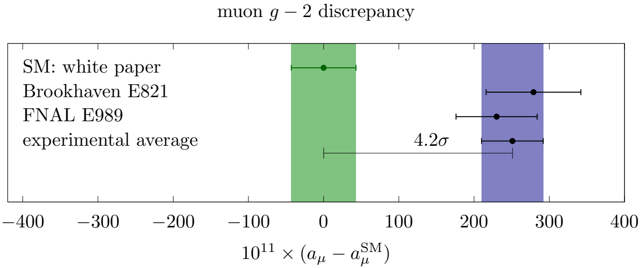

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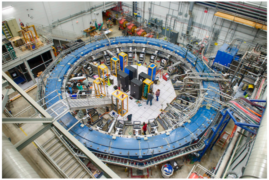

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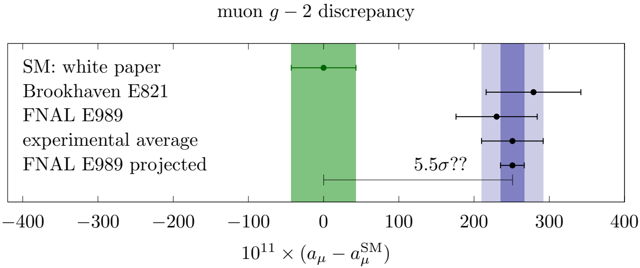

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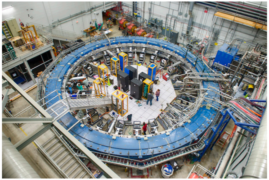
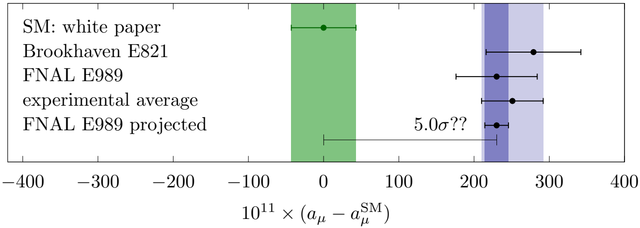


muon $g-2$ discrepancy



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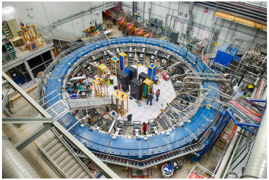

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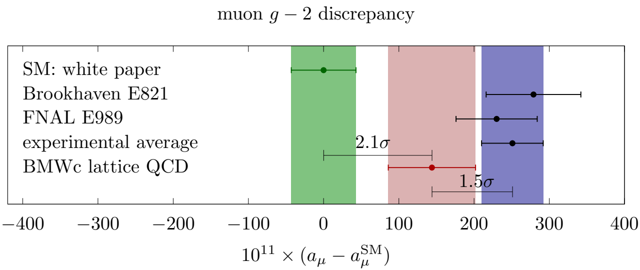


SM theory white paper

$\rightarrow$ T. Aoyama *et al.* (Muon $g-2$ Theory Initiative), Phys. Rept. **887** (2020) 1-166

- community white paper on current status of **SM calculation**
- new consensus on SM prediction, used for **comparison with FNAL result**
- many improvements on **hadronic contributions**

$(g-2)_\mu$: theory vs. experiment

- discrepancy between SM theory white paper and experiment 4.2σ
- hint to new physics?
- size of discrepancy points at **electroweak scale** $\Rightarrow$ heavy new physics needs some enhancement mechanism
- theory error completely dominated by **hadronic effects**
- how to interpret lattice-QCD result by BMWc?

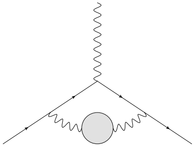
QED and electroweak contributions

- full $\mathcal{O}(\alpha^5)$ calculation by Kinoshita et al. 2012 (involves 12672 diagrams!)
- EW contributions (EW gauge bosons, Higgs) calculated to two loops (three-loop terms negligible)

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.931$	0.104
EW	153.6	1.0
theory total	$116\,591\,810$	43

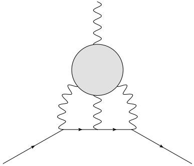
Hadronic contributions

- quantum corrections due to the strong nuclear force
- much smaller than QED, but **dominate uncertainty**



- hadronic vacuum polarization (HVP)

$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$



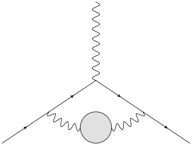
- hadronic light-by-light scattering (HLbL)

$$a_\mu^{\text{HLbL}} = 92(18) \times 10^{-11}$$

Theory vs. experiment

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.931$	0.104
EW	153.6	1.0
HVP	$6\,845$	40
HLbL	92	18
SM total	$116\,591\,810$	43
experiment (E821+E989)	$116\,592\,061$	41
difference theory−exp	251	59

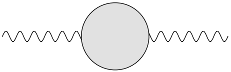
Hadronic vacuum polarization (HVP)



- at present evaluated via **dispersion relations** and cross-section input from $e^+e^- \to$ hadrons
- intriguing discrepancies between e^+e^- experiments $\Rightarrow$ treated as additional systematic uncertainty
- lattice QCD making fast progress
- $\mathbf{2.1\sigma}$ **tension** between dispersion relations and latest lattice results $\to$ S. Borsanyi *et al.*, Nature (2021)

Hadronic vacuum polarization (HVP)

photon HVP function:



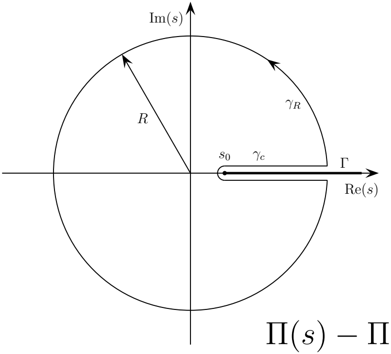
$$= i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma(e^+e^- \to \mathrm{hadrons})$$

Dispersion relation

causality implies **analyticity**:



Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s'-s} ds'$$

deform integration path:

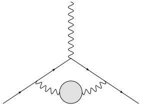
$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s'-s-i\epsilon)s'} ds'$$

HVP contribution to $(g-2)_\mu$

$$a_\mu^{\text{HVP}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\text{thr}}}^{\infty} ds\, \frac{\hat{K}(s)}{s}\, \sigma(e^+e^- \to \text{hadrons})$$

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section $\sigma(e^+e^- \to \text{hadrons})$
- dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE, SND)

Hadronic vacuum polarization



- final white paper number: data-driven evaluation

$$a_\mu^{\text{LO HVP, pheno}} = 6\,931(40) \times 10^{-11}$$

- previous average of published lattice-QCD results

$$a_\mu^{\text{LO HVP, lattice average}} = 7\,116(184) \times 10^{-11}$$

- newest lattice-QCD result

$\rightarrow$ S. Borsanyi *et al.*, Nature (2021)

$$a_\mu^{\text{LO HVP, lattice}} = 7\,075(55) \times 10^{-11}$$

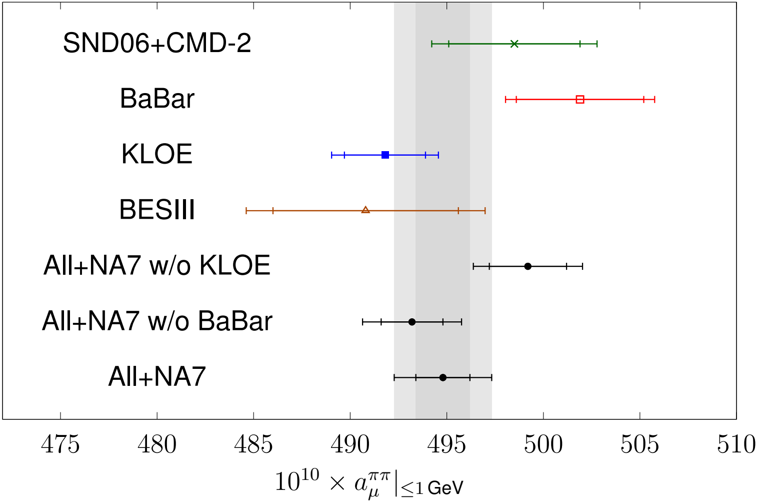
Two-pion contribution to HVP

- $\pi\pi$ contribution amounts to more than 70% of HVP contribution
- responsible for a similar fraction of HVP uncertainty
- can be expressed in terms of **pion vector form factor** $\Rightarrow$ constraints from analyticity and unitarity

  $\rightarrow$ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006

Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



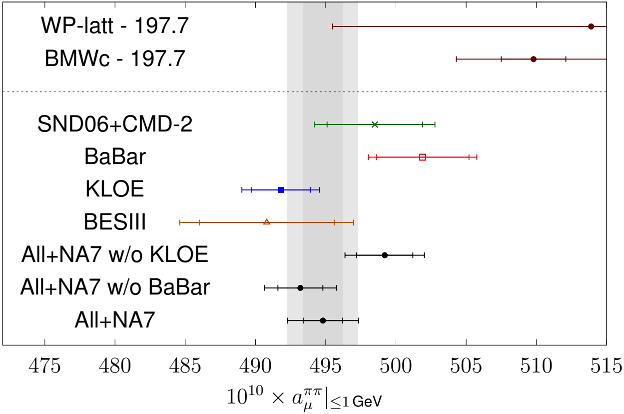
Tension with lattice QCD

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- implications of changing HVP?
- modifications at high energies affect **hadronic running of $\alpha_{\mathrm{QED}}^{\mathrm{eff}}$** $\Rightarrow$ clash with global EW fits

  → Passera, Marciano, Sirlin (2008), Crivellin, Hoferichter, Manzari, Montull (2020), Keshavarzi, Marciano, Passera, Sirlin (2020), Malaescu, Schott (2020)
- lattice studies point at region < 2 GeV
- $\pi\pi$ **channel** dominates
- relative changes in other channels would be prohibitively large

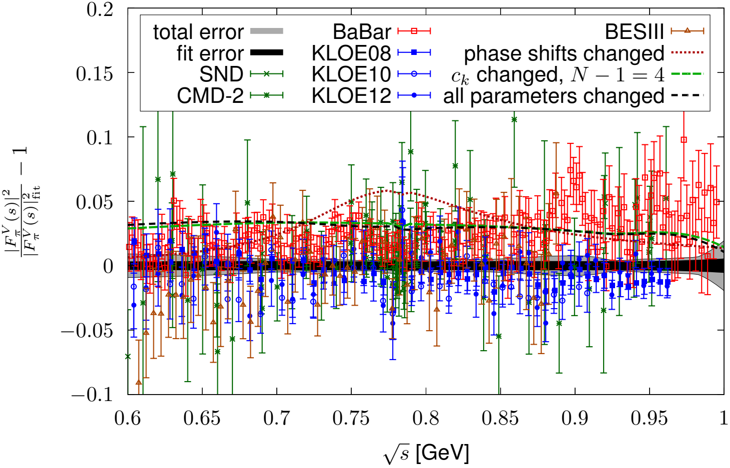
Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV



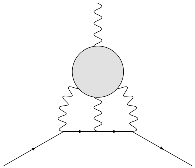
Assumption: suppose all changes occur in $\pi\pi$ channel < 1 GeV

$$\Rightarrow a_\mu^{\mathrm{total}}[\mathrm{WP20}] - a_\mu^{2\pi,<1\,\mathrm{GeV}}[\mathrm{WP20}] = 197.7 \times 10^{-10}$$

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

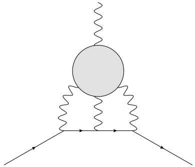


Hadronic light-by-light (HLbL)



- previously based only on hadronic models
- our work: **dispersive framework** based on unitarity and analyticity, replacing hadronic models step by step
- **hadronic models** only for subdominant contributions
- matching to **asymptotic constraints**

Hadronic light-by-light scattering



- dispersion relations + hadronic models (LO, without charm)

$$a_\mu^{\text{HLbL, pheno}} = 89(19) \times 10^{-11}$$

- first lattice-QCD results

$a_\mu^{\text{HLbL, lattice}} = 79(35) \times 10^{-11}$ → T. Blum *et al.*, PRL **124** (2020) 132002

$a_\mu^{\text{HLbL, lattice}} = 106.8(15.9) \times 10^{-11}$ → E.-H. Chao *et al.*, EPJC **81** (2021) 651

HLbL overview

→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
π^0, η, η'-poles	93.8	4.0
pion/kaon box	-16.4	0.2
S-wave $\pi\pi$ rescattering	-8	1
scalars, tensors	-1	3
axials	6	6
light quarks, short distance	15	10
c-loop	3	1
HLbL total (LO)	92	19

HLbL: recent progress

- asymptotic constraints: OPE and two-loop QCD corrections to symmetric limit $Q_{1,2,3} \gg \Lambda_{\text{QCD}}$

  $\to$ Bijnens et al., JHEP **10** (2020) 203; JHEP **04** (2021) 240

- scalar contributions: $\pi\pi/\bar{K}K$ S-wave rescattering up to 1.3 GeV plus $a_0(980)$ in NWA:

$$a_\mu^{\text{HLbL}}[\text{scalars}] = -9(1) \times 10^{-11}$$

  $\to$ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- first steps towards including axials in dispersive framework: $\to$ Zanke, Hoferichter, Kubis, JHEP **07** (2021) 106, Colangelo, Hagelstein, Hoferichter, Laub, Stoffer, EPJC **81** (2021) 702

- holographic-QCD models point to rather large axial contribution $\to$ Cappiello et al., PRD **102** (2020) 016009, Leutgeb, Rebhan, PRD **101** (2020) 114015; arXiv:2108.12345 [hep-ph]

Summary

- both lattice-QCD and dispersive methods making progress on hadronic contributions to $(g-2)_\mu$
  $\Rightarrow$ white paper
- **achieved precision matches** the experimental one
- new FNAL result increases tension with SM to $\boldsymbol{4.2\sigma}$
- final FNAL precision goal calls for **further improvement** in HLbL and HVP

Summary: HVP

- long-standing discrepancy between BaBar/KLOE
  $\Rightarrow$ wait for new e^+e^- data
- intriguing tension with lattice-QCD
  $\Rightarrow$ unitarity/analyticity enable **independent checks** via pion VFF and $\langle r_\pi^2 \rangle$, in addition to further direct lattice results on HVP

Summary: HLbL

- precise **dispersive evaluations** of dominant contributions
- models reduced to sub-dominant contributions, but **dominate uncertainty**
- recent progress on scalar contributions, ongoing work on axial-vector and tensor resonances and asymptotic matching