Quantum Decoherence in Neutrino Oscillations

Emilio Ciuffoli

QMRC, IMP, CAS
email: emilio@impcas.ac.cn

NuFact201, September 9th, 2021

3 Flavor Oscillations

One of the assumptions in the standard, textbook derivation of neutrino oscillations is clearly wrong

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} \quad \Rightarrow \quad |\nu_a\rangle = \sum_\alpha U_{a\alpha} |\nu_\alpha\rangle$$

$$a = e, \mu, \tau, \ \alpha = 1, 2, 3$$

$|\nu_\alpha\rangle$ are eigenstates of the mass matrix

$$|\nu_a(t)\rangle = \sum_\alpha U_{a\alpha} |\nu_\alpha(t)\rangle = \sum_\alpha U_{a\alpha} e^{-iE_\alpha t} |\nu_\alpha\rangle = \sum_{\alpha, a'} U_{a\alpha} e^{-iE_\alpha t} U^\dagger_{\alpha a'} |\nu_{a'}\rangle$$

Here we are assuming that each mass eigenstate is an eigenstate of the Hamiltonian as well

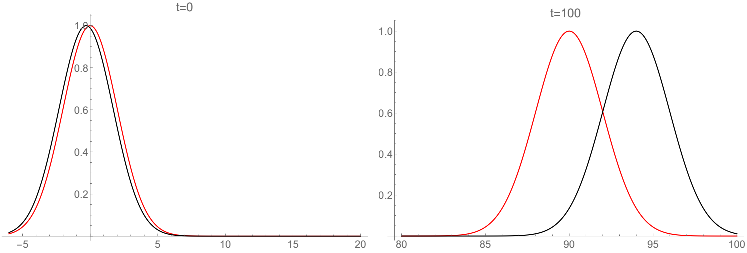
BUT this is an eigenstate of the momentum $\Rightarrow$ plane waves, completely delocalized: they cannot propagate!

Decoherence

In reality, each mass eigenstate can be described using wavepackets, which are localized

$$|\nu\rangle = \int \mathrm{d}p \; f(p)|p\rangle$$

However different masses → different velocities: while propagating, the mass eigenstates will be separated; if this distance is larger than the spatial dimension of the wavepackets, there is no interference between their phases and no oscillations anymore



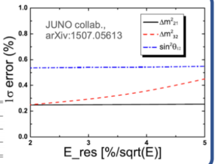
Why Is Decoherence Important?

- Decoherence has never been observed, and there is still no solid theoretical description of such an effect
- We are entering in the precision era of neutrino physics: the current or the next generation of experiment will measure most of the mixing parameters to the sub-percent precision. Even a small effect that can modify the oscillation probability could affect the results of experiments; this could be true for JUNO, for example **(Chan,Chu,Tsui,Wong and Xu, Eur. Phys. J. C 76 (2016) no.6, 310)**.

Neutrino oscillation parameters

- Sub-percent accuracy for θ_{12}, Δm^2_{21} and Δm^2_{31}
- Current precision

	Δm^2_{21}	$\lvert\Delta m^2_{31}\rvert$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ
Dominant Exps.	KamLAND	T2K	SNO+SK	Daya Bay	NOνA	T2K
Individual 1σ	2.4%	2.6%	4.5%	3.4%	5.2%	70%
Nu-FIT 4.0	2.4%	1.3%	4.0%	2.9%	3.8%	16%



Yue Meng, Neutrino2020

Different Approaches

State of the Art: no solid theoretical description of decoherence so far; many approaches in literature, however not even agreement on whether or not it should be observed.

Quantum Mechanic approach: all particles are described using rigid wavepackets, which are not created nor evolved dynamically according to QFT. All the relevant parameters (like the shape and dimension of the wavepackets) must be introduced by hand, usually are estimated using "order-of-magnitude" arguments, which can lead to very different answers.

Quantum Field Theory (QFT) is the formally correct approach (the QM method is just an approximation): the production and the detection of the neutrinos are described consistently by the time-evolution operator. We are developing a model using this approach.

$$|\Omega(0)\rangle = \int \mathrm{d}q \ldots f(q) \ldots |q \ldots\rangle \qquad \Rightarrow \qquad |\Omega(t)\rangle = e^{-iHt}|\Omega(0)\rangle$$

$$A = \langle F|\Omega(t)\rangle \qquad \Rightarrow P = |A|^2$$

Our Model

Started by considering very simplified scenarios, increasing gradually the complexity

- 1+1 dimension, real scalar fields
- First, only creation process **Eur.Phys.J. C79 (2019) no.6, 491**; we have shown that some of the assumptions commonly used in literature, such as the covariance of wavepackets, are not consistent **Nucl.Phys. B 953 (2020), 114972**
- Later, detection process as well, (however, still no environmental interactions) **Nucl.Phys. B958 (2020), 115113; Eur.Phys.J. C81 (2021) no.4, 325**
- No assumptions on the neutrinos, they are created by the time-evolution of the initial state, all the particles are described by dynamical fields

Long term goal: given a (simplified) model for the environmental interactions, be able to compute the coherence length

Our Model

We consider the following processes for the neutrino creation and detection

$$S_H \to S_L + \nu_i \qquad D_L + \nu_j \to D_H$$

$$|0(t)\rangle = e^{-iHt} \int \mathrm{d}p\mathrm{d}q f(p) g(q) |S_H, p; D_L, q\rangle \qquad H = H_0 + \lambda H_I$$

$$\mathcal{H}_I = (\phi_{S,H}(x)\phi_{S,L}(x) + \phi_{D,H}(x)\phi_{D,L}(x))(\psi_1(x) + \psi_2(x))$$

$$A(k,l) = \langle S_L, l; D_H, k|0(t)\rangle = \int \mathrm{d}p\mathrm{d}q[...]\langle S_L, l; D_H, k|e^{-iHt}|S_H, p; D_L, q\rangle$$

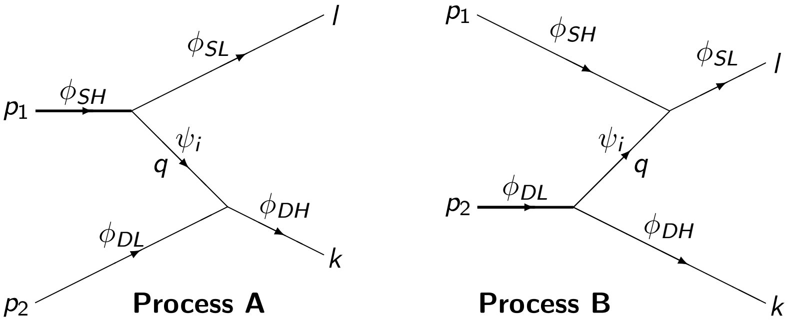
NOTE: NOT S-matrix, finite t!

$$P(k) = \int \mathrm{d}l |A(k,l)|^2 \qquad P = \int \mathrm{d}k P(k)$$

Tree-level approximation (justified when the timescale of the experiment is shorter than the lifetime of the source particle)

$$e^{-iHt} = \sum_{k=0}^{\infty} \frac{(-iHt)^k}{k!} \supset \sum_{k=0}^{\infty} \sum_{n=0}^{k-2} \sum_{m=0}^{k-n-2} \frac{(-it)^k}{k!} H_0^n H_I H_0^m H_I H_0^{k-n-m-2}$$

Our Model - Tree-level approximation



There are two processes that give a non-zero transition amplitude, but the second one is always off shell, so we can ignore it

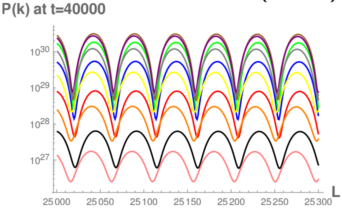
$$\langle S_L, l; D_H, k | e^{-iHt} | S_H, p; D_L, q\rangle \to \frac{e^{-i\mathcal{E}_2 t}}{\Delta\mathcal{E}_{01}} \left(\frac{e^{-i\Delta\mathcal{E}_{02} t} - 1}{\Delta\mathcal{E}_{02}} - \frac{e^{-i\Delta\mathcal{E}_{12} t} - 1}{\Delta\mathcal{E}_{12}} \right)$$

$$\to F(q, l) = \int_0^t \mathrm{d}t_1 \int_0^{t-t_1} \mathrm{d}T e^{-i(\mathcal{E}_0 t_1 + \mathcal{E}_1 T + \mathcal{E}_2 (t - t_1 - T))}$$

No Decoherence in Vacuum

In the QM approach there is a maximum deocherence length. Model-dependent, for example in **C. Giunti, C.W. Kim Phys. Rev. D 58, 017301 (1998)** we have

$$L_{max} \simeq \frac{16\pi^2 E^3}{(\Delta m^2)^2} \simeq 2,500 \qquad \begin{array}{l} E \simeq 1 \\ \Delta m^2 \simeq 0.25 \end{array}$$



At L=25,000 we still have oscillations! Why? In our model neutrino decay in vacuum, *i.e. no environmental interactions*. Without them, the state is the **coherent** sum over all the possible production times **Eur.Phys.J. C81 (2021) no.4, 325**

Environmental interaction localize in time and space the neutrino production **W. H. Zurek Phys. Rev. D 26 (1982) 1862**, which is crucial for the localization of the wavepacket **E. K. Akhmedov and A. Y. Smirnov, Found. Phys. 41 (2011) 1279**. This can be described by the overlapping function, or localizing the daughter particles **C. Giunti, JHEP 11 (2002) 017**, but it is a crude approximation

Are Environmental Interactions Crucial?

Are environmental interactions the only way to localize the neutrino production?

Not really: for example, if the lifetime of the source particles is **smaller** than the timescale of the experiment, the decay probability itself constrain the neutrino production.

If we manage to compute the non-perturbative transition amplitude, we should be able to see how this would affect the decoherence.

These calculations are similar to the ones involved in the computation of the Quantum Zeno Effect (see, for example, **P. Facchi and S. Pascazio, Chaos Solitons Fractals 12 (2001) 2777**), using similar techniques we are trying to tackle this problem.

Other Results

- Some of the assumptions commonly used in literature, such as the covariance of wavepackets (**D.V. Naumov, V.A. Naumov, J. Phys. G 37 (2010) 105014; F. P. An et al., Eur. Phys. J. C 77 (2017) no.9, 606**) are inconsistent, since the time evolution breaks the Lorentz invariance **Nucl.Phys. B 953 (2020), 114972**
- New quantum effect: in a very short time windows after the first neutrinos arrives, the oscillations have not started yet, if the detector is placed at the oscillation minimum, with sufficient time resolution is should be possible to see the detection probability to go down with time. Most likely the requirements for its observation are well beyond the current technical possibilities, however it is worth of more investigation **Eur.Phys.J. C81 (2021) no.4, 325**

Summary and Future Prospects

For a solid theoretical understanding of quantum decoherence it is crucial to have a consistent description of the time-evolution of the fields; we are developing a model that satisfy such a condition.

- We considered simplified scenarios, finding nonetheless interesting results
- The entanglement of the source particle with neutrino and with environment is crucial for decoherence: in vacuum, no decoherence because wavepackets emitted at different times can interact coherently one with the other. Decoherence can emerge from the finite lifetime of the source as well (working on this right now)
- We plan to study more in details the new quantum effect, to see what are precisely the requirements for its observation and if there are some regimes where it is easier
- The long-term goal is to introduce (some simplified model of) environmental interactions, to see how decoherence can arise from the localization of the neutrino production

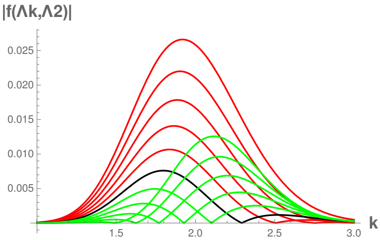
Thank You!

Backup Slides

In some works in the literature it is assumed that the neutrino wavefunction is covariant **D.V. Naumov, V.A. Naumov, J. Phys. G 37 (2010) 105014**; such an assumption was used also to compute the Daya Bay constrains on the decoherence parameters **F. P. An et al., Eur. Phys. J. C 77 (2017) no.9, 606**.

$$f(k,p) \propto e^{-(p-k)_\mu (p-k)^\mu / 2\sigma^2}$$

We have shown that such an assumption is inconsistent: even if, at time t_0, the state is covariant, the time evolution would break the Lorentz invariance



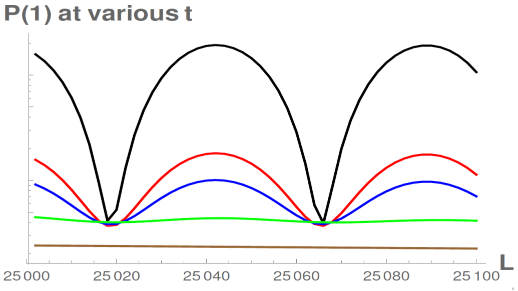
The red, black and green curves correspond to the boosted wavefunction with $\beta < 0$, $\beta = 0$ and $\beta > 0$, respectively

New Quantum Effect

For a very short window of time after the first neutrinos arrive, they have not oscillated yet; if the detector is placed at the oscillation minimum, it would be possible to see the detection probability to increase with time

$$A(k,l) \propto \int \mathrm{d}T[\ldots]e^{-\mu(T-T_0)^2} \to [\ldots]\delta(T-T_0)$$

To see this effect is equivalent to probe the shape of the Gaussian.



Most likely the requirements for its observation are well beyond the current technical possibilities, however it is worth of more investigation

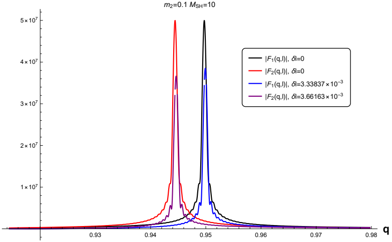
Quantum Zeno Effect

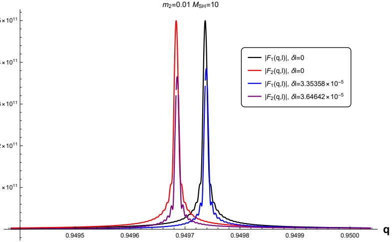
The decay probability follow an exponential behavior only at intermediate times, at very small (and very large) timescales it behave polynomially $\propto t^n$, with $n > 1$. This means that if we take an unstable system and, over a time T, we measure it n times to check whether or not it is decayed, for $n \to \infty$, $P(n) \to 0$. Usually the transition probability is calculated using the resolvent, namely

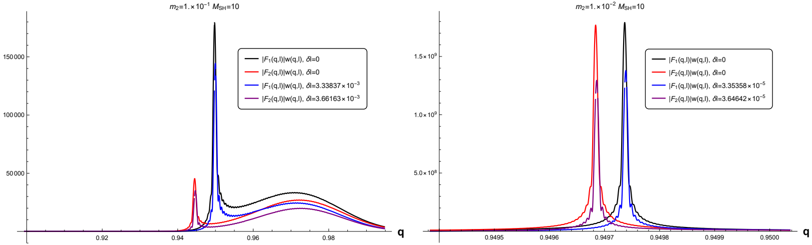
$$A(E) = \langle + | \frac{1}{H - E} | + \rangle = \frac{1}{E - \omega_+ - \Sigma(E)}$$

On-Shell

The factor $F(q, l)$, which imposes the on-shell condition

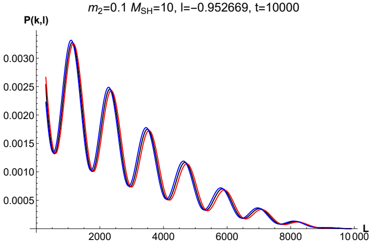


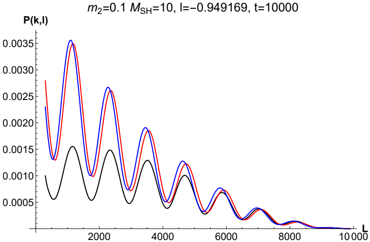




Considering the source and detector wavepackets as well, $\sigma_x = 0.015$

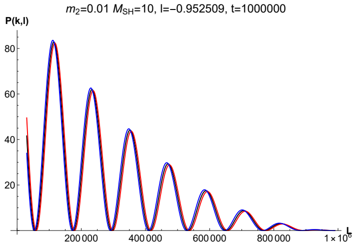
Oscillations

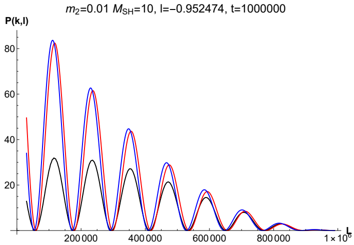




$$\sigma_x = 0.015, \; m_2 = 0.1$$

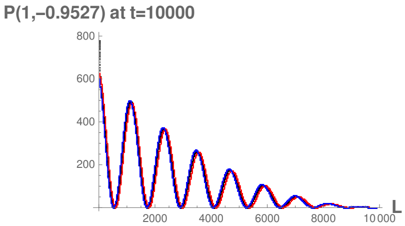
Oscillations

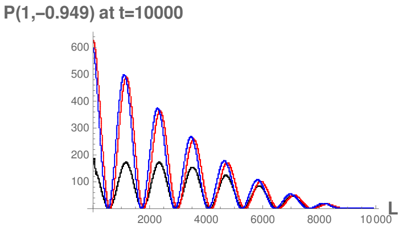




$$\sigma_x = 0.015, \; m_2 = 00.1$$

Oscillations





$$\sigma_x = 0.15, \; m_2 = 0.1$$