Update of Bodek-Yang Model 2021

(accounting for difference between Vector and axial structure functions)

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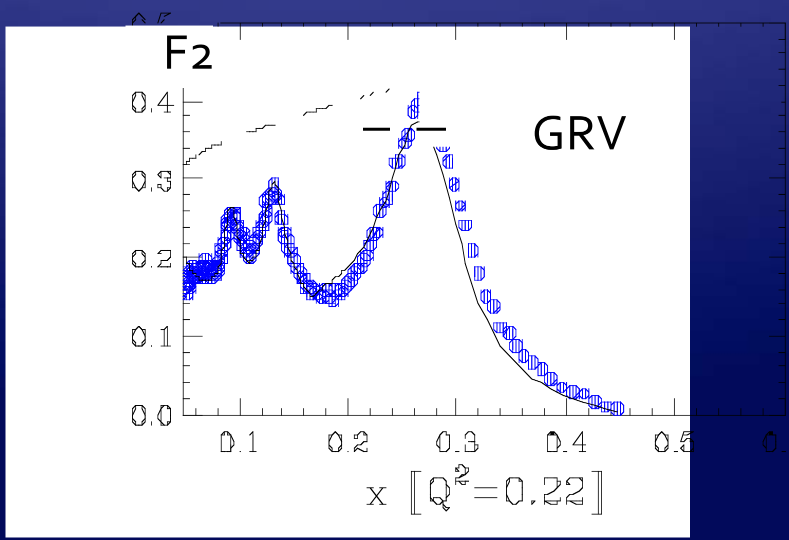
https://indico.cern.ch/event/855372/contributions/4491770/

Wed. Sept. 8 , 2021 7:52 AM (US Eastern) 18+2 min

https://indico.cern.ch/event/855372/timetable/#all.detailed

Bodek-Yang Model

- Bodek-Yang model aims for describing inelastic cross section in all Q^2 regions in terms of effective LO PDFs
- LO PDFs **do not blow** up at low Q^2 . In contrast NLO and NNLO PDFs blow up.
- Use GRV98 LO PDFs with modified scaling variable designed to take care of target mass effects and non-perturbative higher twist.
- K (Q^2) factors to describe PDFs down to Q^2 =0
- K(ν) factors to describe average cross sections in resonance region (W>1.4)



A model in terms of quark-parton model (easy to convert charged lepton scattering to neutrino scattering) works for quarks (isospin=1/2). Not for Δ I=3/2

- Understand PDFs at low Q^2? Wealth of SLAC, JLAB data.
- Understanding of resonance scattering in terms of quark-parton model? (duality works, many studies by JLAB)

Lessons from previous QCD studies

- NLO & NNLO analyses with DIS data: PRL 82, 2467 (1999), Eur. Phys. J. **C13**, 241 (2000) by Bodek and Yang concludes:
  - Kinematic higher twist (target mass) effects are large and must be included in the form of Georgi & Politzer x scaling. And Most of dynamic higher twist corrections (in NLO analysis) are similar to missing NNLO higher order terms. So NNLO pQCD+TM with NNLO PDFs can describe the non- perturbative QCD effects at low Q^2 (But not $Q^2 = 0$)
- Thus, we reverse the approach to build the model:
  - Use LO PDFs and "effective target mass and final state masses" to account for initial target mass, final target mass, and even missing higher orders

We use LO PDFs and K Factors to be able to go to $Q^2 = 0$ (NLO PDF blow up at low Q^2). The equations below are derived accounting for quark PT and effective bound quark initial and final state masses

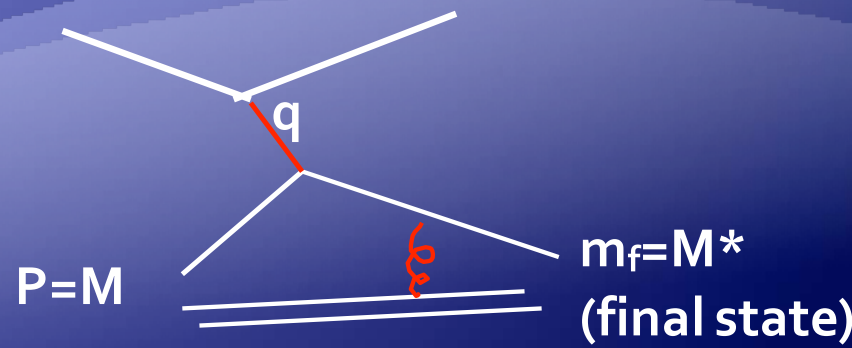
$$\xi = \frac{2xQ'^2}{Q^2(1+\sqrt{1+4M^2x^2/Q^2})},$$

A

$$2Q'^2 = [Q^2 + M_f{}^2 - M_i{}^2] + \sqrt{(Q^2 + M_f{}^2 - M_i{}^2)^2 + 4Q^2(M_i{}^2 + P_T^2)}.$$

Modeling neutrino cross sections

- NNLO pQCD +TM approach: describes the DIS region and resonance data very well



- Bodek-Yang LO approach: (pseudo NNLO)
  - Use effective LO PDFs with a new scaling variable, ξ_w to absorb target mass,Quark PT, initiai+final quark masses , higher twist, missing QCD higher orders

$$x_{Bj} = \frac{Q^2}{2M\nu} \Longrightarrow \xi_W = \frac{Q^2 + B}{\{M\nu[1+\sqrt{(1+Q^2/\nu^2)}] + A\}}$$

  - Multiply all PDFs by K factors for photo prod. Limit $Q^2 = 0$ and non-perturbative effects at low Q.

$$F_2(x, Q^2) \rightarrow \frac{Q^2}{Q^2 + C} F_2(\xi_w, Q^2)$$

B Make work at $Q^2 = 0$, and account for quark PT

A an enhanced target mass term for higher twist

Bodek-Yang Effective LO PDFs Model

1. Start with GRV98 LO (Q^2_{min}=0.80)
2. Replace x_{bj} with a new scaling, ξ_w
3. Multiply all PDFs by K factorfor photo prod. limit and higher twist [$\sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)$]

   Ksea = $Q^2/[Q^2+Csea]$

   Kval = $[1- G_D{}^2 (Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

   motivated by Adler Sum rule

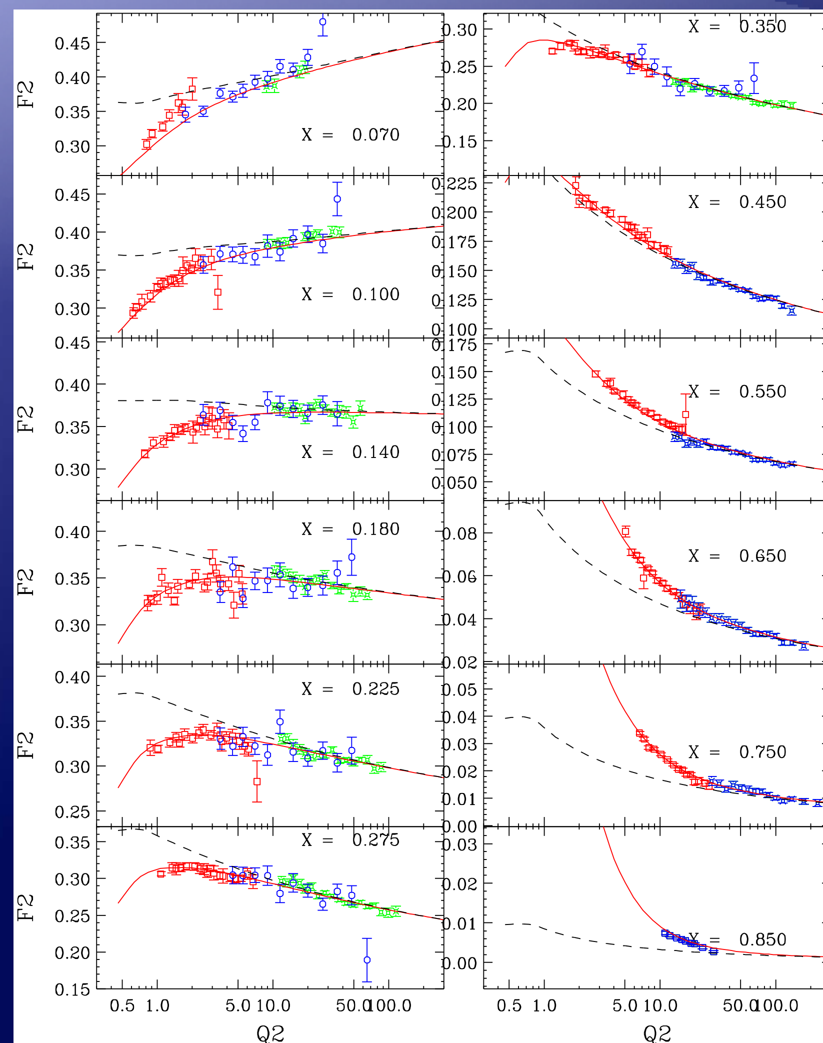
   where $G_D{}^2(Q^2) = 1/ [1+Q^2 / 0.71]^4$

4. Freeze the evolution at $Q^2 = Q^2_{min}$

   - $F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(\xi w, Q^2=0.8)$

5. Fit all DIS F_2(p/D) data: with *W>2 GeV*

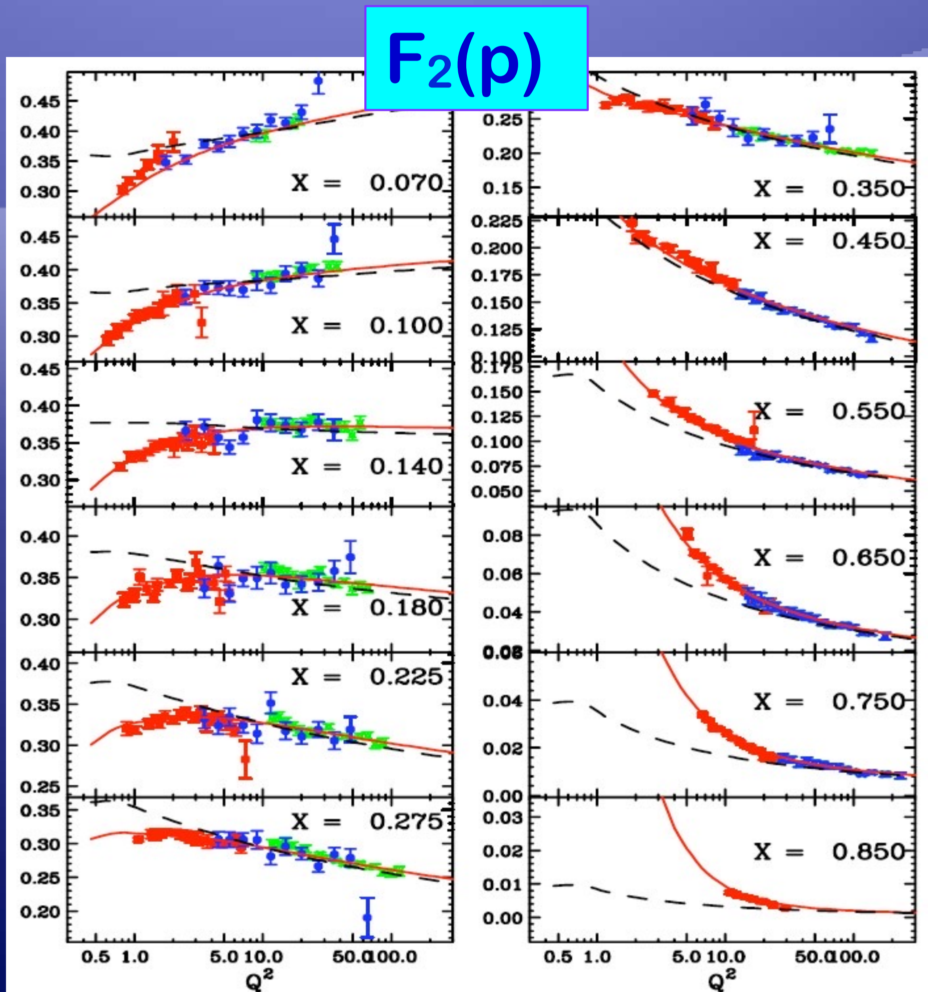
   SLAC/BCDMS/NMC/HERA data

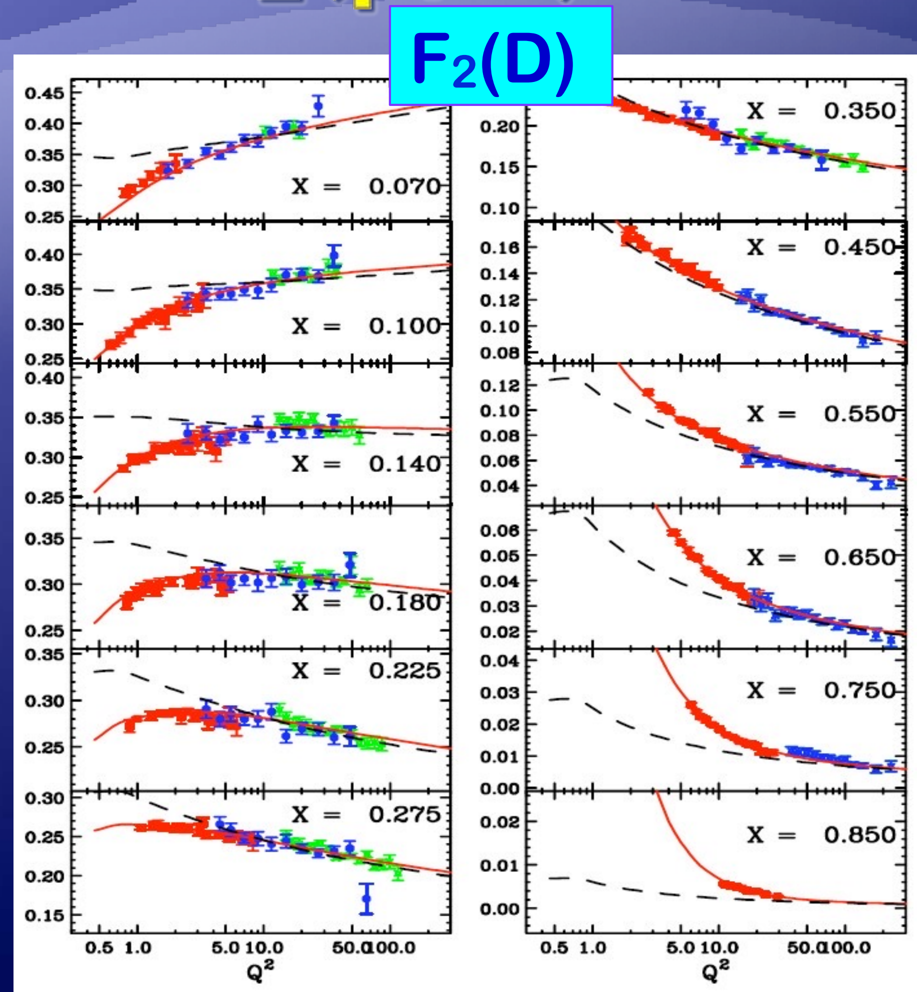


F_2(p)

$\chi^2/DOF = 1235/1200$

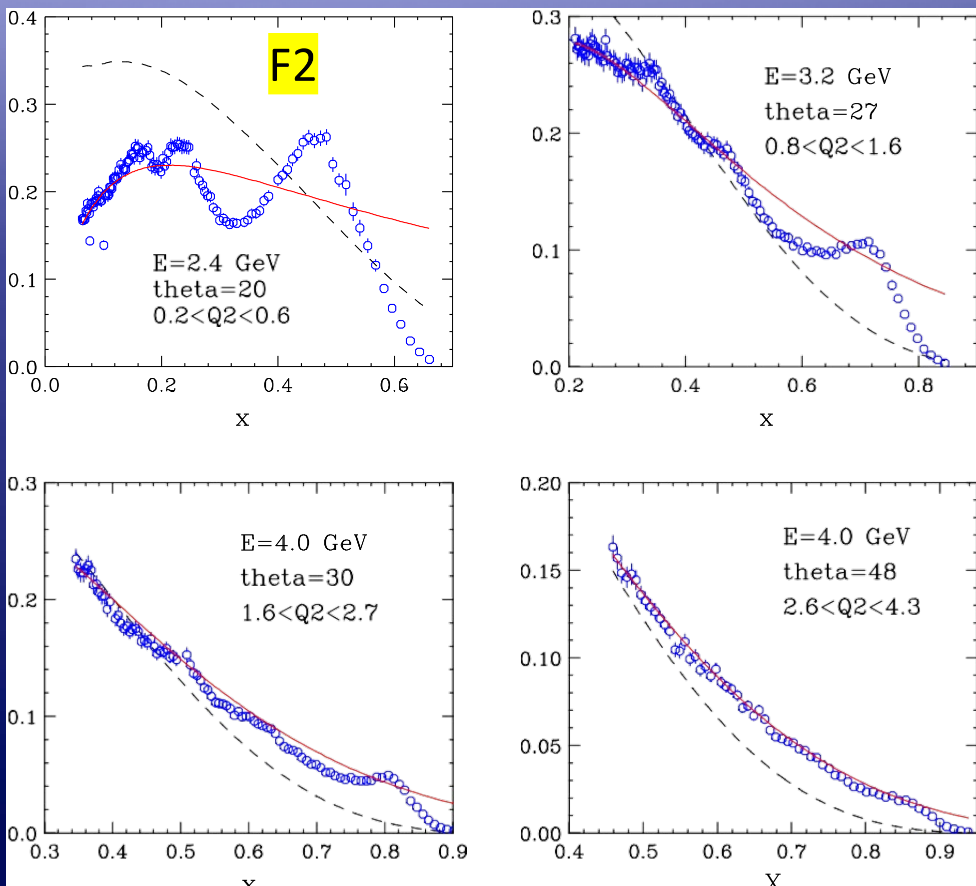
Fit Results on DIS F_2(p/D) data





- Excellent Fits:
  - red solid line: effective LO using ξw
  - black dashed line: GRV98 with x_{bj}

Predictions for Resonance, F2 Photo-production data



F₂(d) resonance

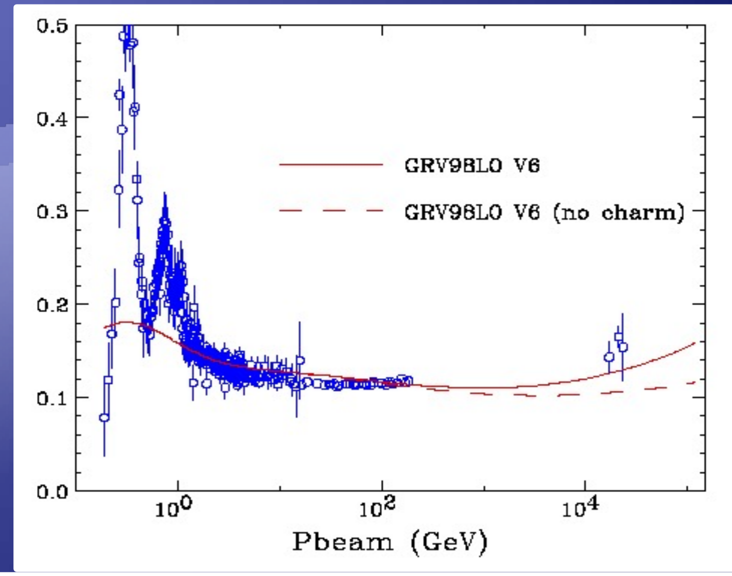


Photo-production cross section (P)

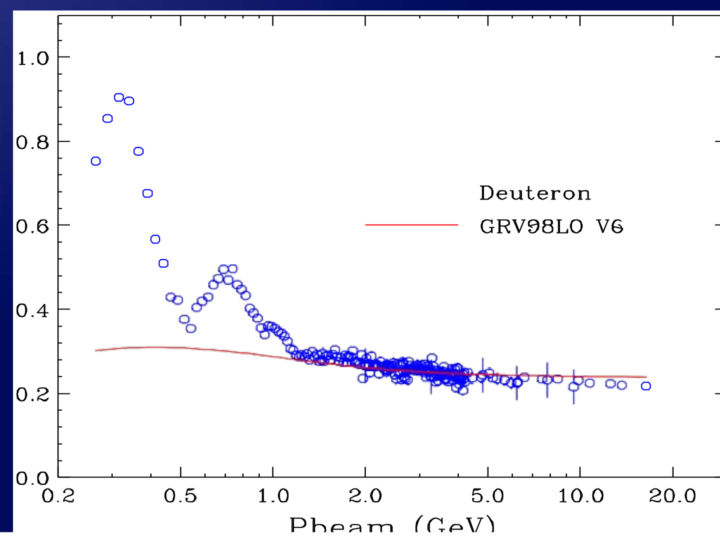


Photo-production cross section (d)

Photo-production data (Cross sections)

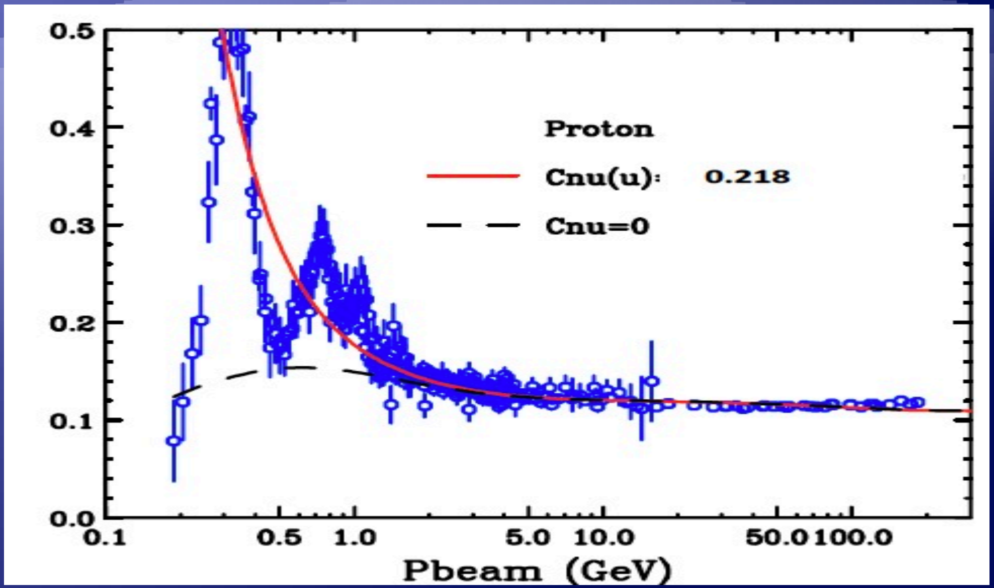
- **Improve model in low Q resonance region**:
- Additional K^{LW} factor for valence quarks:

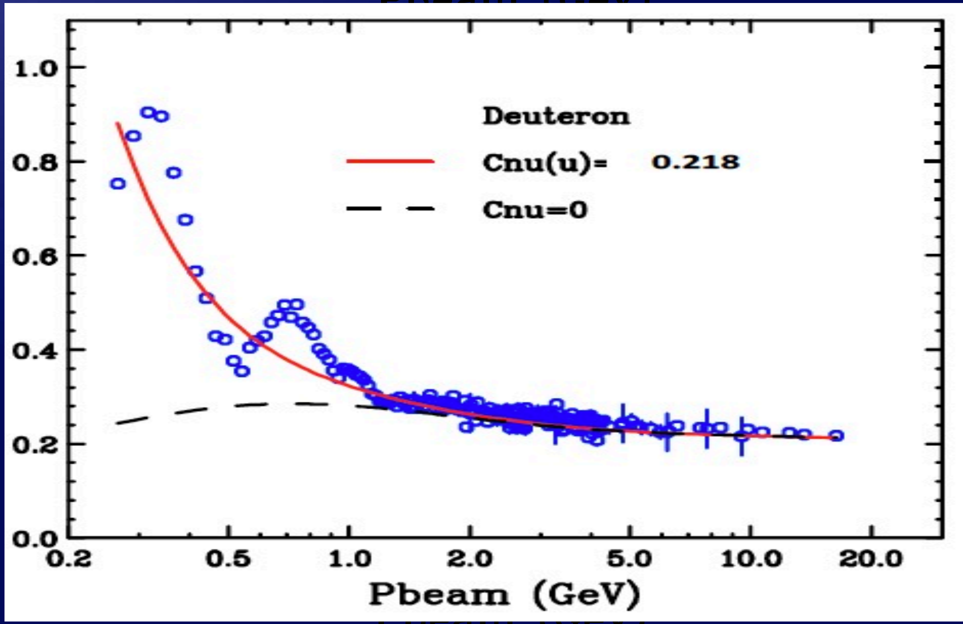
$$Kval = K^{LW} * [1 - G_D^{\ 2}(Q^2)] * [Q^2 + C_{2V}] / [Q^2 + C_{1V}]$$

$$K^{LW} = (\nu^2 + C^{\nu}) / \nu^2$$

This makes a duality work all the way down to Q2=0 (for charged leptons, W>1.4 GeV)

- Photo-production data with ν(Pbeam)>1 GeV included in the fit





Bodek-Yang Effective LO PDFs Model

- Include the photo-production data
- Use different K factors for up and down quark type separately

Kval (u,d) = $[1- G_D^{\ 2}(Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

Ksea (u,d,s) = $Q^2/[Q^2+Csea]$

- Additional K^{LW} factor for valence quarks:

Kval = $K^{LW} * [1- G_D^{\ 2}(Q^2)] * [Q^2+C_{2V}] / [Q^2+C_{1V}]$

where $K^{LW} = (\nu^2+C^{\nu})/\nu^2$

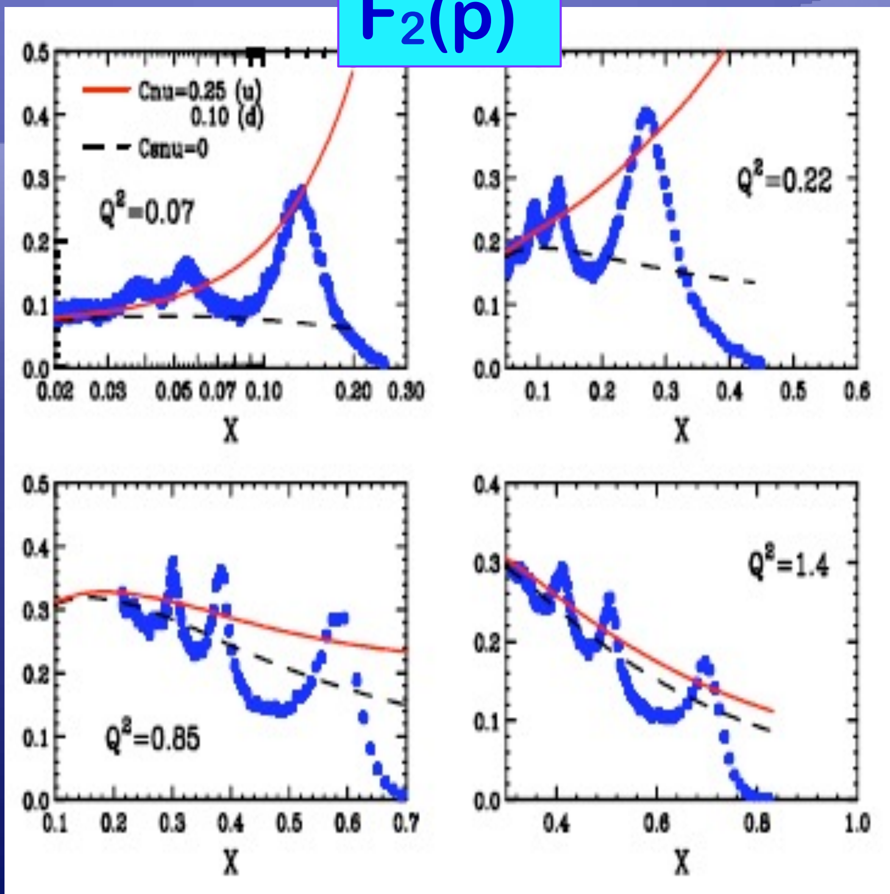
A 0.621	B 0.380	C_{v2d} 0.323	C_{v2u} 0.264
C_{sea}^{down} 0.561	C_{sea}^{up} 0.369	C_{v1d} 0.341	C_{v1u} 0.417
$C_{sea}^{strange}$ 0.561	$C^{low-\nu}$ 0.218	$\mathcal{F}_{valence}$ $[1-G_D^2(Q^2)]$	N 1.026

e/**μ**-N and **ν**-N Vector Structure Functions.

F2 & F_L Resonance data

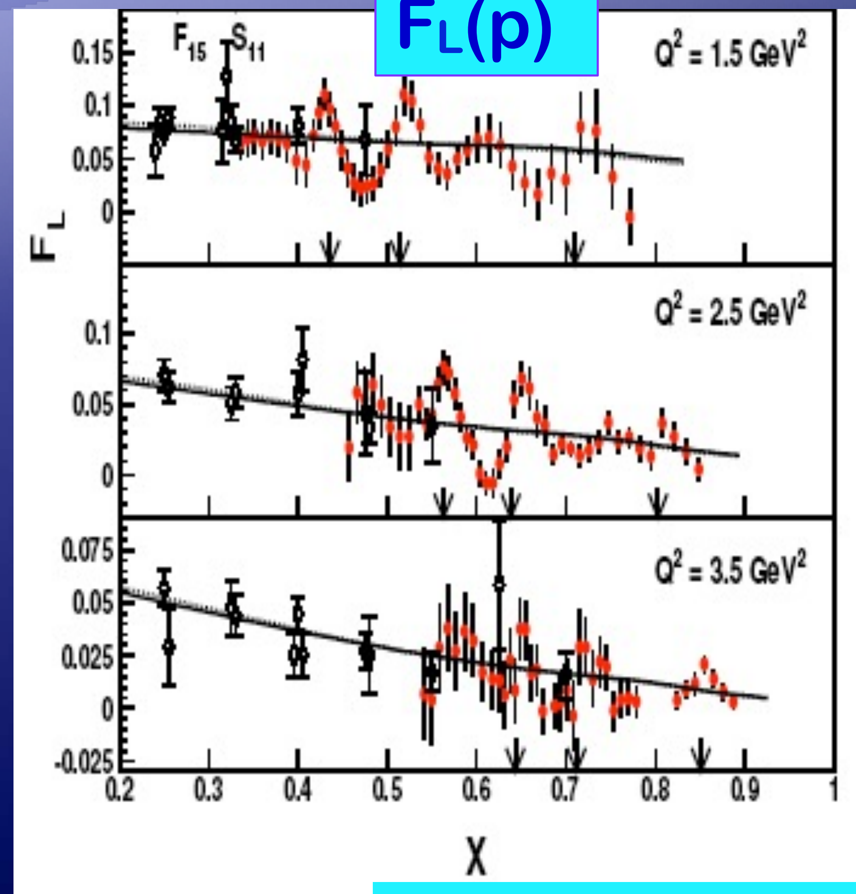
F2(p)

F2



F2

F_L(p)



$$F_L = F_2(1+4M^2x^2/Q^2)\frac{R}{(1+R)}$$

- Predictions are in good agreement (not included in the fit) duality works in resonance region W>1.4 GeV
- F_L was calculated using F2 and R_{1998}

Neutrino cross sections

- Effective LO model with ξw describe all DIS and resonance F_2 data as well as photo-production data ($Q^2=0$ limit): vector contribution works well F_2 and $2xF_1$
- Neutrino Scattering:
  - Effective LO model works for xF_3 (use NLO QCD for F_2 /2 xF_3)
  - Nuclear correction using e/μ scattering data
  - Use $R=R_{1998}$ to get $2xF_1$
  - Implement charm mass effect through ξw slow rescaling algorithm for F_2, $2xF_1$, and xF_3

- Axial vector contribution at low Q^2 (2021 update)

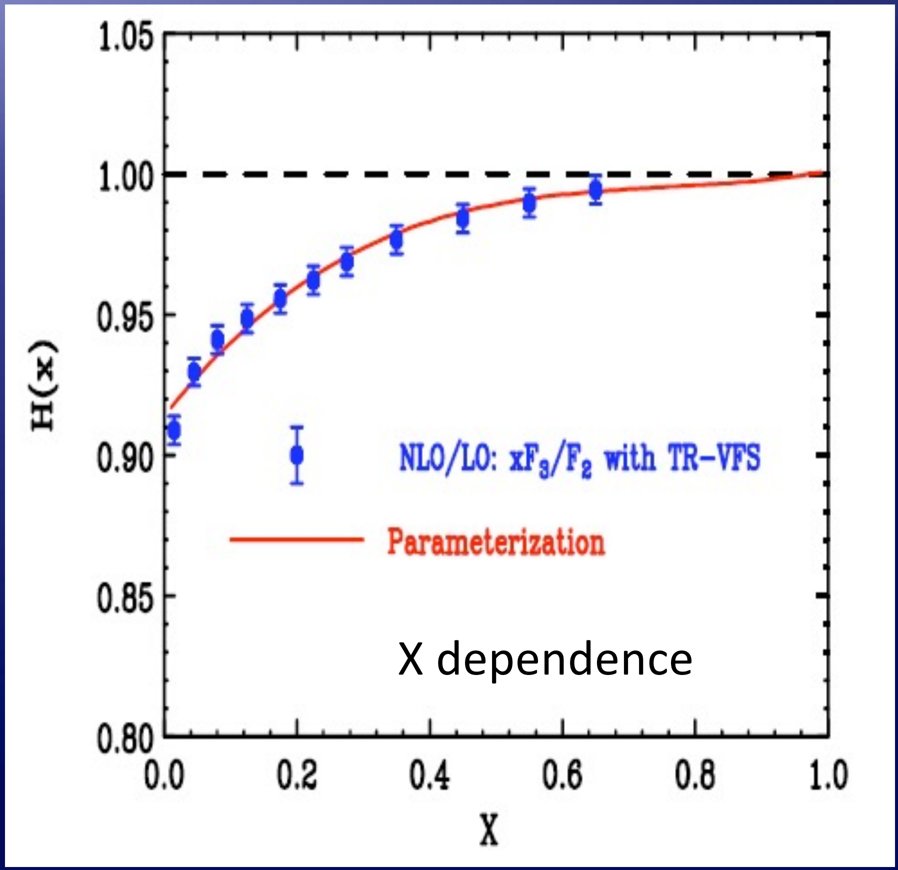
Effective LO model for xF_3?

- Scaling variable, ξw absorbs higher order effect for F_2, but the higher order effects for F_2 and xF_3 are not the same
- Use NLO QCD to get double ratio

$$H(x) = \frac{xF_3(NLO)}{xF_3(LO)} / \frac{F_2(NLO)}{F_2(LO)}$$

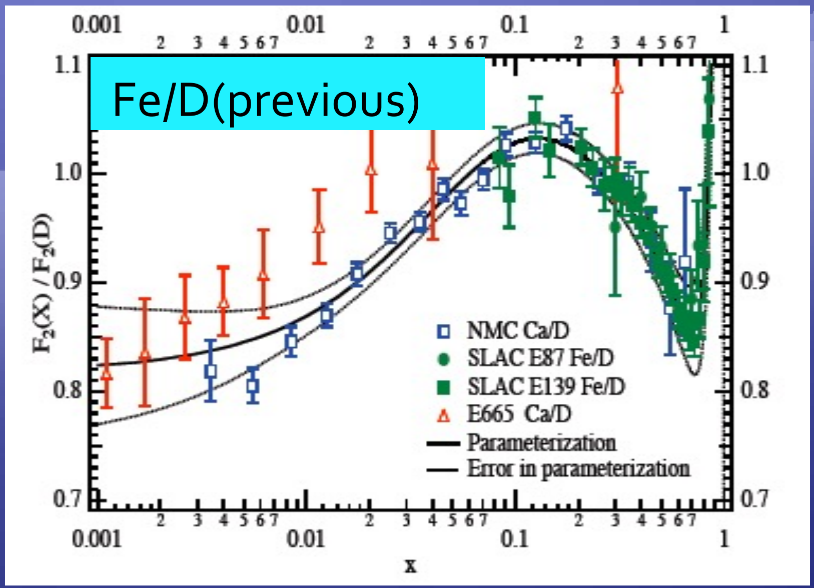
not 1 but almost indep. of Q^2

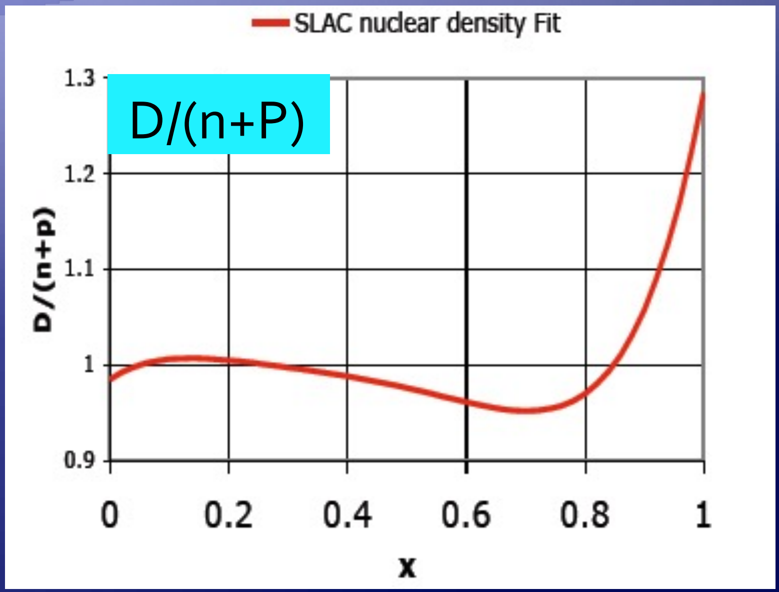
- Enhances anti-neutrino cross section by 3%

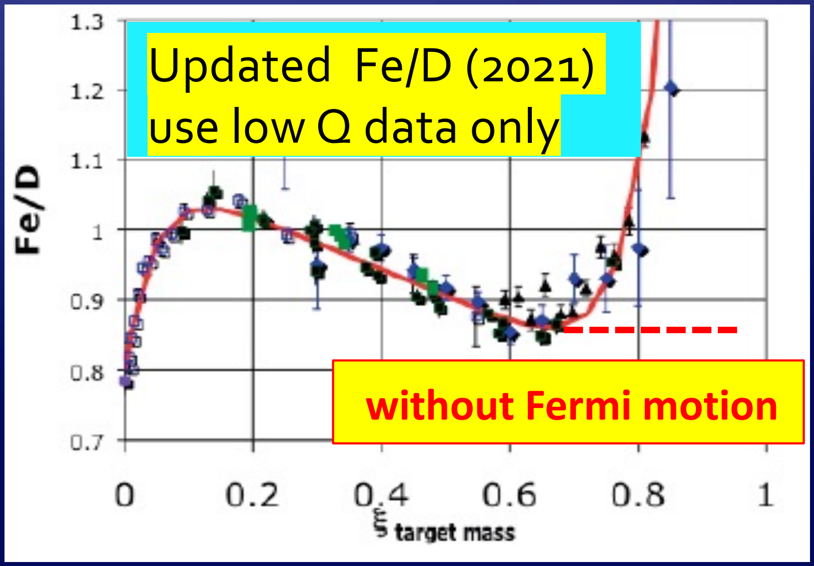


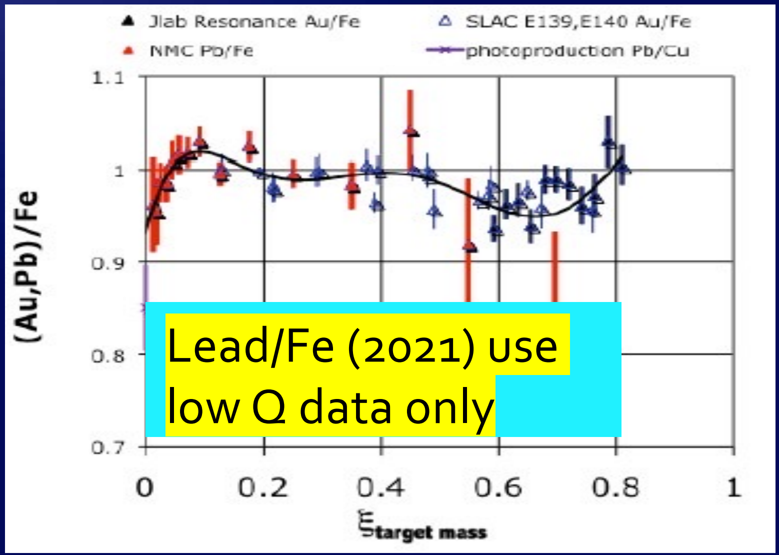
if Fermi motion already included in MC generator - Note: Do not use high x enhancement –

Nuclear Effects: use e/μ data









For comparison to CCFR

For comparison to CHORUS

Axial Vector Structure Functions

- At high Q^2, vector and axial vector contribution are same, but not at low Q2. Previous assumption **Type I (axial=vector)**
- 2021: K factors for axial contributions: type Ⅱ **(Axial>Vector)**

$$K^{vector}_{sea} = \frac{Q^2}{Q^2 + C} \bullet K^{axial}_{sea} = \frac{Q^2 + 0.55 C^{axial}_{sea}}{Q^2 + C^{axial}_{sea}} \qquad K^{axial}_{val} = \frac{Q^2 + 0.3 C^{axial}_{val}}{Q^2 + C^{axial}_{val}}$$

$$\text{where } C^{axial}_{sea} = 0.75, \; C^{axial}_{val} = 0.18$$

Only for F_2 and xF_3

- 0.55 was chosen to satisfy the prediction from PCAC by Kulagin, agrees with CCFR data for F_2 extrapolation to (Q^2=0)
- The non-zero PCAC component of F_2^{axial} at low Q^2: mostly longitudinal so:

$$2xF_1^{axial} = 2xF_1^{vector}$$

Small modification to GRV98 u and d quark sea

To better describe ratio of antineutrino and neutrino cross sections increase GRV98 u and d sea by 5% and decrease valence quarks by same amount, thus leaving $F_2(x, Q^2)$ unchanged, but slightly increasing antineutrino cross sections.

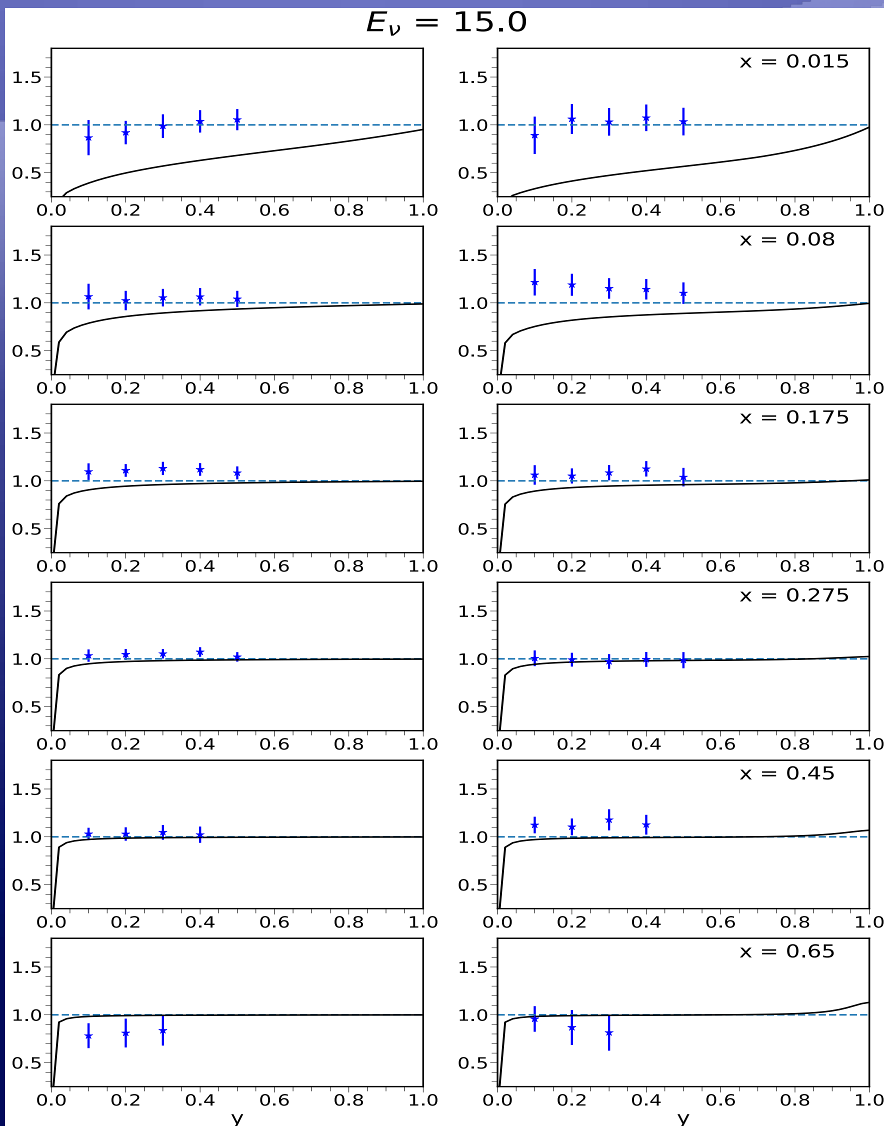
$$
\begin{aligned}
d_{sea} &= 1.05\ d_{sea}^{grv98} \\
\bar{d}_{sea} &= 1.05\ \bar{d}_{sea}^{grv98} \\
u_{sea} &= 1.05\ u_{sea}^{grv98} \\
\bar{u}_{sea} &= 1.05\ \bar{u}_{sea}^{grv98} \\
d_{valence} &= d_{valence}^{grv98} - 0.05\ (d_{sea}^{grv98} + \bar{d}_{sea}^{grv98}) \\
u_{valence} &= u_{valence}^{grv98} - 0.05\ (u_{sea}^{grv98} + \bar{u}_{sea}^{grv98})
\end{aligned}
$$

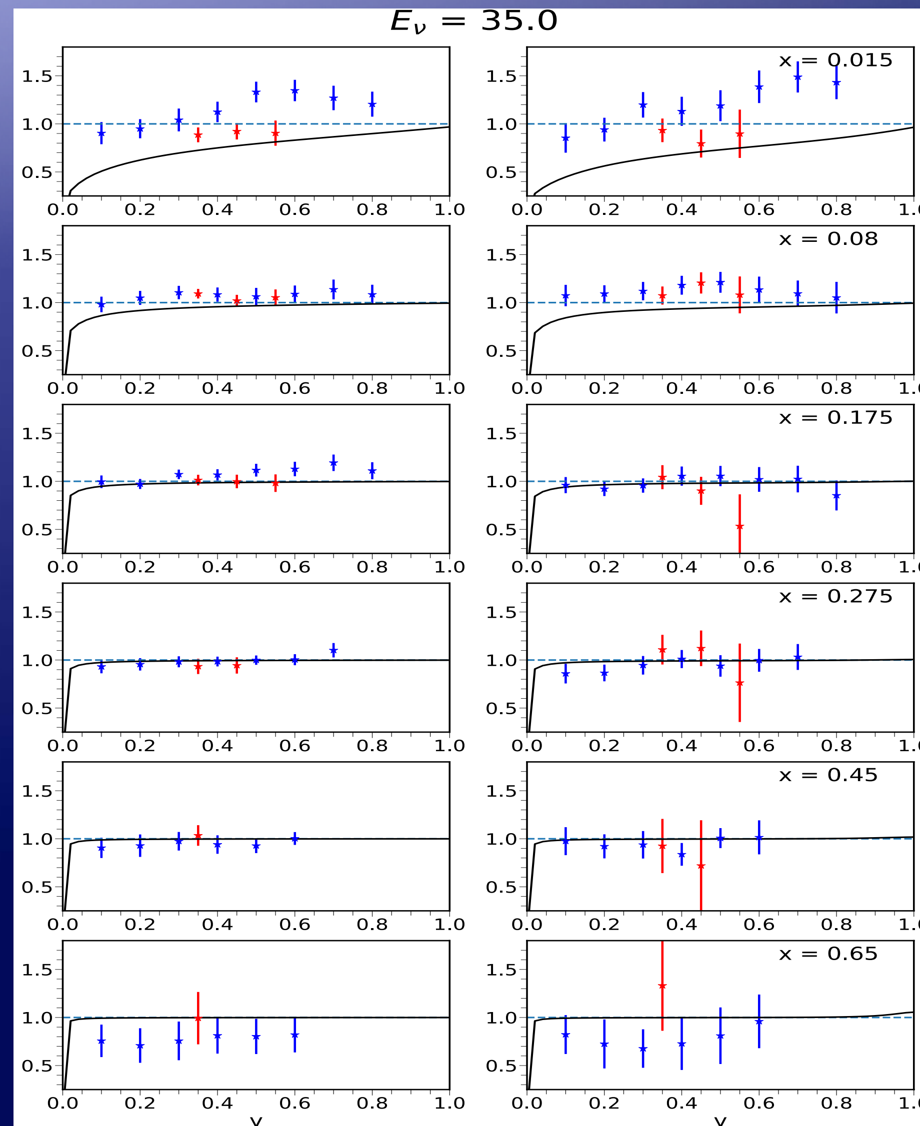
Red point: CCFR/ Model type II (V>A)

Blue point: CHORUS/ Model type II (V>A)

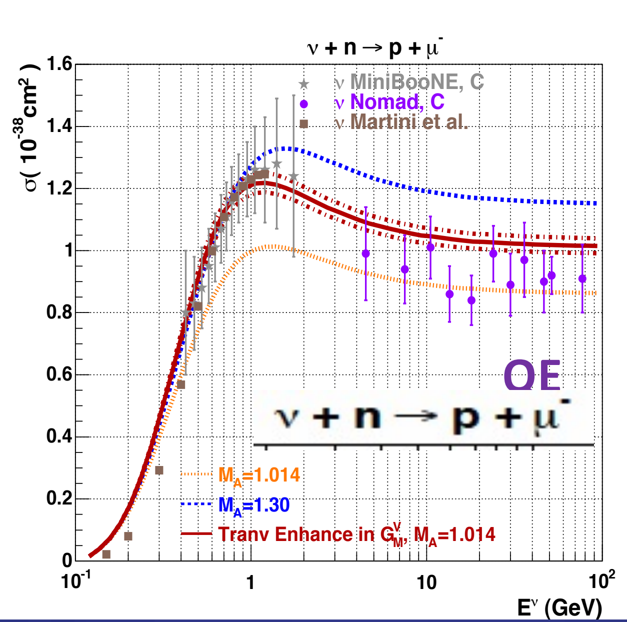
Solid line ---- Type II (A>V Default Model) / Type I Model (V=A)

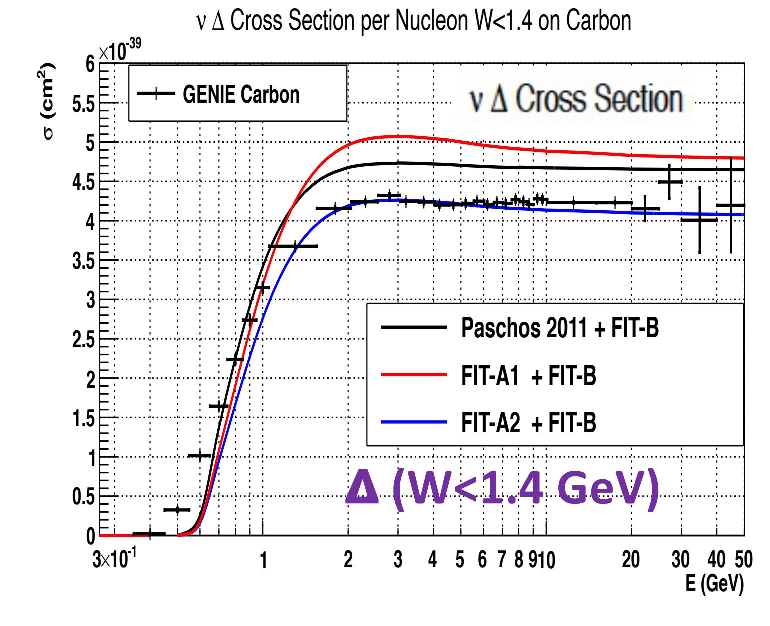
Type II (V>A Default Model) accounts for the difference in Vector vs Axial

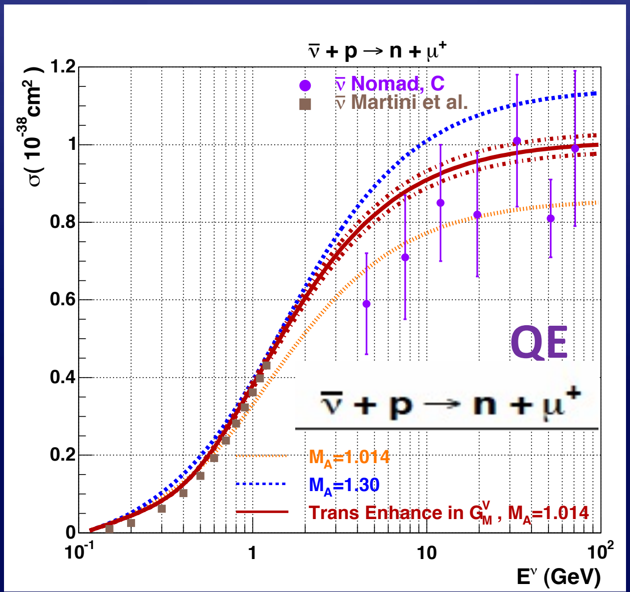


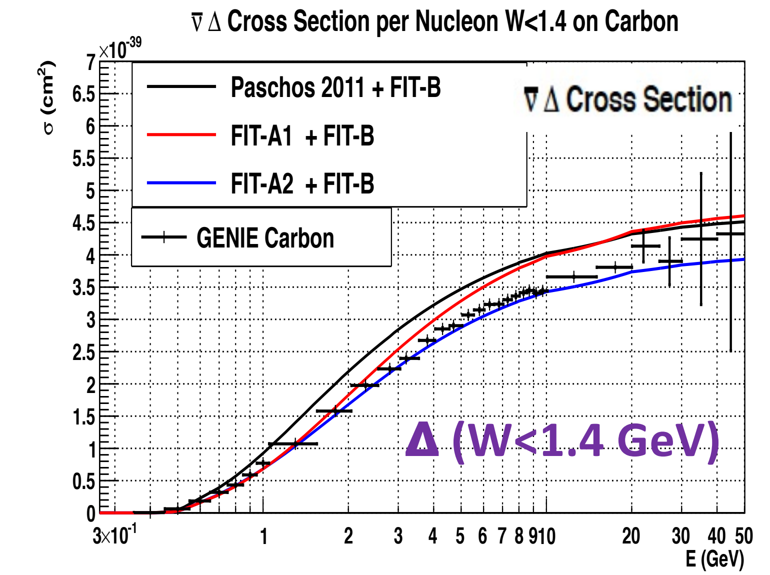


To compare to total cross section data: use BY for W>1.4 GeV and add QE and Δ (W<1.4 GeV) cross section per-nucleon









QE and Δ (W<1.4) GeV) cross section Fits From:

A. Bodek, U. Sarica, D. Naples, L. Ren
Eur. Phys. Journal C - Particles and Fields, Volume 72, 1973 (2012)
arXiv:1201.3025

Find that to model cross sections at low energy. We model the cross sections in the <u>resonance region</u> using low ν Axial Parameters which different from the Vector and are also different for neutrinos and antineutrinos

$$K^{LW}_{vector} = \frac{\nu^2 + C^{low-\nu}_{vector}}{\nu^2}$$

$$K^{LW-(\nu,\overline{\nu})}_{axial} = \frac{\nu^2 + C^{low-\nu(\nu,\overline{\nu})}_{axial}}{\nu^2}$$

$C^{low-\nu}_{vector}$
0.218

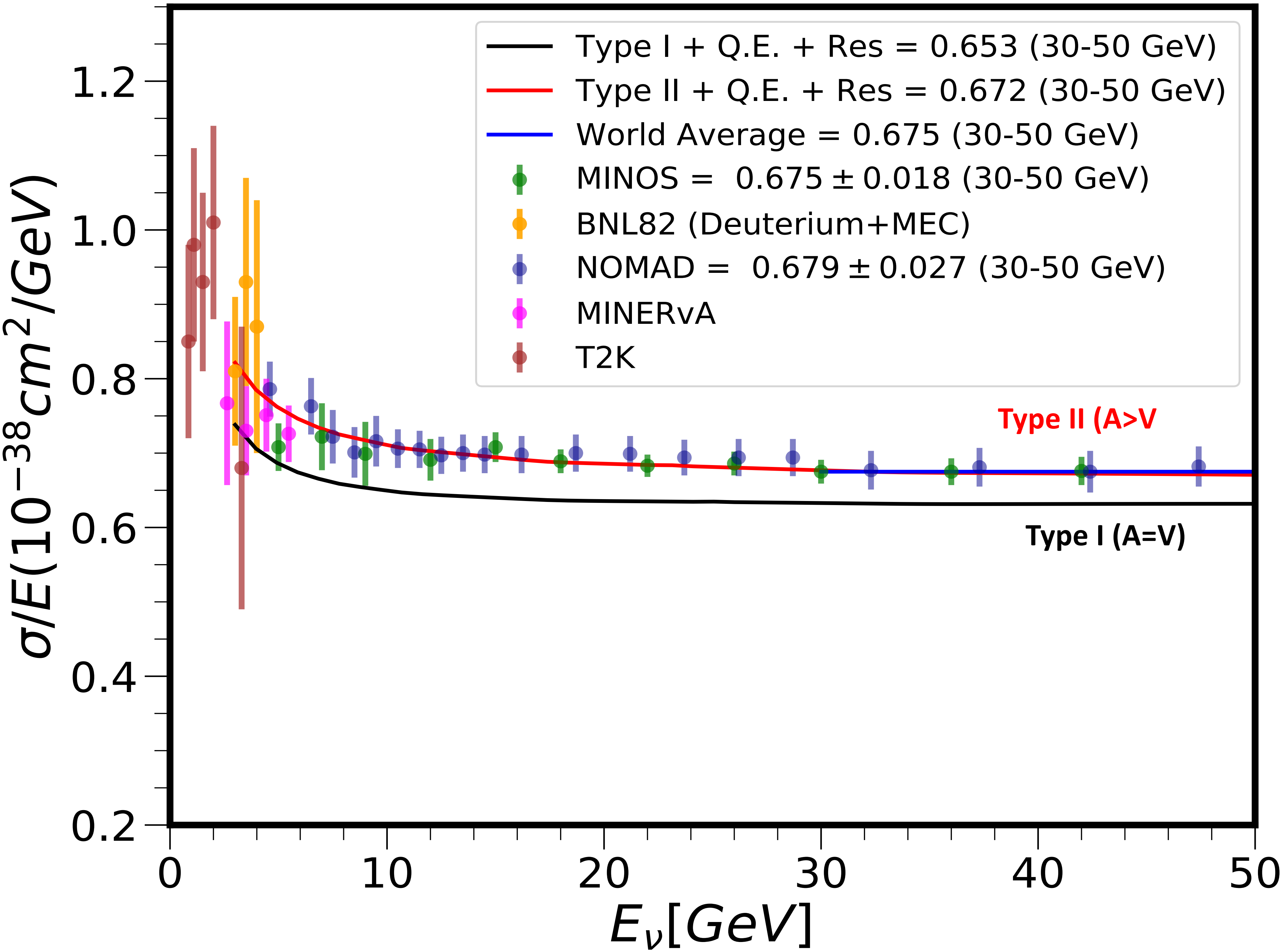
$$C^{low-\nu(\overline{\nu})}_{axial} = 0.654.$$

$$C^{low-\nu(\nu)}_{axial} = 0.436.$$

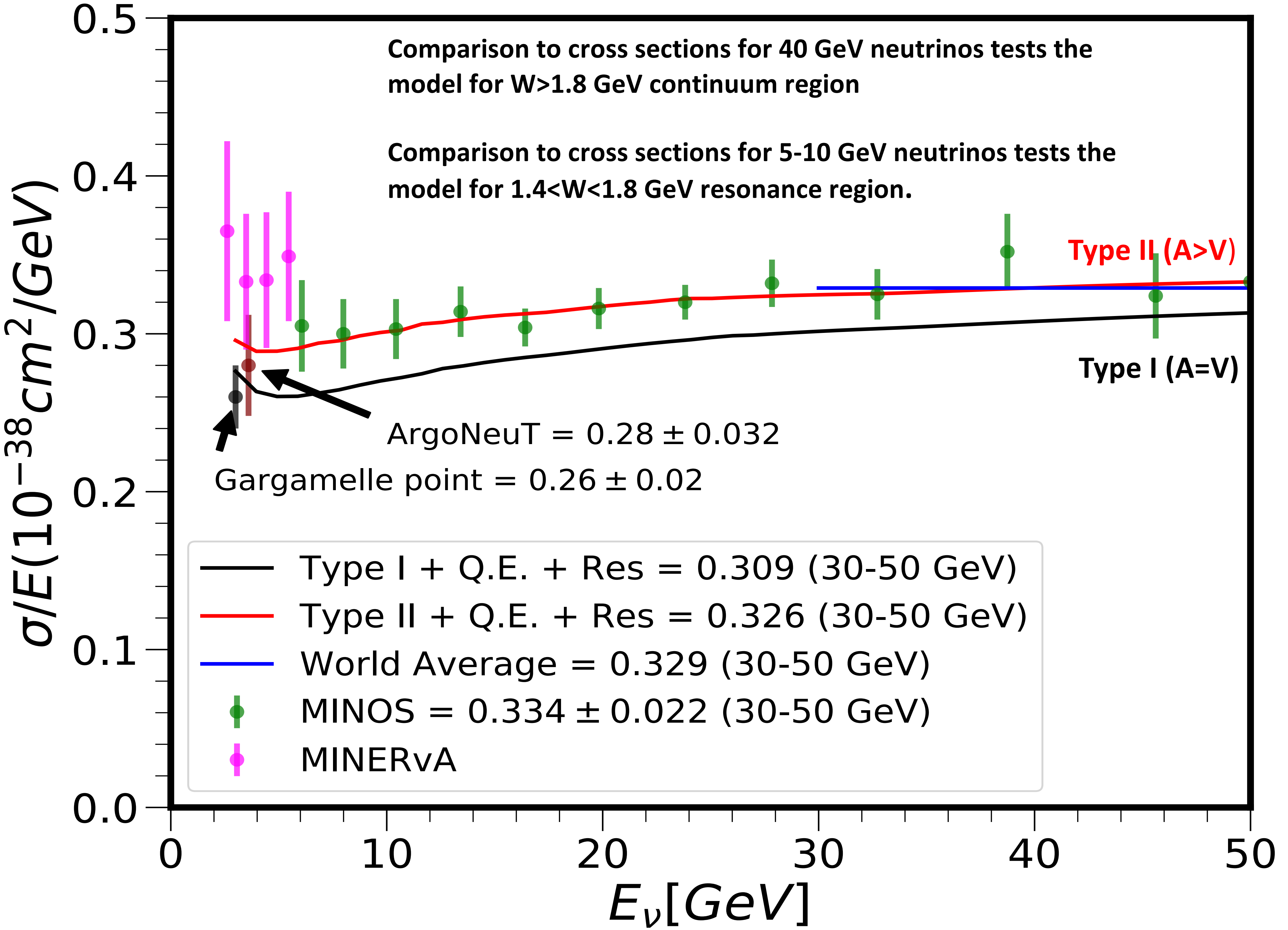
E_ν GeV	QE	$W < 1.4$ GeV $\Delta(1238)$	$1.4 < W < 1.8$ GeV resonances	$W > 1.8$ GeV inelastic
3	23.8%	19.7%	31.3%	25.2%
5	16.2%	12.5%	22.2%	48.1%
10	7.2%	6.5%	13.4%	72.8%
40	1.5%	1.6%	6.5%	90.4%
$E_{\overline{\nu}}$ GeV	QE	$W < 1.4$ GeV $\Delta(1238)$	$1.4 < W < 1.8$ GeV resonances	$W > 1.8$ GeV inelastic
3	40.7%	27.1%	25.6%	6.6%
5	27.8%	20.8%	33.5%	17.9%
10	15.0%	11.7%	25.2%	48.1%
40	3.1%	3.4%	7.1%	86.4%

Table 4. Percent contributions to the total cross section of QE, $\Delta(W < 1.4)$ GeV, higher resonances $1.4 < W < 1.8$ GeV and inelastic continuum $W > 1.8$ GeV.

Comparison to cross sections for 40 GeV neutrinos tests the model for W>1.8 continuum region. W>1.8: 90% (ν) 86% (anti ν). Comparison to cross sections for 3 GeV neutrinos tests the model for 1.4<W<1.8 resonance region. 31% (ν) 26% (anti ν)



Type I + Q.E. + Res = 0.653 (30-50 GeV)
Type II + Q.E. + Res = 0.672 (30-50 GeV)
World Average = 0.675 (30-50 GeV)
MINOS = 0.675 ± 0.018 (30-50 GeV)
BNL82 (Deuterium+MEC)
NOMAD = 0.679 ± 0.027 (30-50 GeV)
MINERvA
T2K
Type II (A>V
Type I (A=V)
$\sigma/E(10^{-38}cm^2/GeV)$
$E_\nu[GeV]$
0.2
0.4
0.6
0.8
1.0
1.2
0
10
20
30
40
50

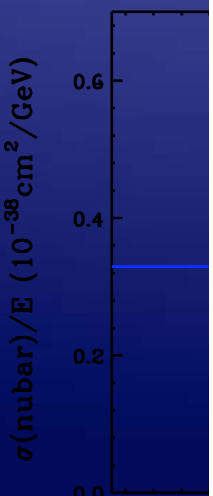


Comparison to cross sections for 40 GeV neutrinos tests the model for W>1.8 GeV continuum region
Comparison to cross sections for 5-10 GeV neutrinos tests the model for 1.4<W<1.8 GeV resonance region.
Type II (A>V)
Type I (A=V)
ArgoNeuT = 0.28 ± 0.032
Gargamelle point = 0.26 ± 0.02
Type I + Q.E. + Res = 0.309 (30-50 GeV)
Type II + Q.E. + Res = 0.326 (30-50 GeV)
World Average = 0.329 (30-50 GeV)
MINOS = 0.334 ± 0.022 (30-50 GeV)
MINERvA
$\sigma/E(10^{-38}cm^2/GeV)$
$E_\nu[GeV]$

Total cross sections

BY(DIS, W>1.4)+ +
+Q.E. (measured)
+Δ Resonance
(W<1.4) measured
At 40 GeV energy

	Type I (V=A)	Type II (A>V)	World Average
σ_ν /E	0.656 ±0.024	0.674 ±0.024	0.675 ±0.006
$\sigma_{\bar{\nu}}$/E	0.311 ± 0.016	0.327 ± 0.016	0.329 ± 0.011
$\sigma_{\bar{\nu}}/\sigma_\nu$	0.474 ± 0.012	0.487 ± 0.012	0.485 ± 0.005



source	change (error)	change in σ_ν	change in $\sigma_{\bar{\nu}}$	change in $\sigma_{\bar{\nu}}/\sigma_\nu$
R	+0.1	-1.3%	-2.7%	-1.4%
$f_{\overline{Q}}$	+5%	-0.4%	+0.9%	+1.4%
$K^{axial} - K^{vector}$	+ 50%	+1.3%	+2.4%	+1.1%
N	+3%	+3%	+3%	0
Total		±3.6%	±4.8%	±2.5%

Systematics

Test of Duality for W<1.4 GeV Does the Model Work for QE and Δ (1238) (W<4) resonance region

E_ν GeV	QE	$W < 1.4$ GeV $\Delta(1238)$	$1.08 < W < 1.4$ GeV BY II model	Ratio BY II/(QE+Δ)
3	1.83	1.57	3.72	1.09±0.15
5	1.10	0.92	2.25	1.1±0.16
10	0.53	0.45	1.13	1.16±0.16
40	0.13	0.11	0.30	1.21±0.17

$E_{\overline{\nu}}$ GeV	QE	$W < 1.4$ GeV $\Delta(1238)$	$1.08 < W < 1.4$ GeV BY II model	Ratio BY II/(QE+Δ)
3	1.20	0.80	2.15	1.08±0.15
5	0.81	0.60	1.56	1.11 ±0.16
10	0.46	10.35	0.90	1.11±0.16
40	0.13	0.10	0.27	1.20±0.17

Table 8. Test of Duality: Comparison of the sum of the measured σ/E (in units of $10^{-39}\ cm^2/GeV$) for QE and $\Delta(W < 1.4)$ GeV, to the prediction of the Type II $(A > V)$(=BY II) model for $1.08 < W < 1.4$ GeV. The experimental errors for the QE and Δ cross sections are assumed to be 10%. The model predictions for the integrated cross section in the $1.08 < W < 1.4$ GeV region appears describe the *sum* of the QE and the $\Delta(W < 1.4)$ GeV measured cross sections.

It appears that the **model total cross section over the range 1.08<W<1.4 GeV**

Also describes the

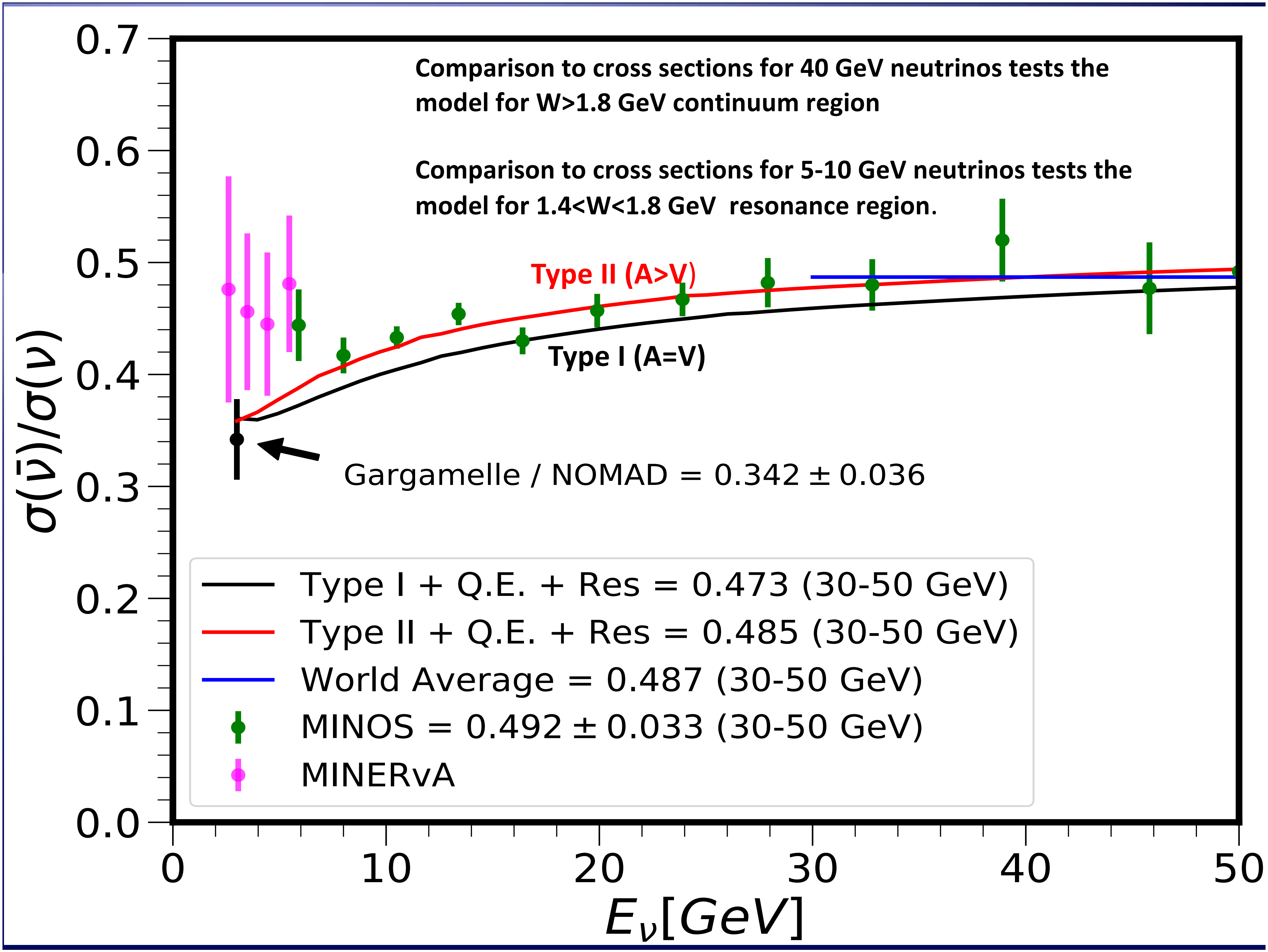
sum the measured QE and Δ (1238)(W<1.4) Production cross sections to 10%

Summary & Discussions

- BY Effective LO model with ξw describe all e/μ DIS and resonance data as well as photo-production data (down to $Q^2=0$): provide a good reference for vector SF for neutrino cross section
- $d\sigma/dxdy$ data (CCFR,CHORUS) favor updated BY(DIS) type II model
- K factors for axial sea vectors in BY(DIS) type II model are based on PCAC and agree with CCFR F2 Q2=0 measurement.
- BY(DIS) type II model (low Q^2: axial>vector) provide a good reference for both neutrino and anti-neutrino cross sections (W>1.8).
- Model also works on-average down to W>1.4 GeV, thus providing some overlap with resonance models (and should be used for W>1.8).

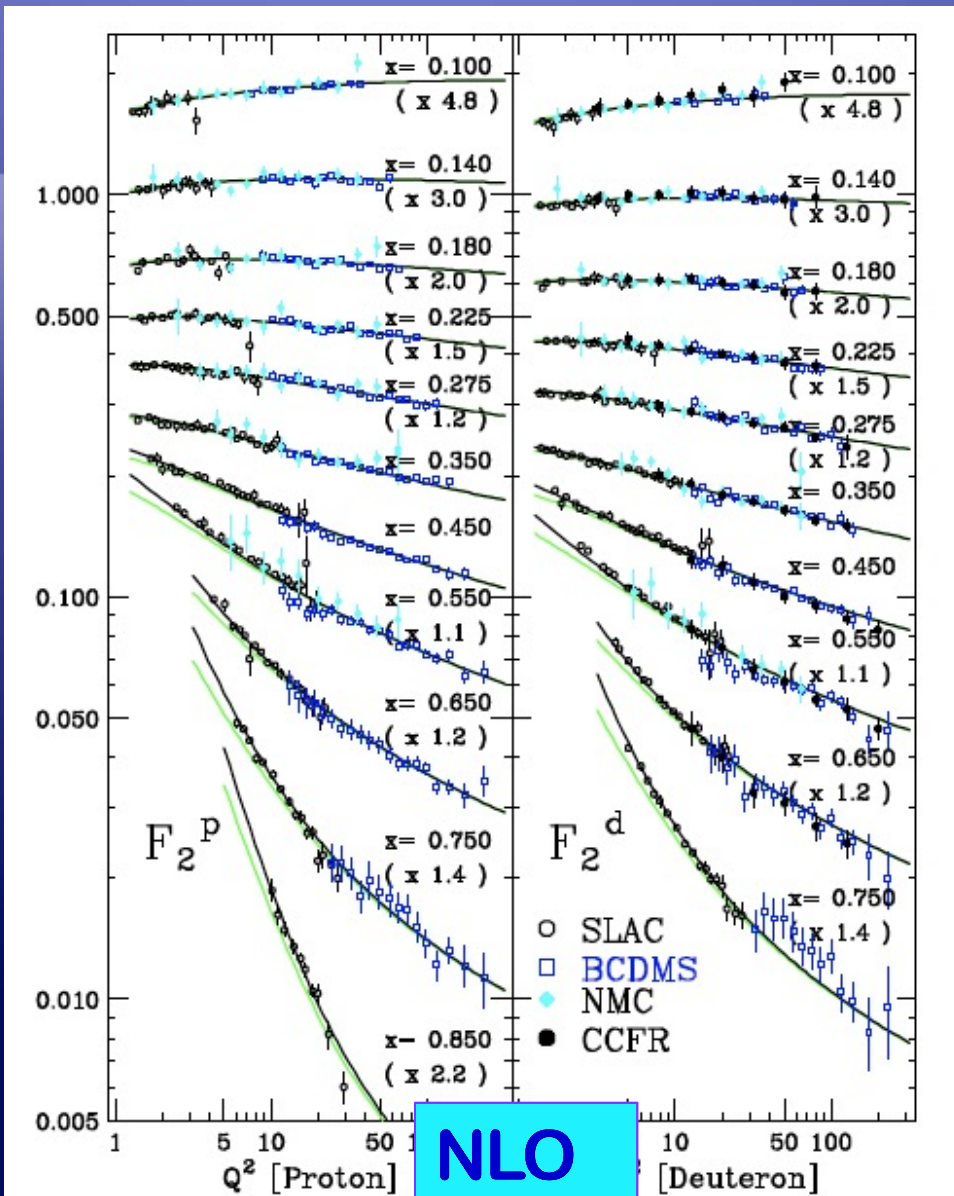
However, it appears that the model also describes the SUM of the QE and $\Delta(1238)$ production cross sections,

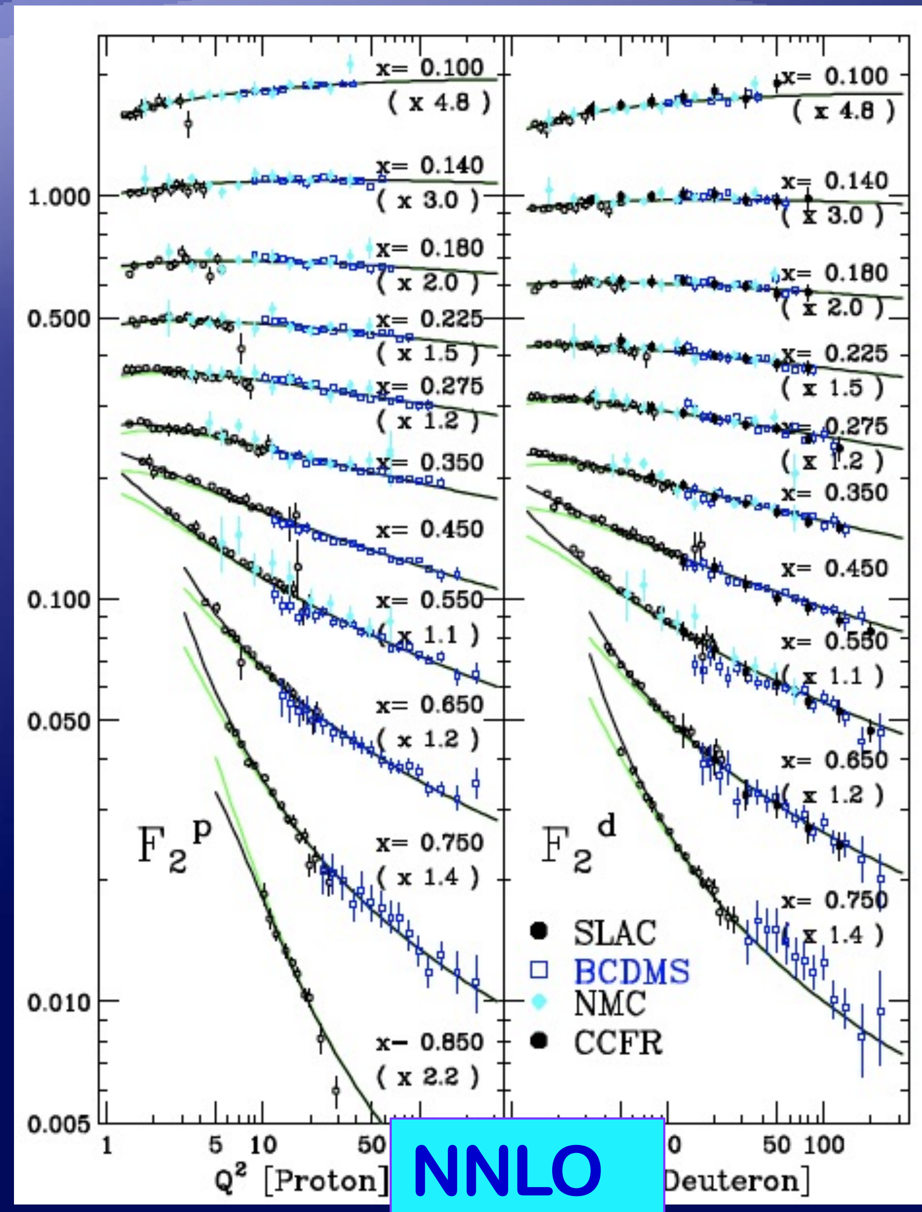
Supplementary Slides



Comparison to cross sections for 40 GeV neutrinos tests the model for W>1.8 GeV continuum region
Comparison to cross sections for 5-10 GeV neutrinos tests the model for 1.4<W<1.8 GeV resonance region.
Type II (A>V)
Type I (A=V)
Gargamelle / NOMAD = 0.342 ± 0.036
Type I + Q.E. + Res = 0.473 (30-50 GeV)
Type II + Q.E. + Res = 0.485 (30-50 GeV)
World Average = 0.487 (30-50 GeV)
MINOS = 0.492 ± 0.033 (30-50 GeV)
MINERvA
$\sigma(\bar{\nu})/\sigma(\nu)$
$E_\nu[GeV]$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
10
20
30
40
50

NLO vs NNLO Analyses



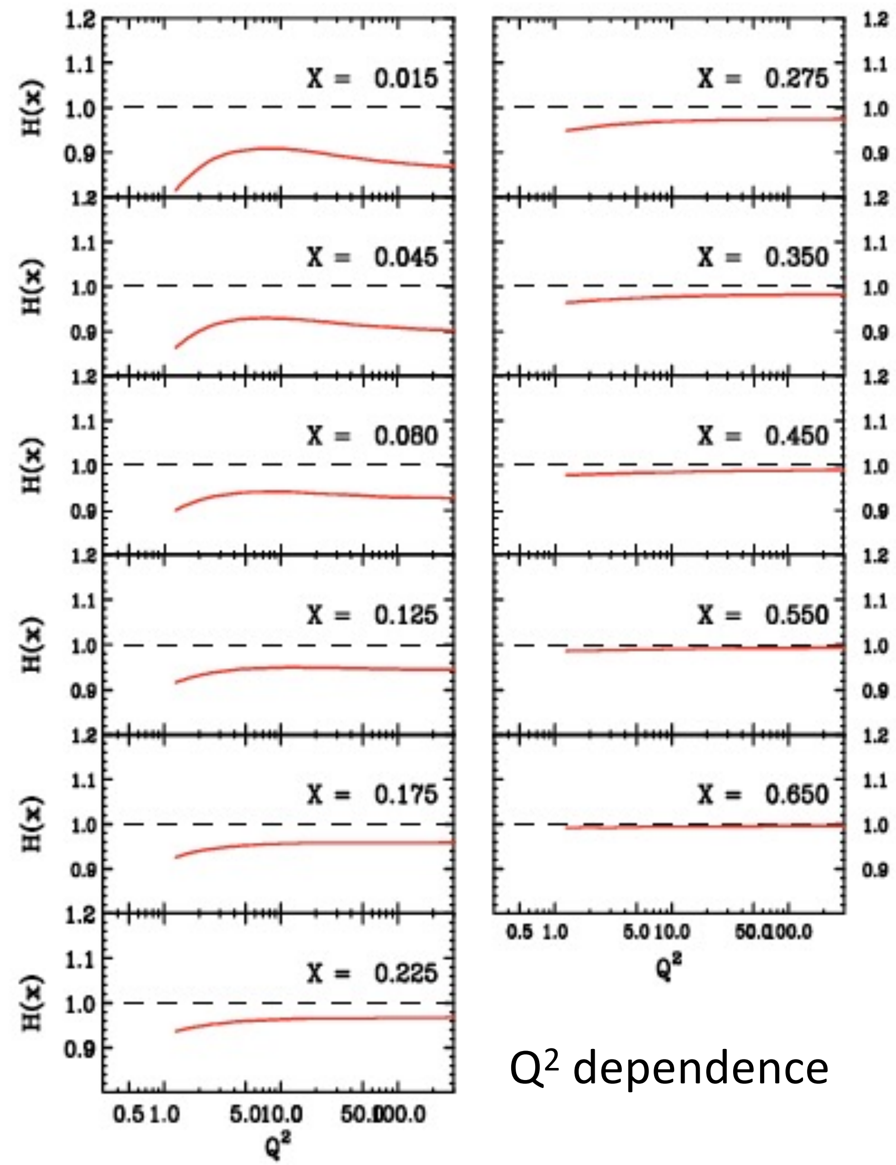


Effective LO model for xF_3?

- $H(x,Q^2)$?

$$H(x) = \frac{xF_3(NLO)}{xF_3(LO)} / \frac{F_2(NLO)}{F_2(LO)}$$

- $H(x,Q^2)$ is almost independent of Q^2



Q^2 dependence