Monte carlo challenges for non pert QED (particle physics in intense background fields)

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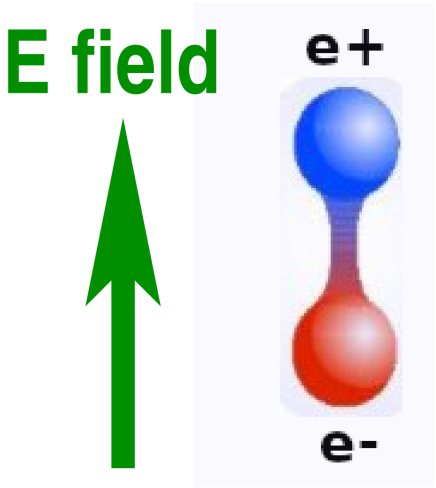
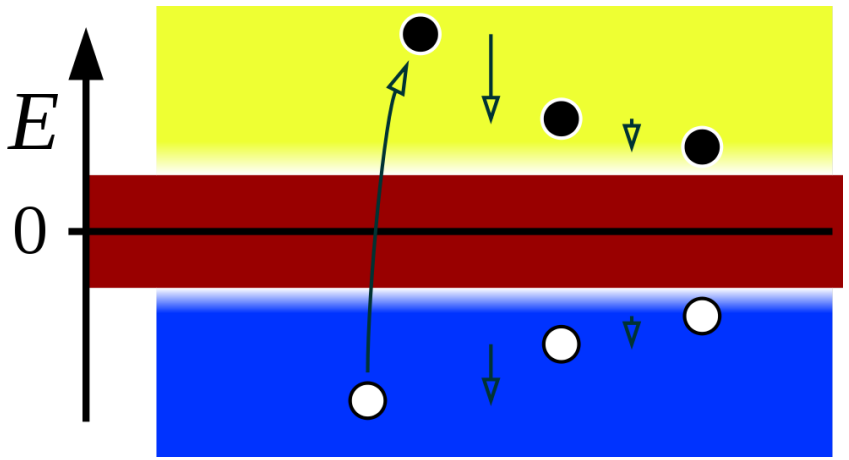
The strong field scale: Schwinger pair creation



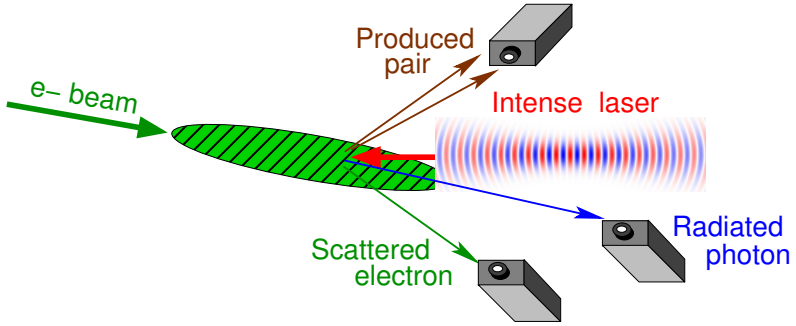


" Schwinger pairs created if virtual pairs separated by Compton wavelength $\lambda = \hbar/m_e c$ within the virtual pair lifetime $\Delta t = \hbar/m_e c^2$. "
" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

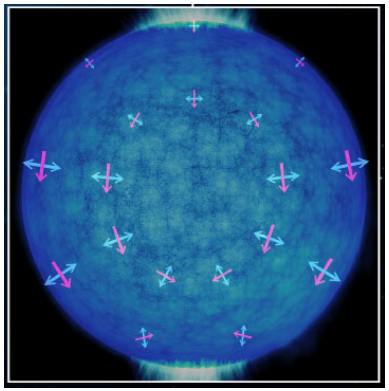
- The Schwinger critical field ($E_{\text{cr}} = m_e^2 c^3/e\hbar = 1.32 \times 10^{18}$ V/m)
- What novel experimental effects can we expect as $E \to E_{\text{cr}}$
- How do we incorporate these vacuum changes into our theories (SQED/SQFT)?

Where might we expect non perturbative effects?



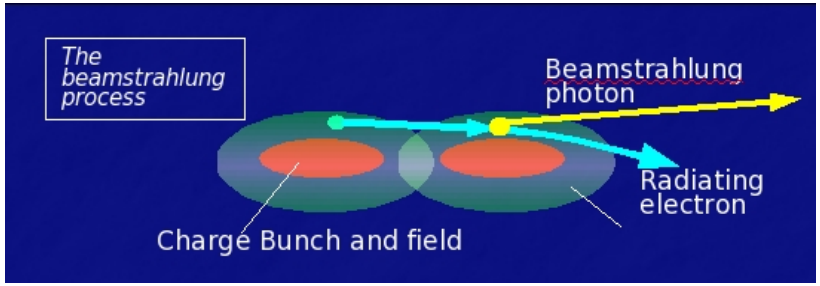
Electron/laser interactions

E_{cr} in the e-beam rest frame



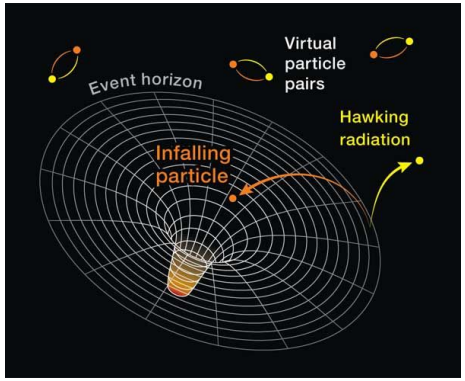
Magnetar B_{cr} near surface

Vacuum birefringence



q+q- particle collider

E_{cr} in each bunch's rest frame



Hawking radiation

G_{cr} equivalent critical gravitational field

Furry Picture: a non perturbative, semi classical QFT

Furry Pic Lagrangian, background field A^{ext}

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}} A\!\!\!/\,\psi^{\text{FP}}$$

$\hookrightarrow$ **Bound Dirac equation**

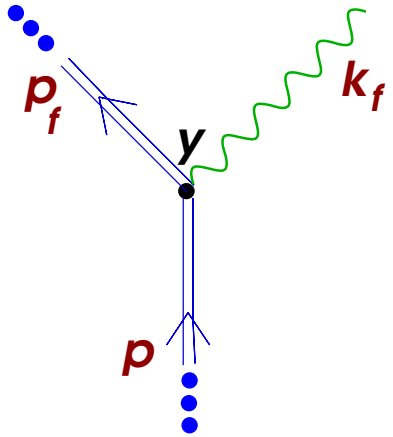
$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

$\hookrightarrow$ **Dressed wave functions (Volkov 1935)**

$$\psi^{\text{FP}} = \mathbf{E}_p\, e^{-ip\cdot x}\, u_p$$

$$\mathbf{E}_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}} k\!\!\!/ + i2e(A^e\cdot p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Dressed Feynman vertex



- Exact solutions (double straight lines in Feynman diagrams) for plane waves, Coulomb fields, gravitational fields, non collinear fields
- Background field contributes momentum, so odd vertex diagrams permitted
- Effective coupling constant, $f(\alpha, \chi)$

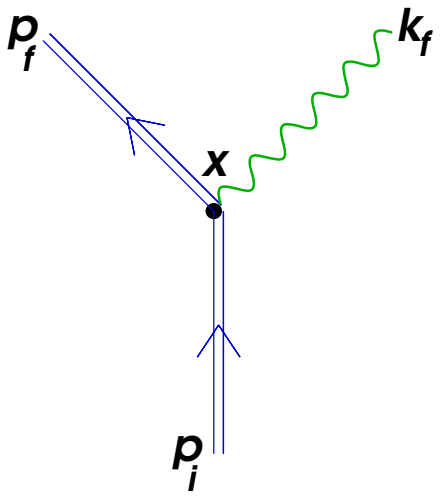
Alternative Volkov solutions

[*PRD 94, 073002 (2016)*]

- Alternative form using commutation properties of γ_μ & $(\not{p} - m)u_{\text{pr}} = 0$ in terms of the canonical momentum, $\Pi_{\text{px}\mu}$

$$\Psi^{\text{V}}_{\text{prx}} = n_p \left[\not{\Pi}_{px} + m\right] \frac{\not{k}}{2k\cdot p}\, u_{\text{pr}}\, e^{-i\Delta_{\text{px}}}$$

$$\Pi_{\text{px}\mu} \equiv p_\mu - A^e_{\text{x}\mu} + \frac{2A^e_{\text{x}}\cdot p - A^{e\,2}_{\text{x}}}{2k\cdot p} k_\mu$$



- The Π vectors are conserved and have nice properties

$$\left(i\partial_\mu - A^e_{\text{x}\mu}\right)\Delta_{\text{px}} = \Pi_{\text{px}\mu}, \quad \Pi^2_{\text{px}} = p^2 = m^2$$

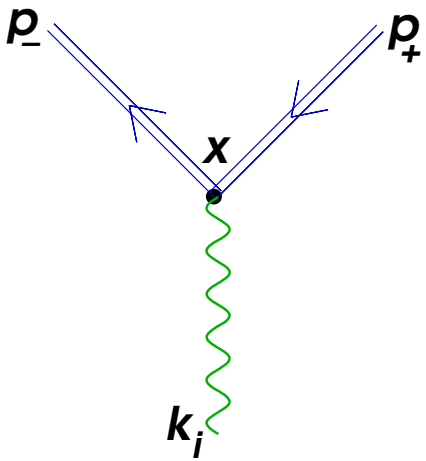
- Conservation of energy-momentum in dressed vertex

$$\int_{-\infty}^{\infty} \text{d}x_\mu \left(\Pi_{\text{fx}} + k_{\text{f}} - \Pi_{\text{ix}}\right)_\mu e^{i\left(\Delta_{\text{fx}} + k_f\cdot x - \Delta_{\text{ix}}\right)} = 0$$

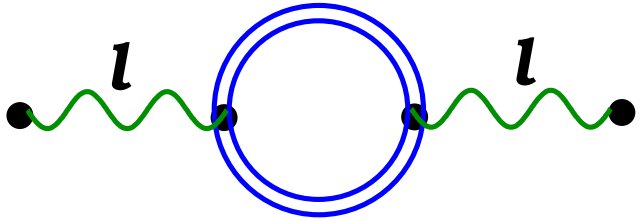
- Simple scalar products

$$\Pi_{\text{ix}}\cdot\Pi_{\text{fx}} = m^2, \quad k\cdot\Pi_{\text{px}} = k\cdot p$$

Some non perturbative processes

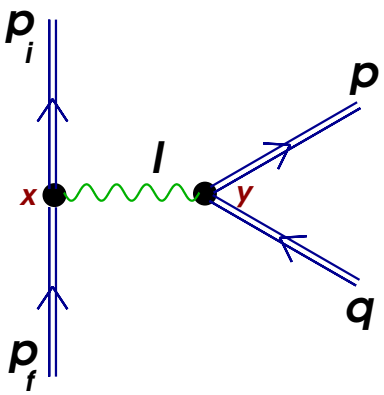


One photon pair prod^{n} (photon decay)

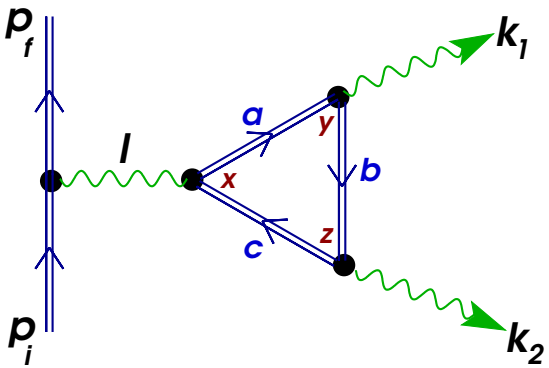


propagator $= \dfrac{1}{l^2 + iM(l^2, \chi)}$

Complex mass (resonant propagators)

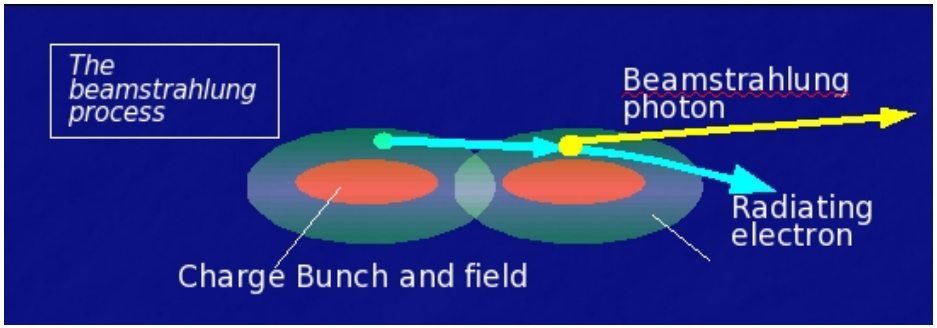


Trident process (resonant production)



Photon splitting (vacuum birefringence)

Strong fields at the collider Interaction Point



- SQED χ depends on collider bunch parameters and the pinch effect
- **All collider processes are SQFT processes**: backgrounds **and** signal
- Coherent e+e- pair production, depolarisation, WW pair production

[A. Hartin, IJMPA **33**, 1830011 (2018)]

Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
χ_{av}	0.00015	0.001	0.24	4.9

Field strength parameter, χ

Schwinger critical field, E_c

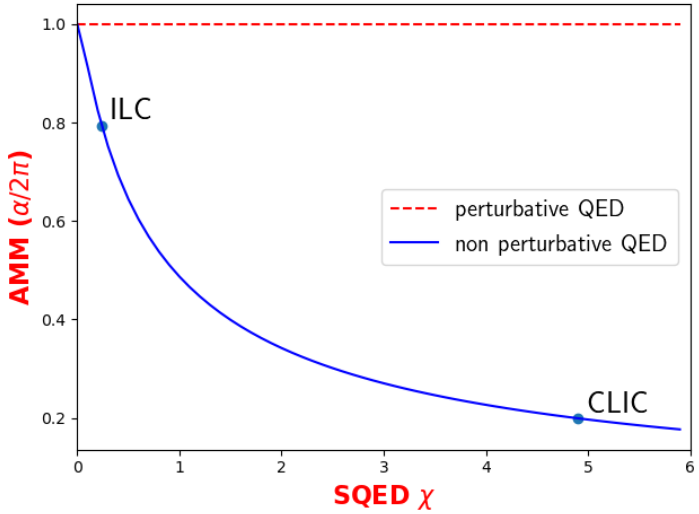
$$\chi = E_{\text{rest}}/E_{\text{cr}}, \quad E_c = 1.3 \times 10^{18}\ \text{V/m}$$

precision spin physics/IP depolarisation needs:

e- anomalous magnetic moment

perturbative: $\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}$

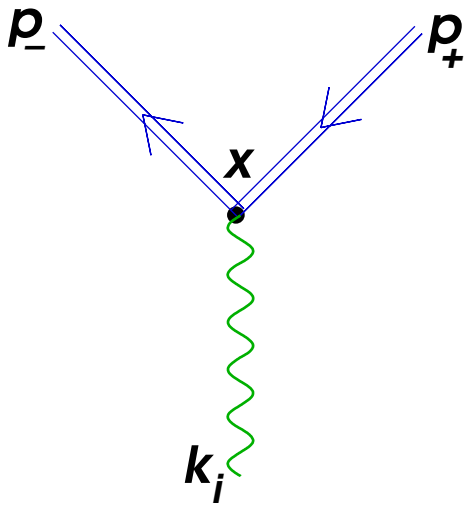
$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^\infty \frac{2\pi\, dx}{(1+x)^3}\left(\frac{x}{\chi}\right)^{\frac{1}{3}} \mathrm{Gi}\left(\frac{x}{\chi}\right)^{\frac{1}{3}}$$



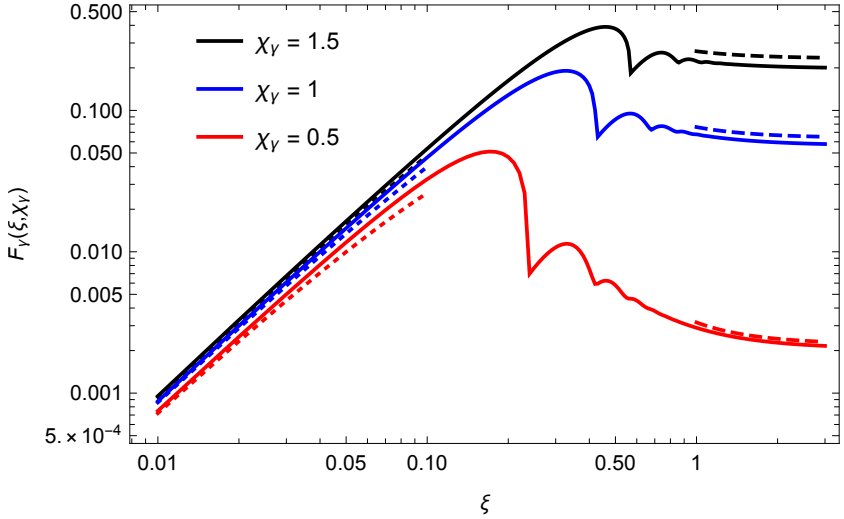
OPPP and Schwinger critical field measurement

[*PRD 99, 036008 (2019)*]

One photon pair production (OPPP)



OPPP rate, non perturbative regime



$$\Gamma_{\text{OPPP}} = \frac{\alpha m^2}{2\omega_{\text{i}}} \sum_{s>s_0}^{\infty} \int_1^{v_s} \frac{dv}{v\sqrt{v(v-1)}} \left[\mathrm{J}_s^2 + \frac{\xi^2}{2}(2v-1)\left(\mathrm{J}_{s+1}^2 + \mathrm{J}_{s\text{-}1}^2 - 2\,\mathrm{J}_s^2\right)\right] \propto \frac{\alpha m^2}{2\omega_{\text{i}}} \frac{E}{E_c} \exp\left[-\frac{8mE_c}{3\omega_i E}\right]$$

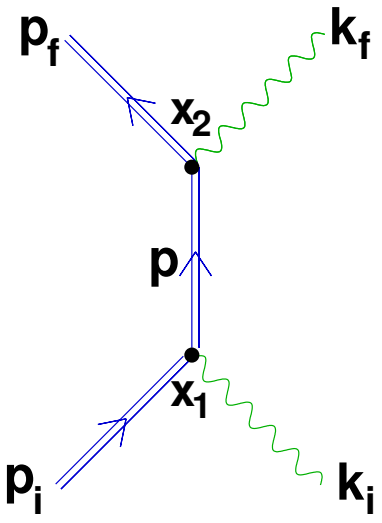
OPPP Rate at constant χ reaches non perturbative asymptote for $\xi \geq 1$

Note the similarity with the rate of Schwinger pair creation (from the vacuum):

$$\Gamma_{\text{Schwinger}} = \frac{m^4}{(2\pi)^3} \frac{E^2}{E_c^2} \exp\left[-\pi \frac{E_c}{E}\right]$$

An experiment to measure the non perturbative OPPP process also informs us about Schwinger pair creation

Challenge: Dressed vertices & higher order traces



SQED Feynman diagrams are easy...

- Double fermion lines are Volkov-type solutions
- Volkov E_p functions can be grouped around the vertex
- only need one new Feynman picture element - the dressed vertex

$$\gamma^{\mu}_{\text{fpx}_2} = \int d^4x_2\, \bar{E}_{p_f}(x_2)\gamma^{\mu} E_{p_i}(x_2)\, e^{i(p_f+k_f-p)\cdot x_2}$$

... but the calculations, not

- 2nd order trace has 4 dressed vertices. How many terms?

$$\sum |M_{\text{fi}}|^2 \propto \text{Tr}\left[(\not{p}_{\text{f}}+m)\gamma^{\mu}_{\text{fpx}_2}(\not{p}+m)\gamma^{\nu}_{\text{ipx}_2}(\not{p}_{\text{i}}+m)\bar{\gamma}_{\text{fpx}_1\nu}(\not{p}+m)\bar{\gamma}_{\text{fpx}_1\mu}\right]$$

- 4 channels x 4 γ x (2x2) E_{p} x (4x2) spin sum = 512 terms
- Higher order terms become intractable
- Aesthetic: Analytic expressions should be simple!
- Need schema for Furry pic trace simplification to any order

Fierz transformation for Volkov spinors

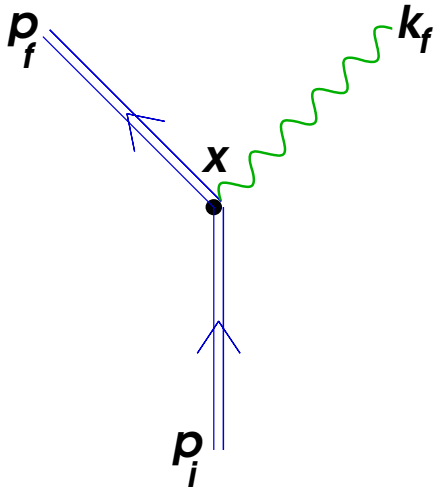
[*PRD 94, 073002 (2016)*]

New bound Dirac solutions. Canonical momentum Π_{px}. Define the Volkov spinor V_{px}

$$\Psi^{\mathsf{V}}_{\mathsf{prx}} = n_p \left[\not{\Pi}_{\mathsf{px}} + m\right] \frac{\not{k}}{2k\cdot p}\, u_{\mathsf{pr}}\, e^{-i\Delta_{\mathsf{px}}} \equiv V_{\mathsf{pxr}}\, e^{-i\Delta_{\mathsf{px}}}$$

Extend Fierz transformations to Volkov spinors

$$\sum_{\mathsf{rsr's'}} [\bar{V}_{\mathsf{frx}}\, \Gamma^j_{\mathsf{J}}\, V_{\mathsf{isx}}][\bar{V}_{\mathsf{is'x'}}\, \Gamma_{\mathsf{J}\ \mathsf{j}} V_{\mathsf{fr'x'}}] = \sum_{\mathsf{rsr's'K}} F_{\mathsf{JK}} \left[\bar{V}_{\mathsf{frx}}\, \gamma^\mu \Gamma^k_{\mathsf{K}} \gamma_\mu\, V_{\mathsf{fr'x'}}\right] \left[\bar{V}_{\mathsf{is'x'}}\, \Gamma_{\mathsf{k}\,\mathsf{K}}\, V_{\mathsf{isx}}\right]$$



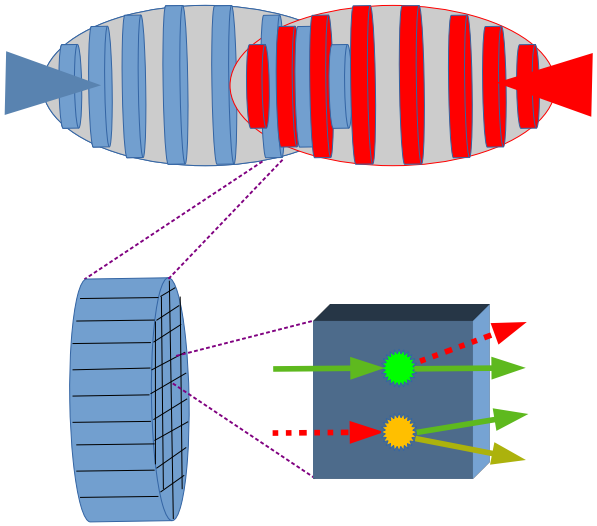
example: amplitude for HICS

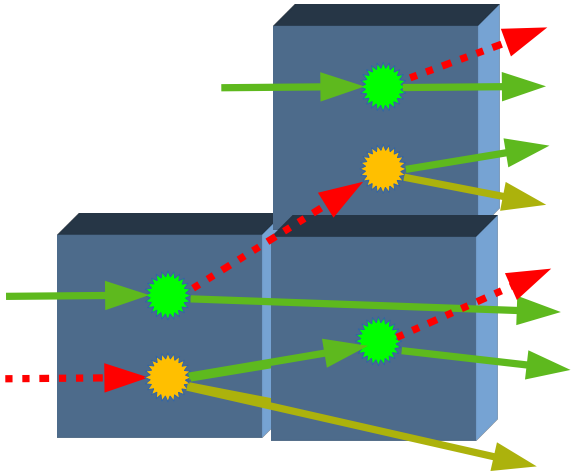
$$M_{\mathsf{fi}} = -ie \int d^4x\, \bar{\psi}^{\mathsf{V}}_{\mathsf{frx}}\, \not{A}_{\mathsf{fx}}\, \psi^{\mathsf{V}}_{\mathsf{isx}}$$

squared amplitude splits into two traces

$$|M_{\mathsf{fi}}|^2 \propto \sum_{\mathsf{K}} F_{\mathsf{SK}}\ \mathrm{Tr}\left[\bar{V}_{\mathsf{fx}}\, \gamma^\mu \Gamma^k_{\mathsf{K}} \gamma_\mu\, V_{\mathsf{fx'}}\right] \mathrm{Tr}\left[\bar{V}_{\mathsf{ix'}}\, \Gamma_{\mathsf{k}\,\mathsf{K}}\, V_{\mathsf{ix}}\right]$$

Challenge: Charge bunch interactions





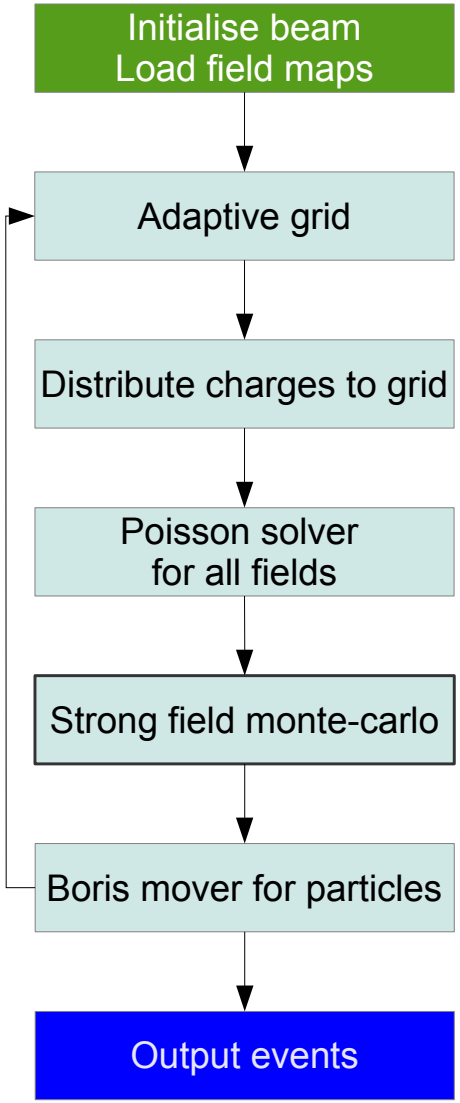
Discretise the interaction

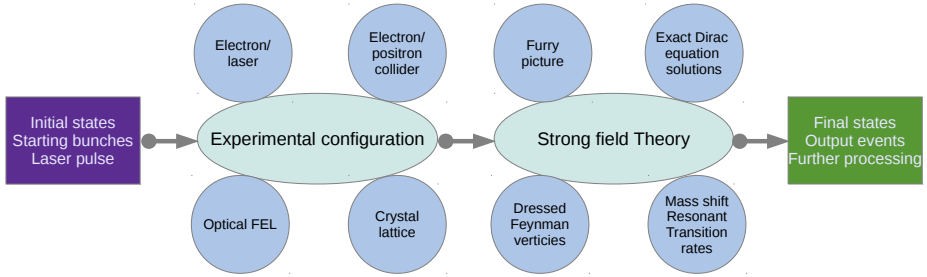
- transform to head on collision
- Divide into overlapping slices
- Divide slices into mc voxels
- Calculate SQED parameters (ξ, χ) in each voxel
- Monte carlo for each SQED process (rarest first)

Macro vs Micro

- Real particles enter/leave voxel
- Higher order processes are tested **within** each voxel
- Distinguish between analytic rate within one voxel, and the effective global rate from sampling across whole bunch/pulse
- final particle ensemble built up over successive voxel monte carlo + time step through the whole collision (typically 5σ separation)

IPstrong - SQED monte carlo PIC simulation code



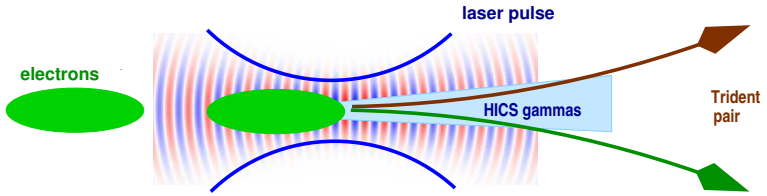


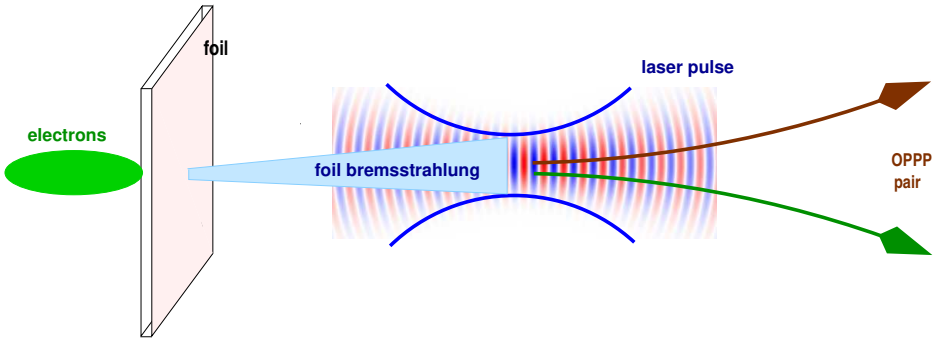
- Furry picture SQED interactions, via monte carlo, embedded in a 3D PIC electromagnetic solver
- e-/laser, e+e-, higher order interactions, optical FEL, crystal lattices
- Internally generated or externally loaded bunches
- Fortran 2008 with openMPI (extend for GPU)
- 3D poisson solver (PSPFFT, MPI)
- Currently contains - 1st order SQED, 2 step processes, circularly polarised lasers
- More information:
  https://anthonyhartin.wixsite.com/physics/software
  Int.J.Mod.Phys.A 33 (2018) 13, 1830011

BACKUP

Relativistic electron/intense laser interactions

- Near head on collision between high energy electrons and focussed laser
- Field strength of laser relativistically boosted, parameters ξ, χ
- Several complementary experiments, spread of SQED parameters
- Different experimental configs allow several SQED processes
- Real experiment has gaussian pulse with varying intensity (ξ)
- Monte carlo shows a good match for low ξ but varies thereafter
- Experimental effects need to be "unpacked" - angular spread, gaussian pulse, crossing angle
- Need to include full rate to compare with exponential



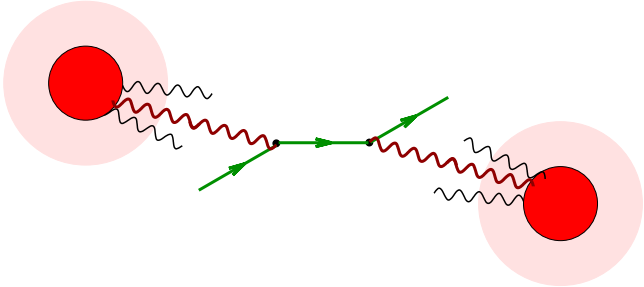


$$\chi = \xi \frac{k \cdot p}{m^2}$$ **recoil parameter**

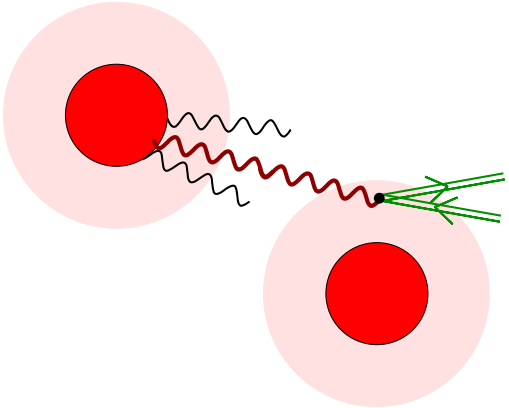
$$\xi (= a_0) = \frac{e|\vec{A}|}{m}$$ **intensity parameter**

Experiment	$\lambda(nm)$	E_{laser} (J)	focus (μm^2)	pulse (fs)	E_{e^-}(GeV)	ξ	χ
SLAC E144	527/1053	2	50	1880	46.6	0.66	2.7
LUXE Phase0	800	0.35	100	35	17.5	1.54	0.29
LUXE Phase1	800	7	100	35	17.5	6.9	1.29
FACET II	800	0.7	64	35	10	2.3	0.29
ELI-NP	1053	2.2	100	22	0.750	6.4	0.04
AWAKE	800	3	64	20	50	7.45	4.0

Prospective: SQED in UP heavy ion collisions



heavy ion UPC: 2 equivalent photons



SQED heavy ion UPC: 1 equivalent photon + intense field

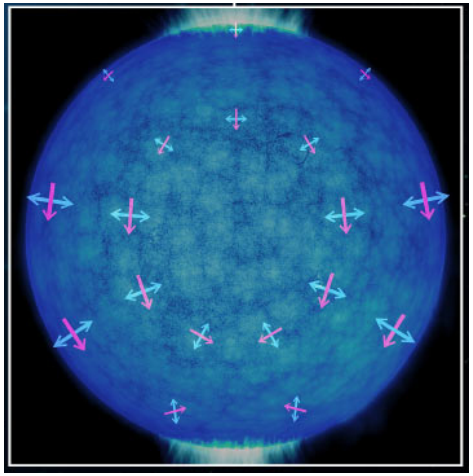
The "usual" EPA approach

- Approaching ions considered equivalent photons
- Search for low activity collisions, no QCD
- Gamma-gamma physics with Coulomb corrections
- Recent ATLAS Pb-Pb photon scattering search, hep-ex:1904.03536
- **CURIOUS:** unexplained resonances in heavy ion positron spectra at GSI, Darmstadt

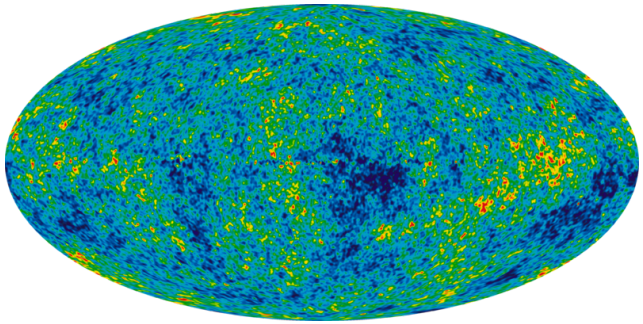
An SQED approach (new possible studies)

- Extremely strong fields operating over very short time scale, use SQED
- Equivalent gammas from one ion pass through the field of oncoming ion
- Ion field adjusts screening charge. photon has an effective mass
- SQED assisted Schwinger production
- SQED trident pair production, has SQED resonance in effective propagator (resonant positron spectrum)

Strong field Astrophysical/Cosmological arena



Magnetar Vacuum birefringence



CMB signatures

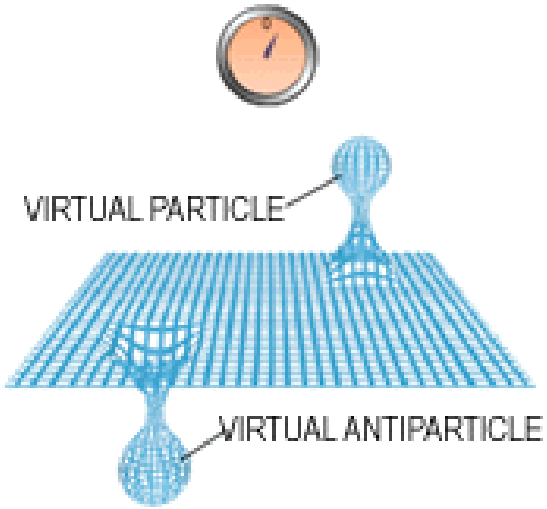
Vacuum birefringence

- Strong field effects in observation of polarised light from Magnetar
- Possible evidence of strong field vacuum birefringence (10^{13} G)
- Polarisation should be correlated with magnetic field relative to Earth
- Reported by arxiv:1610.08323

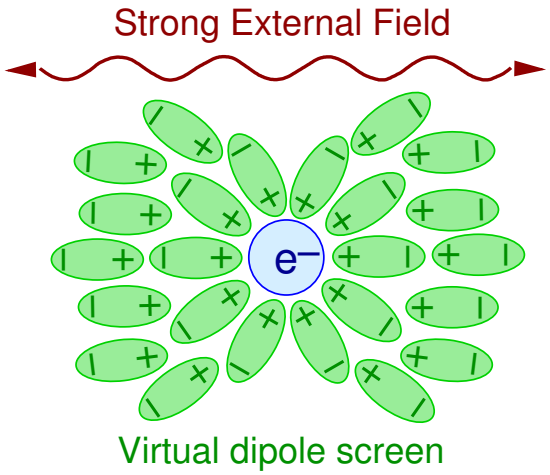
Cosmological Schwinger effect

- Hawking radiation is Schwinger pair creation in strong gravitational field
- Non perturbative QED $\leftrightarrow$ QED in curved space-times (Hollands, Wald arxiv:1401.2026)
- Strong field provided by gravity in early universe (Martin arxiv:0704.3540)
- Two point correlation function linked to CMB fluctuations

The polarisable quantum vacuum



Heisenberg uncertainty



Bare/Dressed charge

- Casimir force implies virtual particles have real physical effects
- Strong background field couples to charged virtual particles and polarises the vacuum
- The screening charge is rearranged, leading to possibly large effects even at modest field strengths
- At Schwinger critical field strength, vacuum decays into real pairs
- New phenomenology results - odd vertex diagrams, resonant propagators, different manifestations of IR divergences
- Polarisable vacuum applicable to strong gravity as well as strong EM fields
- Need to investigate experimental signatures within reach using today's and upcoming technology