TRACK RECONSTRUCTION WITH QUANTUM COMPUTERS AT LUXE

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LUXE EXPERIMENT

• LUXE (Laser Und XFEL Experiment) is a proposed experiment at DESY aiming to study QED in the strong-field regime where it becomes non-perturbative.

• Use European XFEL electron beam and high-power laser.


Field intensity parameter

$$\xi = \sqrt{4\pi\alpha} \left( \frac{\varepsilon_L}{\omega_L m_e} \right) = \frac{m_e \varepsilon_L}{\omega_L \varepsilon_{cr}}$$

- e-laser mode: Non-linear Compton scattering
- γ-laser mode: Non-linear Breit-Wheeler pair creation
**LUXE EXPERIMENT SETUP**

**e-laser setup (Not in scale)**

- **Dipole magnet 1**
- **Laser pulse**
- **Compton $\gamma$**
- **$e^- - e^+$**
- **$\gamma$ dump**
- **$\gamma$-profiler**
- **Dipole magnet 2**
- **Scint. screen**
- **Shielding**
- **Backscattering calorimeter**
- **Cherenkov counter behind a Scint. screen**
- **$\gamma_{ALPs}$ detector (TBD)**
- **ALPs**
- **$\gamma$-converter**
- **Compton $\gamma$’s**
- **Pixel tracker**
- **Calorimeter**
- **Electron beam dump**
- **Electron beam from the XFEL**
- **$\gamma_{ALPs}$**
- **$\gamma$-profiler**

Yee Chinn Yap
One of the main measurements at LUXE is the positron flux vs $\xi$.

Two challenges:
- Good linearity up to a multiplicity of $O(10^6)$.
- Background rate needs to be below $10^{-3}/BX$ at low $\xi$.

Study the use of quantum computing.

LUXE has a two-phase approach (2nd phase with an upgraded laser).

$\approx 10$ orders of magnitude!
SIMULATION

• Signal interactions at the IP are generated with a custom MC (T. G. Blackburn, A. J. MacLeod, B. King, arXiv:2103.06673).

• The resulting positrons are propagated through the dipole magnet and tracking detector using a simplified simulation.

• For simplicity, consider four detection layers without gap/overlap.

• Ability to turn on/off the detector resolution effect, parametric multiple scattering, etc.
TRACKING PROBLEM

• Study $\xi=3, 4, 5$ and $7$ in the e-laser phase-1 scenario. Number of positrons ranges from 800 to 500,000.

• Limit to the 500 tracks closest to the beam line (typically densest region) such that the size of the problem is constant.
  • But the complexity increases due to increasing track density with $\xi$.

• Starting point: doublets (triplets) which is a set of two (three) hits in consecutive layers.
**PRE-SELECTION**

- Pre-selection is applied on the initial doublet/triplet candidates to reduce the combinatorial candidates at ~100 % efficiency.

- Triplets are formed starting from doublets.

- Pre-selection based on the expected angles from geometry (doublet level) and the straightness of the triplet candidates.

- Triplets are formed from 1\textsuperscript{st} to 3\textsuperscript{rd} layer, and 2\textsuperscript{nd} to 4\textsuperscript{th} layer.
CLASSICAL BENCHMARK

- As benchmark, we use an ACTS*-based tracking with combinatorial Kalman Filter (CKF) technique for the track finding and fitting.
- Seeding using the first three layers, similar to the triplet pre-selection.
- Initial estimate of track parameters from seed is used to predict next hit and updated progressively, with the measurement search performed at the same time as the fit.
- Ambiguity solving applied to remove tracks with shared hits from the initial track collection.

*ACTS: A Common Tracking Software https://acts.readthedocs.io
GRAPH NEURAL NETWORK

- Graph constructed from doublets.
- Hits are nodes and the connections between them are edges. The doublet structure is called a segment.
- All nodes of consecutive layers are connected, and only the ones that satisfy pre-selection cuts are kept.
THE QUANTUM APPROACH

• The triplets are identified to form tracks by expressing the problem as a quadratic unconstrained binary optimisation (QUBO), problem similar to https://doi.org/10.1007/s41781-019-0032-5.

• Minimising the QUBO is equivalent to finding the ground state of the Hamiltonian.

\[
O(a, b, T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T_i, T_j \in \{0, 1\}
\]

- \(a_i\): quantify the quality of the triplets.
- \(b_{ij}\): quantify the compatibility between triplet pairs.
  - \(b_{ij} = 0\), if no shared hit
  - \(b_{ij} = +1\), if in conflict
  - \(b_{ij} = -S(T_i, T_j)\), if two hits are shared

QUBO can be mapped to Ising Hamiltonian and solved using Variational Quantum Eigensolver (VQE).

\[ \mathcal{H} = - \sum_{n=1}^{N} \sigma_n^x \sigma_{n+1}^x - \alpha \sum_{n=1}^{N} \sigma_n^x \]

- Use Qiskit from IBM.
- Two sets of results:
  - Exact solution using matrix diagonalisation (NumPy Eigenvalues) for benchmarking
  - VQE (without QC noise) using one choice of Ansatz and optimiser.
- As \( \xi \) increases, the track density increases and the number of interactions of a triplet with other triplets too increases.
**SUB-QUBO**

- Need as many qubits as there are triplets.
- Due to the limited number of qubits, the QUBO is split into sub-QUBOs (of size 7) to be solved.
  - After the sub-QUBOs are solved, the results are combined and a tabu search performed.
- Repeat for \( n \) iterations.

![Image from Lucy Linder's thesis](image_url)
PERFORMANCE

• Track must contain four hits, found either with classical CKF tracking or combining selected triplet pairs into quadruplets if they share two hits between them.
  • A correct track has all four hits matched to the same generated particle.

• Performance metrics:

\[
\text{Efficiency} = \frac{N_{\text{matched tracks}}}{N_{\text{generated tracks}}} \quad \text{and} \quad \text{Fake rate} = \frac{N_{\text{fake tracks}}}{N_{\text{reconstructed tracks}}}
\]

• Compare classical (CKF and GNN) and quantum (VQE and exact solution) approaches.

• Noise and real quantum device also tested but at a smaller scale.
RESULTS

• Conventional tracking as benchmark shows the performance that can be realistically achieved. Room for improvement for other tracking methods (preliminary results shown).

*GNN performance limited by training data size.
SUMMARY AND NEXT STEPS

• Tracking challenge in LUXE presented.

• Study the use of a hybrid quantum-classical algorithm in track reconstruction along with conventional tracking method as well as GNN-based tracking.

• A first implementation of track reconstruction in LUXE using quantum devices is in place.
  • Preliminary study shows performance similar to traditional algorithms, however limited by the size of the device.

• Next:
  • Study the performance in more extreme environments, take into account the QC noise and explore regions where QC could outperform the traditional methods.
VQE

- VQE ansatz: TwoLocal with $R_Y$, and circular CNOT entangler.

- Optimiser: Constrained Optimization by Linear Approximation (COBYLA).
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QUANTUM DEVICE

- Comparison of real quantum hardware and ideal noise-free simulation.
- 2 tracks, 5 triplets, 5 qubits.
- Correct triplet identified.