TRACK RECONSTRUCTION WITH QUANTUM COMPUTERS AT LUXE

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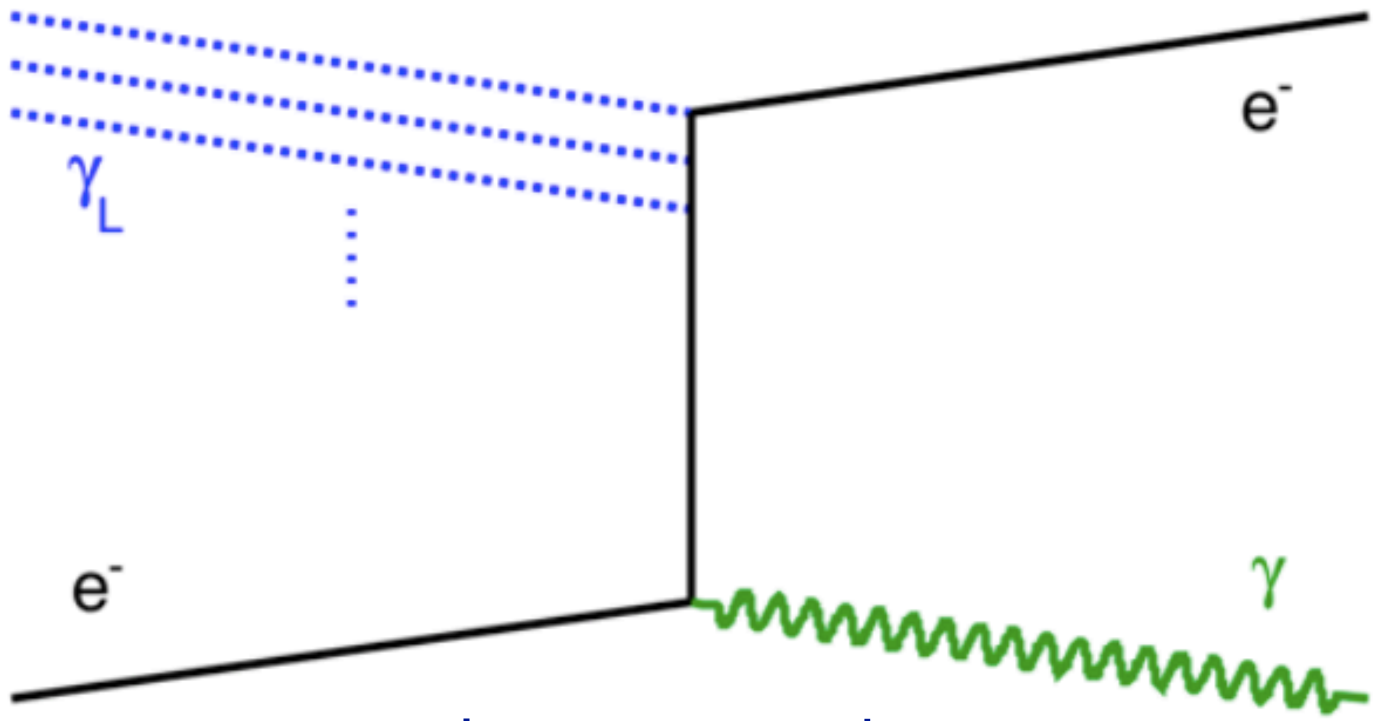
LUXE EXPERIMENT

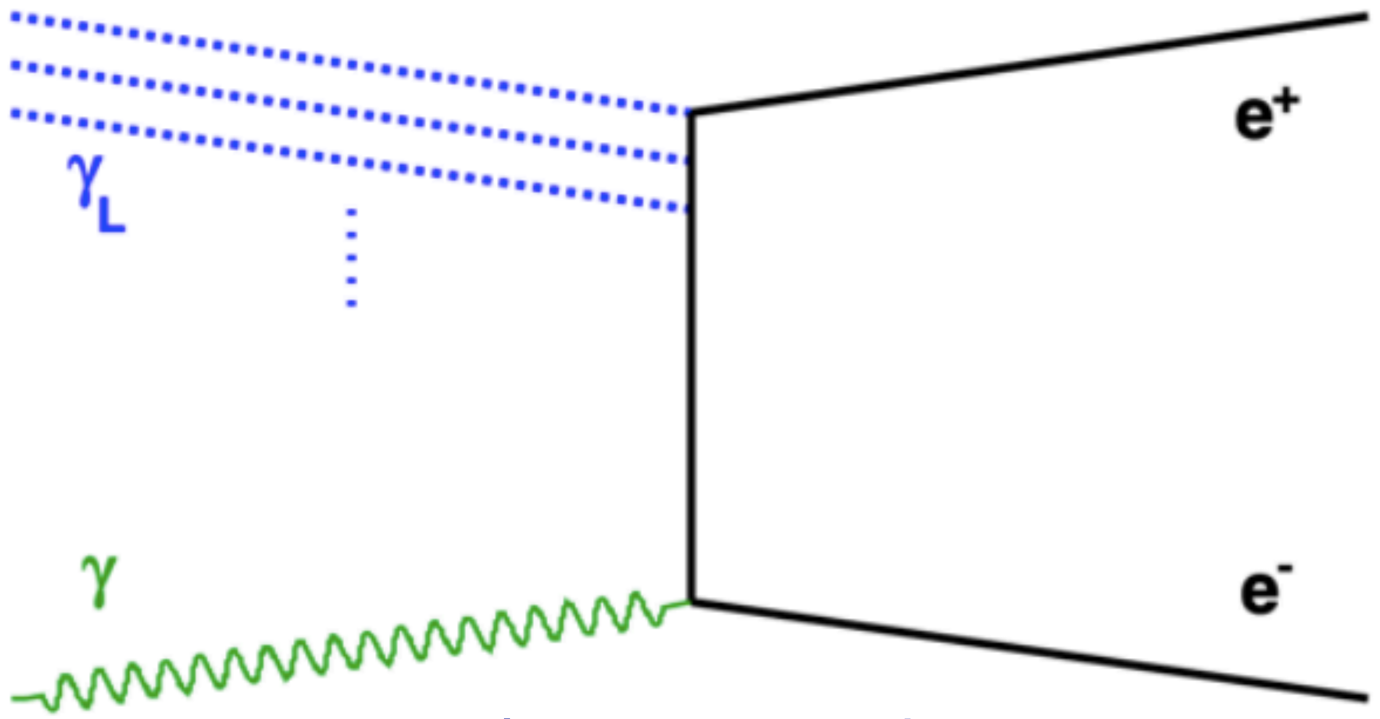
- LUXE (Laser Und XFEL Experiment) is a proposed experiment at DESY aiming to study QED in the strong-field regime where it becomes non-perturbative.
- Use European XFEL electron beam and high-power laser.
- CDR: arXiv:2102.02032, website: https://luxe.desy.de/

Field intensity parameter

$$\xi = \sqrt{4\pi\alpha}\left(\frac{\varepsilon_L}{\omega_L m_e}\right) = \frac{m_e \varepsilon_L}{\omega_L \varepsilon_{cr}}$$

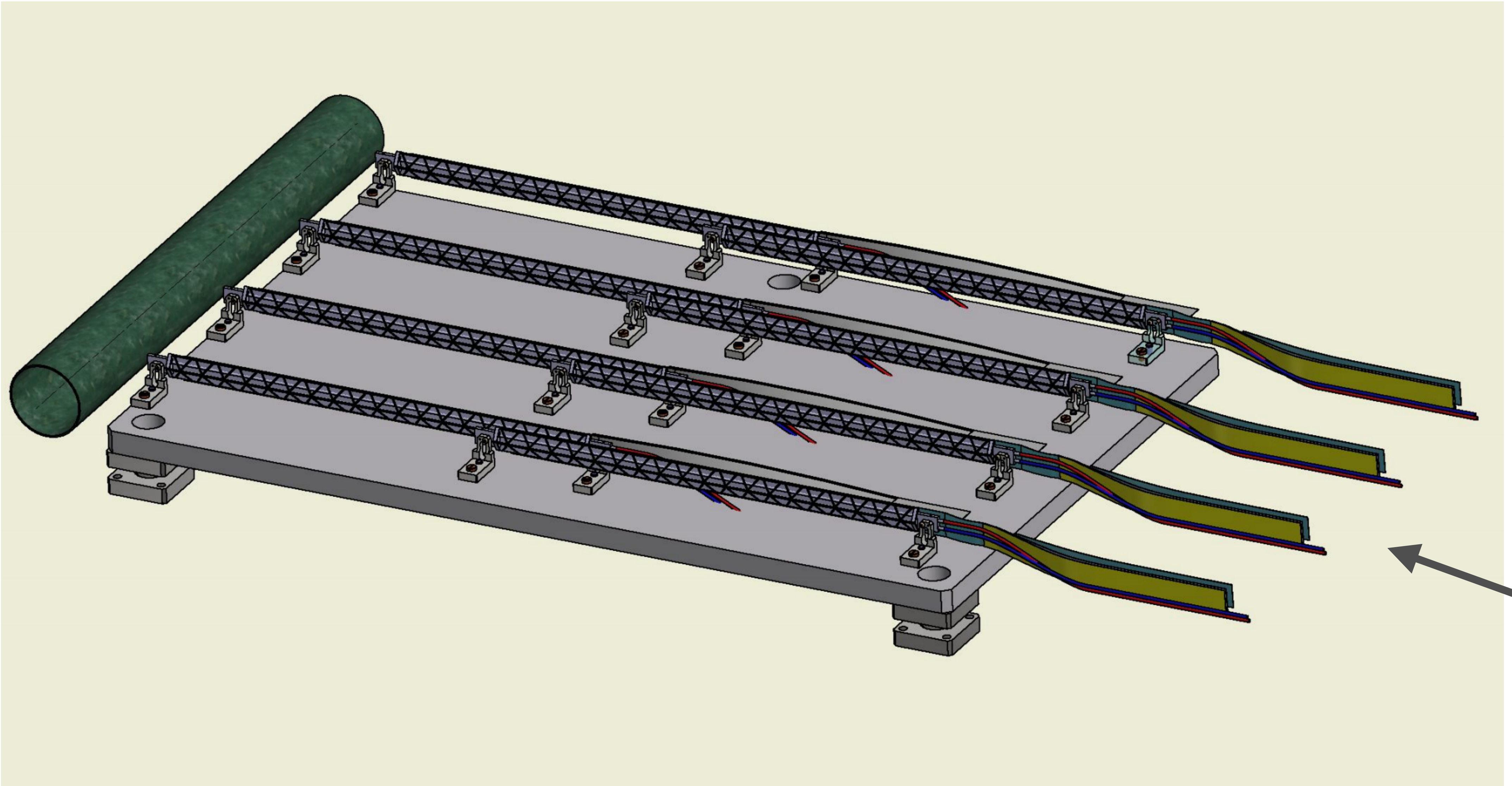


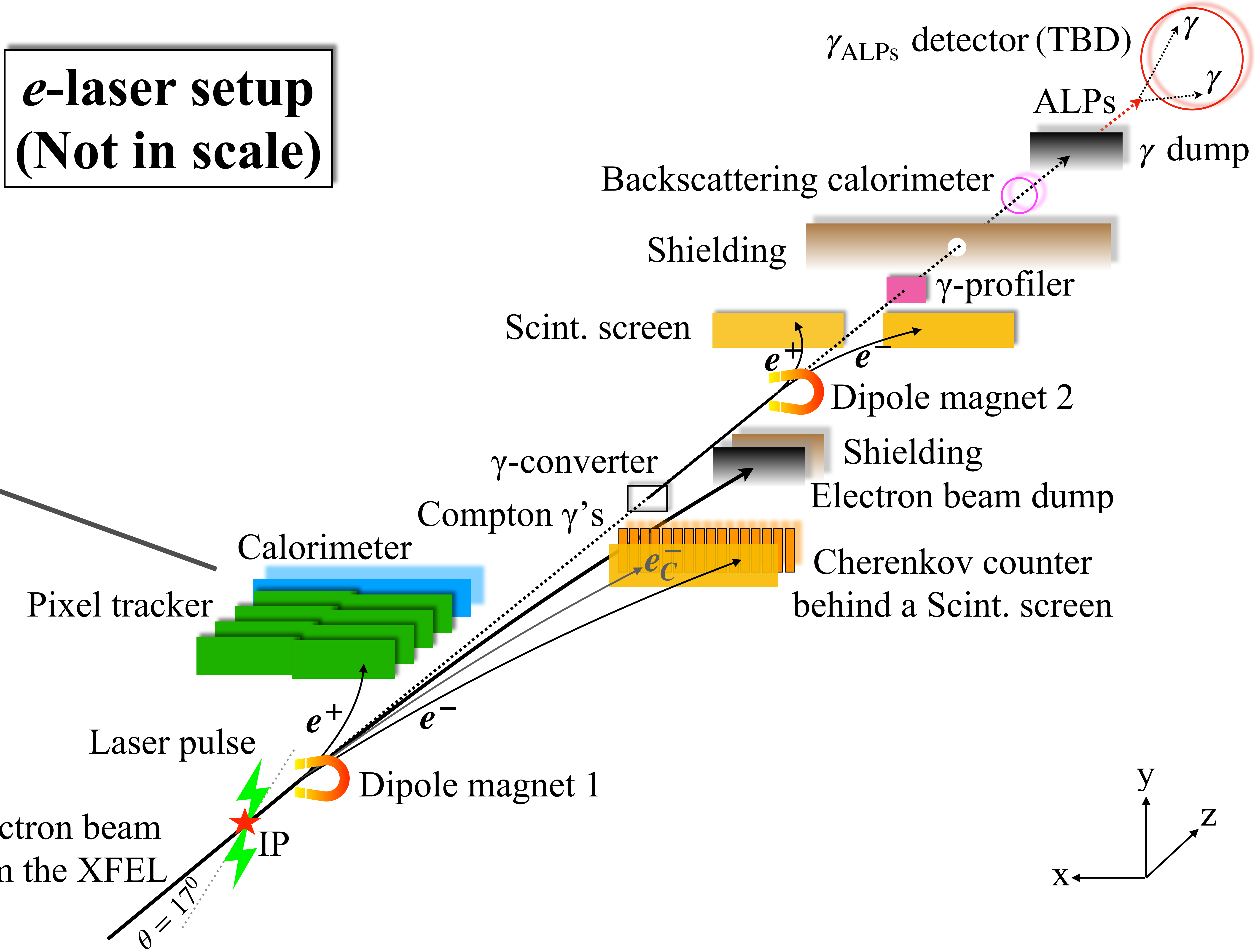
e-laser mode:
Non-linear Compton scattering



γ-laser mode:
Non-linear Breit-Wheeler pair creation

LUXE EXPERIMENT SETUP

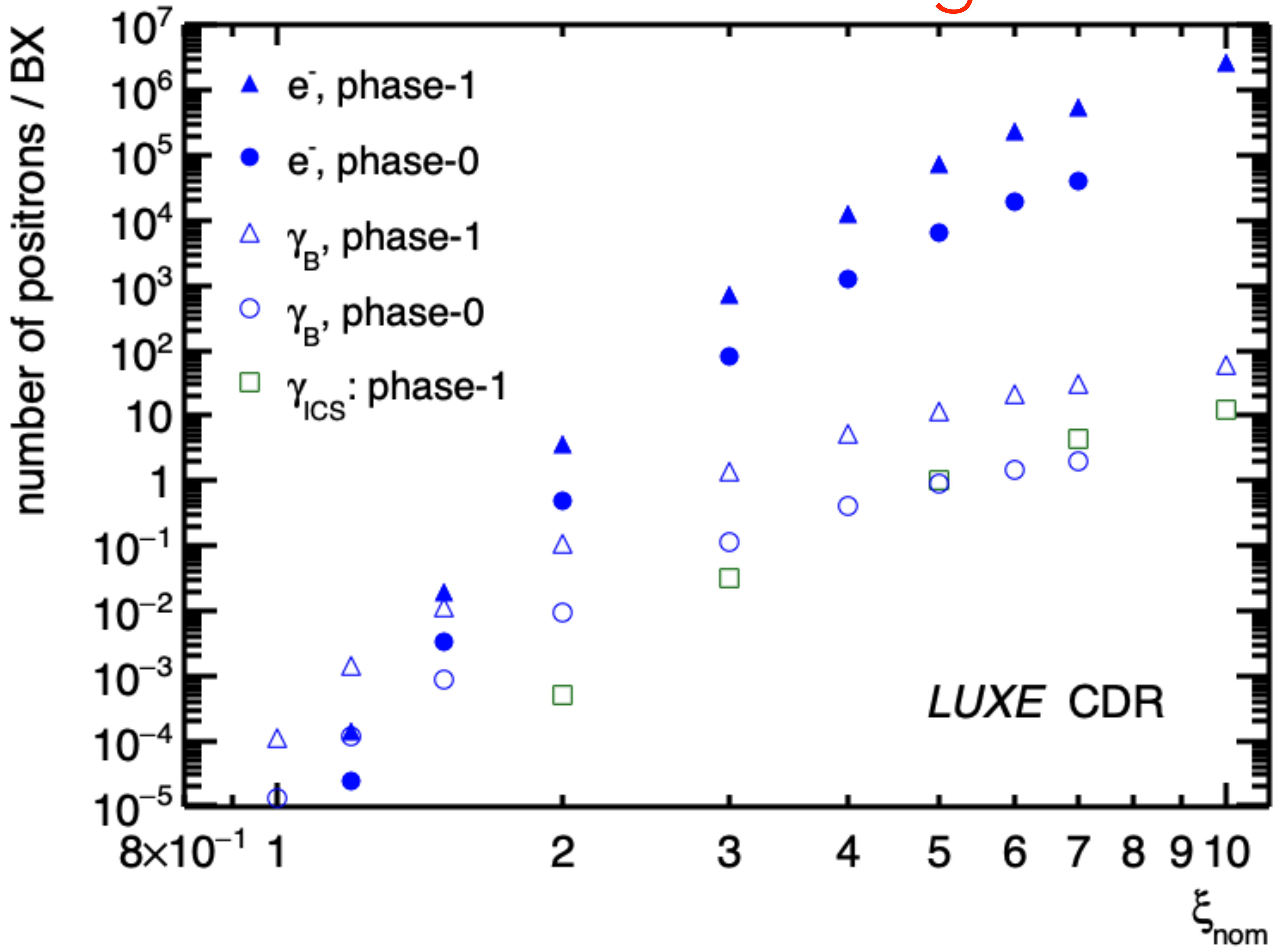




TRACKING CHALLENGE

- One of the main measurements at LUXE is the positron flux vs ξ.
- Two challenges:
  - Good linearity up to a multiplicity of $\mathcal{O}(10^6)$.
  - Background rate needs to be below 10^{-3}/BX at low ξ.
- Study the use of quantum computing.

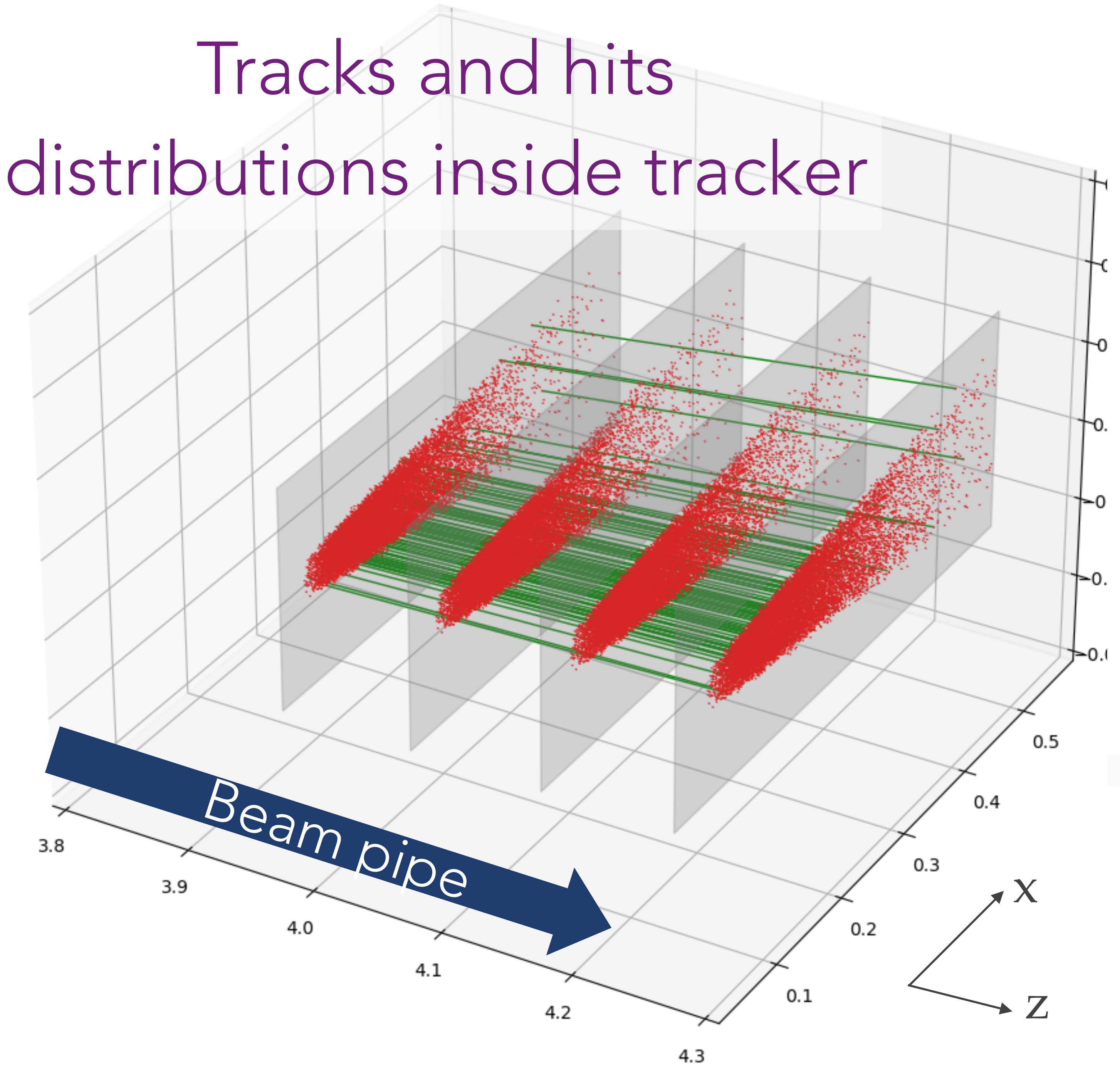
≳10 orders of magnitude!

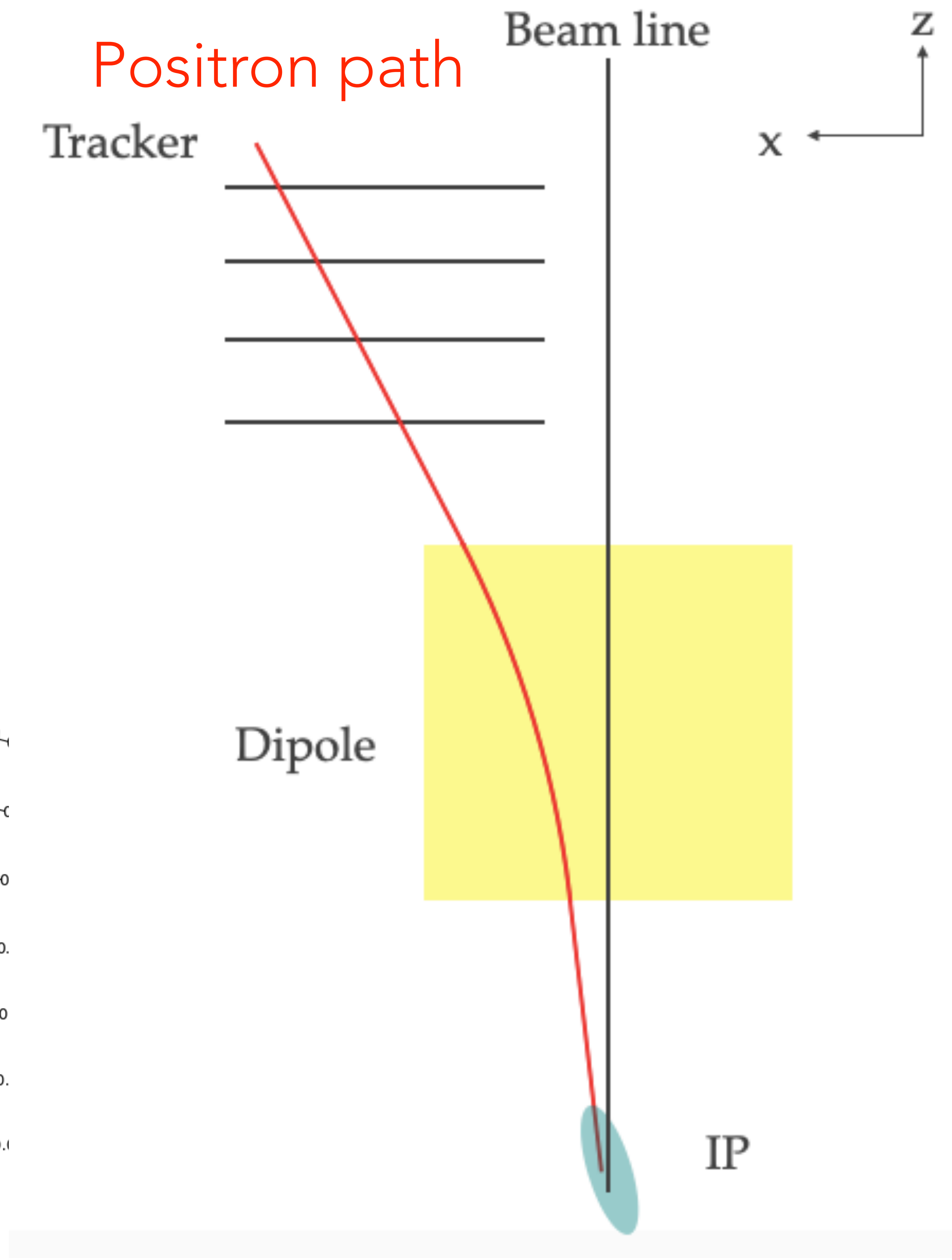


LUXE has a two-phase approach (2nd phase with an upgraded laser).

SIMULATION

- Signal interactions at the IP are generated with a custom MC (T. G. Blackburn, A. J. MacLeod, B. King, arXiv:2103.06673).
- The resulting positrons are propagated through the dipole magnet and tracking detector using a simplified simulation.
- For simplicity, consider four detection layers without gap/overlap.
- Ability to turn on/off the detector resolution effect, parametric multiple scattering, etc.



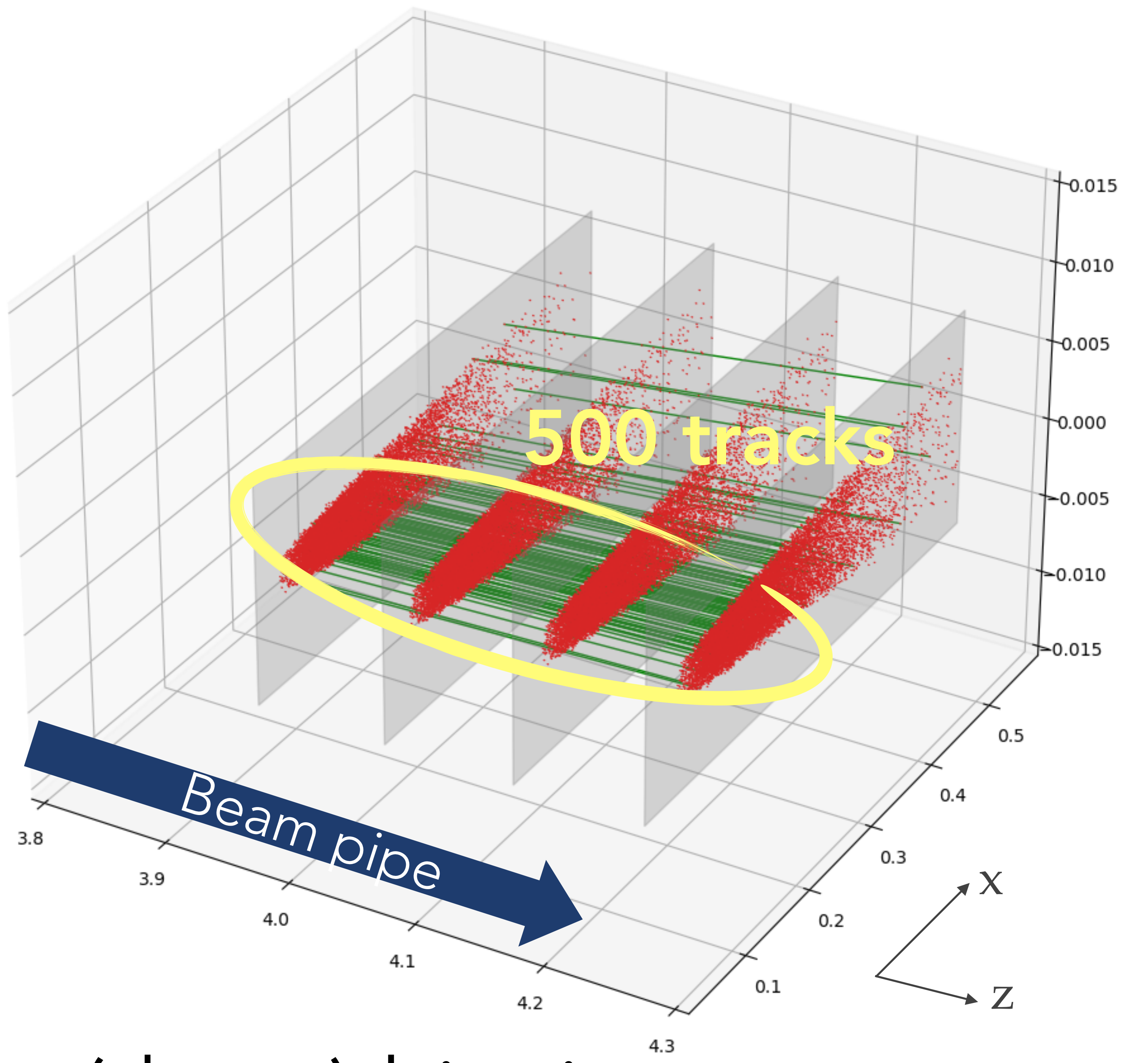


*Sketch not to scale

TRACKING PROBLEM

- Study ξ=3, 4, 5 and 7 in the e-laser phase-1 scenario. Number of positrons ranges from **800 to 500,000**.

- Limit to the **500 tracks** closest to the beam line (typically densest region) such that the size of the problem is constant.
  - But the complexity increases due to increasing track density with ξ.



- Starting point: doublets (triplets) which is a set of two (three) hits in consecutive layers.

PRE-SELECTION

- Pre-selection is applied on the initial doublet/triplet candidates to reduce the combinatorial candidates at ~100 % efficiency.

- Triplets are formed starting from doublets.

- Pre-selection based on the expected angles from geometry (doublet level) and the straightness of the triplet candidates.

- Triplets are formed from 1st to 3rd layer, and 2nd to 4th layer.

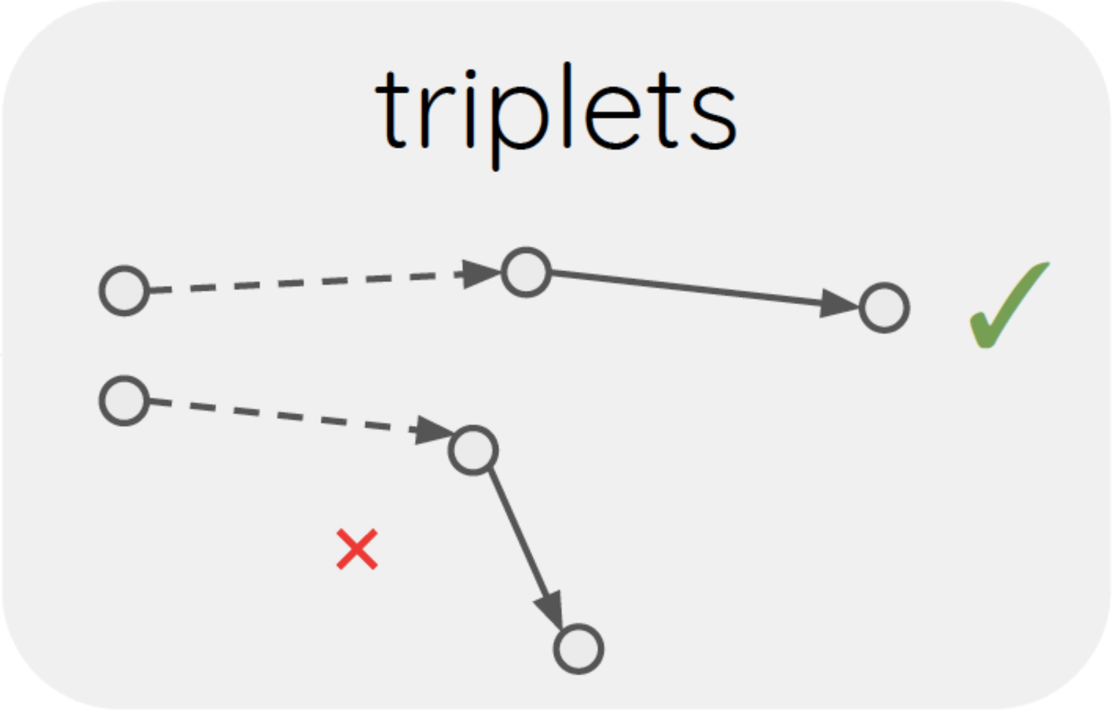
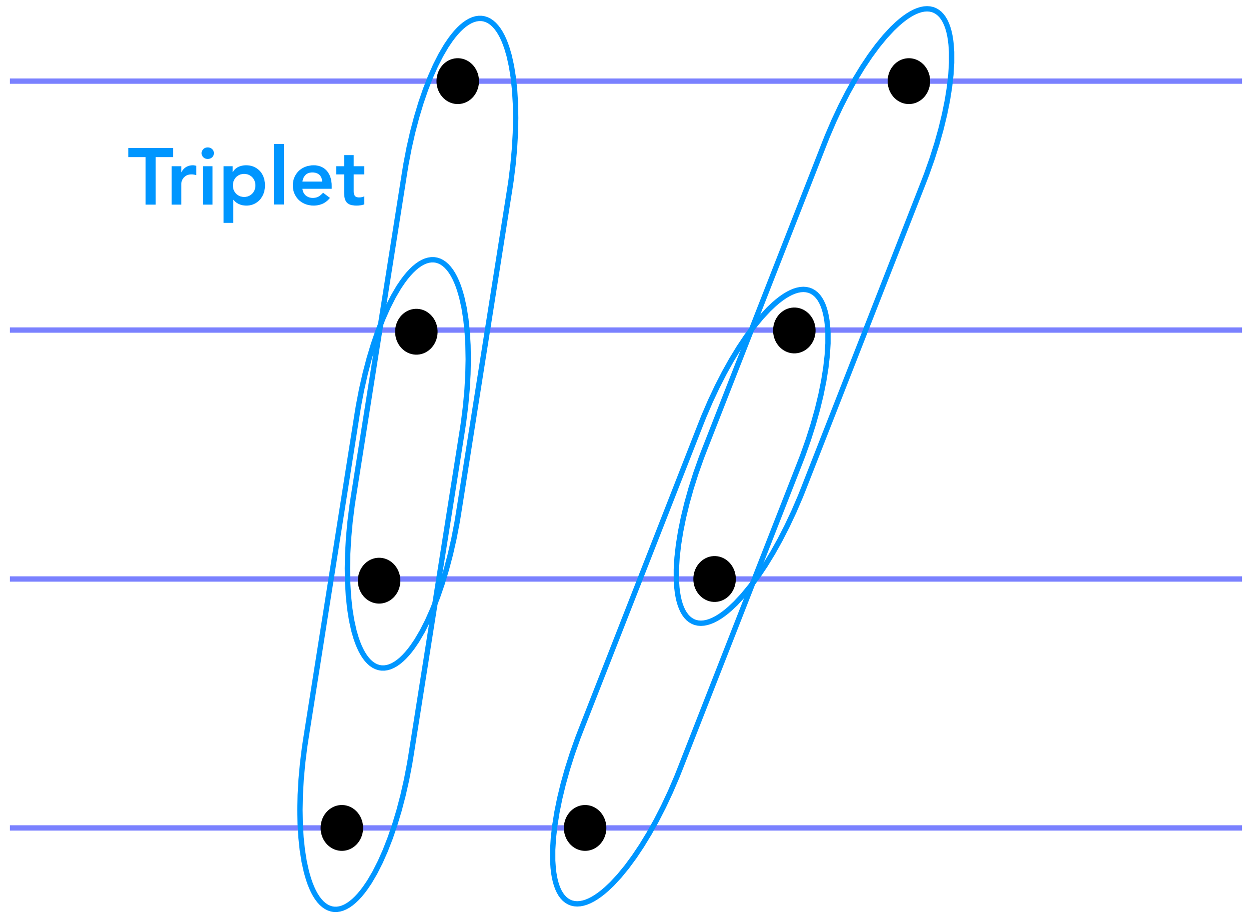
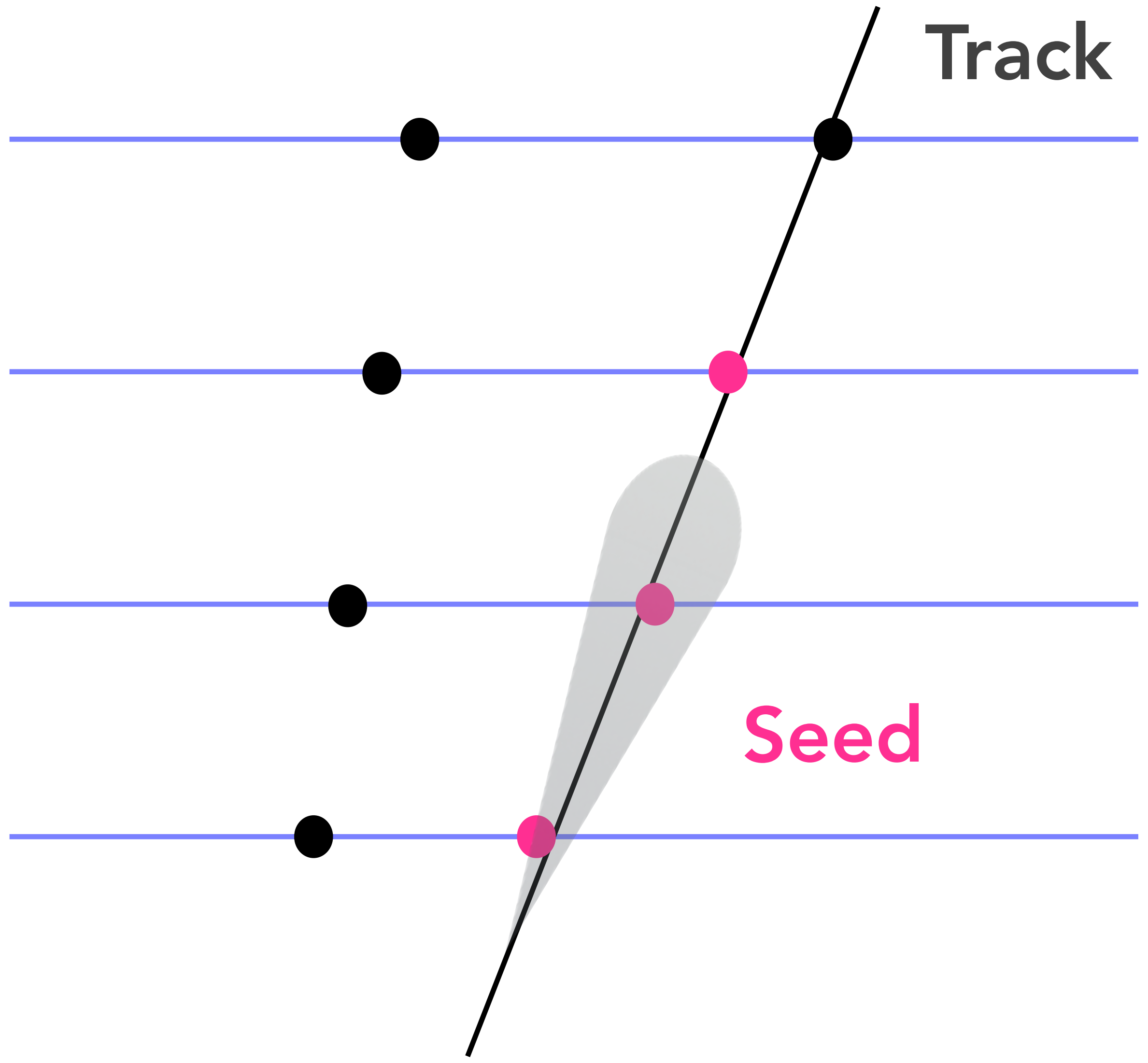


Image from Lucy Linder's thesis



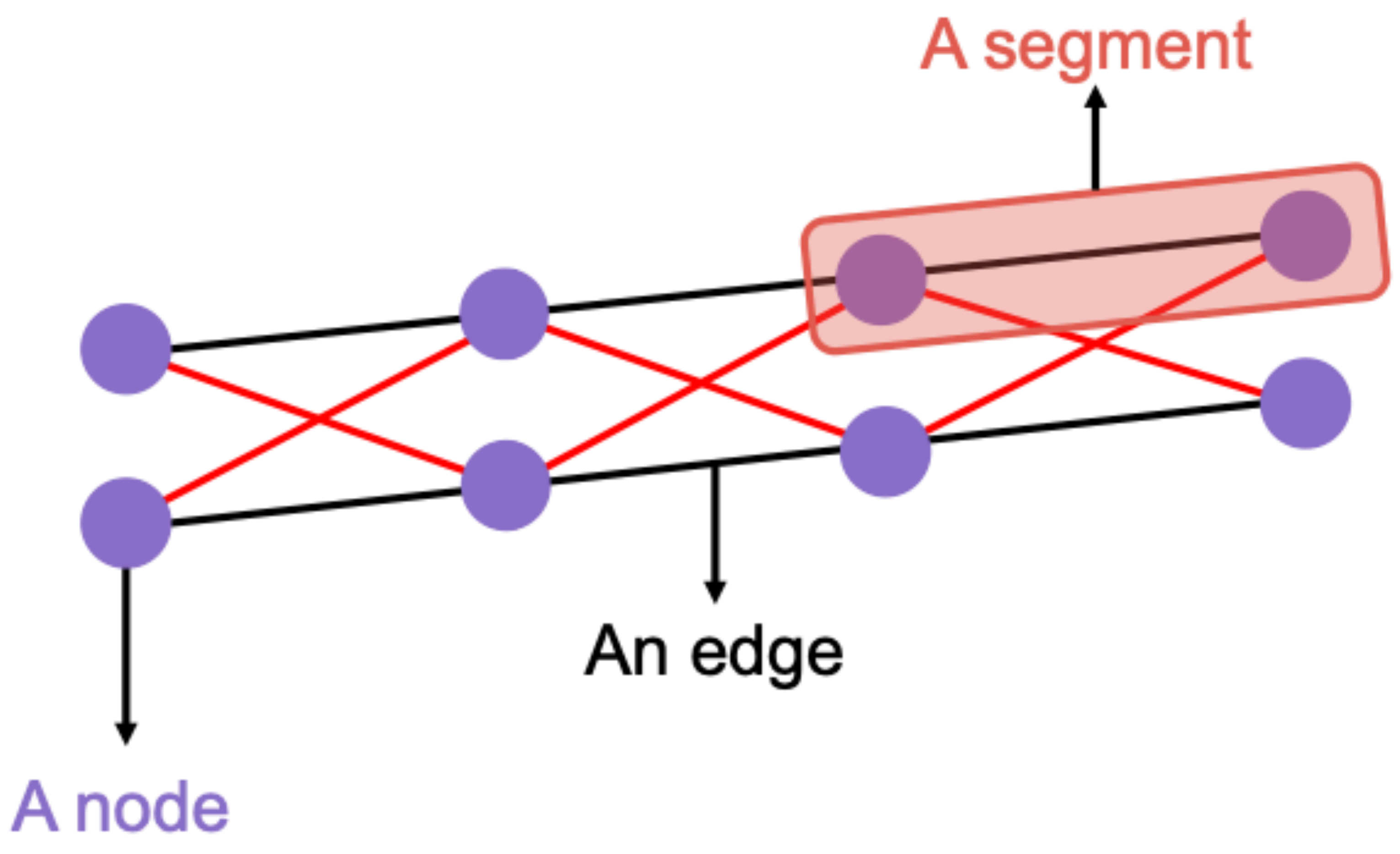
CLASSICAL BENCHMARK

- As benchmark, we use an ACTS*-based tracking with combinatorial Kalman Filter (CKF) technique for the track finding and fitting.
  - Seeding using the first three layers, similar to the triplet pre-selection.
  - Initial estimate of track parameters from seed is used to predict next hit and updated progressively, with the measurement search performed at the same time as the fit.
- Ambiguity solving applied to remove tracks with shared hits from the initial track collection.

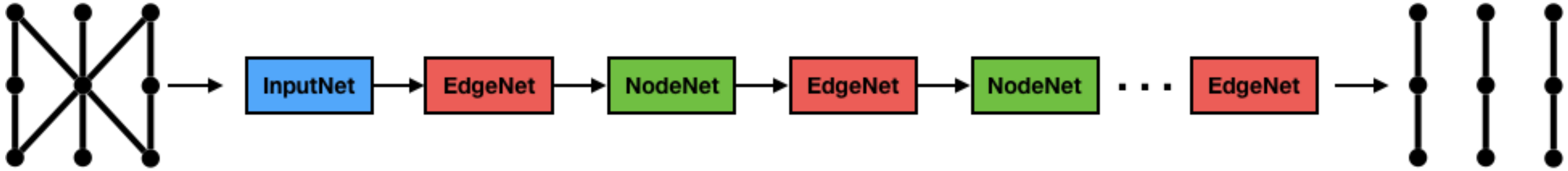


*ACTS: A Common Tracking Software https://acts.readthedocs.io

GRAPH NEURAL NETWORK



- Graph constructed from doublets.
- Hits are nodes and the connections between them are edges. The doublet structure is called a segment.
- All nodes of consecutive layers are connected, and only the ones that satisfy pre-selection cuts are kept.



- Developed in HEP.TrkX project (arXiv:1810.06111) and further extended in Exa.TrkX (arXiv:2103.06995). Hybrid quantum-classical version also exists (Q.TrkX, arXiv:2109.12636)

THE QUANTUM APPROACH

- The triplets are identified to form tracks by expressing the problem as a quadratic unconstrained binary optimisation (QUBO), problem similar to https://doi.org/10.1007/s41781-019-0032-5*.

- Minimising the QUBO is equivalent to finding the ground state of the Hamiltonian.

$$O(a,b,T) = \underbrace{\sum_{i=1}^{N} a_i T_i}_{\text{Quality of triplets}} + \underbrace{\sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j}_{\text{Compatibility between triplet pairs}} \quad T_i, T_j \in \{0,1\}$$

a_i quantify the quality of the triplets.

b_{ij} quantify the compatibility between triplet pairs.

$b_{ij} = 0$, if no shared hit

$= +1$, if in conflict

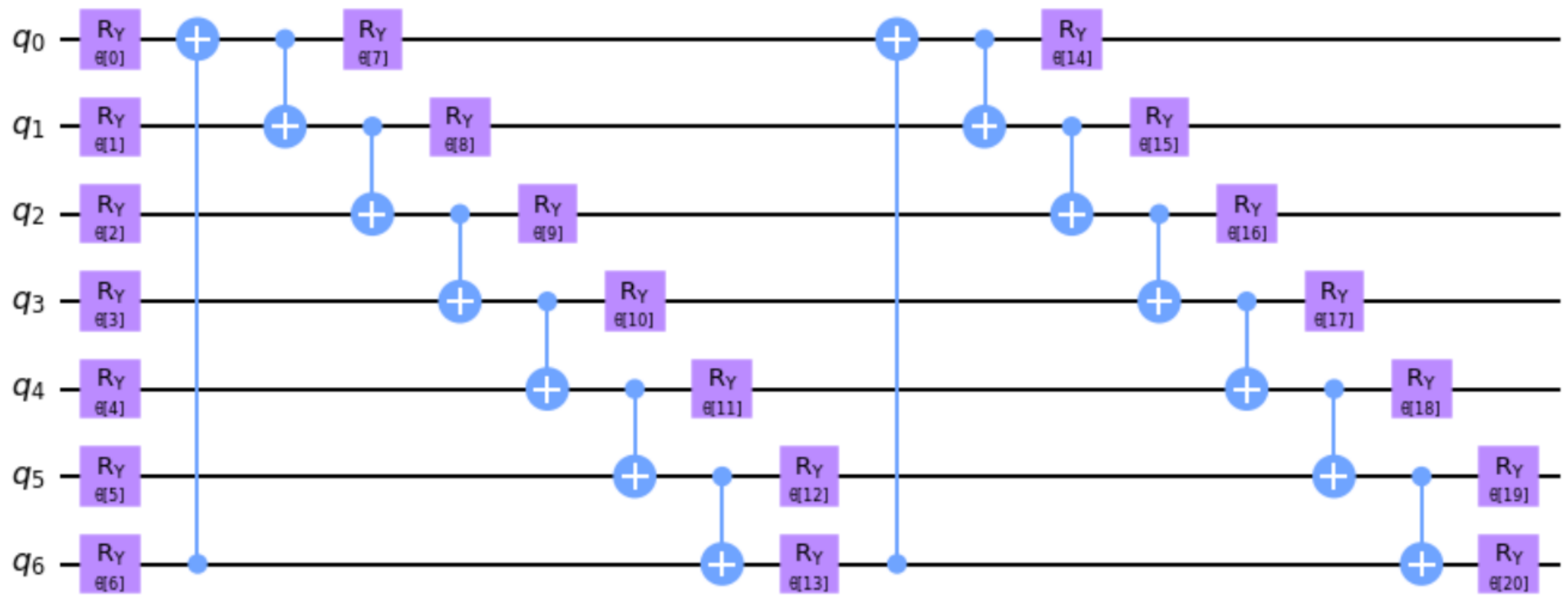
$= -S(T_i, T_j)$, if two hits are shared

*Bapst, F., Bhimji, W., Calafiura, P. *et al.* A Pattern Recognition Algorithm for Quantum Annealers. *Comput Softw Big Sci* **4,** 1 (2020).

SOLVING QUBO

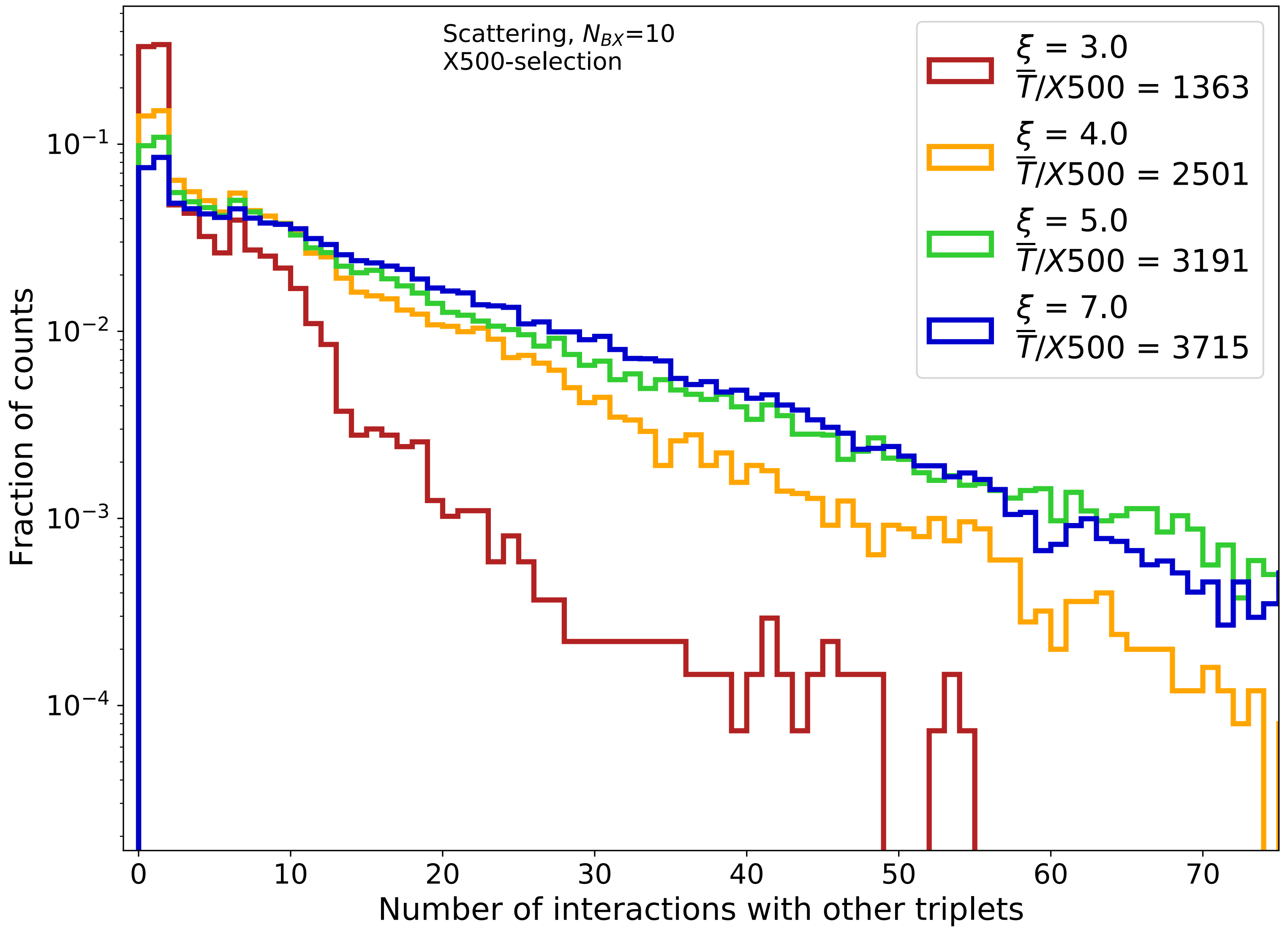
- QUBO can be mapped to Ising Hamiltonian and solved using Variational Quantum Eigensolver (VQE).

$$\mathcal{H} = -\sum_{n=1}^{N} \sigma_n^x \sigma_{n+1}^x - \alpha \sum_{n=1}^{N} \sigma_n^x$$



- Use Qiskit from IBM.

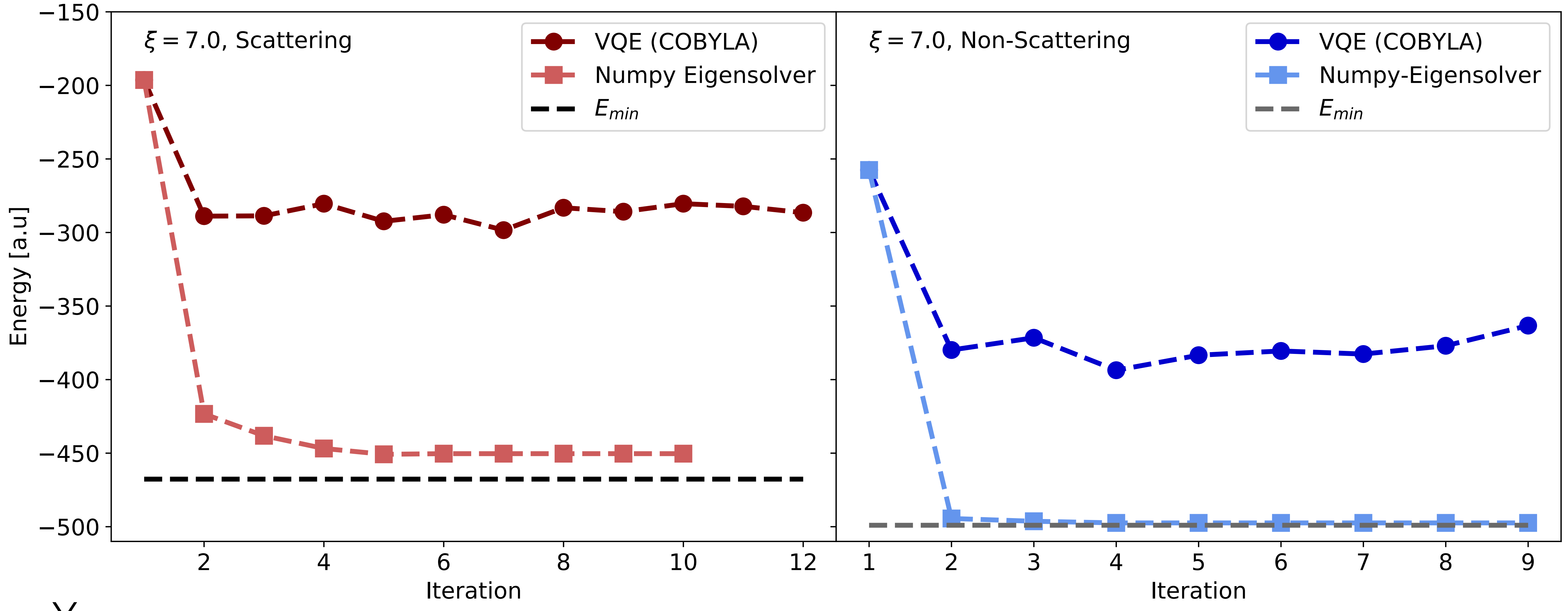
- Two sets of results:
  - Exact solution using matrix diagonalisation (NumPy Eigensolver) for benchmarking
  - VQE (without QC noise) using one choice of Ansatz and optimiser.

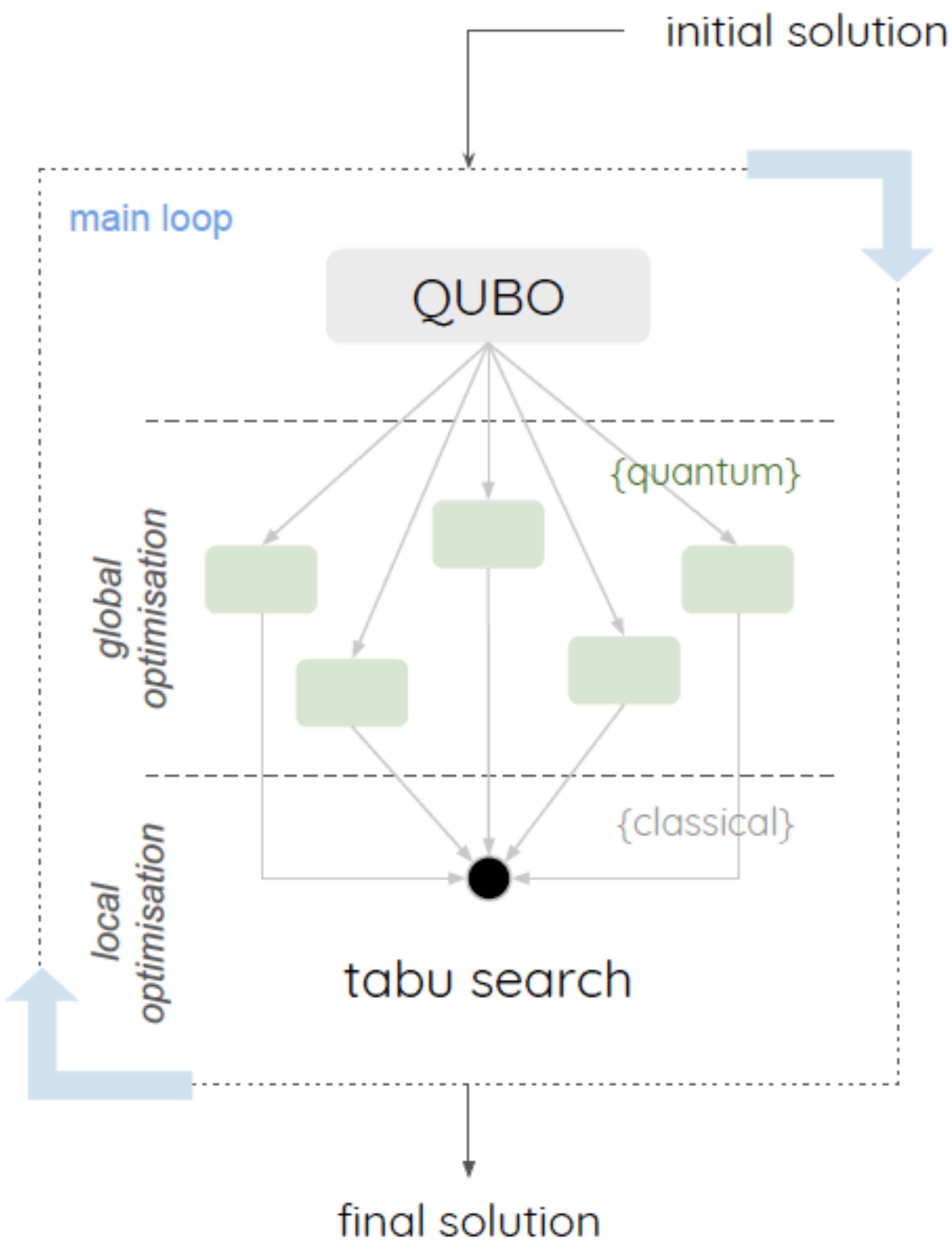


- As ξ increases, the track density increases and the number of interactions of a triplet with other triplets too increases.

SUB-QUBO

- Need as many qubits as there are triplets.
- Due to the limited number of qubits, the QUBO is split into sub-QUBOs (of size 7) to be solved.
  - After the sub-QUBOs are solved, the results are combined and a tabu search performed.
  - Repeat for *n* iterations.





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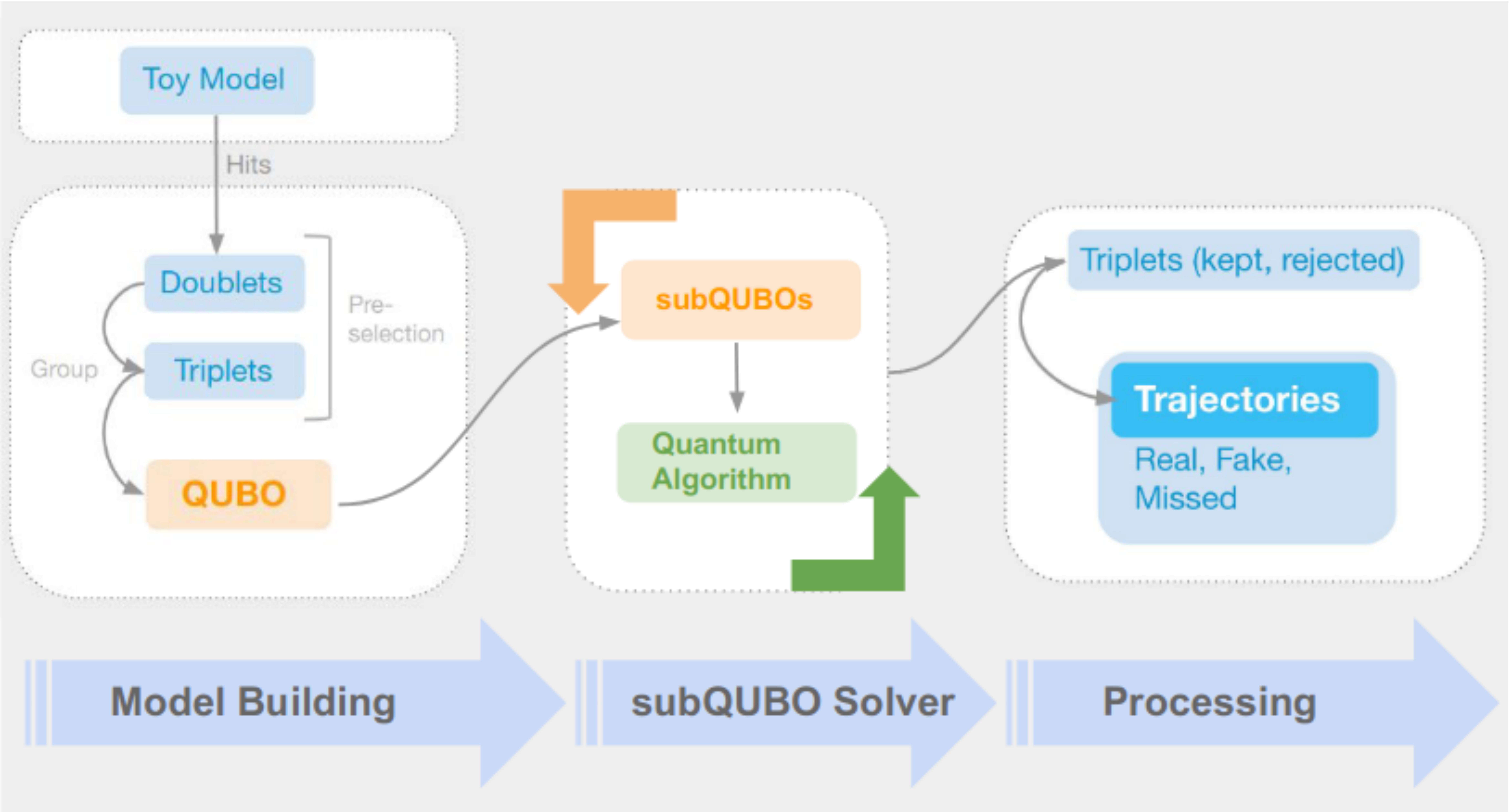

Toy Model
Hits
Doublets
Group
Triplets
Pre-selection
QUBO
subQUBOs
Quantum Algorithm
Triplets (kept, rejected)
Trajectories
Real, Fake, Missed
Model Building
subQUBO Solver
Processing

PERFORMANCE

- Track must contain four hits, found either with classical CKF tracking or combining selected triplet pairs into quadruplets if they share two hits between them.
  - A correct track has **all four hits matched** to the same generated particle.

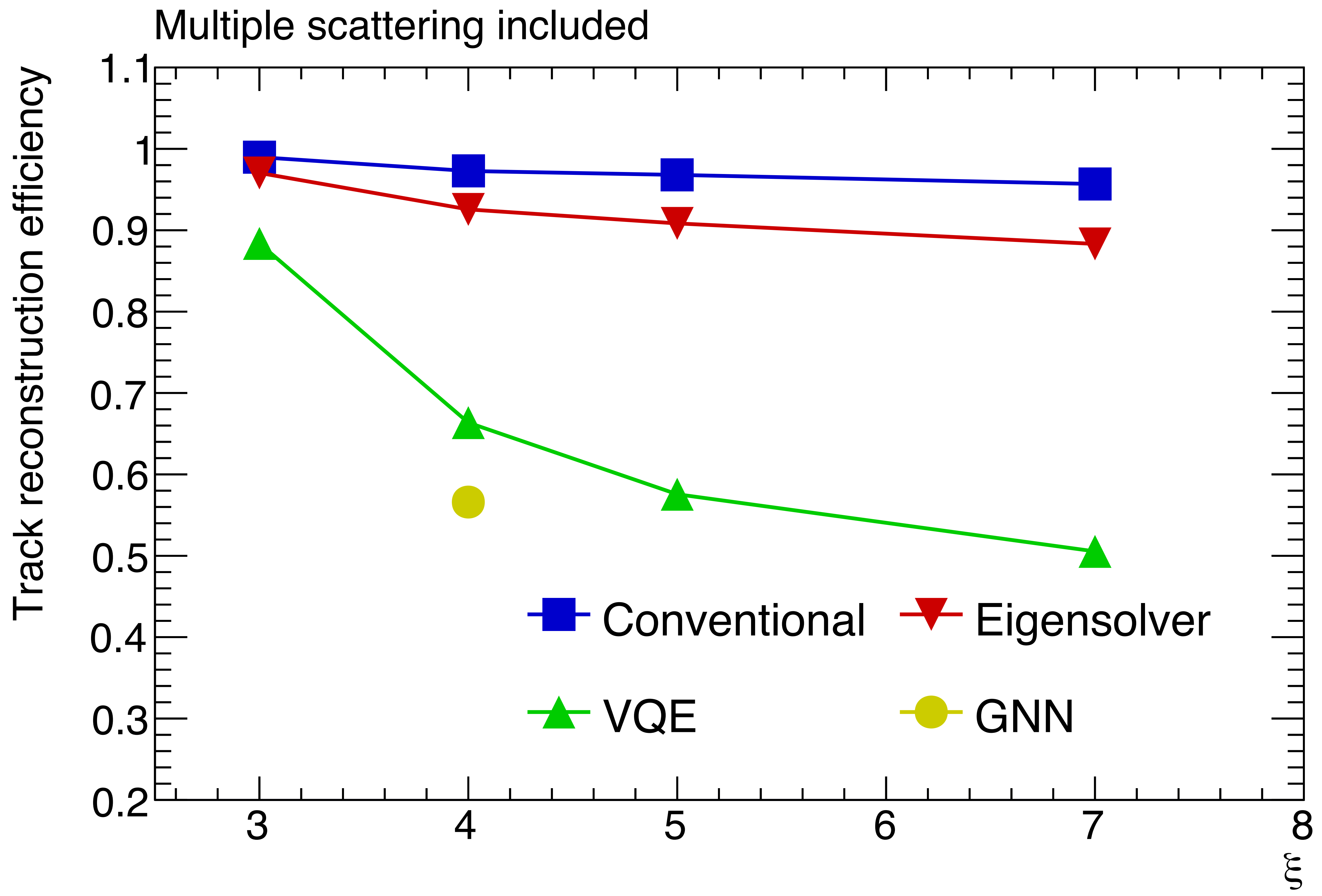
- Performance metrics:

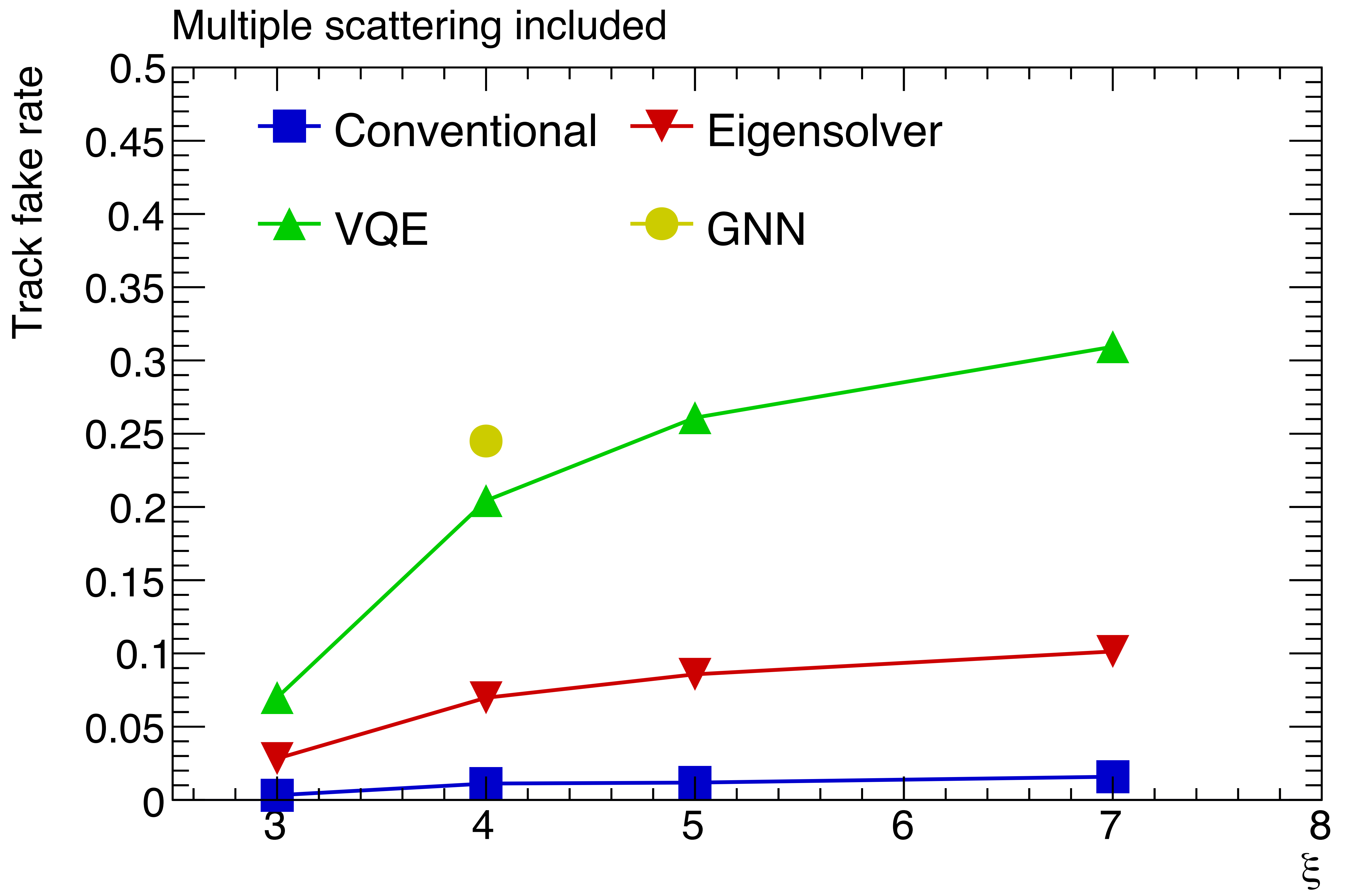
$$\text{Efficiency} = \frac{N^{\text{matched}}_{\text{tracks}}}{N^{\text{generated}}_{\text{tracks}}} \text{ and Fake rate} = \frac{N^{\text{fake}}_{\text{tracks}}}{N^{\text{reconstructed}}_{\text{tracks}}}$$

- Compare classical (CKF and GNN) and quantum (VQE and exact solution) approaches.

- Noise and real quantum device also tested but at a smaller scale.

RESULTS





- Conventional tracking as benchmark shows the performance that can be realistically achieved. Room for improvement for other tracking methods (preliminary results shown).

*GNN performance limited by training data size.

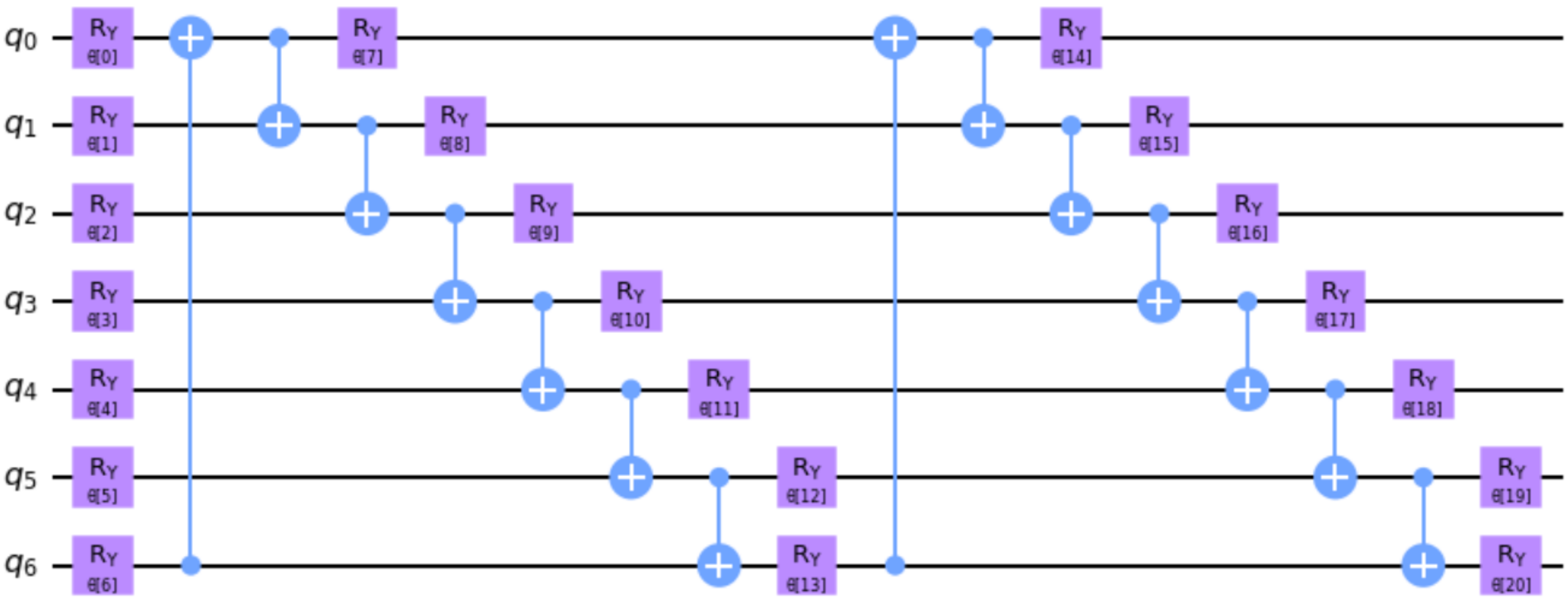
SUMMARY AND NEXT STEPS

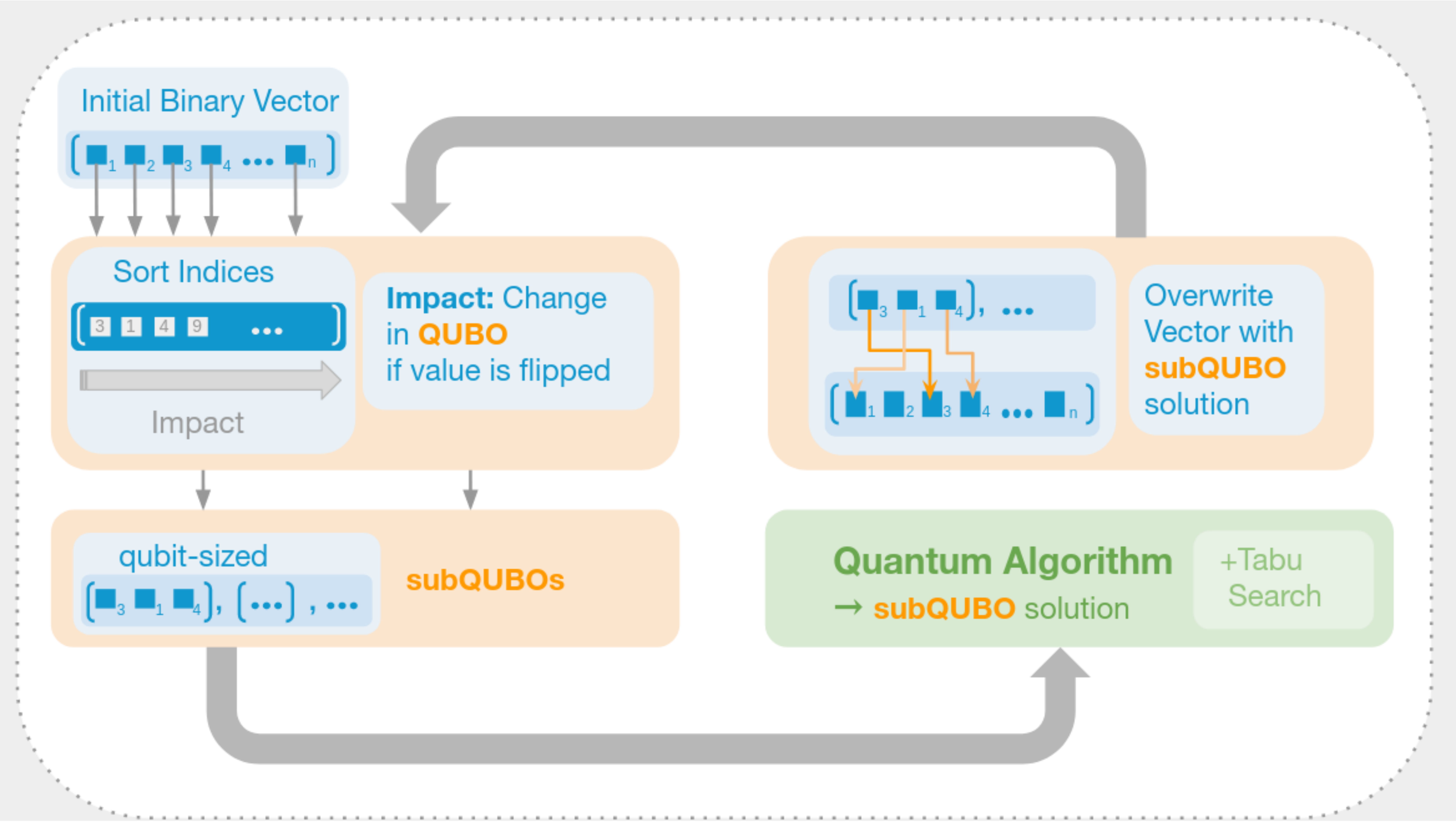
- Tracking challenge in LUXE presented.
- Study the use of a hybrid quantum-classical algorithm in track reconstruction along with conventional tracking method as well as GNN-based tracking.
- A first implementation of track reconstruction in LUXE using quantum devices is in place.
  - Preliminary study shows performance similar to traditional algorithms, however limited by the size of the device.
- Next:
  - Study the performance in more extreme environments, take into account the QC noise and explore regions where QC could outperform the traditional methods.

BACK-UP SLIDES

VQE

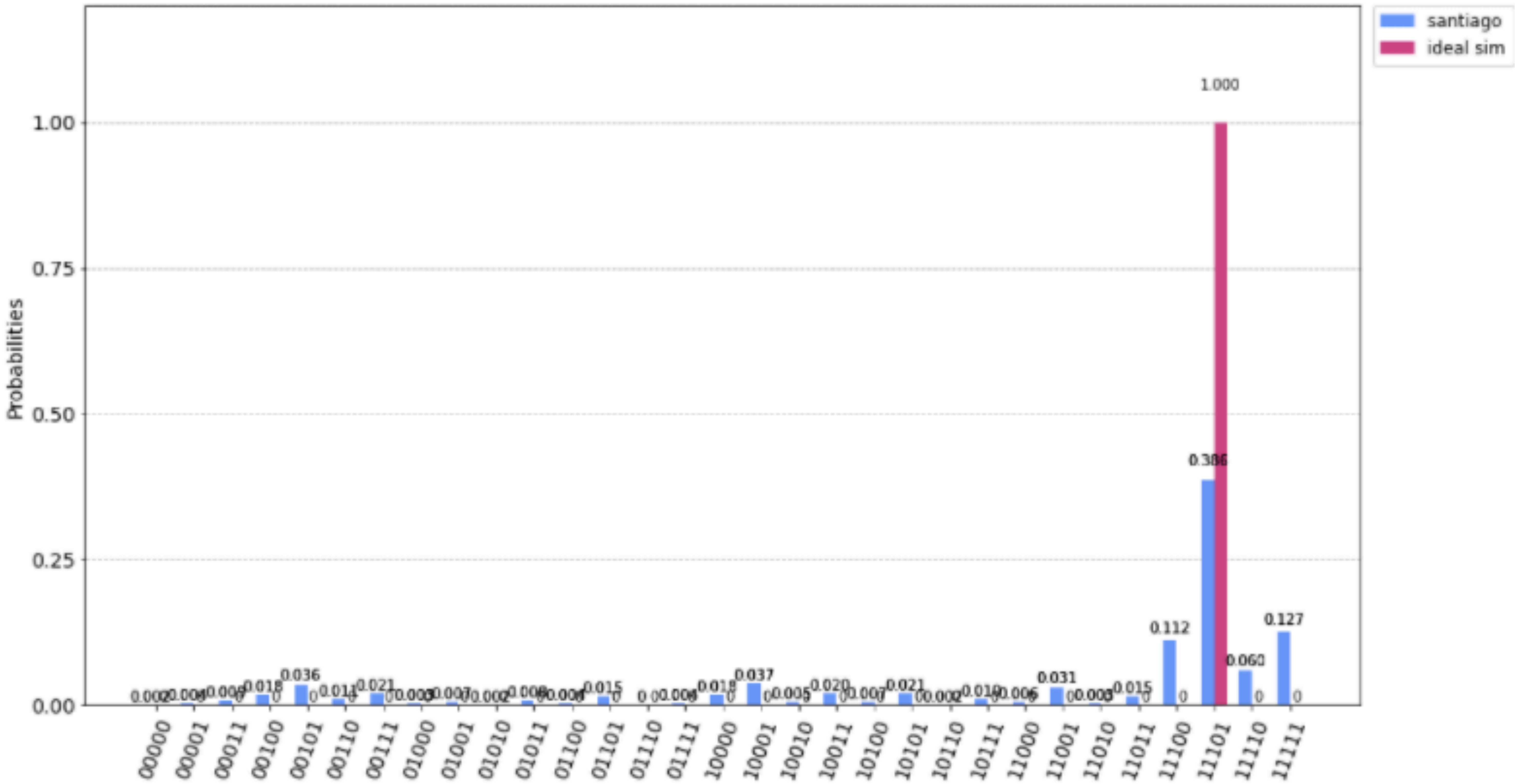
- VQE ansatz: TwoLocal with R_Y, and circular CNOT entangler.
- Optimiser: Constrained Optimization by Linear Approximation (COBYLA).

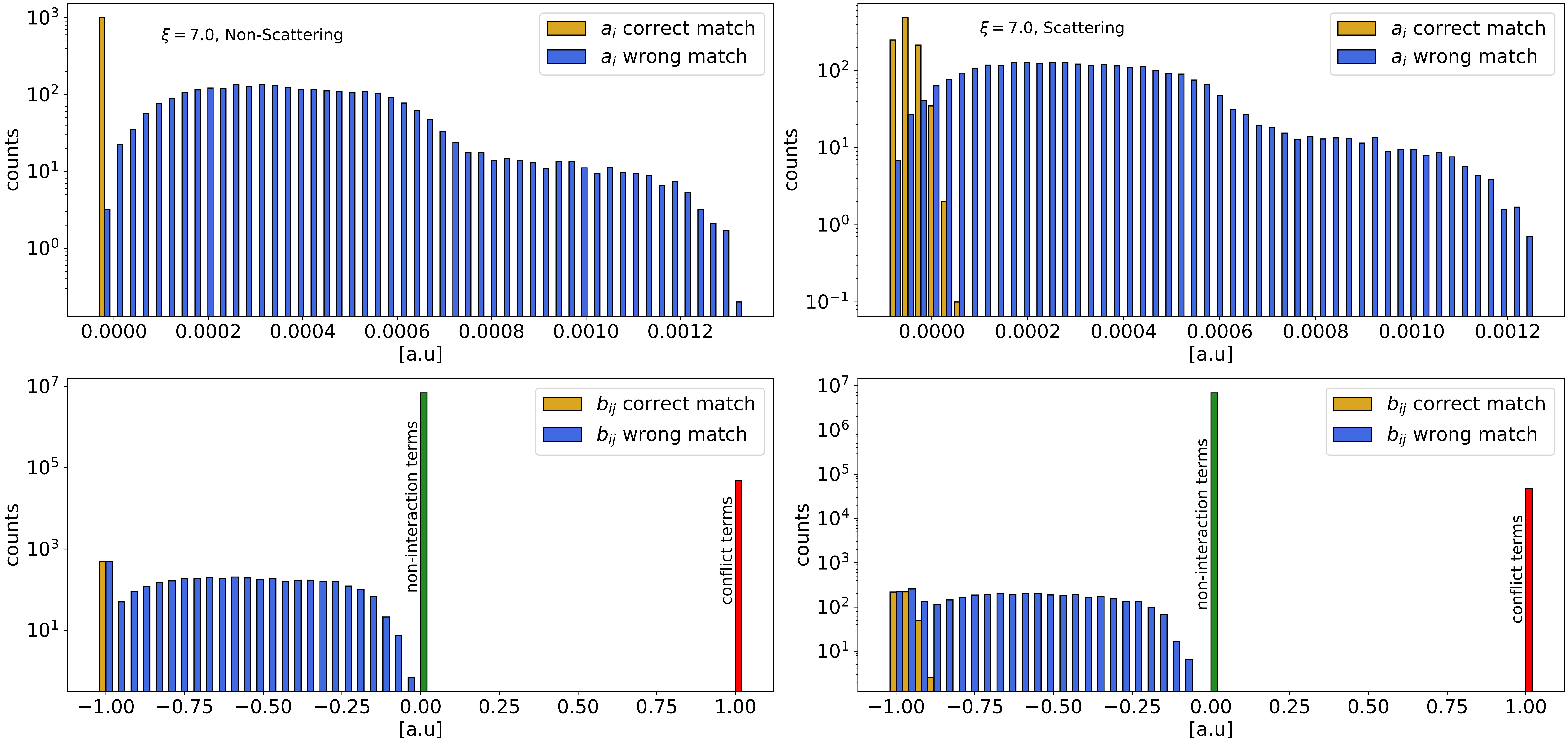



Initial Binary Vector
Sort Indices
3 1 4 9 ...
Impact
Impact: Change in QUBO if value is flipped
qubit-sized
subQUBOs
Overwrite Vector with subQUBO solution
Quantum Algorithm
→ subQUBO solution
+Tabu Search

QUANTUM DEVICE

- Comparison of real quantum hardware and ideal noise-free simulation.
- 2 tracks, 5 triplets, 5 qubits.
- Correct triplet identified.





$\xi = 7.0$, Non-Scattering
a_i correct match
a_i wrong match
counts
[a.u]
$\xi = 7.0$, Scattering
a_i correct match
a_i wrong match
counts
[a.u]
b_{ij} correct match
b_{ij} wrong match
non-interaction terms
conflict terms
counts
[a.u]
b_{ij} correct match
b_{ij} wrong match
non-interaction terms
conflict terms
counts
[a.u]