Automatic Differentiation in C++ and CUDA using Clad

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Motivation

In mathematics and computer algebra, **automatic differentiation** (AD) is defined as a set of techniques used for numerically evaluating the derivative of a function specified by a computer program.

**Automatic differentiation** is an alternative technique to Symbolic differentiation or Numerical differentiation (the method of finite differences) and **powers gradient-based optimisation** algorithms used in applications such as Deep Learning, Robotics, High Energy Physics, etc.

The aim of Clad is to provide automatic differentiation for C/C++ which works without code modification.

Deep Learning use-case: Gradient Descent = gradient of the cost function with respect to the neural network parameters.
Differentiable Programming

Goal: quantify the sensibility of the program and its outputs with respect to its parameters

Differentiable Programming includes:
- neural networks
- programs using automatic differentiation
- gradient based optimisation to approximate a loss function

![computation graph]

weights, biases ← Δloss

output → loss

Auto Diff

Neural ODEs

Neural TMs

VAEs

Neural Networks

Ioana Ifrim - GPU Acceleration of Automatic Differentiation in C++ with Clad - 20th International Workshop on Advanced Computing and Analysis Techniques in Physics Research - Nov 2021
**Differentiation**

**Numerical Differentiation**

- precision errors
- high computational complexity
- higher order problem (formula approximated by missing higher order terms)

\[
\frac{\partial f(x)}{\partial x_i} \approx \frac{f(x + h e_i) - f(x)}{h}
\]

**Symbolic Differentiation**

- can produce exponentially large symbolic expressions which take correspondingly long times to evaluate
- may require transcribing result back into code
- works on single mathematical expressions (no control flow)

\[
\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)
\]

\[
\frac{d}{dx} (f(x) g(x)) = \left(\frac{d}{dx} f(x)\right) g(x) + f(x) \left(\frac{d}{dx} g(x)\right)
\]

**Automatic Differentiation**

- automatically generate a C++ program to compute the derivative of a given function
- the procedure involves applying the chain rule of differential calculus throughout the semantics of the original program

```cpp
double relu3(double x) {
    double result;
    if (x > 0)
        result = pow(x, 3);
    else
        result = 0;
    return result;
}

double diffe_relu3(double x) {
    return autodiff<relu3>(x);
}
```

```cpp
double relu3(double x) {
    double result;
    if (x > 0)
        result = pow(x, 3);
    else
        result = 0;
    return result;
}

double diffe_relu3(double x) {
    return autodiff<relu3>(x);
}
```
AD Approaches

Implementation approaches in AD can be classified based on the amount of work done at compile time. Thus, we can identify several approaches: Domain Specific Languages (DSL), Tracing / Taping and Source Transformation.

By keeping all the intricate knowledge of the original source code, source transformation approaches enable optimisation.
Clad: An approach to source transformation AD

Clad uses the source transformation approach by statically analysing the original code to produce a gradient function in the source code language. Clad:

- Mitigates the difficulties related to custom C++ parsers
- Has full access to the Clang compiler’s internals; it means that Clad is able to follow the high-level semantics of algorithms and can perform domain-specific optimisations
- It can automatically generate code (re-targeting C++) on accelerator hardware with appropriate scheduling
- Has a direct connection to compiler diagnostics engine and thus can produce precise and expressive diagnostics positioned at desired source locations

```cpp
#include "clad/Differentiator/Differentiator.h"
#include <iostream>

double f(double x, double y) { return x * y; }

int main() {
    auto f_dx = clad::differentiate(f, "x");

    // derivative of 'f' - (x, y) = (3, 4)
    std::cout << f_dx.execute(3, 4) << std::endl;
    // prints: 4

    f_dx.dump(); // prints:
    //
    // double f_darg0(double x, double y) {
    //     double _d_x = 1;
    //     double _d_y = 0;
    //     return _d_x * y + x * _d_y;
    // }
    //
    //
    * /
}
```

Clad Example
Clad.AD Plugin for Clang

Clad is a compiler plugin extending Clang able to produce derivatives in both forward and reverse mode:

- Requires no code modification for computing derivatives of existing codebase
- Features both reverse mode AD (backpropagation) and forward mode AD
- Computes derivatives of functions, member functions, functors and lambda expressions
- Supports a large subset of C++ including if statements, for, while loops
- Provides direct functions for the computation of Hessian and Jacobian matrices
- Supports array differentiation, that is, it can differentiate either with respect to whole arrays or particular indices of the array
- Features numerical differentiation support, to be used where automatic differentiation is not feasible

Clang Compilation Pipeline. Clad

Clad is a Clang Plugin transforming the AST of the supported languages : C++, CUDA, C, ObjC

Clad can:
- Produce the AST and pipe it to the backend
- Decompile that AST into code - which is code one can compile with any other compilation pipeline (gcc/ msvc/ etc) and use it by plugging it into one’s library

```cpp
double f(double x) {
    return x * x;
}
```

```cpp
double f_darg0(double x) {
    double _d_x = 1;
    return _d_x * x + x * _d_x;
}
```
__device__ __host__ double gauss(double* x, double* p, double sigma, int dim) {
    double t = 0;
    for (int i = 0; i < dim; i++)
        t += (x[i] - p[i]) * (x[i] - p[i]);
    t = -t / (2 * sigma * sigma);
    return std::pow(2 * M_PI, -dim / 2.0) * std::pow(sigma, -0.5) * std::exp(t);
}

void gauss_grad(double* x, double* p, double sigma, int dim, clad::array_ref<double> _d_x, clad::array_ref<double> _d_p, clad::array_ref<double> _d_sigma, clad::array_ref<double> _d_dim) __attribute__((device)) __attribute__((host)) {
    double _d_t = 0;
    unsigned long _t2;
    int _d_i = 0;
    clad::tape<double> _t3 = {};
    clad::tape<int> _t4 = {};
    for (; _t2; _t2–) {
        double _r_d0 = _d_t;
        _d_t += _r_d0;
        double _r0 = _r_d0 * clad::pop(_t3);
        _d_x[clad::pop(_t4)] += _r0;
        _d_p[clad::pop(_t5)] += _r0;
        double _r1 = clad::pop(_t6) * _r_d0;
        _d_x[clad::pop(_t7)] += _r1;
        _d_p[clad::pop(_t8)] += _r1;
        _d_t -= _r_d0;
    }
}

Clad can compute the gradient of host/ device functions

CUDA computation kernels can now call Clad defined derivatives

Currently working on:

- enabling automatic offloading of gradient computations to GPU
- differentiating CUDA kernels
The demo shows cling usage of clad as a plugin to produce a derivative on the fly and send it to a CUDA kernel for execution.
Clad as a Service

AD Tutorial - CLAD & Jupyter Notebook

xeus-cling provides a Jupyter kernel for C++ with the help of the C++ interpreter cling and the native implementation of the Jupyter protocol xeus.

Within the xeus-cling framework, Clad can enable automatic differentiation (AD) such that users can automatically generate C++ code for their computation of derivatives of their functions.

```cpp
In [ ]: #include "clad/Differentiator/Differentiator.h"
2 #include <iostream>
```

Forward Mode AD

For a function $f$ of several inputs and single (scalar) output, forward mode AD can be used to compute (or, in case of Clad, create a function) computing a directional derivative of $f$ with respect to a single specified input variable. Moreover, the generated derivative function has the same signature as the original function $f$, however its return value is the value of the derivative.

```cpp
In []: double fn(double x, double y) {
2     return x*x + y*y;
3 }
```

```cpp
In [ ]: auto fn_dx = clad::differentiate(fn, "x");
```

```cpp
In [ ]: fn_dx.execute(5)
```

Reverse Mode AD

Usage of CLAD within the Jupyter Notebook with the help of “xeus-cling” (a Jupyter kernel for C++ based on the C++ interpreter cling)
ROOT is a data analysis software package used to process data in the field of high-energy physics.

Clad has replaced numerical gradient calculations for formula based functions.

The Clad gradient is then used to compute the gradient of the objective function ($\chi^2$ or negative log-likelihood function) when fitting:

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{Y_i - f(x_i, p)}{\sigma_i^2} \right)^2$$

Thus, ROOT fitting class computes $\nabla_p (\chi^2)$ from $\nabla_p (f(x, p))$ obtained using Clad.

**Clad Hessian Mode in ROOT**
(GSoC 2021 - Baidyanath Kundu)

Comparison of fitting time using Clad VS Numerical Diff of objective function - fitting sum of gaussians

* current implementation still requires one numerical gradient call for second derivatives (when seeding) - higher speedups will be possible when introducing second derivatives computation using Clad.
Summary

• Clad uses the source transformation approach by statically analysing the original code to produce a gradient function in the source code language

• Clad can produce the AST and pipe it to the backend as well as decompile that AST into code. Moreover, one can compile the produced code with any other preferred compilation pipeline (gcc/ msvc/etc), then plug it in one’s library and use it

• Continuous effort is put into expanding the support subset of C++, such as support for differentiating continue and break statements

• The new CUDA support means generated Clad derivatives are now supported for computations on CUDA kernels thus allowing for further optimisation

• The performance results in ROOT show good improvement, however work is ongoing on a set of general benchmarks

• Currently the scheduling procedure requires a certain degree of user input to make it suitable for a hybrid CPU/GPU setup. Our current aim is to fully automate this last step for complete CUDA integration, where the full toolchain process needs to be formalised with both scheduling optimisation and global memory constraints in mind
Thank you!
Backup - Clad Features Showcase
The independent parameter can be specified either using the parameter name or the parameter index; `d_fn_1.execute` returns the computed derivative.

If no parameter is specified, then the function is differentiated w.r.t all the parameters.
Clad Features Showcase

**Hessian**

```cpp
#include "clad/Differentiator/Differentiator.h"
#include <iostream>

double kinetic_energy(double mass, double velocity) {
    return mass * velocity * velocity * 0.5;
}

int main() {
    // Can manually specify independent arguments
    auto hessian = clad::hessian(kinetic_energy, "mass, velocity");

    // Creates an empty matrix to store the Hessian in
    // 2 independent variables require 4 elements (2^2=4)
    double matrix[4];

    // Substitutes these values into the Hessian function
    // pipes the result into the matrix variable.
    hessian.execute(10, 2, matrix);

    std::cout << "Hessian matrix:
";  
    for (int i=0; i<2; ++i) {
        for (int j=0; j<2; ++j) {
            std::cout << matrix[i*2 + j] << " ";
        }
        std::cout << "\n";
    }
}
```

Both support differentiating w.r.t multiple parameters. Moreover, in both cases, the array which will store the computed Hessian or Jacobian matrix should be passed as the last argument to the call to CladFunction::execute.

**Jacobian**

```cpp
#include "clad/Differentiator/Differentiator.h"
#include <iostream>

void fn(double i, double j, double *res) {
    res[0] = i*i;
    res[1] = j*j;
    res[2] = i*j;
}

int main() {
    auto d_fn = clad::jacobian(fn);
    double res[3] = {0, 0, 0};

    // Creates a matrix to store the Jacobian in
    // It will store in this case 6 derivatives
    double matrix[6] = {0, 0, 0, 0, 0, 0};

    // Substitutes these values into the Jacobian function
    // pipes the result into the derivatives variable.
    d_fn.execute(3, 5, res, matrix);

    std::cout << "Jacobian matrix:
";  
    for (int i=0; i<3; ++i) {
        for (int j=0; j<2; ++j) {
            std::cout << matrix[i*2 + j] << " ";
        }
        std::cout << "\n";
    }
}
```
Newly Supported C++ Constructs

Functors

- functor objects are stateful
- can be used to create configurable algorithms
- calls to functor objects are often inlined by compilers - better performance

```cpp
#include "clad/Differentiator/Differentiator.h"

// A class type with user-defined call operator
class Equation {
  double m_x, m_y;

public:
  Equation(double x, double y) : m_x(x), m_y(y) {}
  double operator()(double i, double j) {
    return m_x*i*j + m_y*i*j;
  }
  void setX(double x) {
    m_x = x;
  }
};

Equation E(3,5);

// differentiate `E` wrt parameter `i`
// `E` is saved in the `CladFunction` object `d_E`
auto d_E = clad::differentiate(E, "i");

// differentiate `E` wrt parameter `i`
// `E` is saved in the `CladFunction` object `d_E_ptr`
auto d_E_ptr = clad::differentiate(&E, "i");
```

Differentiating functor objects in Clad
(GSoC 2021- Parth Arora)
Newly Supported C++ Constructs

Lambda Expressions

- defining an anonymous function object (a *closure*) at the location where it's invoked or passed as an argument to a function

```cpp
#include "clad/Differentiator/Differentiator.h"

auto momentum = [](double mass, double velocity) {
    return mass * velocity;
};
```

```cpp
//both ways are equivalent
auto d_momentum = clad::differentiate(&momentum, "velocity");
auto d_momentumRef = clad::differentiate(momentum, "velocity");

//compute derivatives wrt 'velocity' when (mass, velocity) = (5,7)
std::cout<<d_momentum.execute(5, 7)<< "\n";

auto d_momentumGrad = clad::gradient(&momentum);
double d_mass=0, d_velocity=0;

//compute derivatives wrt 'mass' and 'velocity'
//given (mass, velocity) = (5,7)
d_momentumGrad.execute(5, 7, &d_mass, &d_velocity);
std::cout<<d_mass<< "\n";"