Two-loop QCD amplitudes for Higgs production with a bottom quark pair

(based on hep-ph/2107.14733)

Jakub Kryś
Durham University, Università di Torino

With: Simon Badger, Heribertus Bayu Hartanto and Simone Zoia

ACAT 2021
Outline

1. Introduction
   - Background
   - Finite fields

2. Results
   - $pp \rightarrow b \bar{b} H$

3. Conclusion
Introduction

Background

Finite fields

Results

\[ pp \rightarrow \bar{b} \bar{b}H \]

Conclusion
Scattering amplitudes cannot be calculated exactly $\rightarrow$ need for high accuracy \textbf{loop corrections}.

\[ d\sigma = d\sigma^{\text{LO}} + \alpha_s d\sigma^{\text{NLO}} + \alpha_s^2 d\sigma^{\text{NNLO}} + \ldots \]

\[ \text{10--30\%} \quad \text{1--10\%} \]
Typical workflow

Calculating loop corrections to scattering amplitudes has several typical steps:

1. Draw all relevant Feynman diagrams
2. Write down the integrand
3. Reduce the amplitude onto a set of master integrals
4. Evaluate the result at a chosen phase-space point
Complexity

Feynman diagrams

LARGE

intermediate expressions

C-number
Complexity

- Complexity increases with **loop order** and **multiplicity**.
- Current QCD frontier: $2 \rightarrow 3$ scattering at NNLO.

- Massless case: results for all relevant Feynman integrals available.
- One external mass: results for all planar + **some** non-planar integrals now available.

<table>
<thead>
<tr>
<th>Type</th>
<th>Date</th>
<th>Authors</th>
<th>Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>one-mass, planar</td>
<td>Nov '15</td>
<td>Papadopoulos, Tommasini, Wever</td>
<td>(one penta-box, MPLs)</td>
</tr>
<tr>
<td></td>
<td>May '20</td>
<td>Abreu, Ita, Moriello, Page, Tschernow, Zeng</td>
<td>(DEs+numerical sols)</td>
</tr>
<tr>
<td></td>
<td>Sep '20</td>
<td>Canko, Papadopoulos, Syrrakos</td>
<td>(MPLs)</td>
</tr>
<tr>
<td></td>
<td>Dec '20</td>
<td>Syrrakos</td>
<td>(1L pentagon, MPLs)</td>
</tr>
<tr>
<td>one-mass, non-planar</td>
<td>Oct '19</td>
<td>Papadopoulos, Wever</td>
<td>(one hexa-box, MPLs)</td>
</tr>
<tr>
<td></td>
<td>July '21</td>
<td>Abreu, Page, Ita, Tschernow</td>
<td>(hexa-box, DEs+numerical sols)</td>
</tr>
</tbody>
</table>
Recent work

- $pp \to W/H + b\bar{b}$ at 2L (leading colour, massless $b$ quarks)

  $W^\pm$

  [Badger, Hartanto, Zoia, Feb ’21]

  [Badger, Hartanto, Kryš, Zoia, July ’21]

- $W(\rightarrow \ell\ell') + 4$-partons at 2L (leading colour, massless quarks)

  [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, Oct ’21]
Finite fields

- To avoid analytic complexity in intermediate steps, use numerical evaluations over finite fields
- We work with rational numbers modulo a large prime number:
  \[ q = \frac{a}{b} \longrightarrow q \mod p \equiv (a \times (b^{-1} \mod p)) \mod p \]
  \[ \frac{3}{7} \equiv 2 \mod 11 \]
- One can reconstruct the analytic result from its many numerical evaluations
- FiniteFlow [Peraro, ’19]
1. Introduction
   Background
   Finite fields

2. Results
   \( pp \rightarrow b\overline{b}H \)

3. Conclusion
$pp \to b\bar{b}H$

- Three channels are relevant:
  - $0 \to \bar{b}(p_1) + b(p_2) + g(p_3) + g(p_4) + H(p_5)$
  - $0 \to \bar{b}(p_1) + b(p_2) + \bar{q}(p_3) + q(p_4) + H(p_5)$
  - $0 \to \bar{b}(p_1) + b(p_2) + \bar{b}(p_3) + b(p_4) + H(p_5)$

- After colour-decomposition, amplitude can be written as:

$$A = \sum_{T \in \text{topologies}} \int d^4k_1 d^4k_2 \frac{\sum_i c_i(\{p\})\text{mon}_i(\{k, p\})}{\prod_{j \in T} D_j(\{k, p\})}$$

- Coefficients $c_i(p(x))$ are given a rational parametrisation using momentum twistors $x$

(Begin finite field sampling)
The amplitude is mapped onto scalar integrals within 15 maximal topologies.
$pp \rightarrow b\bar{b}H$
\[ pp \rightarrow b\bar{b}H \]

- The amplitude is mapped onto scalar integrals within 15 maximal topologies
- Scalar integrals are IBP-reduced onto a master integral basis
  
  \[ A = \sum_i d_i(\epsilon, p(x)) \times M\!L_i(\epsilon, p) \]

- We work with MIs that satisfy:
  
  \[ d\mathcal{M}I = \epsilon \left( \sum_{i=1}^{58} a_i \times d\log w_i \right) \mathcal{M}I \]

  [Henn, '13], [Abreu et al., '20]
• Subtract the poles to get the finite remainder:

\[ F^{(L)} = \sum_{i} r_i (p(x)) m_i(f), \]

where \( m_i(f) \) are monomials formed from elements of the finite remainder function basis

• Reconstruct the coefficients, now free of \( \epsilon \)

(End finite field sampling)
Evaluating special functions

- The finite remainder function basis is written in terms of Chen’s iterated integrals [Chen, ’77]
- The iterated integrals expose cancellations in finite remainders
- They satisfy differential equations as well, order-by-order up to $O(\epsilon^4)$
- Solve them numerically in DiffExp using the method of generalised series expansions [Moriello, ’19], [Hidding, ’20]
The finite remainders of the $gg$ channel interfered with tree-level amplitudes, evaluated at a univariate phase-space slice.

$$p_1 = \frac{y_1 \sqrt{s}}{2} (1, 1, 0, 0) \quad p_2 = \frac{y_2 \sqrt{s}}{2} (1, \cos \theta, -\sin \theta \sin \phi, -\sin \theta \cos \phi)$$

$$p_3 = \frac{\sqrt{s}}{2} (-1, 0, 0, -1) \quad p_4 = \frac{\sqrt{s}}{2} (-1, 0, 0, 1)$$
Outline

1. Introduction
   Background
   Finite fields

2. Results
   \[ pp \rightarrow b\bar{b}H \]

3. Conclusion
Conclusion

- Calculated two-loop QCD amplitudes for $pp \rightarrow b\bar{b}H$
- Developed a Mathematica + FORM + FiniteFlow routine that can be adapted to other processes as needed
- Implemented several tools to overcome the complexity
- Integrals for non-planar topologies needed for $pp \rightarrow H + 2j$ and for $pp \rightarrow V + 2j$ beyond leading colour
Details of reconstruction

Linear relations between rational coefficients:

- Coefficients $r_i$ are not independent
- Find relations between them and choose the independent ones based on the lowest polynomial degree

\[ F^{(L)} = \sum_i r_i(p(x)) m_i(f) \]
Details of reconstruction

1. Linear relations between rational coefficients: \[ F^{(L)} = \sum_i r_i (p(x)) m_i(f) \]
   - Coefficients \( r_i \) are not independent
   - Find relations between them and choose the independent ones based on the lowest polynomial degree

2. Factor matching:
   - Aid the reconstruction by providing an ansatz of factors related to the letters
     \[ \{ \langle ij \rangle, [ij], \langle i | p_5 | j \rangle, s_{ij}, s_{ij} - s_{kl}, s_{i5} - p_5^2, p_5^2, tr_5, \Delta_1, \Delta_2, \]  
     \[ s_{15}(s_{13} + s_{34}) - p_5^2 s_{34}, s_{25}(s_{24} + s_{34}) - p_5^2 s_{34} \} \]
   - All denominator factors guessed + some of the numerator
Details of reconstruction

1. Linear relations between rational coefficients: \( F^{(L)} = \sum_i r_i (p(x)) m_i(f) \)
   - Coefficients \( r_i \) are not independent
   - Find relations between them and choose the independent ones based on the lowest polynomial degree

2. Factor matching:
   - Aid the reconstruction by providing an ansatz of factors related to the letters
     \[
     \{ \langle ij \rangle, [ij], \langle i|p_5|j \rangle, s_{ij}, s_{ij} - s_{kl}, s_{i5} - p_5^2, p_5^2, tr_5, \Delta_1, \Delta_2, \\
     s_{15}(s_{13} + s_{34}) - p_5^2 s_{34}, s_{25}(s_{24} + s_{34}) - p_5^2 s_{34} \}
     \]
   - All denominator factors guessed + some of the numerator

3. Univariate partial fractioning
   - Having guessed the denominator, construct a partial-fractioned ansatz
### Details of reconstruction

<table>
<thead>
<tr>
<th>$\bar{b}bggH$ configurations</th>
<th>$r_i(x)$</th>
<th>independent $r_i(x)$</th>
<th>partial fraction in $x_5$</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^{(2),1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+++$</td>
<td>$63/57$</td>
<td>$52/46$</td>
<td>$20/6$</td>
<td>$3361$</td>
</tr>
<tr>
<td>$+++$</td>
<td>$135/134$</td>
<td>$119/120$</td>
<td>$28/22$</td>
<td>$24901$</td>
</tr>
<tr>
<td>$++-$</td>
<td>$105/111$</td>
<td>$105/111$</td>
<td>$22/12$</td>
<td>$4797$</td>
</tr>
<tr>
<td>$F^{(2),n_f}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+++$</td>
<td>$45/41$</td>
<td>$45/41$</td>
<td>$16/6$</td>
<td>$1381$</td>
</tr>
<tr>
<td>$+++$</td>
<td>$94/95$</td>
<td>$94/95$</td>
<td>$17/6$</td>
<td>$1853$</td>
</tr>
<tr>
<td>$++-$</td>
<td>$89/95$</td>
<td>$62/69$</td>
<td>$18/3$</td>
<td>$2492$</td>
</tr>
<tr>
<td>$F^{(2),n_f^2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+++$</td>
<td>$12/8$</td>
<td>$9/7$</td>
<td>$0/0$</td>
<td>$3$</td>
</tr>
<tr>
<td>$+++$</td>
<td>$11/16$</td>
<td>$11/16$</td>
<td>$3/0$</td>
<td>$22$</td>
</tr>
<tr>
<td>$++-$</td>
<td>$12/20$</td>
<td>$8/16$</td>
<td>$8/0$</td>
<td>$242$</td>
</tr>
</tbody>
</table>

Maximum numerator/denominator polynomial degrees and the sample points needed for the reconstruction of the finite remainder coefficients.

$$F^{(L)} = \sum_{i} r_i(p(x)) m_i(f)$$
Chen’s iterated integrals

Defined as:

\[
[w_{i_1}, \ldots, w_{i_n}]_{s_0} (s) = \int_0^1 \frac{d}{dt} \log w_{i_n} (\gamma(t)) [w_{i_1}, \ldots, w_{i_{n-1}}]_{s_0} (\gamma(t)), \quad [ ]_{s_0} := 1
\]

The number of integration kernels \( w_i \) is known as \textit{transcendental weight}.

They have several advantages:

1. Automate implement functional relations
2. Singularities or branch points only where one of the letters vanishes or diverges
3. Simplify the finite remainder function basis through analytic cancellations
Our workflow

1. Feynman diagrams
2. Colour decomposition
3. Collect in topologies

- QGRAF
- Mathematica/FORM

- Expansion of MIs onto special function basis
- IBP reduction
- Integrand reduction onto maximal topologies

finite fields

- Pole subtraction
- $\epsilon \rightarrow 0$
- Finite remainder

$d = 4 - 2\epsilon$