Possible studies on generalized parton distributions and gravitational form factors by high-energy LBNF neutrino beam

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https://indico.cern.ch/event/855372/

with Roberto Petti (Univ of South Carolina)

September 9, 2021

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- **Introduction**

  Origin of nucleon spin and mass

  Hadron tomography and 3D structure functions

- **Generalized distribution amplitudes [= timelike GPDs (generalized parton distributions)] and extraction of gravitational form factors from KEK-B data**

- **GPDs and gravitational form factors in neutrino reactions**

- **Prospects and summary**

I may skip some slides.

References

- General introductory review on GPDs (Generalized Parton Distributions)

  Generalized parton distributions,
  M. Diehl, Phys. Rept. 388 (2003) 41.

- Timelike GPDs (GDAs) and KEKB-data analysis

  Hadron tomography by generalized distribution amplitudes in pion-pair production process $\gamma^*\gamma \to \pi^0\pi^0$ and gravitational form factors for pion,
  S. Kumano, Qin-Tao Song, O. V. Teryaev, Phys. Rev.D 97 (2018) 014020.

- Updated information on GPDs in neutrino reactions

  Hard exclusive neutrino production of a light meson,
  B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 95, 114029 (2017) and references therein.

- High-energy neutrino interactions and GPDs

  High-energy neutrino-nucleus interactions,
  S. Kumano, EPJ Web Conf. 208 (2019) 07003.

- Synergies between EIC (Electron-Ion Collider) project and neutrino reactions

  EIC yellow report, A. Accardi *et al.*, arXiv:2103.05419,
  see Sec. 7.5.2, Neutrino phyiscs by S. Kumano and R. Petti.

- Fermilab-LBNF neutrino-beam information

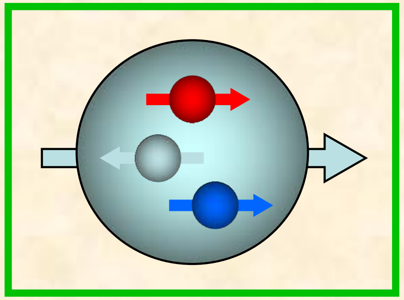
  Impact of high energy beam tunes on the sensitivities to the standard unknowns at DUNE,
  J. Rout, S. Roy, M. Masud, M. Bishai, P. Mehta, Phys. Rev. D 102 (2020) 116018.

Introduction

- Origins of nucleon spin and mass
- Hadron tomography
  and 3D structure functions

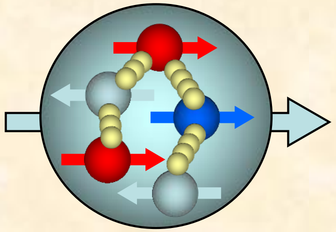
Origin of nucleon spin

“old” standard model

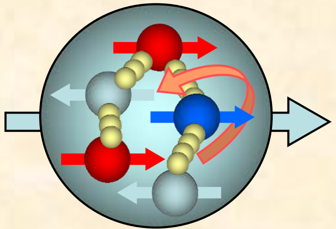


$$p_\uparrow = \frac{1}{3\sqrt{2}}\left(uud\left[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right] + \text{permutations}\right)$$

$$\boxed{\begin{aligned}&\Delta q(x) \equiv q_\uparrow(x) - q_\downarrow(x)\\ &\Delta\Sigma = \sum_i \int dx\left[\Delta q_i(x) + \Delta\bar{q}_i(x)\right] \to 1\ (100\,\%)\end{aligned}}$$

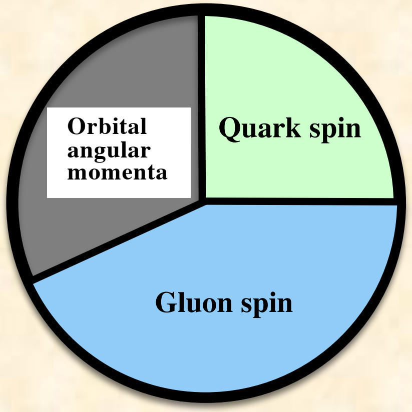


gluon spin



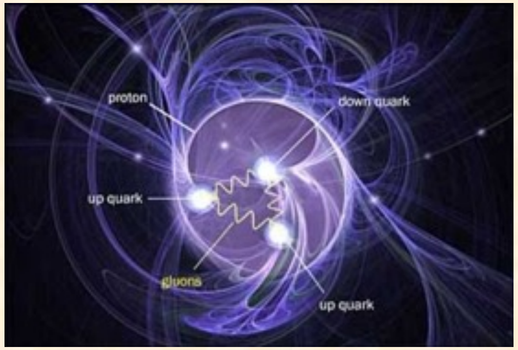
angular momentum

$$\frac{1}{2} = \frac{1}{2}\underbrace{\left(\Delta u_v + \Delta d_v + \Delta q_{sea}\right)}_{\Delta\Sigma} + \Delta G + L_q + L_g$$



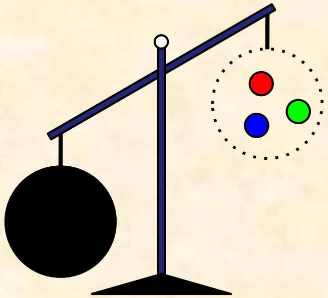
“A possible” spin decomposition

Possibly a large gluon-spin contribution (2014)



Scientific American (2014)

Origin of hadron masses

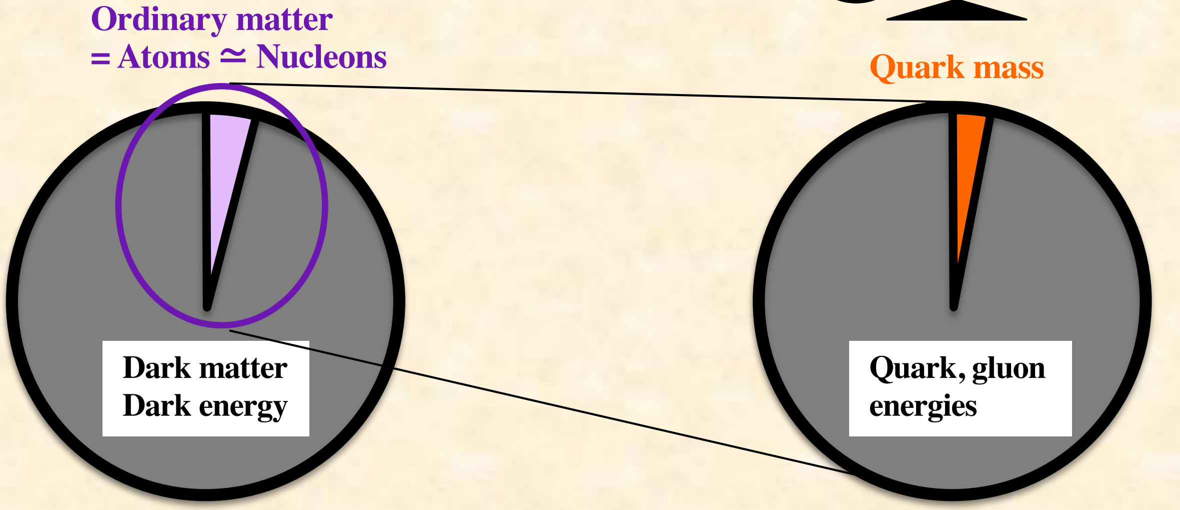


Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \middle| \int d^3x \, T^{00}(x) \middle| p \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

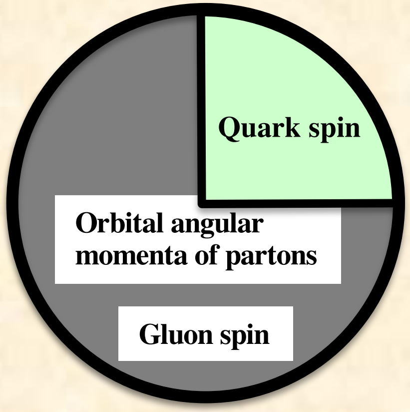


Dark matter | **Origin of nucleon mass**

Nucleon spin: $\frac{1}{2} = \left\langle p \middle| J^3 \middle| p \right\rangle$

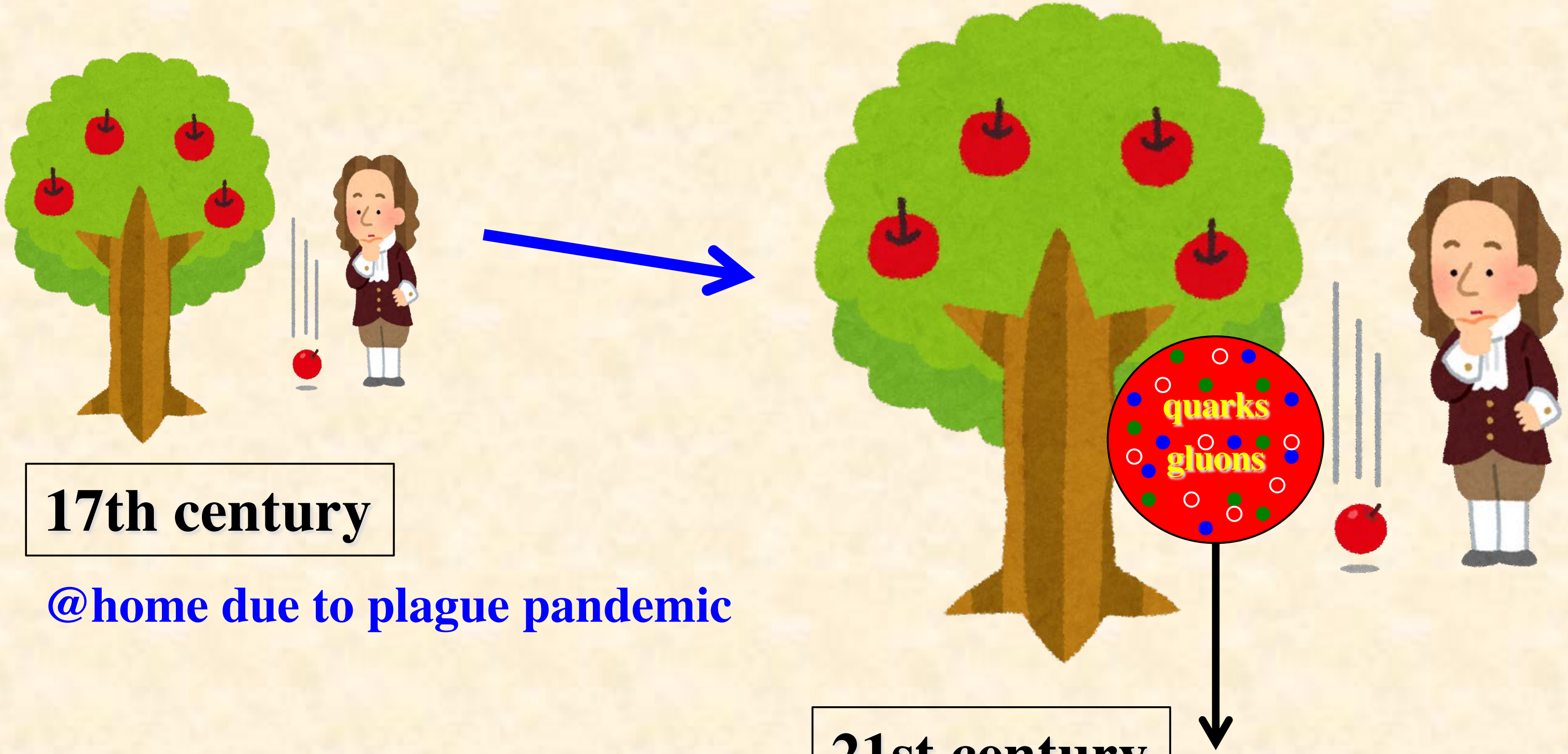
3rd component of total angular momentum: $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$



Origin of nucleon spin
("Dark spin")

Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic) world in terms of quark and gluon degrees of freedom.



17th century

@home due to plague pandemic

21st century

@home due to coronavirus pandemic

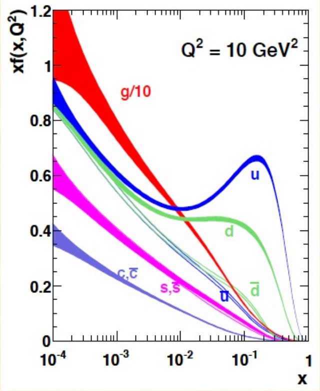
Hadron tomography: Wigner distribution and various structure functions

$$\begin{tikzcd}
\boxed{\text{Form factor}} & & \boxed{\text{PDF (Parton Distribution Function)}} & \\
& \boxed{\text{GTMD } W(x,\vec{k}_T,\Delta)} \arrow[ul, "\int dx\, d^2k_T"'] \arrow[ur, "\int d^2k_T,\ \Delta\to 0"] \arrow[r, "\Delta^+\to 0"'] \arrow[d, "\int d^2k_T"] \arrow[dl, "\Delta\to 0"'] & \text{Wigner } W(x,\vec{k}_T,\vec{r}_T) & \\
\boxed{\text{TMD (Transverse Momentum Dependent) parton distribution}} & \text{GPD (Generalized Parton Distribution)} \arrow[r, "s\leftrightarrow t"'] & \text{GDA (Generalized Distribution Amplitude)} &
\end{tikzcd}$$

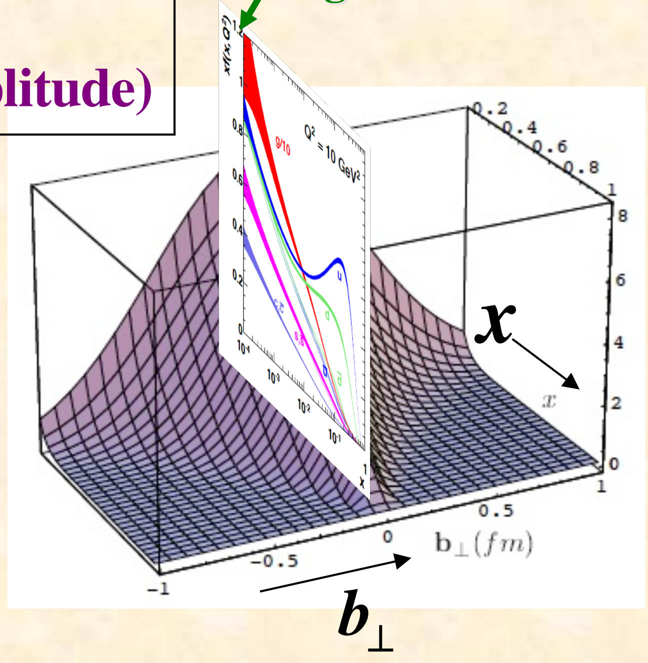
$\Delta = p' - p$

Two-photon process $\gamma^* \gamma \to h\bar{h}$

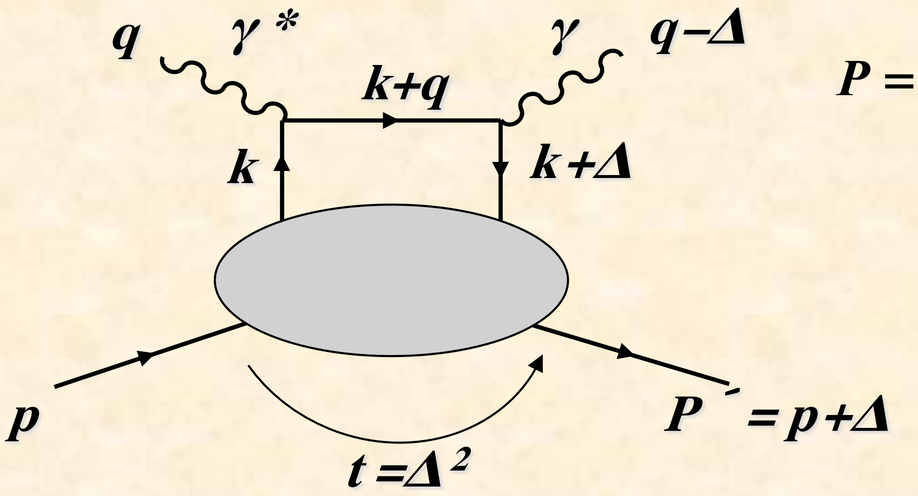
3D world



e.g. HERA studies



Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \dfrac{Q^2}{2p\cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

Forward limit: PDFs $H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

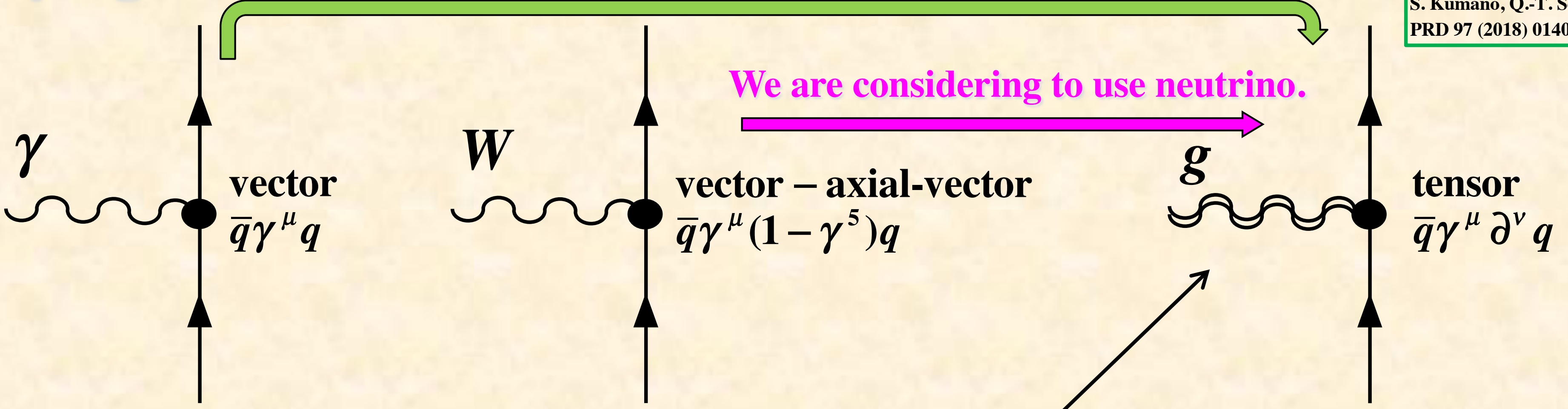
Sum rule: $J_q = \dfrac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \dfrac{1}{2}\Delta q + L_q$

⇒ probe L_q, key quantity to solve the spin puzzle!

Why "gravitational" interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev,
PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\underline{\int\frac{dz^-}{4\pi}e^{ixP^+z^-}\langle p'|\bar{q}(-z/2)\gamma^+q(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0}}=\frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p)+E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

Non-local operator of GPDs/GDAs:

$$(P^+)^n\int dx\,x^{n-1}\underline{\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left[\bar{q}(-z/2)\gamma^+q(z/2)\right]_{z^+=0,\vec{z}_\perp=0}}$$

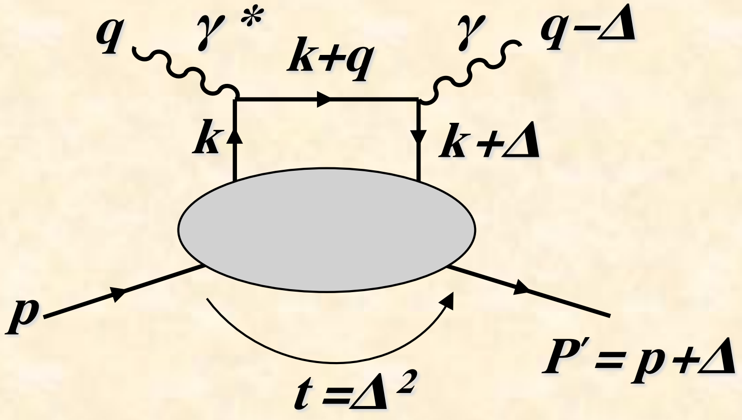
$$=\left(i\frac{\partial}{\partial z^-}\right)^{n-1}\left[\bar{q}(-z/2)\gamma^+q(z/2)\right]_{z=0}$$

$$=\underline{\bar{q}(0)\gamma^+\left(i\overleftrightarrow{\partial}^+\right)^{n-1}q(0)}$$

= energy-momentum tensor of a quark for $n=2$
(electromagnetic for $n=1$)

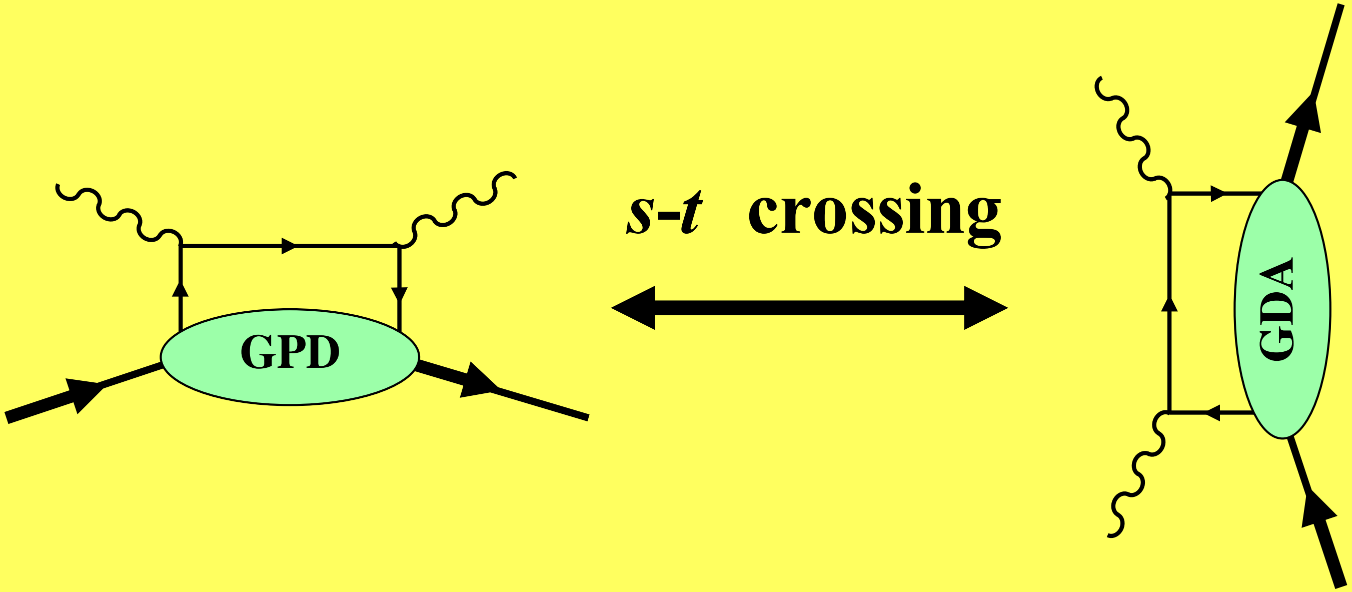
= source of gravitiy

Virtual Compton
or (timelike) two-photon process



Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data

GDA = Timelike GPDs



***s*-*t* crossing**

S. Kumano, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

Cross section for $\gamma^* \gamma \to \pi^0 \pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1-\frac{4m_\pi^2}{s}} \overline{\sum_{\lambda,\lambda'}} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i\int d^4\xi e^{-i\xi\cdot q} \left\langle \pi(p)\pi(p') \middle| T J_{em}^\mu(\xi) J_{em}^\nu(0) \middle| 0 \right\rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2)$$

GDA: $\Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p)\pi(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1-\frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2)$$

- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1)\zeta(1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

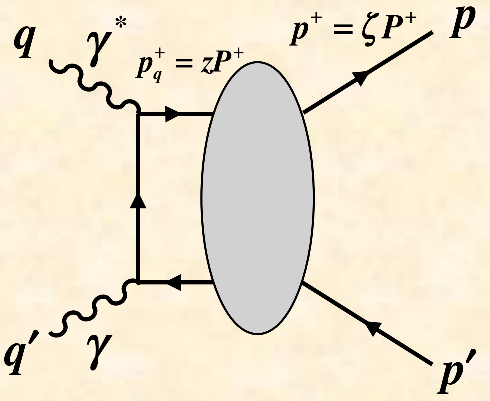
- **Resonances: Tthere exist resonance contributions to the cross section.**

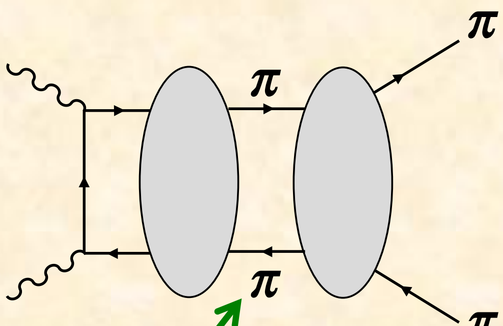
$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1) \left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde{B}_{10}(W) = \text{resonance } \left[f_0(500), f_0(980)\right] + \text{continuum}$$

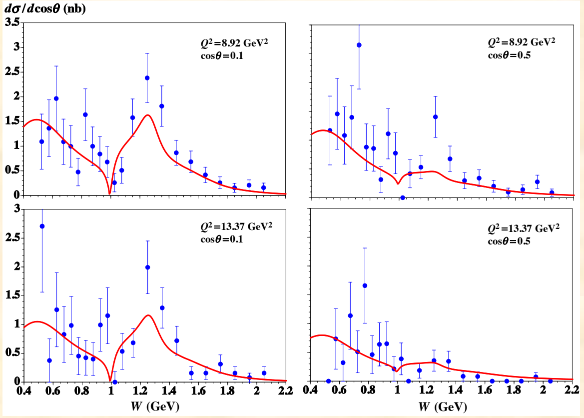
$$\tilde{B}_{12}(W) = \text{resonance} \left[f_2(1270)\right] + \text{continuum}$$



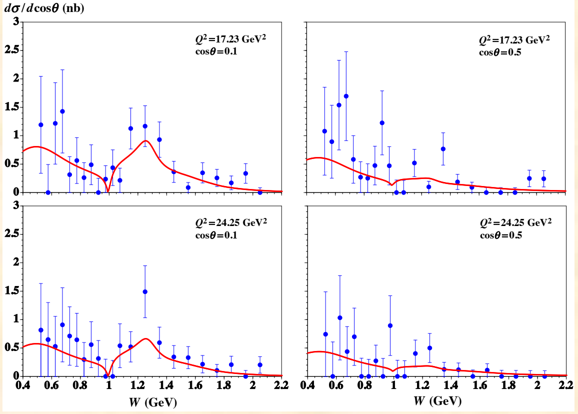


Including intermediate resonance contributions

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$



$Q^2 = 17.23,\ 24.25\ \text{GeV}^2$



Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

Gravitational form factors and radii for pion

$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

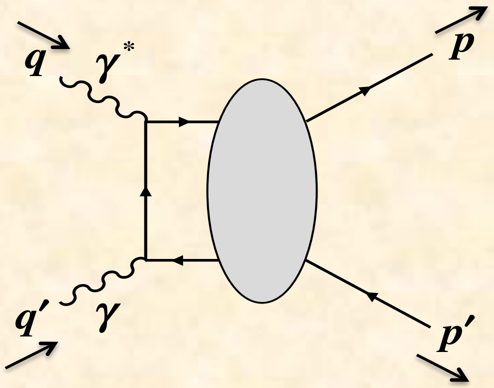
$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu \Delta^\nu \Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p$$

See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion



Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

⇒ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

⇒ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

⇒ Gravitational radii of pion

Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009.

Spacelike gravitational form factors and radii for pion

$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds\,\frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q\,e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\,e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

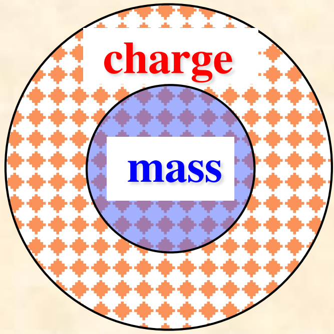
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim0.39\ \text{fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim0.88\ \text{fm}$$

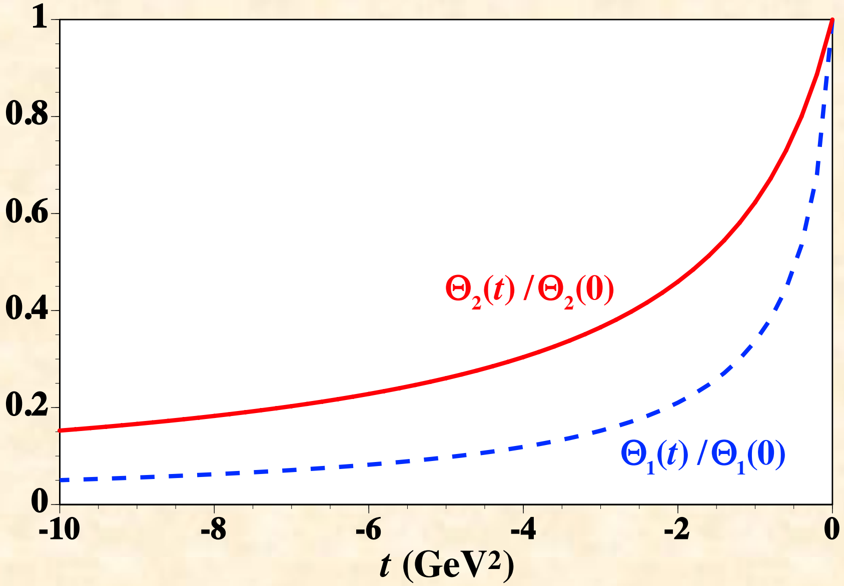
←

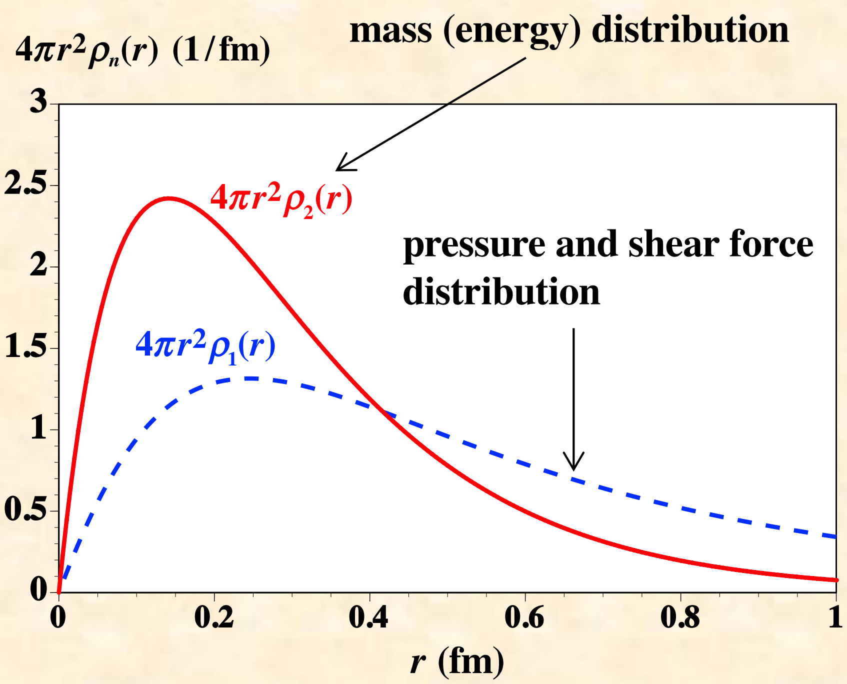
First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow\ \sqrt{\langle r^2\rangle_{\text{charge}}}=0.672\pm0.008\ \text{fm}$$



Hadron mass radius puzzle?





mass (energy) distribution

pressure and shear force distribution

Possible studies on
GPDs and gravitational form factors
in neutrino reactions

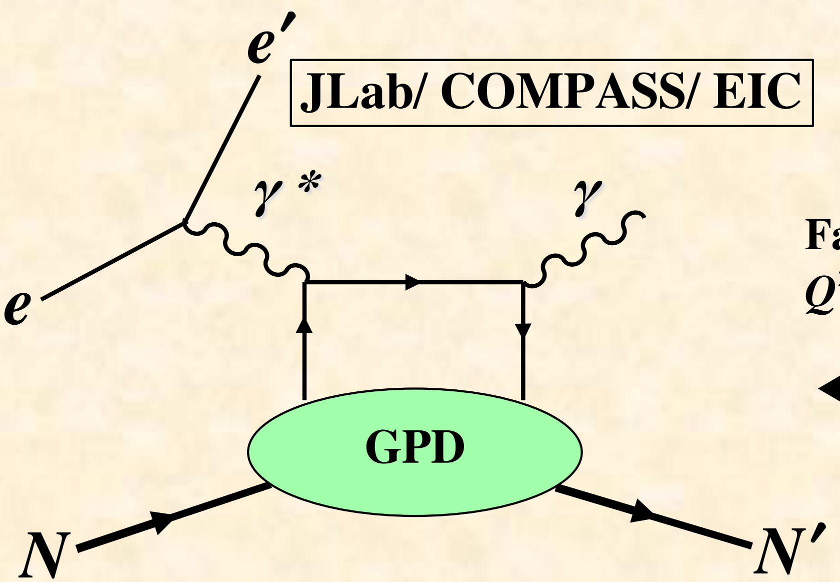
References on GPDs in ν reactions

may skip

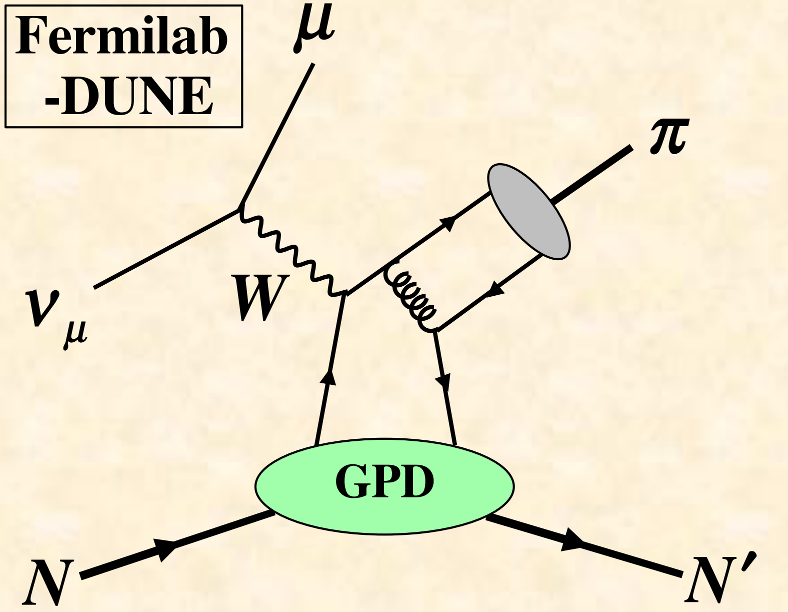
- B. Lehmann-Dronke and A. Schafer, Phys. Lett. B 521, 55 (2001).
  D_s production
- P. Amore, C. Coriano, and M. Guzzi, J. High Energy Phys. 02 (2005) 038.
  DVNS (Deeply Virtual Neutrino Scattering): Neutral current
- C. Coriano and M. Guzzi, Phys. Rev. D 71, 053002 (2005).
  DVNS (Deeply Virtual Neutrino Scattering): Charged current
- A. Psaker, W. Melnitchouk, and A. V. Radyushkin, Phys. Rev. D 75, 054001 (2007).
  Detailed GPD formalism and numerical analysis on DVNS
- G. R. Goldstein, O. G. Hernandez, S. Liuti, and T. McAskill, AIP Conf. Proc. 1222, 248 (2010).
  π^0 production formalism and nuclear target
- B. Z. Kopeliovich, I. Schmidt, and M. Siddikov, Phys. Rev. D 86, 113018 (2012).
  Meson (π, K, η) productions and GPDs
- B. Z. Kopeliovich, I. Schmidt, and M. Siddikov, Phys. Rev. D 89, 053001 (2014).
  Higher-twist effects in π production
- B. Pire and L. Szymanowski, Phys. Rev. Lett. 115, 092001 (2015).
  $D^{+/-}$ production and chiral-odd GPDs
- M. Siddikov and I. Schmidt, Phys. Rev. D 95, 013004 (2017).
  NLO corrections in π and K productions
- B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 95, 094001 (2017).
  D production with gluon (in addition to quark) contributions
- B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 95, 114029 (2017).
  π and ρ production with gluon (in addition to quark) contributions

↑ * The most updated information is obtained in this 2017 publication for the pion production.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

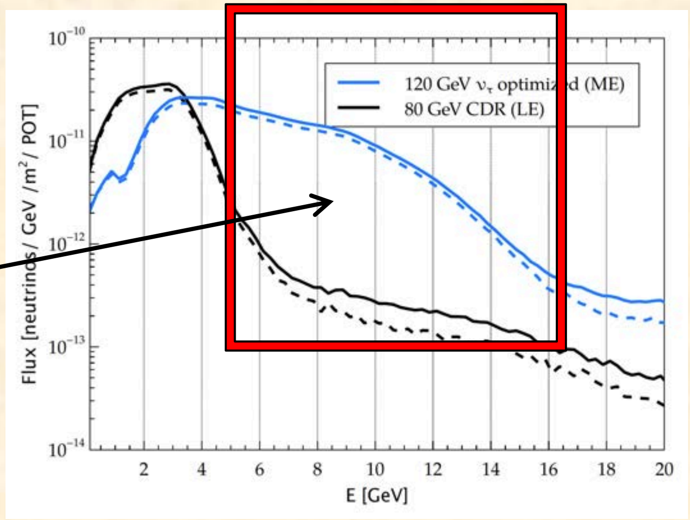


Factorization condition:
$Q^2 \gg |t|, \Lambda^2_{\rm QCD}$



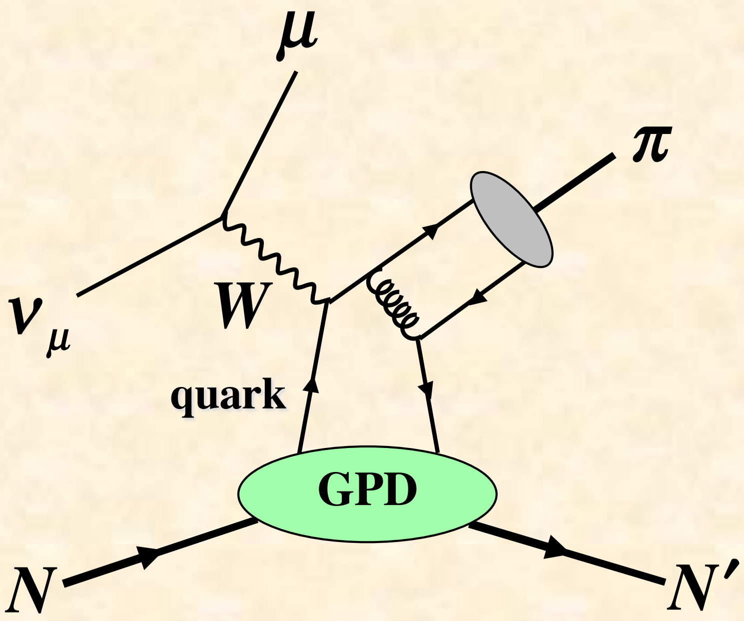
Deep Underground Neutrino Experiment (DUNE) at Long-Baseline Neutrino Facility (LBNF)

High-energy part of the LBNF ν beam can be used for the GPD studies.



J. Rout *et al.*, PRD 102 (2020) 116018

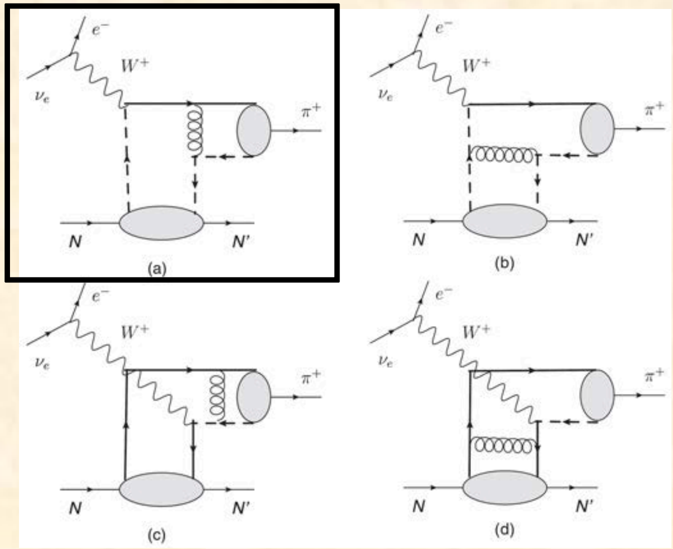
Recent work on pion production in neutrino reaction for GPD studies



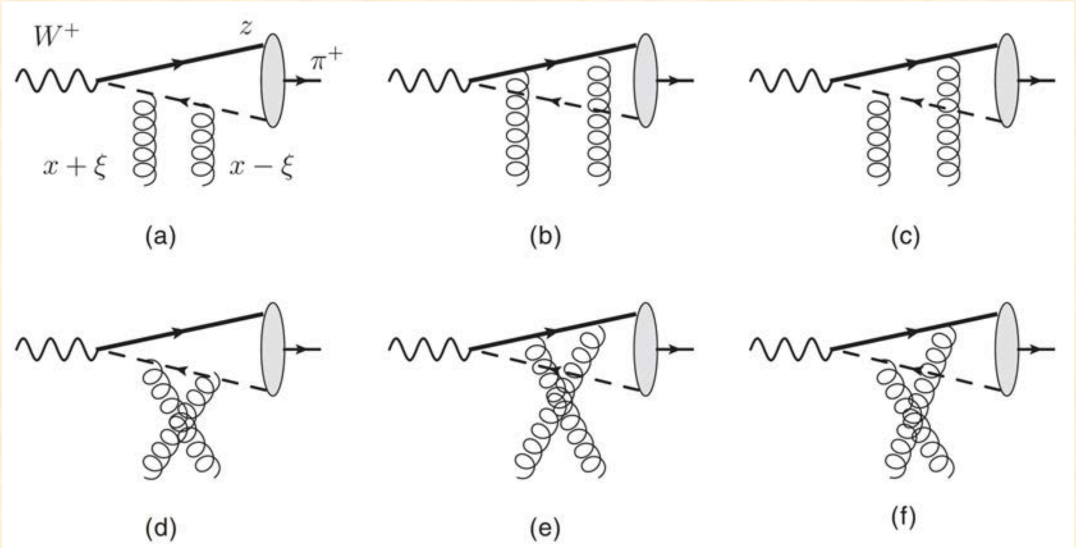
B. Pire, L. Szymanowski, and J. Wagner,
Phys. Rev. D 95, 114029 (2017).

There are several processes to contribute to the pion-production cross section, including the gluon GPD terms.

Quark GPDs



Gluon GPDs



Cross section formalism

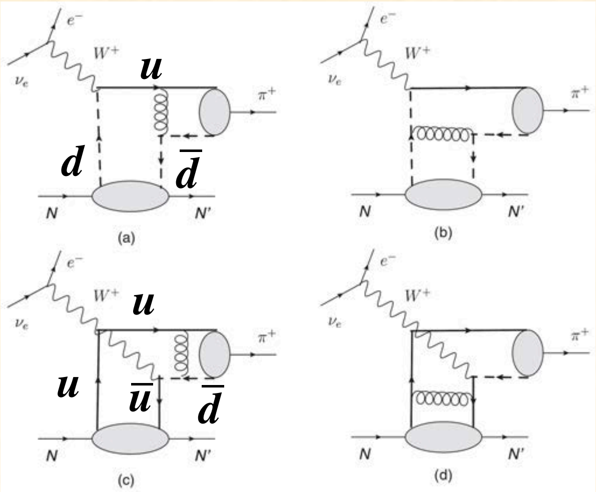
B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} = \Gamma\, \varepsilon\, \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32\,(2\pi)^4 (s-m_N^2)^2\, y\,(1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^{\nu} = \frac{1}{Q^2}\Bigg[(1-\xi^2)\left\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\widetilde{\mathcal{H}}_q\right|^2\right\} + \frac{\xi^4}{1-\xi^2}\left\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\widetilde{\mathcal{E}}_q\right|^2\right\}$$
$$-2\xi^2\,\mathrm{Re}\left\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\right\} - 2\xi^2\,\mathrm{Re}\left\{C_q\widetilde{\mathcal{H}}_q(C_q\widetilde{\mathcal{E}}_q)^*\right\}\Bigg]$$

Quark contributions



$$T_q = -i\frac{C_q}{2Q} N(p')\left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \widetilde{\mathcal{H}}_q \hat{n}\gamma_5 - \widetilde{\mathcal{E}}_q \frac{\gamma_5 n\cdot\Delta}{2m_N}\right] N(p)$$

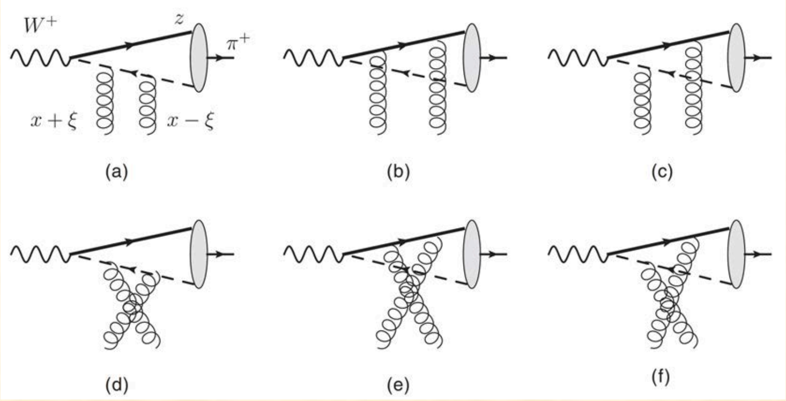
$$\mathcal{F}_q = 2f_\pi \int \frac{dz\,\phi_\pi(z)}{1-z}\int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

= (pion distribution amplitude) · (quark GPD)

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

$$F = H, E, \tilde{H}, \tilde{E}$$

Gluon contributions



$$T_g = -i\frac{C_g}{2Q} N(p')\left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N}\right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi}\int \frac{dz\,\phi_\pi(z)}{z(1-z)}\int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

Cross section estimates

proton: $\nu p \to \ell^- \pi^+ p$ **neutron: $\nu n \to \ell^- \pi^+ n$**

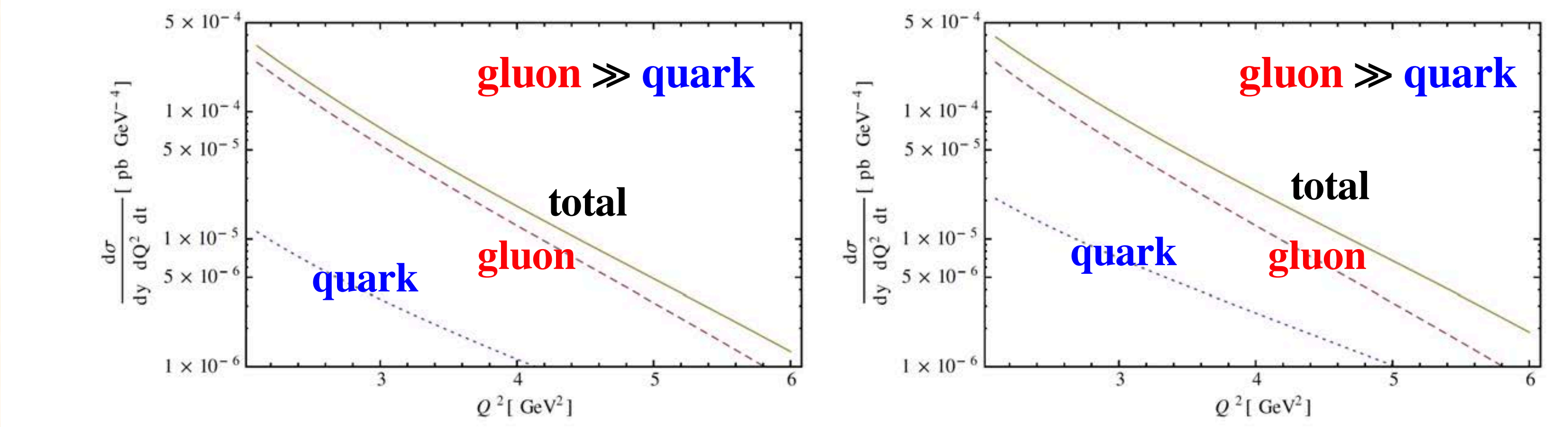


FIG. 3. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu N\to l^- N\pi^+)}{dydQ^2dt}$ (in pb GeV^{-4}) for $y = 0.7$, $\Delta_T = 0$ and $s = 20$ GeV^2, on a proton (left panel) and on a neutron (right panel). The quark contribution (dotted curves) is significantly smaller than the gluon contribution (dashed curves). The solid curves are the sum of the (quark + gluon + interference) contributions.

neutron → proton: $\nu n \to \ell^- \pi^0 p$

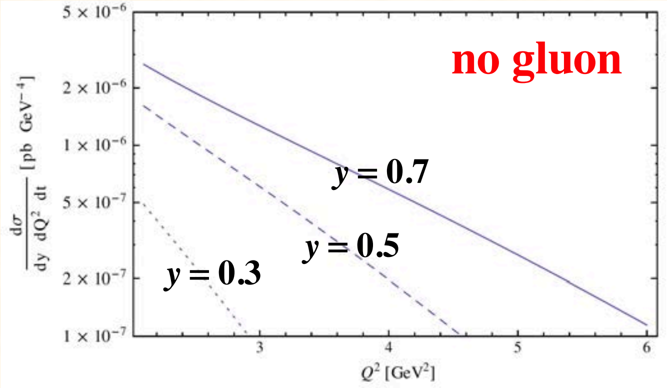
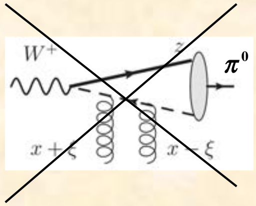


FIG. 6. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu n\to l^- p\pi^0)}{dydQ^2dt}$ (in pb GeV^{-4}) for $\Delta_T = 0$ and $s = 20$ GeV^2. The solid, dashed, and dotted lines correspond to $y = 0.7$, 0.5, and 0.3, respectively. There is no gluon contribution to this amplitude.

Neutrino GPD studies are complementary to the charged-lepton projects.

- **Gluon GPDs could be probed in charged-pion production.**
- **Flavor dependece of quark GPDs could be investigated.**

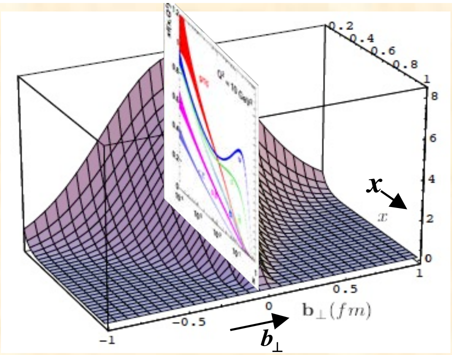


no gluon for π^0

Prospects & Summary

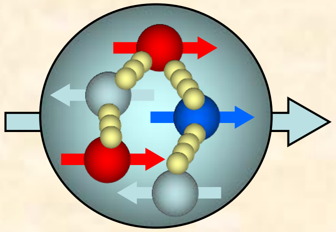
By hadron tomography

3D view of hadrons

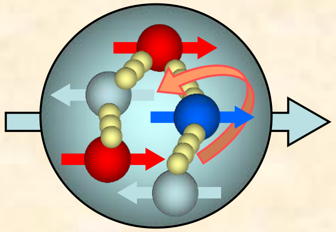


Origin of nucleon spin

By the tomography, we determine



or



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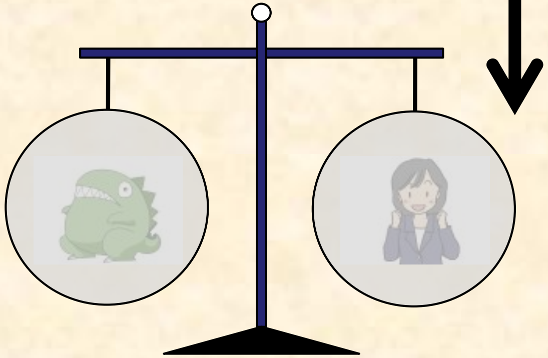
Exotic hadrons



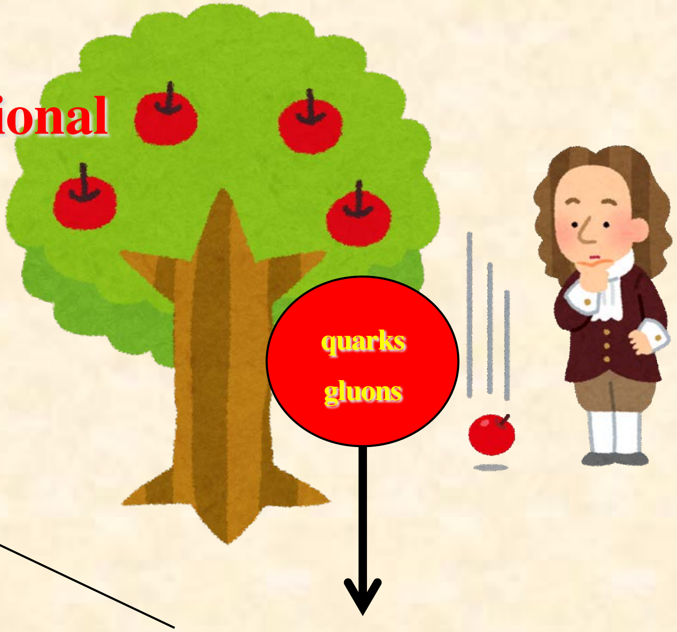
By tomography, we determine

or

.

Origin of gravitational source (mass)

By tomography, we determine gravitational sources in terms of quarks and gluons.



Origin of nucleon spin: decomposition

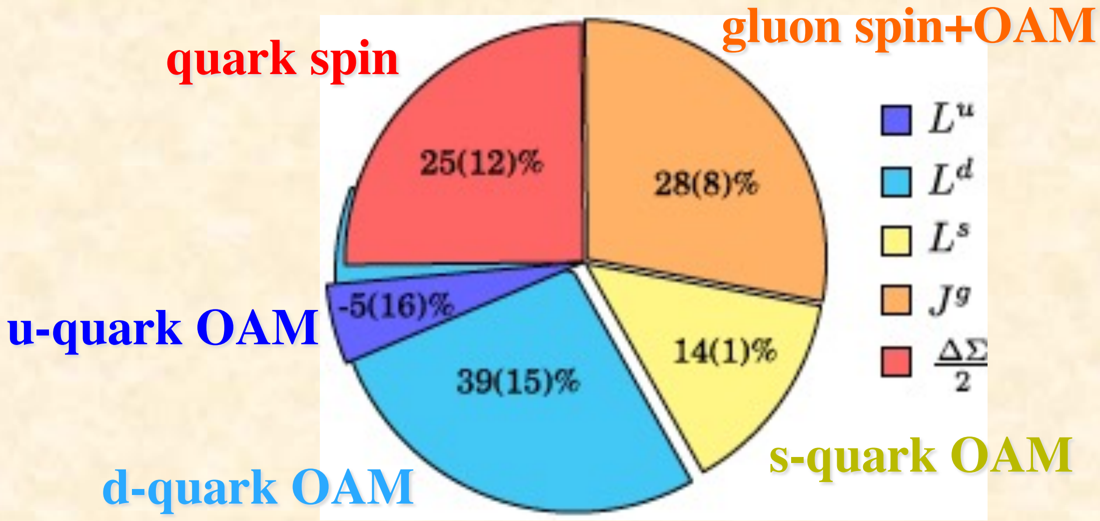
may skip

$$\frac{1}{2}=\left\langle p\middle|J^{3}\middle|p\right\rangle,\quad J^{3}=\frac{1}{2}\varepsilon^{3jk}\int d^{3}x\, M^{3jk}(x),\quad M^{\alpha\mu\nu}(x)=T^{\alpha\nu}(x)x^{\mu}-T^{\alpha\mu}(x)x^{\nu}$$

Gauge invariant decomposion: see review papers of M. Wakamatsu, Int. J. Mod. Phys. A29 (2014) 1430012;
E. Leader and C. Lorce, Phys. Rept. 541 (2014) 163;
and Y. Hatta (and S. Yoshida, K. Tanaka), Phys. Rev. D84 (2011) 041701;
Phys. Lett. B 708 (2012) 186; JHEP 1210 (2012) 080; 1302 (2013) 003.

$\frac{1}{2}=\frac{1}{2}\Delta\Sigma+\Delta g+L_q+L_g$, $\Delta\Sigma$ = **quark spin contribution,** Δg = **gluon spin contribution,**
L_q = **quark orbital-angular-momentum (OAM) contribution,**
L_g = **gluon orbital-angular-momentum (OAM) contribution**

Lattice QCD estimate in M. Deke *et al.*, Phys. Rev. D 91 (2015) 0145505



Spin decomposion

- **quark spin 25%**
- **quark OAM 45%**
- **gluon spin + OAM 30%**

Origin of nucleon mass

may skip

Nucleon mass: $M = \langle p | H | p \rangle, \quad H = \int d^3x \, T^{00}(x)$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}{}_{\alpha}(x)$$

We need theoretical and experimental efforts
to decompose nucleon mass for finding its origin.

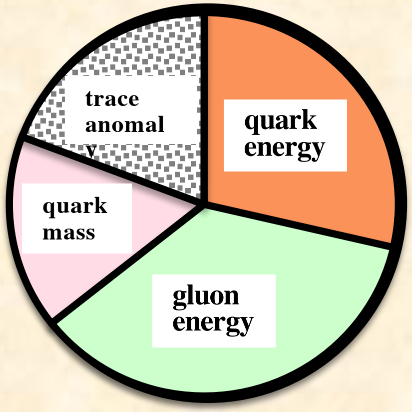
X. Ji, PRL 74 (1995) 1071.

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \bar{T}^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}\right)_{\text{traceless}} + \left(-\frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}\right)_{\text{trace}}, \quad T^{\alpha}{}_{\alpha} = \bar{q}\, m\, q + \frac{\beta(g)}{2g} F^2$$

$$H = H_q(\text{quark energy}) + H_g(\text{gluon energy}) + H_m(\text{quark mass}) + H_a(\text{trace anomaly})$$

$$H_q = \int d^3x \, \bar{q}(x)\left(-i\vec{D}\cdot\vec{\alpha}\right) q(x), \quad H_g = \int d^3x \, \frac{1}{2}\left(\vec{E}^2 + \vec{B}^2\right)$$

$$H_m = \int d^3x \, \bar{q}(x)\, m\, q(x), \quad H_s = \int d^3x \, \frac{9\alpha_s}{16\pi}\left(\vec{E}^2 + \vec{B}^2\right)$$



Recent progress on trace-anomaly, gravitational form factor, scale depdence in perturbative QCD:

Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;
K.Tanaka, JHEP 01 (2019) 120.

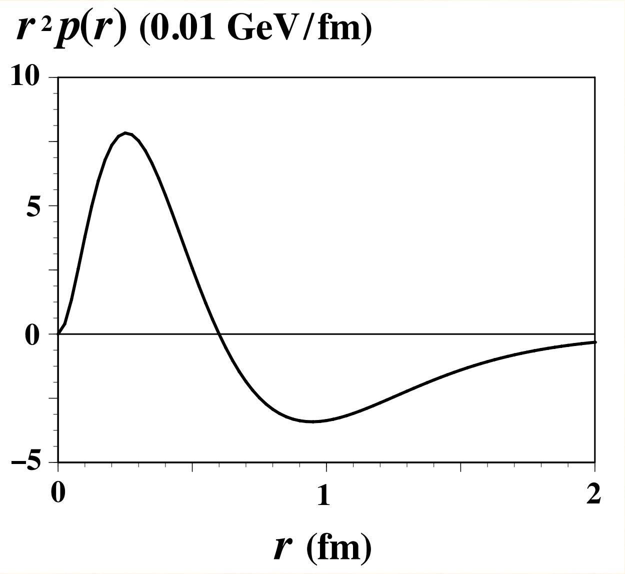
Nucleon pressure

may skip

$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

Recent progress

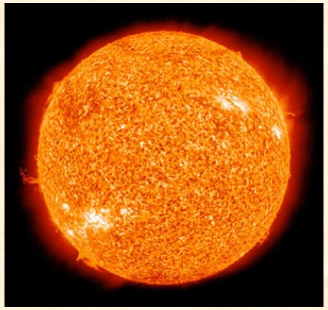
V. D. Burkert, L. Elouadrhiri, and F. X. Girod,
Nature 557 (2018) 396;
M. V. Polyakov and P. Schweitzer,
Int. J. Mod. Phys. A 33 (2018) 1830025;
C. Lorce, H. Moutarde, and A. P. Tranwinski,
Eur. Phys. J. C 79 (2019) 89.



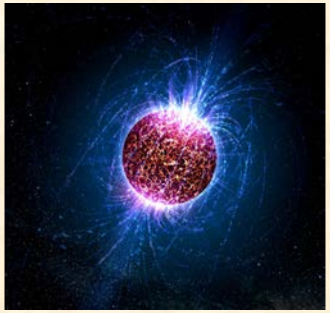
Highest pressure in nature   1 Pa (Pascal) = 1 N/m^2

Earth atmosphere
10^5 Pa =1000 hPa

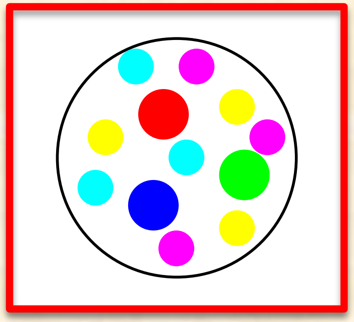
Center of earth
10^{11} Pa = 100GPa



Center of Sun
10^{16} Pa = 10 PPa

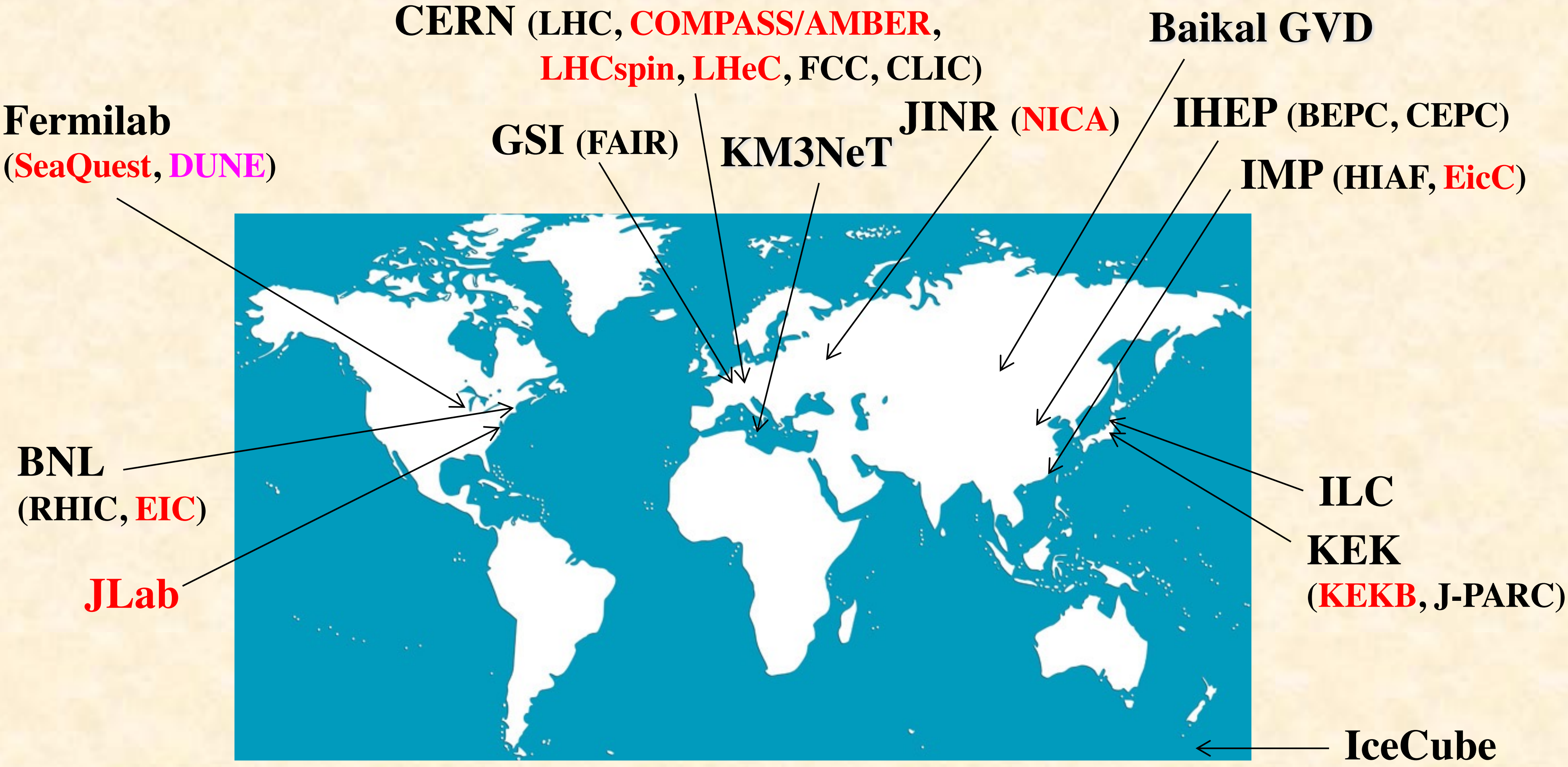


Neutron star
10^{34} Pa



Hadron
10^{35} Pa

GPD studies in neutrino reactions at LBNF is unique!



CERN (LHC, COMPASS/AMBER, LHCspin, LHeC, FCC, CLIC)

Baikal GVD

Fermilab (SeaQuest, DUNE)

GSI (FAIR)

KM3NeT

JINR (NICA)

IHEP (BEPC, CEPC)

IMP (HIAF, EicC)

BNL (RHIC, EIC)

ILC

KEK (KEKB, J-PARC)

JLab

IceCube

Facilities on hadron structure functions.

DUNE could be used for structure function studies (*e.g.* gravitational form factors).

Summary

Hadron-tomography project at neutrino facilities

- **Puzzle to find the origin of nucleon spin**
- **Puzzle to find the origin of hadron masses in terms of quark and gluon degrees of freedom**
- **Neutrino-scattering experiments (LBNF) are valuable and complementary to JLab, COPMASS, KEK-B, and the other facility projects in the sense that the cross sections are sensitive to quark flavor.**

Time has come to understand the gravitational sources and their interactions in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

The End

The End