Monte carlo challenges for non pert QED (particle physics in intense background fields)

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The strong field scale: Schwinger pair creation

"Schwinger pairs created if virtual pairs separated by Compton wavelength \( \lambda = \hbar/m_e c \) within the virtual pair lifetime \( \Delta t = \hbar/m_e c^2 \)."

"In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles."

Greiner and Muller, QED of Strong Fields

- The Schwinger critical field \( E_{cr} = m_e^2 c^3/e\hbar = 1.32 \times 10^{18} \text{ V/m} \)

- What novel experimental effects can we expect as \( E \to E_{cr} \)

- How do we incorporate these vacuum changes into our theories (SQED/SQFT)?
Where might we expect non perturbative effects?

**Electron/laser interactions**

$E_{cr}$ in the e-beam rest frame

**Magnetar** $B_{cr}$ near surface

Vacuum birefringence

**q+q- particle collider**

$E_{cr}$ in each bunch’s rest frame

**Hawking radiation**

$G_{cr}$ equivalent critical gravitational field
Furry Picture: a non perturbative, semi classical QFT

**Furry Pic Lagrangian, background field** $A^{\text{ext}}$

\[ \mathcal{L}^{\text{Int}}_{\text{QED}} = \bar{\psi} (i \partial - m) \psi - \frac{1}{4} (F_{\mu \nu})^2 - e \bar{\psi} (A^{\text{ext}} + A) \psi \]

\[ \mathcal{L}^{\text{FP}}_{\text{QED}} = \bar{\psi}^{\text{FP}} (i \partial - e A^{\text{ext}} - m) \psi^{\text{FP}} - \frac{1}{4} (F_{\mu \nu})^2 - e \bar{\psi}^{\text{FP}} A \psi^{\text{FP}} \]

\[ \leftrightarrow \text{Bound Dirac equation} \]

\[ (i \partial - e A^{\text{ext}} - m) \psi^{\text{FP}} = 0 \]

\[ \leftrightarrow \text{Dressed wave functions (Volkov 1935)} \]

\[ \psi^{\text{FP}} = e^{-i p \cdot x} u_p \]

\[ E_p = \exp \left[ - \frac{1}{2(k \cdot p)} (e A^{\text{ext}} + i 2 e (A^e \cdot p) - i e^2 A^{\text{ext}}^2) \right] \]

- **Dressed Feynman vertex**

- Exact solutions (double straight lines in Feynman diagrams) for plane waves, Coulomb fields, gravitational fields, non collinear fields

- Background field contributes momentum, so odd vertex diagrams permitted

- Effective coupling constant, $f(\alpha, \chi)$
Alternative Volkov solutions

[ PRD 94, 073002 (2016) ]

- Alternative form using commutation properties of $\gamma_\mu$ & $(\phi - m)u_{pr} = 0$
in terms of the canonical momentum, $\Pi_{px_\mu}$

\[ \Psi_{prx}^V = n_p \left[ \Pi_{px} + m \right] \frac{k}{2k \cdot p} u_{pr} e^{-i\Delta_{px}} \]

\[ \Pi_{px_\mu} \equiv p_\mu - A^e_{x_\mu} + \frac{2A^e_x \cdot p - A^e_x}{2k \cdot p} k_\mu \]

- The $\Pi$ vectors are conserved and have nice properties

\[ (i\partial_{\mu} - A^e_{x_\mu}) \Delta_{px} = \Pi_{px_\mu}, \quad \Pi_{px}^2 = p^2 = m^2 \]

- Conservation of energy-momentum in dressed vertex

\[ \int_{-\infty}^{\infty} dx_\mu (\Pi_{fx} + k_f - \Pi_{ix})_{\mu} e^{i(\Delta_{fx} + k_f \cdot x - \Delta_{ix})} = 0 \]

- Simple scalar products

\[ \Pi_{ix} \cdot \Pi_{fx} = m^2, \quad k \cdot \Pi_{px} = k \cdot p \]
Some non perturbative processes

One photon pair production (photon decay)

Trident process (resonant production)

Complex mass (resonant propagators)

Photon splitting (vacuum birefringence)

\[
\text{propagator} = \frac{1}{l^2 + iM(l^2, \chi)}
\]
Strong fields at the collider Interaction Point

- SQED $\chi$ depends on collider bunch parameters and the pinch effect
- All collider processes are SQFT processes: backgrounds and signal
- Coherent $e^+e^-$ pair production, depolarisation, WW pair production

$[A. Hartin, IJMPA 33, 1830011 (2018)]$

Field strength parameter, $\chi$

Schwinger critical field, $E_c$

$$\chi = \frac{E_{\text{rest}}}{E_c}, \quad E_c = 1.3 \times 10^{18} \text{ V/m}$$

precision spin physics/IP depolarisation needs:

e- anomalous magnetic moment

perturbative:

$$\frac{\Delta \mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi dx}{(1 + x)^3} \left( \frac{x}{\chi} \right)^{\frac{1}{3}} G_i \left( \frac{x}{\chi} \right)^{\frac{1}{3}}$$

Machine | LEP2 | SLC | ILC | CLIC
--- | --- | --- | --- | ---
$E (\text{GeV})$ | 94.5 | 46.6 | 500 | 1500
$N (\times 10^{10})$ | 334 | 4 | 2 | 0.37
$\sigma_x, \sigma_y (\mu\text{m})$ | 190, 3 | 2.1, 0.9 | 0.49, 0.002 | 0.045, 0.001
$\sigma_z \text{ (mm)}$ | 20 | 1.1 | 0.15 | 0.044
$\chi_{av}$ | 0.00015 | 0.001 | 0.24 | 4.9

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One photon pair production (OPPP)

\[ \Gamma_{\text{OPPP}} = \frac{\alpha m^2}{2\omega_i} \sum_{s > s_0} \int_{v_s}^{\infty} \frac{dv}{v \sqrt{v(v-1)}} \left[ J^2_s + \frac{\xi^2}{2} (2v-1) (J^2_{s+1} + J^2_{s-1} - 2 J^2_s) \right] \propto \frac{\alpha m^2}{2\omega_i} \frac{E}{E_c} \exp \left[ -\frac{8mE_c}{3\omega_i E} \right] \]

OPPP Rate at constant \( \chi \) reaches non perturbative asymptote for \( \xi \geq 1 \)

Note the similarity with the rate of Schwinger pair creation (from the vacuum):

\[ \Gamma_{\text{Schwinger}} = \frac{m^4}{(2\pi)^3} \frac{E^2}{E_c^2} \exp \left[ -\pi \frac{E_c}{E} \right] \]

An experiment to measure the non perturbative OPPP process also informs us about Schwinger pair creation
SQED Feynman diagrams are easy...
- Double fermion lines are Volkov-type solutions
- Volkov $E_p$ functions can be grouped around the vertex
- only need one new Feynman picture element - the dressed vertex

$$\gamma_{\text{ipx}_2}^\mu = \int d^4x_2 \bar{E}_{pf}(x_2)\gamma^\mu E_{pi}(x_2) e^{i(p_f+k_f-p)\cdot x_2}$$

... but the calculations, not
- 2nd order trace has 4 dressed vertices. How many terms?

$$\sum |M_{fi}|^2 \propto \text{Tr} \left[ (\phi_f+m)\gamma_{\text{ipx}_2}^\mu (\phi+i_m)\gamma_{\text{ipx}_2}^\nu (\phi + m)\bar{\gamma}_{\text{tpx}_1}^\nu (\phi + m)\bar{\gamma}_{\text{tpx}_1}^\mu \right]$$

- 4 channels x 4 $\gamma$ x (2x2) $E_p$ x (4x2) spin sum = 512 terms
- Higher order terms become intractable
- Aesthetic: Analytic expressions should be simple!
- Need schema for Furry pic trace simplification to any order

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Fierz transformation for Volkov spinors

New bound Dirac solutions. Canonical momentum $\Pi_{p\mathbf{x}}$. Define the Volkov spinor $V_{p\mathbf{x}}$

$$
\Psi^V_{p\mathbf{x}} = n_p \left[ \Pi_{p\mathbf{x}} + m \right] \frac{k}{2k \cdot p} u_{p\mathbf{r}} e^{-i\Delta_{p\mathbf{x}}} \equiv V_{p\mathbf{r}} e^{-i\Delta_{p\mathbf{x}}}
$$

Extend Fierz transformations to Volkov spinors

$$
\sum_{rs'r's'} \left[ \bar{V}_{fr\mathbf{x}} \Gamma_j V_{is\mathbf{x}} \right] \left[ \bar{V}_{is'\mathbf{x}'} \Gamma_j V_{fr'\mathbf{x}'} \right] = \sum_{rsr's'K} F_{JK} \left[ \bar{V}_{fr\mathbf{x}} \gamma^\mu \Gamma^K \gamma_\mu V_{fr'\mathbf{x}'} \right] \left[ \bar{V}_{is'\mathbf{x}'} \Gamma_K V_{is\mathbf{x}} \right]
$$

example: amplitude for HICS

$$
M_{fi} = -ie \int d^4x \bar{\psi}^V_{fr\mathbf{x}} A_{fx} \psi^V_{is\mathbf{x}}
$$

squared amplitude splits into two traces

$$
|M_{fi}|^2 \propto \sum_K F_{SK} \text{Tr} \left[ \bar{V}_{fx} \gamma^\mu \Gamma^K \gamma_\mu V_{ix'} \right] \text{Tr} \left[ \bar{V}_{ix'} \Gamma_K V_{ix} \right]
$$
**Challenge: Charge bunch interactions**

**Discretise the interaction**
- transform to head on collision
- Divide into overlapping slices
- Divide slices into mc voxels
- Calculate SQED parameters \((\xi, \chi)\) in each voxel
- Monte carlo for each SQED process (rarest first)

**Macro vs Micro**
- Real particles enter/leave voxel
- Higher order processes are tested within each voxel
- Distinguish between analytic rate within one voxel, and the effective global rate from sampling across whole bunch/pulse
- final particle ensemble built up over successive voxel monte carlo + time step through the whole collision (typically 5\(\sigma\) separation)
IPstrong - SQED monte carlo PIC simulation code

- Initialise beam
- Load field maps
- Adaptive grid
- Distribute charges to grid
- Poisson solver for all fields
- Strong field monte-carlo
- Boris mover for particles
- Output events

- Furry picture SQED interactions, via monte carlo, embedded in a 3D PIC electromagnetic solver
- e-/laser, e+e-, higher order interactions, optical FEL, crystal lattices
- Internally generated or externally loaded bunches
- Fortran 2008 with openMPI (extend for GPU)
- 3D poisson solver (PSPFFT, MPI)
- Currently contains - 1st order SQED, 2 step processes, circularly polarised lasers

More information:
https://anthonyhartin.wixsite.com/physics/software
BACKUP
Relativistic electron/intense laser interactions

- Near head on collision between high energy electrons and focussed laser
- Field strength of laser relativistically boosted, parameters $\xi, \chi$
- Several complementary experiments, spread of SQED parameters
- Different experimental configs allow several SQED processes
- Real experiment has gaussian pulse with varying intensity ($\xi$)
- Monte carlo shows a good match for low $\xi$ but varies thereafter
- Experimental effects need to be "unpacked" - angular spread, gaussian pulse, crossing angle
- Need to include full rate to compare with exponential

\[ \chi = \xi \frac{k \cdot p}{m^2} \]

\[ \xi (= a_0) = \frac{e|\vec{A}|}{m} \]

<table>
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<th>Experiment</th>
<th>$\lambda(nm)$</th>
<th>$E_{\text{laser}}$ (J)</th>
<th>focus ($\mu m^2$)</th>
<th>pulse (fs)</th>
<th>$E_e$ (GeV)</th>
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non perturbative QED
Prospective: SQED in UP heavy ion collisions

The "usual" EPA approach
- Approaching ions considered equivalent photons
- Search for low activity collisions, no QCD
- Gamma-gamma physics with Coulomb corrections
- Recent ATLAS Pb-Pb photon scattering search, hep-ex:1904.03536
- CURIOUS: unexplained resonances in heavy ion positron spectra at GSI, Darmstadt

An SQED approach (new possible studies)
- Extremely strong fields operating over very short time scale, use SQED
- Equivalent gammas from one ion pass through the field of oncoming ion
- Ion field adjusts screening charge. photon has an effective mass
- SQED assisted Schwinger production
- SQED trident pair production, has SQED resonance in effective propagator (resonant positron spectrum)
**Vacuum birefringence**

- Strong field effects in observation of polarised light from Magnetar
- Possible evidence of strong field vacuum birefringence \((10^{13} \text{ G})\)
- Polarisation should be correlated with magnetic field relative to Earth
- Reported by arxiv:1610.08323

**Cosmological Schwinger effect**

- Hawking radiation is Schwinger pair creation in strong gravitational field
- Non perturbative QED ↔ QED in curved space-times (Hollands, Wald arxiv:1401.2026)
- Strong field provided by gravity in early universe (Martin arxiv:0704.3540)
- Two point correlation function linked to CMB fluctuations
The polarisable quantum vacuum

Casimir force implies virtual particles have real physical effects

Strong background field couples to charged virtual particles and polarises the vacuum

The screening charge is rearranged, leading to possibly large effects even at modest field strengths

At Schwinger critical field strength, vacuum decays into real pairs

New phenomenology results - odd vertex diagrams, resonant propagators, different manifestations of IR divergences

Polarisable vacuum applicable to strong gravity as well as strong EM fields

Need to investigate experimental signatures within reach using today’s and upcoming technology

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