Membranes elastic degrees of freedom, a multi-loop approach.

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— Context —

Fluctuating membranes

d-dim extended objects embedded in a D-dim space subject to small quantum and/or thermal fluctuations.

Applications:

- cond-mat: graphene, silicene, phosphorene ...
- bio: living cells surfaces (phospholipid bilayers)
- hep: worldsheet, branes ...

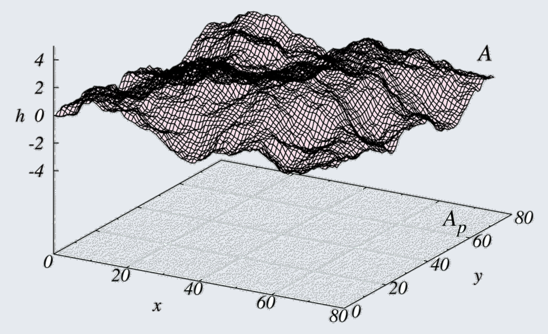


Figure: Generic fluctuating membrane

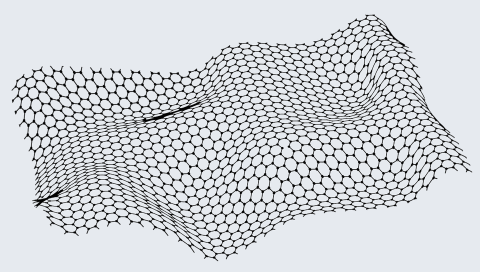


Figure: Fluctuating graphene

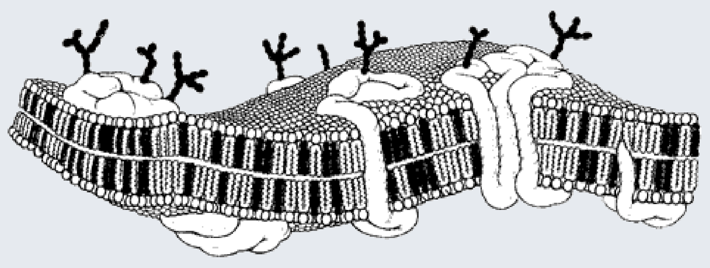
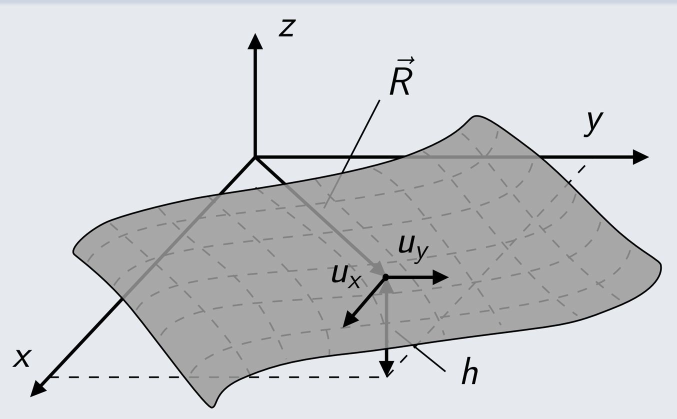


Figure: Cell bi-layered membrane

— Model —

Model parametrization in d=2



Fields parametrization:

- $h(\vec{x}) \equiv$ height displacement (flexuron)
- $\vec{u}(\vec{x}) \equiv$ longitudinal displacement (phonon)
- $\vec{R}(\vec{x}) = (\vec{x} + \vec{u}(\vec{x}), h(\vec{x})) \equiv$ coordinates with $\vec{x} = (x, y)$ and $\vec{u}(\vec{x}) = (u_x(\vec{x}), u_y(\vec{x}))$

$$S[\vec{u}, h] = \int d^2x \left[\frac{1}{2}(\Delta h)^2 + \frac{\mu}{(4\pi)^2}(u_{ij})^2 + \frac{\lambda}{2(4\pi)^2}(u_{ii})^2\right] \quad \text{with} \quad u_{ij} \approx \frac{1}{2}\left[\partial_i u_j + \partial_j u_i + \partial_i h\, \partial_j h\right]$$

$u_{ij} \equiv$ stress tensor ; fluctuations with respect to the flat configuration $\vec{R}^{(0)}(\vec{x}) = (\vec{x}, 0)$

λ, $\mu \equiv$ coupling constants $\equiv$ Lamé coefficients

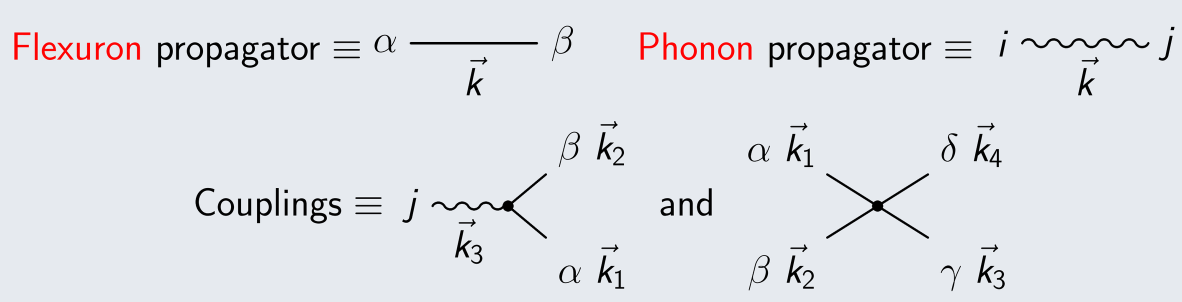
In short: massless and highly derivative scalar two-field and two-coupling theory.

What to compute?

- $\Sigma \equiv$ flexuron self-energy, $\Pi \equiv$ phonon polarization.
- $(\lambda^*, \mu^*) \equiv$ Lamé coefficients at a stable fixed point.
- $\eta \equiv$ elastic critical exponent $\equiv$ field anomalous dimension.
- $\eta = 0$ in Gaussian approx, but do corrections induce $\eta > 0$? (is there a stable phase?)

— Perturbative approach —

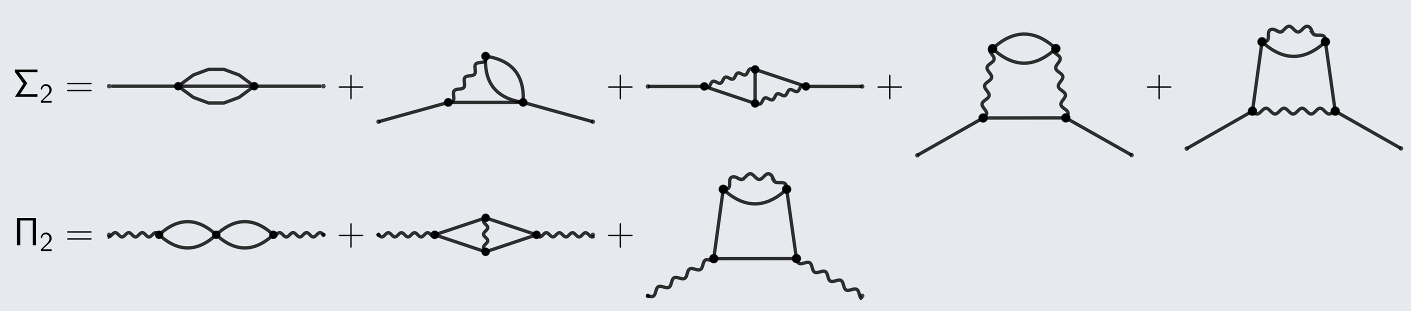
Feynman rules and diagrams



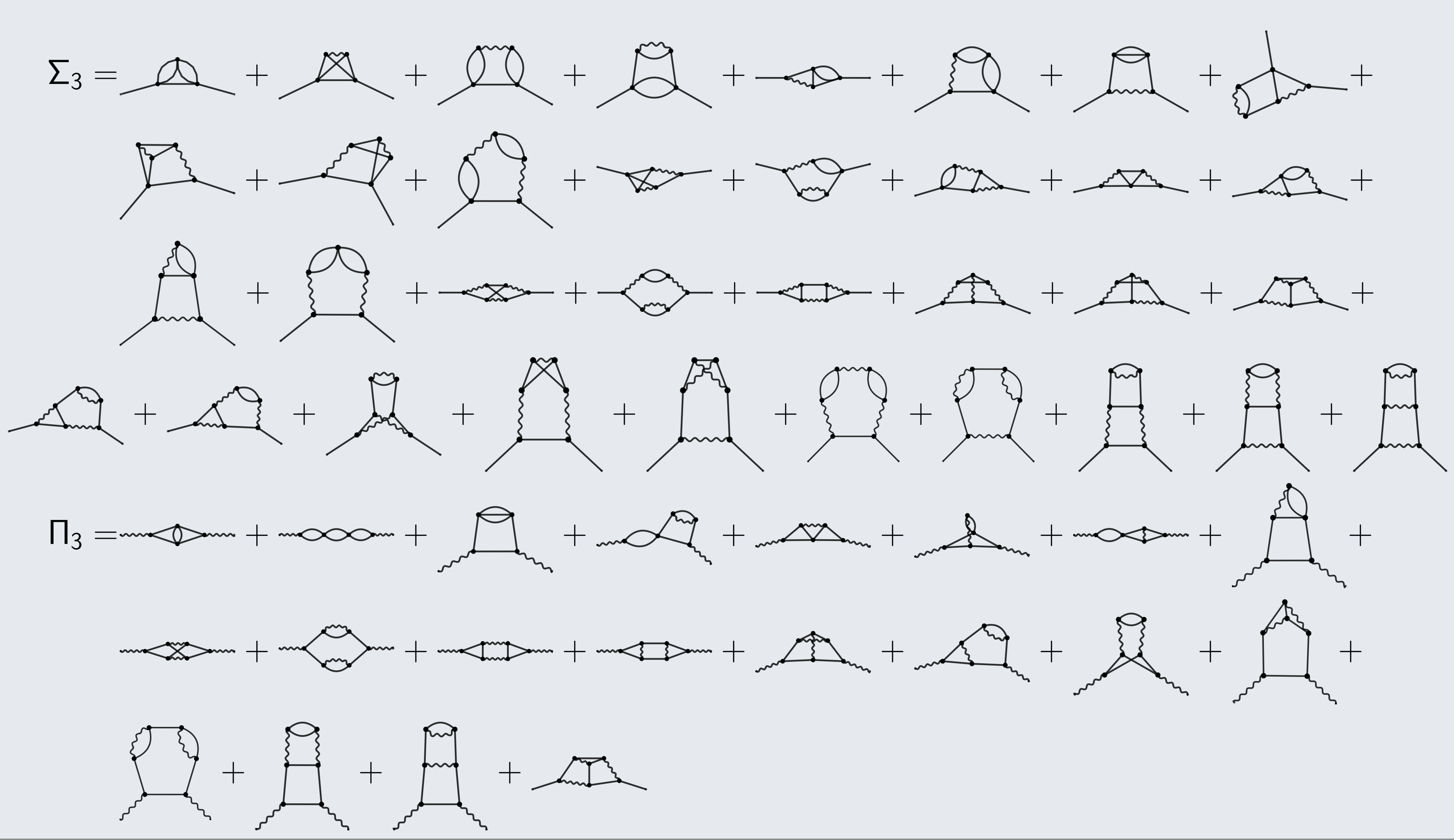
What has been done?

- One-loop (hand computations) [Aronovitz & Lubensky, '88]

- Two-loop (partially automated) [Coquand, Mouhanna, Teber, '20]



- Three-loop (highly automated) [Metayer, Mouhanna, Teber, '21] (this work)



— Technical details —

Conventions and method

Computations were carried out up to 3-loop for an arbitrary d-dim membrane embedded in a D-dim space (co dimension $d_c = D - d \geq 0$) in dim-reg ($d = 4 - 2\varepsilon$). Numerator algebra was performed with Mathematica (double tensorial structure). Integrals were computed via IBP reduction methods using Litered [Lee '13] and master integrals were known from other models. Finally renormalization was achieved using Schwinger-Dyson equations with these conventions:

$$h = Z^{1/2} h_r, \quad u = Z\, u_r, \quad \lambda = M^{2\varepsilon} Z_\lambda\, \lambda_r, \quad \mu = M^{2\varepsilon} Z_\mu\, \mu_r$$

$$\beta_\lambda \sim \frac{\partial Z_\lambda}{\partial M}, \quad \beta_\mu \sim \frac{\partial Z_\mu}{\partial M}, \quad \eta \sim \frac{\partial Z}{\partial M}$$

$$\text{finite} = (p^4 - \Sigma)Z, \quad \text{finite} = (p^2\mu - \Pi_\perp)Z^2, \quad \text{finite} = (p^2(\lambda + 2\mu) - \Pi_\parallel)Z^2 .$$

See [Metayer, Mouhanna, Teber, '21] for more details.

— Results —

Fixed points

$\exists$ points in the (λ, μ) plane where the membrane is scale invariant (fractal deformations). At these fixed points, beta functions vanish while the anomalous dimensions take universal values.

Fixed points:

- $P_1 \equiv$ Gaussian & unstable,
- $P_2 \equiv$ shearless & unstable,
- $P_3 \equiv$ vanishing bulk modulus & unstable,
- $P_4 \equiv$ non-trivial & stable.

At P_4, the couplings are (3-loop):

$$\lambda^* = -0.1018 \quad \text{and} \quad \mu^* = 0.4696$$

P_4 is stable and may govern a low-energy phase!

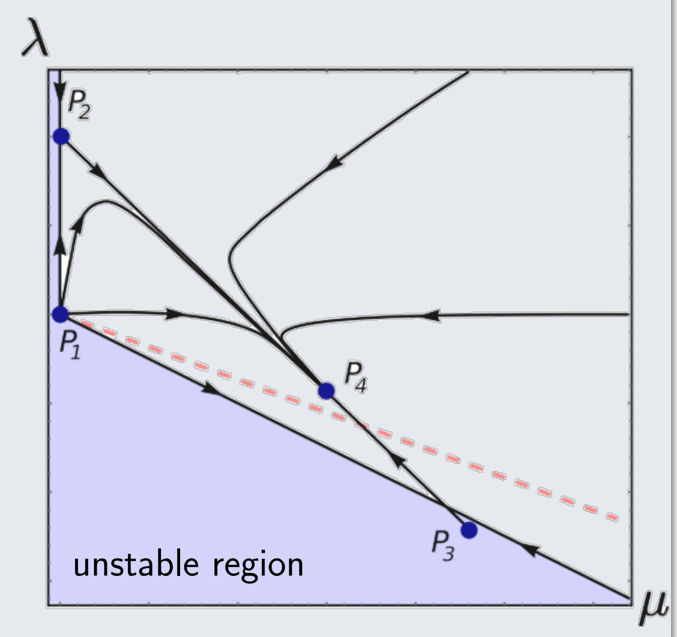


Figure: RG flow diagram

Elastic critical exponent

At the stable fixed point P_4 and in physical conditions ($\varepsilon = 1$):

$$\eta = \underbrace{\frac{24}{25}}_{1-loop} - \underbrace{\frac{144}{3125}}_{2-loop} - \underbrace{\frac{4(1286928\zeta_3 - 568241)}{146484375}}_{3-loop} \quad \text{with} \quad \zeta_3 \approx 1.202$$

$\eta = 0.8872$ up to 3-loop and $\eta > 0 \implies \exists$ stable phase!

Comparisons: η is universal $\implies$ benchmarking other computation methods:

η	Exact ($\varepsilon = 1$)	(Re-)expanded in ε
3-loop (this work)	0.887	$0.96\,\varepsilon - 0.046\,\varepsilon^2 - 0.0267\,\varepsilon^3$
SCSA	0.821	$0.96\,\varepsilon - 0.048\,\varepsilon^2 - 0.0279\,\varepsilon^3$
NPRG	0.849	$0.96\,\varepsilon - 0.037\,\varepsilon^2 - 0.0266\,\varepsilon^3$
Numerical	0.85	

SCSA $\equiv$ Self consistent screening approximation, NPRG $\equiv$ Non-perturbative RG

Overview of other quantities accessible with λ^*, μ^* and η

Quantities derived from λ and μ:

- Young modulus $\equiv Y = \dfrac{2\mu(d\lambda + 2\mu)}{(d-1)\lambda + 2\mu}$
- Poisson ratio $\equiv \nu = \dfrac{\exp_T}{\exp_L} = \dfrac{\lambda}{(d-1)\lambda + 2\mu}$
- Bulk modulus $\equiv B = \lambda + 2\mu/d$
- p-wave modulus $\equiv M = \lambda + 2\mu$
- s-wave sound velocity $\equiv c_s = \sqrt{\mu/\rho}$ ...

Crit. exponents depend only on η:

- $\mu(p) \sim \lambda(p) \sim p^{4-d-2\eta}$
- Bending/rigidity modulus $\equiv \kappa(p) \sim p^{-\eta}$
- Young modulus $\equiv Y(p) \sim p^{4-d-2\eta}$
- Roughness exponent $\equiv \psi = (4 - d - \eta)/2$
- Lower-crit dim $\equiv D_{lc} = 2 - \eta$
- $\nu_\sigma = \dfrac{1}{d - 2 + \eta}$, $\quad \delta_\sigma = \dfrac{2 - \eta}{d - 2 + \eta}$ ...

— Related models —

- **Effective flexural model:** Done at 3-loop in [Metayer, Mouhanna, Teber, '21]. Same physical results with a different approach $\implies$ reinforce results and scheme-independence.
- **Disordered model:** Adding impurities or defects in the membrane. New technical challenges (5 couplings, new replica tensorial structures, factor > 10 in computation time ...). Done at 3-loop in [Metayer, Mouhanna, to be published]. New stable fixed points, new non-perturbative fixed point? ...
- **Other related models**
  Interfaces? Fluid membranes? Elastic-electronic couplings? ...

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