A factorisation-aware matrix element emulator

[arXiv:2107.06625]

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ACAT 2021

November 29 2021

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- High energy collider experiments are becoming increasingly more precise, meaning theoretical predictions need to improve as well.
- Successfully emulating matrix elements will provide a fast and accurate alternative to more traditional matrix element providers.

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- when gluons are *collinear* to the quark or anti-quark, $s_{qg} \to 0$.
- when gluons are *soft*, $E_g \to 0$.
- Singularities make it difficult to model over the phase-space effectively.

Method

Dipole factorisation formula

Catani and Seymour introduced [1] universal dipoles that smoothly interpolates between the soft and collinear limits.

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Dipole factorisation formula

Catani and Seymour introduced [1] universal dipoles that smoothly interpolates between the soft and collinear limits. We use these dipoles to factorise out the IR singularity structure from matrix elements

$$|\mathcal{M}_{n+1}|^2 \to |\mathcal{M}_n|^2 \otimes \mathbf{V}_{ij,k}\,, \tag{1}$$

where all divergences are isolated in the process independent factor $\mathbf{V}_{ij,k}$.

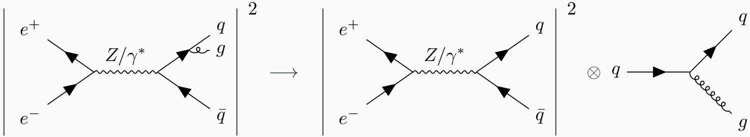

Figure 1: Schematic of dipole factorisation.

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Ansatz

We use the dipole factorisation formula to build an ansatz of the colour and helicity summed $n+1$-body matrix element

$$\langle|\mathcal{M}_{n+1}|^2\rangle = \sum_{\{ijk\}} C_{ijk} D_{ij,k} \,, \quad \text{where} \quad D_{ij,k} = \frac{\langle V_{ij,k}\rangle}{s_{ij}} \,. \tag{2}$$

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The sum over $\{ijk\}$ denotes the sum over relevant permutations of external final state particles.

Neural network architecture

The basis of our neural network model is a dense neural network with eight hidden layers.

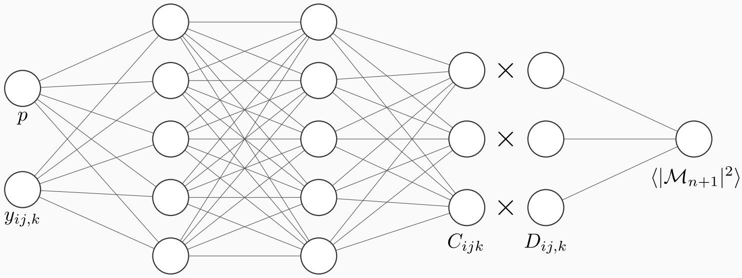

Figure 2: Schematic diagram of our neural network architecture.

Inputs of neural network

Direct inputs to network

- Phase-space points: $p^{(i)} = [E^{(i)}, p_x^{(i)}, p_y^{(i)}, p_z^{(i)}]$

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Global phase-space cuts are applied according to $y_{ij} \geq y_{\text{cut}}$, where we explore three values of $y_{\text{cut}} = [0.01, 0.001, 0.0001]$.

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- Spin-averaged dipoles $D_{ij,k} = \frac{\langle V_{ij,k} \rangle}{s_{ij}}$
- Averaging over spin means that we have lost information about the spin-correlation in $g \to gg$ splitting.
  - Introduce $S_{ij} \sin(2\phi_{ij}) + C_{ij} \cos(2\phi_{ij})$ in the ansatz for pairs of gluons.
  - ϕ_{ij} is the azimuthal angle of the decay particles in the plane perpendicular to the parent particle momentum.

Outputs of neural network

The output of the neural network is the colour and helicity summed matrix element

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During training, the predictions are compared against the target value of the matrix element which we scale according to

$$y = \text{arcsinh}\left(\frac{\langle |\mathcal{M}_{n+1}|^2 \rangle}{S_{\text{pred}}} \right). \tag{3}$$

Custom loss function

We use the mean squared error loss function along with a regularisation term

$$
\begin{aligned}
L &= L_{\text{MSE}} + L_{\text{pen}} \\
L &= \frac{1}{N}\sum_{n=1}^{N}(y - f(\mathrm{x};\theta))^2 + J\sum_i \frac{D_i^{-2}}{\sum_j D_j^{-2}}|C_i D_i|.
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We promote the network to learn about the universal factorisation property in matrix elements, hence becoming *factorisation-aware*.

Results

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We present results for our matrix element emulator:

1. Compare results obtained with our method to those in a previous work by Aylett-Bullock and Badger [2].

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4. Compare MC statistical error to NN accuracy.

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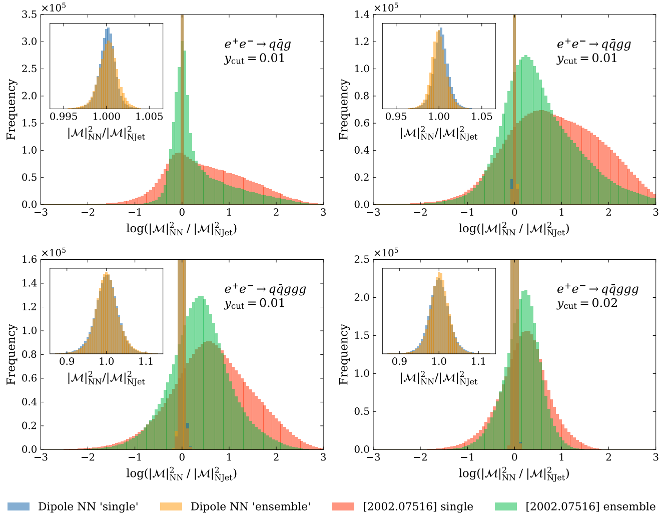
Comparison with n3jet[2]



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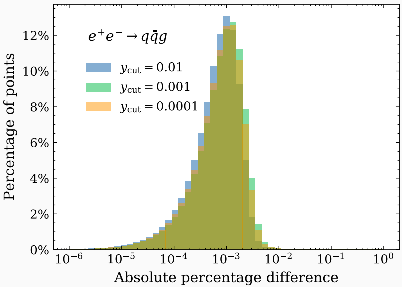
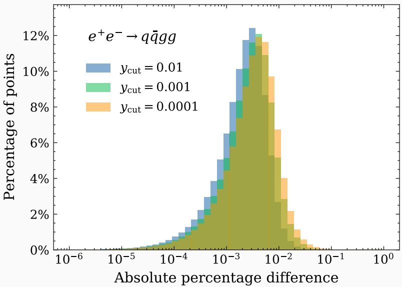
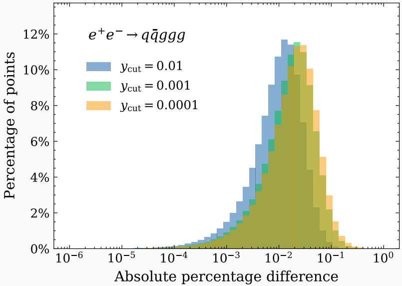
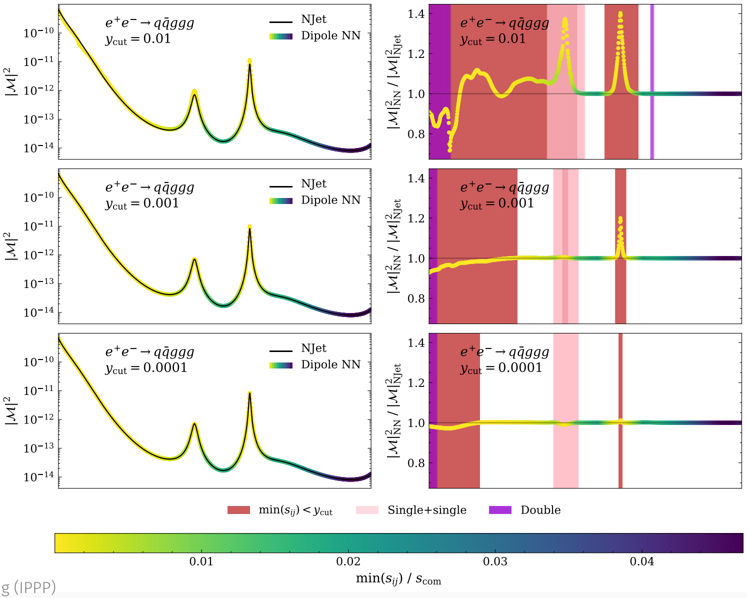
Error distributions



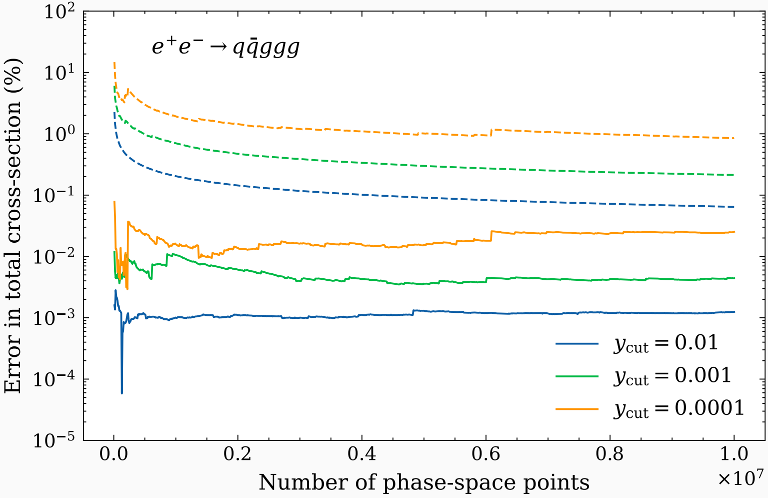




Random phase-space trajectory for 5 jet case



Comparison of errors



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`github.com/htruong0/fame`