Particle generation trough restrictive planes in GEANT4 simulations for potential applications of cosmic ray muon tomography

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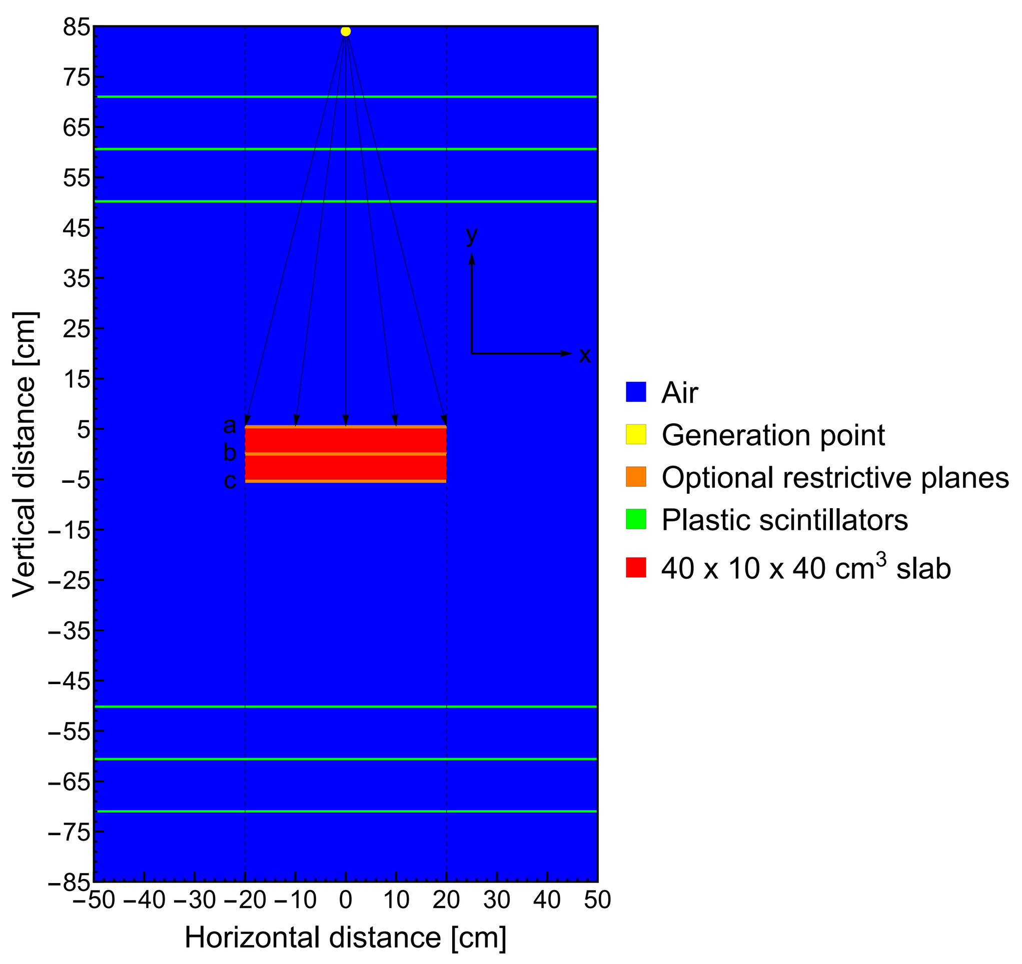
Motivation

In this study, by attempting to resolve the excessive particle loss due to the wide angular distribution during the particle generation, we exhibit an unconventional methodology that is hinged on the direction limitation via the vectorial construction from the generation location to the restriction area rather than using a certain angular distribution or interval. In other words, we favor a momentum direction that is determined by a vector constructed between an initial point chosen on a generative point/plane and a latter point arbitrarily selected on a restrictive plane of the same dimensions with the basal cross section of the target material, thereby tracking as well as optimizing the resulting particle loss.

Tomographic setup and basic parameters



Particle	μ^-
Beam direction	Downward
Momentum direction	Restrictive downward
Beam geometry	Prismatic
Initial position (cm)	y=85
Particle injector	G4ParticleGun
Number of particles	10^5
Energy distribution	Non-linear discrete
Energy interval	[0, 8]
Enegy bin step length (GeV)	0.1
Target geometry	Rectangular prism
Target volume (cm^3)	40×10×40
Material database	G4/NIST
Reference physics list	FTFP_BERT

* Average scattering angle and its standard deviation over N number of the non-absorbed/non-decayed muons:

$$\bar{\theta} \pm \delta\theta = \frac{1}{N}\sum_{i=1}^{N}\theta_i \pm \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\theta_j - \bar{\theta})^2} \quad (1)$$

* Root-mean-square (RMS) of the scattering angle over N number of the non-absorbed/non-decayed muons:

$$\theta^{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\theta_i^2} \quad (2)$$

* Number of absorbed muons within volume-of-interest (VOI):

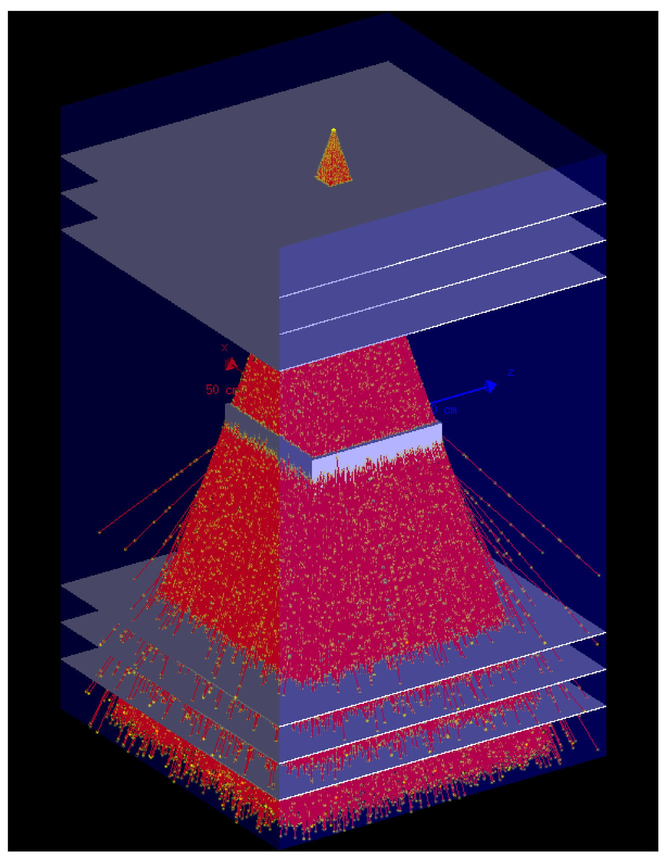
$$\#^{\mathrm{In-target}}_{\mathrm{Capture}} = \#\text{ of muMinusCaptureAtRest in VOI} \quad (3)$$

* Off-target loss:

$$\#^{\mathrm{Off-target}}_{\mathrm{Loss}} \approx \underbrace{\#_{\mathrm{Out-scattering}}}_{\text{Characteristic}} + \underbrace{\#_{\mathrm{Decay}}}_{\text{Negligible}} + \underbrace{\#^{\mathrm{Off-target}}_{\mathrm{Capture}}}_{\text{Negligible}} \quad (4)$$

Implementation of restrictive planes in GEANT4

Generative point - restrictive plane scheme



* Particle generation at height=85 cm:

$$x_0 = 0$$
$$y_0 = 85$$
$$z_0 = 0$$

* Particle restriction on $2L \times 2D$ cm^2:

$$x_1 = -L + 2 \times L \times \mathrm{G4UniformRand()}$$
$$y_1 = \mathrm{constant}$$
$$z_1 = -D + 2 \times D \times \mathrm{G4UniformRand()}$$

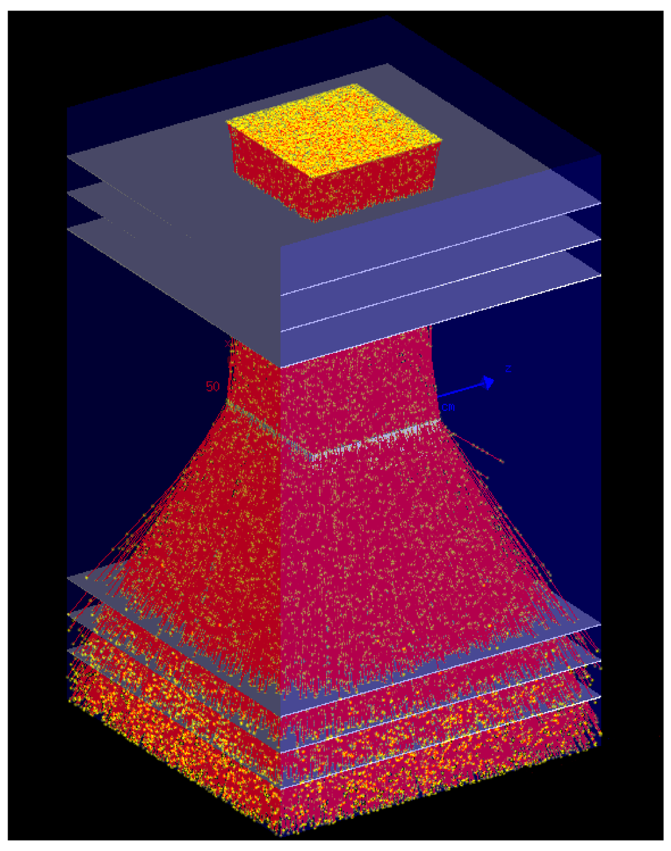
* Vector construction:

$$px = x_1 - x_0 = x_1 \quad py = y_1 - y_0 \quad pz = z_1 - z_0 = z_1$$

* Components of selective momentum direction:

$$P_x = \frac{px}{\sqrt{px^2+py^2+pz^2}}$$
$$P_y = \frac{py}{\sqrt{px^2+py^2+pz^2}}$$
$$P_z = \frac{pz}{\sqrt{px^2+py^2+pz^2}}$$

Generative - restrictive planar interplay



* Particle generation at height=85 cm:

$$x_0 = -L + 2 \times L \times \mathrm{G4UniformRand()}$$
$$y_0 = 85$$
$$z_0 = -D + 2 \times D \times \mathrm{G4UniformRand()}$$

* Particle restriction on $2L \times 2D$ cm^2:

$$x_1 = -L + 2 \times L \times \mathrm{G4UniformRand()}$$
$$y_1 = \mathrm{constant}$$
$$z_1 = -D + 2 \times D \times \mathrm{G4UniformRand()}$$

* Vector construction:

$$px = x_1 - x_0 \quad py = y_1 - y_0 \quad pz = z_1 - z_0$$

* Components of selective momentum direction:

$$P_x = \frac{px}{\sqrt{px^2+py^2+pz^2}}$$
$$P_y = \frac{py}{\sqrt{px^2+py^2+pz^2}}$$
$$P_z = \frac{pz}{\sqrt{px^2+py^2+pz^2}}$$

Outcomes of generative point - restrictive plane scheme for thickness=10 cm

Restrictive plane a

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	14.890±25.741	29.738	-	516
Copper	37.376±55.515	66.924	1083	616
Iron	32.980±47.420	57.761	1073	541
Lead	59.486±81.898	101.222	1135	1215
Uranium	73.649±91.114	117.158	3267	1542

Restrictive plane b

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	15.771±26.427	30.775	-	54
Copper	39.545±56.941	69.326	1179	216
Iron	35.306±50.117	61.304	1172	133
Lead	63.172±84.172	105.241	1220	833
Uranium	78.160±93.551	121.904	3604	1187

Restrictive plane c

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	16.142±27.368	31.774	-	35
Copper	40.355±58.022	70.676	1216	193
Iron	35.916±50.635	62.080	1215	107
Lead	64.542±85.965	107.497	1287	793
Uranium	79.700±96.102	124.850	3764	1059

Results of generative - restrictive planar interplay for thickness=10 cm

Restrictive plane a

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	15.196±26.036	30.146	-	1196
Copper	37.454±55.612	67.049	1118	1728
Iron	33.375±48.047	58.502	1092	1575
Lead	59.927±83.320	102.633	1206	2624
Uranium	74.073±92.787	118.728	3352	3299

Restrictive plane b

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	16.103±27.566	31.925	-	138
Copper	39.897±57.927	70.337	1220	581
Iron	35.380±50.142	61.367	1206	430
Lead	63.335±84.573	105.659	1327	1423
Uranium	78.399±94.631	122.888	3699	1926

Restrictive plane c

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	16.279±27.365	31.841	-	88
Copper	40.386±57.627	70.370	1258	389
Iron	36.135±50.751	62.300	1249	263
Lead	64.517±86.095	107.586	1358	1164
Uranium	80.087±96.225	125.193	3833	1537

Effect of thickness on restrictive planes: contrast between two schemes

Generative point - restrictive plane

Restrictive plane b - thickness=20 cm

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	20.815±29.601	36.187	1091	68
Copper	50.148±63.551	80.954	4959	483
Iron	48.013±65.588	81.283	3585	320
Lead	79.095±91.046	120.604	5311	1202
Uranium	100.978±112.582	151.233	8867	2004

Restrictive plane b - thickness=40 cm

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	27.849±37.186	46.458	3046	93
Copper	65.133±75.969	100.068	11072	588
Iron	58.208±67.893	89.429	10365	528
Lead	102.566±112.951	152.570	11036	2210
Uranium	121.060±121.502	171.517	20371	3084

Generative plane - restrictive plane

Restrictive plane b - thickness=20 cm

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	20.914±29.492	36.155	1132	187
Copper	49.705±62.124	79.561	5262	858
Iron	47.122±63.939	79.428	3742	694
Lead	78.553±90.927	120.159	5430	2010
Uranium	100.737±113.638	151.860	9285	2979

Restrictive plane b - thickness=40 cm

Material	$\bar{\theta}_{\mathrm{D}\neg\mathrm{CRY}} \pm \delta\theta$ [mrad]	$\theta^{\mathrm{D}\neg\mathrm{CRY}}_{\mathrm{RMS}}$ [mrad]	$\#^{\mathrm{In-target}}_{\mathrm{Capture}}$	$\#^{\mathrm{Off-target}}_{\mathrm{Loss}}$
Aluminum	28.022±37.620	46.910	3080	272
Copper	65.229±77.147	101.026	11341	1184
Iron	58.373±68.363	89.894	10599	1086
Lead	101.906±113.230	152.335	11341	3341
Uranium	120.089±121.872	171.097	20867	4181

Concluding remarks

All in all, by setting out our restrictive generation scheme, we optimize the particle loss by keeping an angular disparity that is directly dependent on the VOI geometry as well as the vertical position of the restrictive plane for a tomographic system of a finite size. Upon our simulation outcomes, we show that the particle generation by means of restrictive planes is an effective strategy that is flexible towards a variety of computational objectives in GEANT4. Into the bargain, we explicitly observe that the off-target loss is a characteristic parameter that varies in an ascending order from aluminum to uranium.