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Algorithms for Amplitude Evolution

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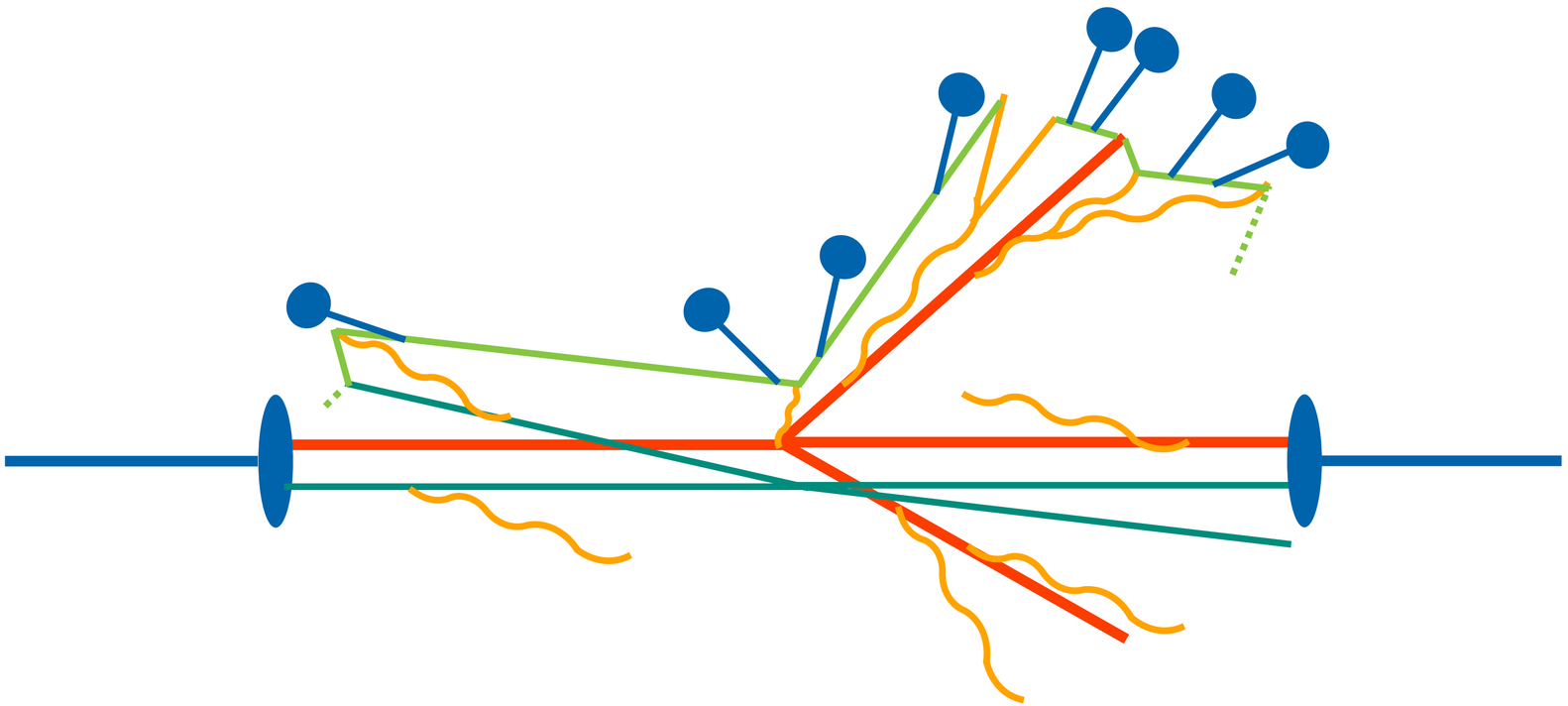
Particle Physics — University of Vienna

At the
ACAT 2021 Conference
Daejeon/Online | 29 November 2021

QCD cross sections

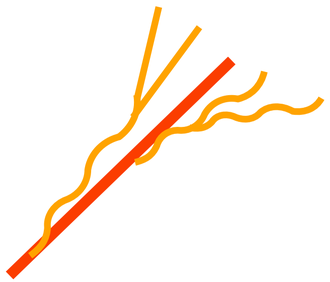


$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Setting the scene

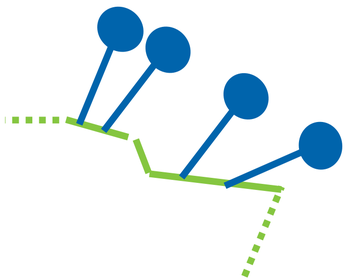
QCD description of collider reactions:
Complexity challenges precision.

Hard partonic scattering:
NLO QCD routinely



Jet evolution — parton branching:
NLL sometimes, mostly unclear

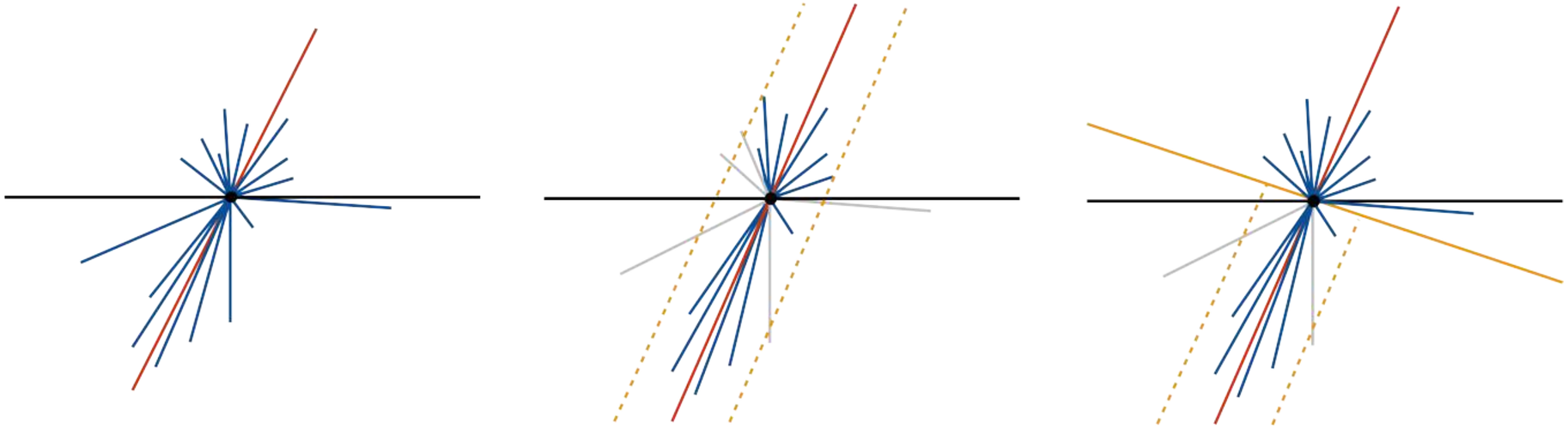
[Salam et al. — JHEP 09 (2018) 033]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014]



Multi-parton interactions
Hadronization

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

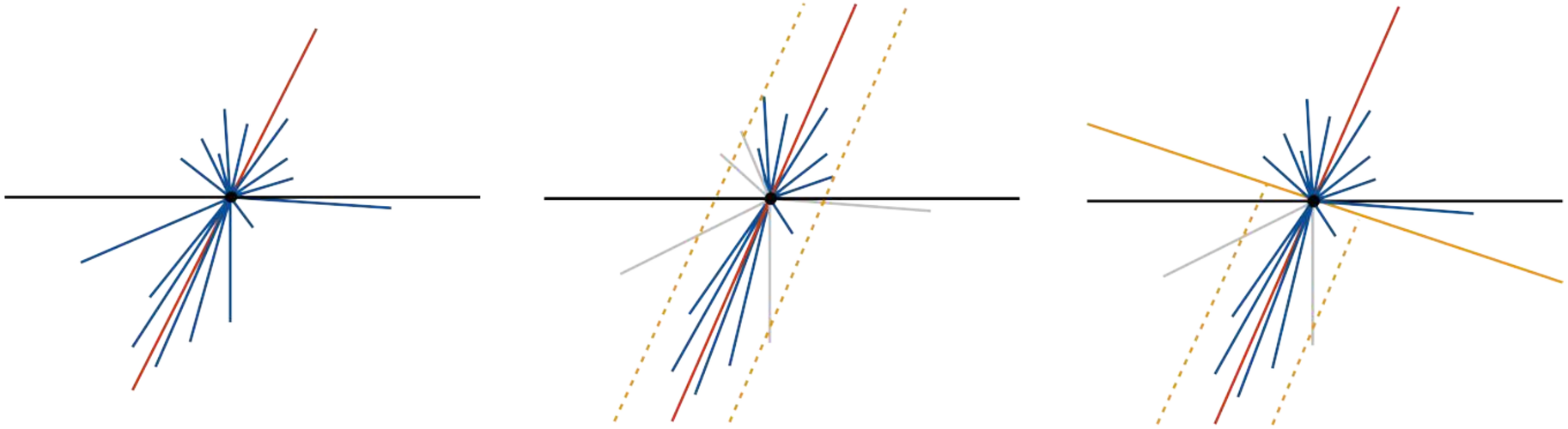
(Non-)global Observables



Measure deviation from jet topology only in patch of phase space.
Coherent branching breaks down, full complexity of QCD amplitudes strikes back.
If non-global bit is isolated can use dipole cascades to resum.

[Dasgupta, Salam, Banfi, Marchesini, Smye, Becher et al. …]

Pressing issues in parton showers



NLO with matching

NLL with coherent branching
Issues in dipole showers

Issues in coherent branching
LL with dipole showers

Understand and decide on accuracy of (existing) parton shower algorithms, take as a starting point for incremental improvements.

[Dasgupta, Dreyer, Hamilton, Monni, Salam et al. — JHEP 09 (2018) 033, …]
[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]
[Bewick, Ferrario, Richardson, Seymour — JHEP 04 (2020) 019]

$$\sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l \leq 2k} c_{nkl} \; \alpha_s^k(Q) \; \ln^l \frac{1}{\tau}$$

Pressing issues in parton showers

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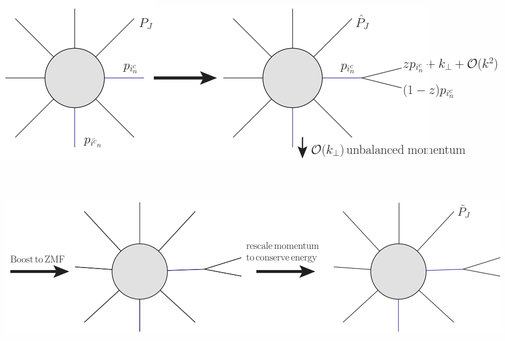
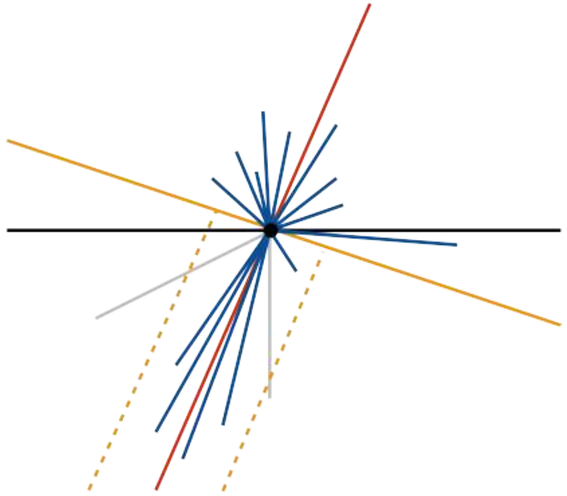
$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\, p_{j_n}\cdot q_n} \longrightarrow$$

$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\, p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}$$



[Dasgupta, Dreyer, Hamilton, Monni, Salam —PRL 125 (2020) 5]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014]



Dipole showers reproducing coherent branching:
NLL & NLC global, LL & LC non-global

Understand and decide on accuracy of (existing) parton shower algorithms, take as a starting point for incremental improvements.

[Dasgupta, Dreyer, Hamilton, Monni, Salam et al. — JHEP 09 (2018) 033, …]
[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]
[Bewick, Ferrario, Richardson, Seymour — JHEP 04 (2020) 019]

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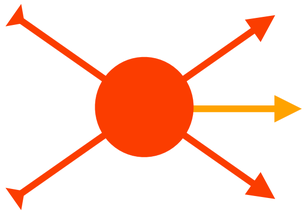
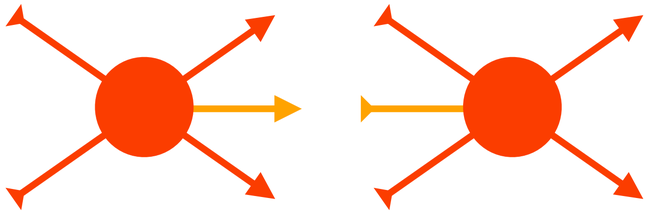
Cross Sections and Amplitudes

Cross Sections and Amplitudes



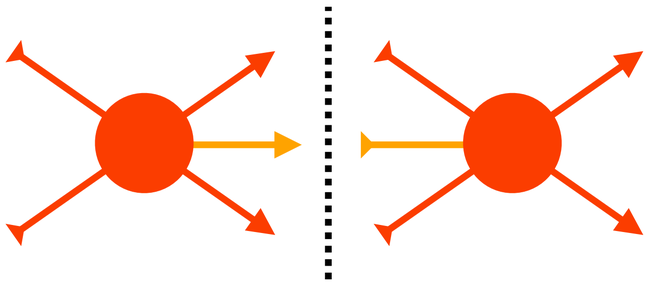
$$\langle f | \hat{S} | i \rangle$$

Cross Sections and Amplitudes



$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle$$

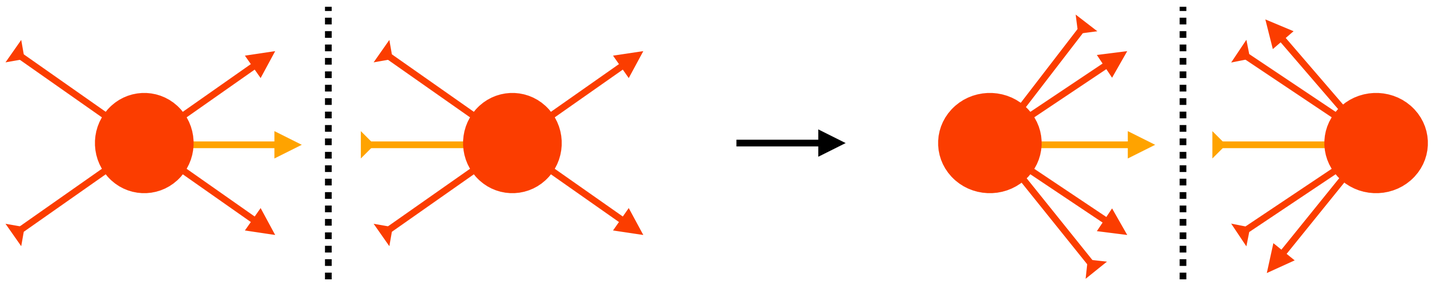
Cross Sections and Amplitudes



$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle$$

$$p_i \delta(o - o_f) \mathrm{d}\phi_f$$

Cross Sections and Amplitudes



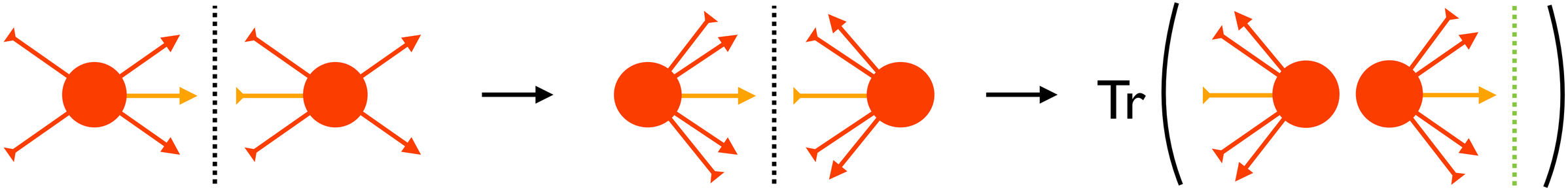
$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle \qquad \{\mathcal{M}|i,f\} \qquad \{i,f|\mathcal{M}\}$$

$$p_i\delta(o-o_f)\mathrm{d}\phi_f$$

Correlation function of field operators.

External wave functions.

Cross Sections and Amplitudes



$$\langle f|\hat{S}|i\rangle \qquad \langle i|\hat{S}^\dagger|f\rangle \qquad \{\mathcal{M}|i,f\} \qquad \{i,f|\mathcal{M}\} \qquad |\mathcal{M}\}\{\mathcal{M}| \quad |i,f\}\{i,f|$$

$$p_i\delta(o-o_f)\mathrm{d}\phi_f$$

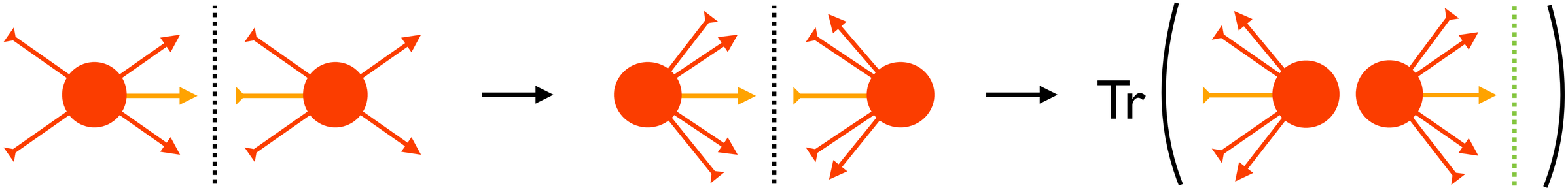
Correlation function of field operators.

External wave functions.

Cross section density operator.

Measurement projector.

Cross Sections and Amplitudes



$\langle f|\hat{S}|i\rangle$ $\langle i|\hat{S}^\dagger|f\rangle$ $\{\mathcal{M}|i,f\}$ $\{i,f|\mathcal{M}\}$ $|\mathcal{M}\}\{\mathcal{M}|$ $|i,f\}\{i,f|$

$p_i\delta(o-o_f)\mathrm{d}\phi_f$

Correlation function of field operators.

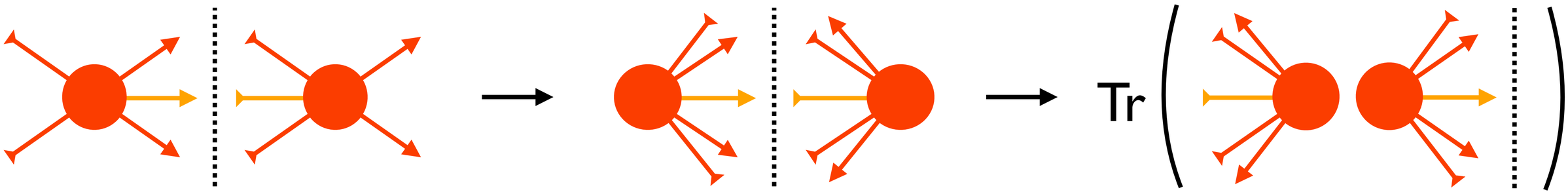
External wave functions.

Cross section density operator.

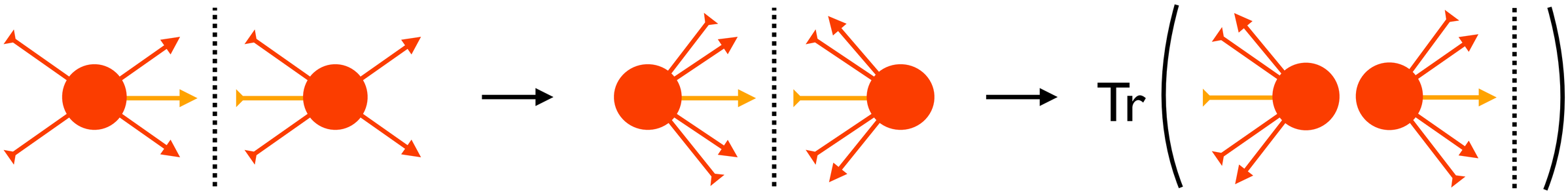
Measurement projector.

Unless stated otherwise: $|\mathcal{M}\} \to |\mathcal{M}\rangle$

Cross Sections and Amplitudes



Cross Sections and Amplitudes



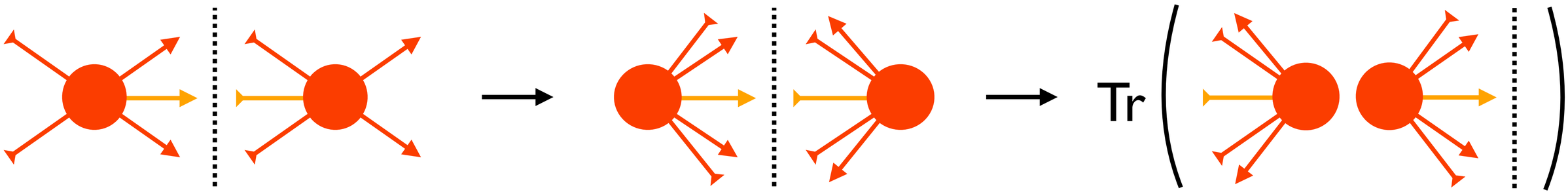
$$\sigma[u] = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n\right] u(q_1, \ldots, q_n) \mathrm{d}\phi(q_1, \ldots, q_n)$$

sum over emissions

'density operator' ~ amplitude amplitude$^+$

observable and phase space

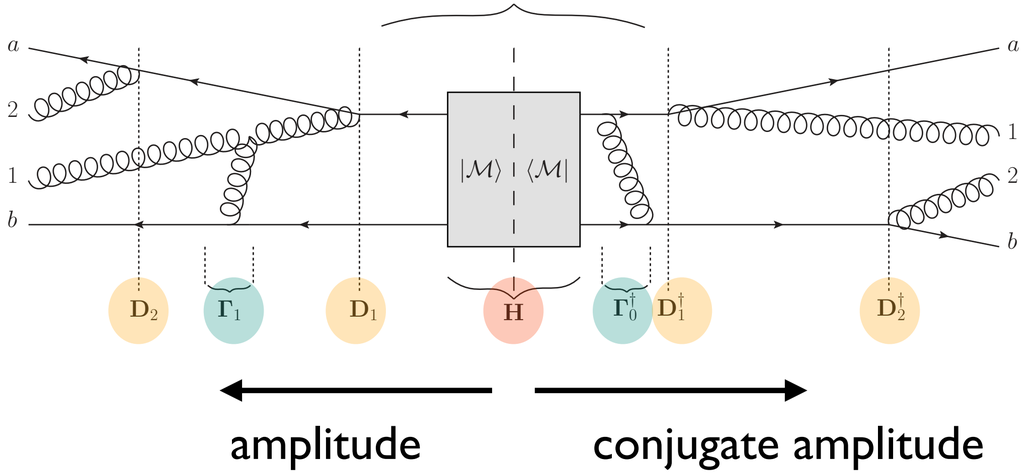
Cross Sections and Amplitudes



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.

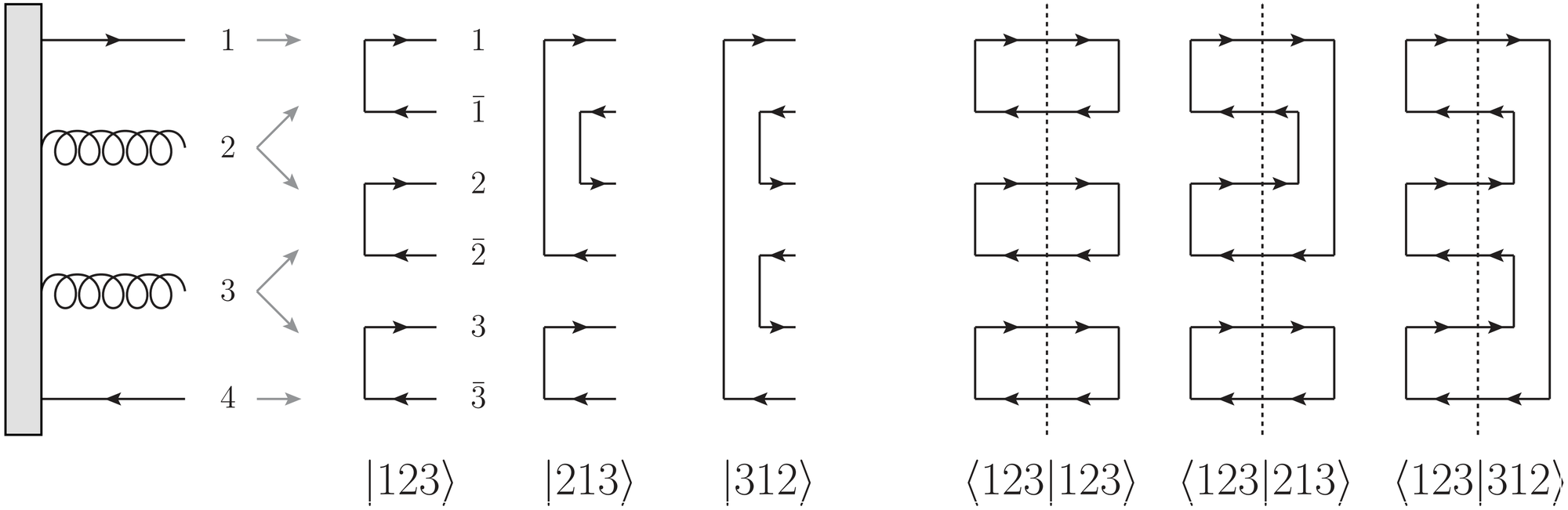


[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Tracking colour

Decompose amplitudes in flow of colour charge.

$$\mathrm{Tr}\left[\mathbf{A}_n\right] = \sum_{\sigma,\tau} A_{\tau\sigma} \langle\sigma|\tau\rangle$$



$$\mathbf{1} = \sum_{\tau} |\tau]\langle\tau| = \sum_{\tau,\sigma} S^{-1}_{\tau\sigma} |\tau\rangle\langle\sigma| \qquad S_{\tau\sigma} = \langle\tau|\sigma\rangle$$

N^3 N^2 N

[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Tracking colour

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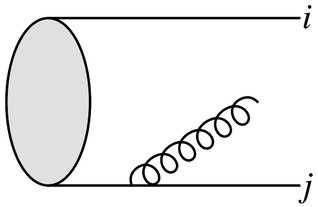
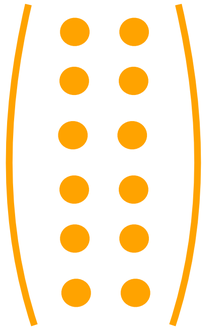
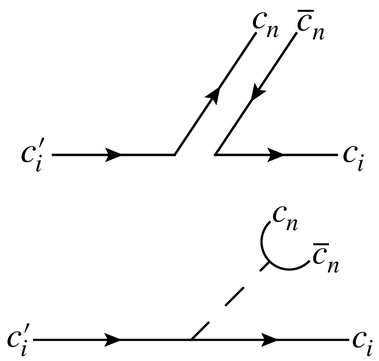
Gluon emission

$$\mathbf{D}_n(k) = \sum_i D_{ni}(k)\mathbf{T}_i$$



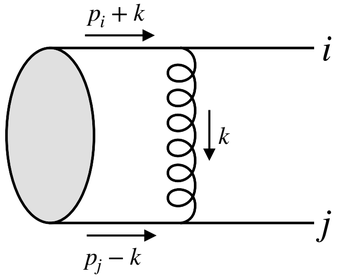




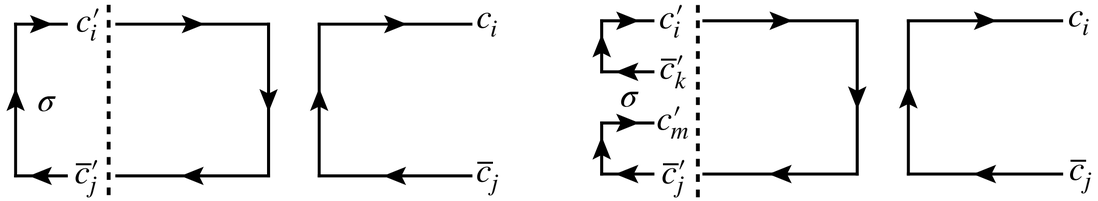
Explicit suppression in 1/N

Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$



$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

[Plätzer – EPJ C 74 (2014) 2907] — diagrams from [Ruffa, MSc thesis 2020]

Tracking colour

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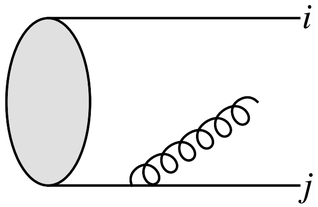
Gluon emission



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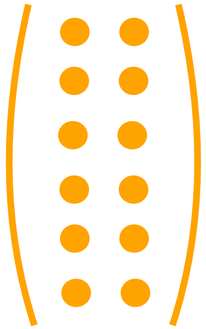
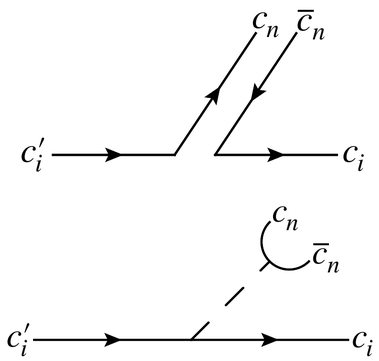
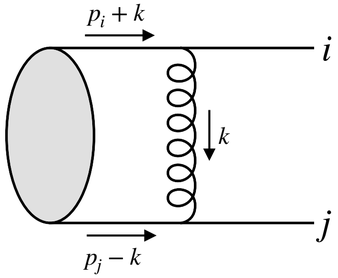




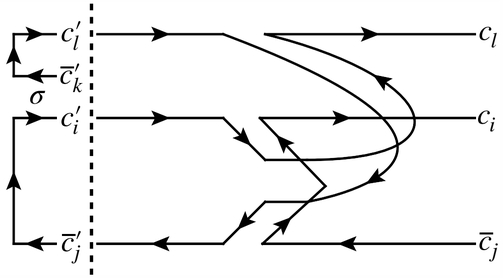
Explicit suppression in 1/N

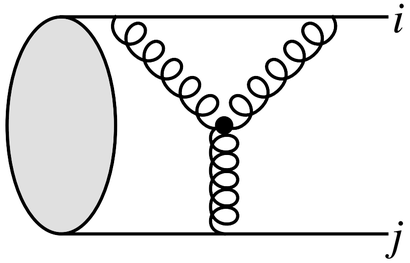
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$



$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$





[Plätzer, Ruffa — JHEP 06 (2021) 007]

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

[Plätzer – EPJ C 74 (2014) 2907] — diagrams from [Ruffa, MSc thesis 2020]

Colour Evolution Beyond Leading Order

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[Plätzer, Ruffa — JHEP 06 (2021) 007 & work in progress]

Include simultaneously unresolved emissions and higher loop structures

$$E\frac{\partial}{\partial E}\mathbf{A}_n(E) = \mathbf{\Gamma}_n(E)\mathbf{A}_n(E) + \mathbf{A}_n(E)\mathbf{\Gamma}_n^\dagger(E) - \sum_k \mathbf{R}_n^{(k)}(E)\mathbf{A}_{n-k}(E)\mathbf{R}_n^{(k),\dagger}(E)$$

combination of purely virtual and unresolved real corrections, point-by-point in phase space

resolved real emissions and virtual/ unresolved corrections to emissions

Similar in origin to a fixed-order calculation with the subtraction method:

Subtract (unresolved) real emissions — cast virtual corrections into phase-space type integrals instead of integrating subtraction terms.

Beyond Leading Colour

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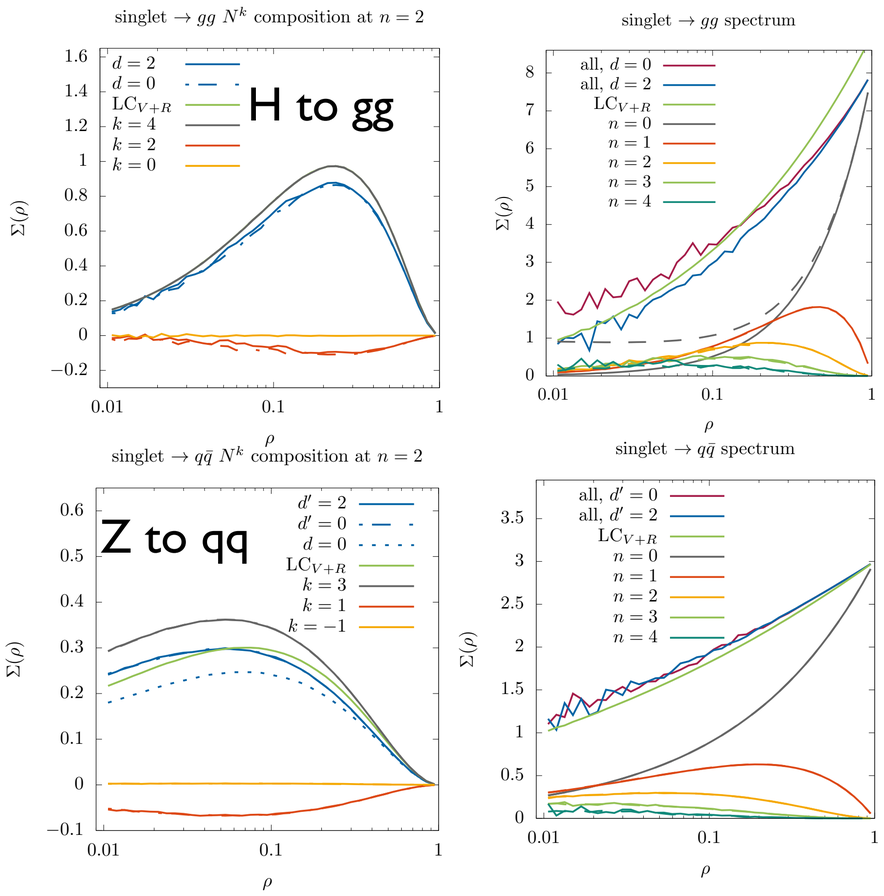
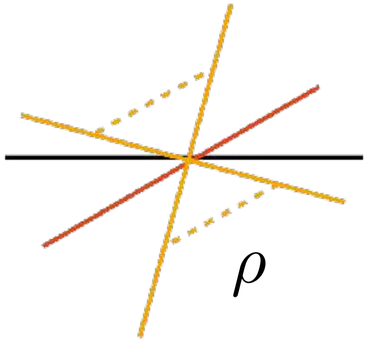
CVolver library implements numerical evolution in colour space.

origins in [Plätzer – EPJ C 74 (2014) 2907]

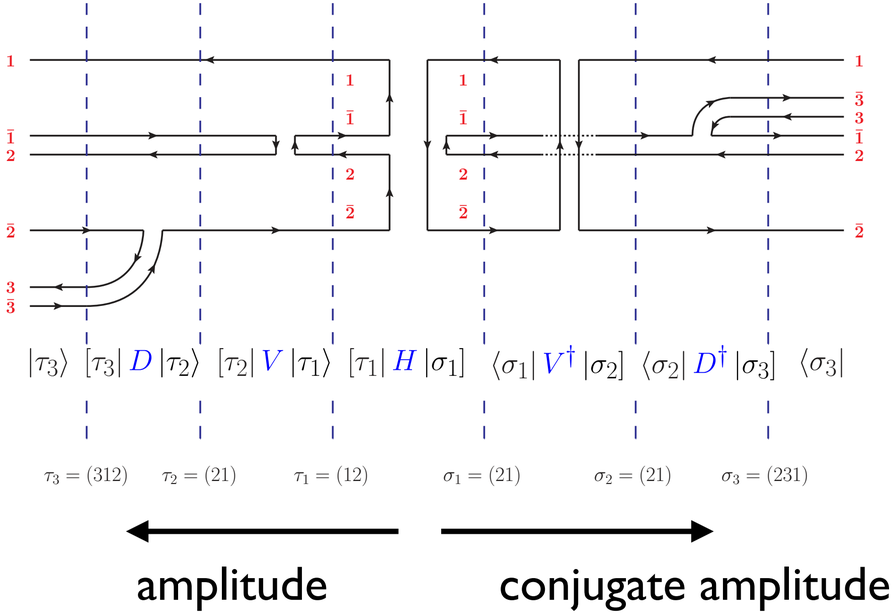
Resummation of non-global logarithms at full colour:





$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$

[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]

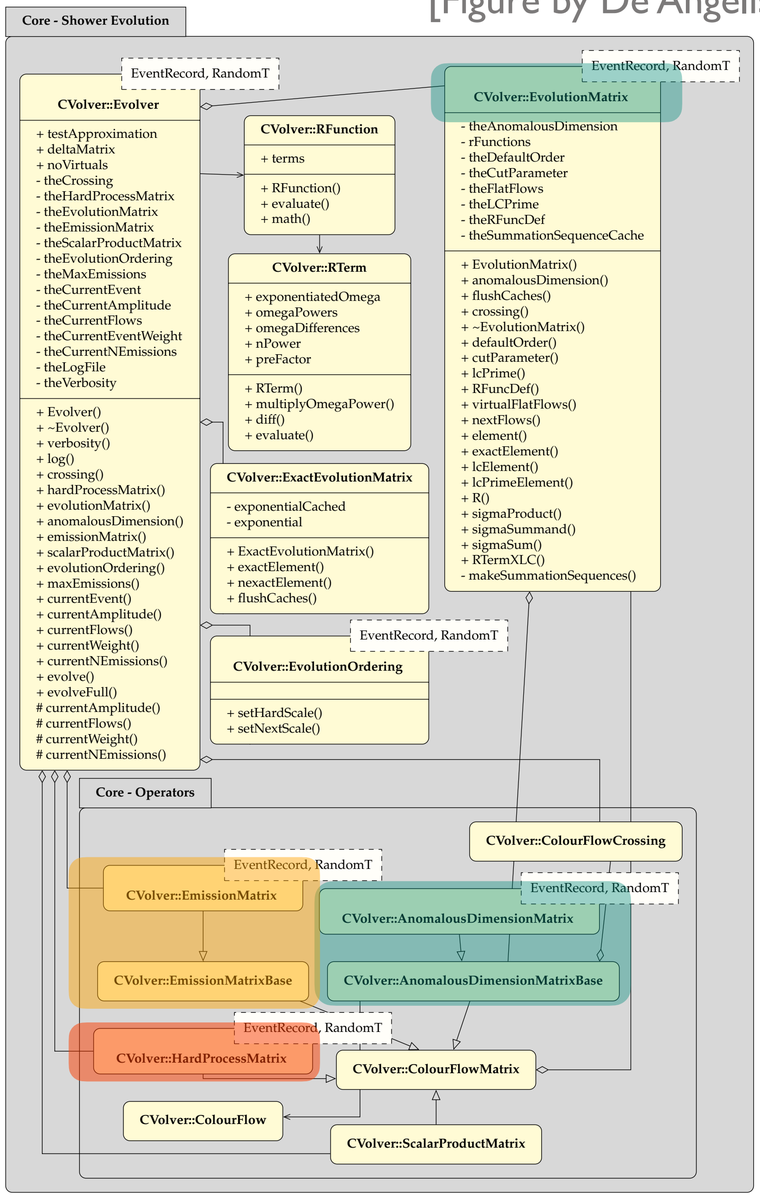


Avoid complexity which grows with colour space dimensionality:

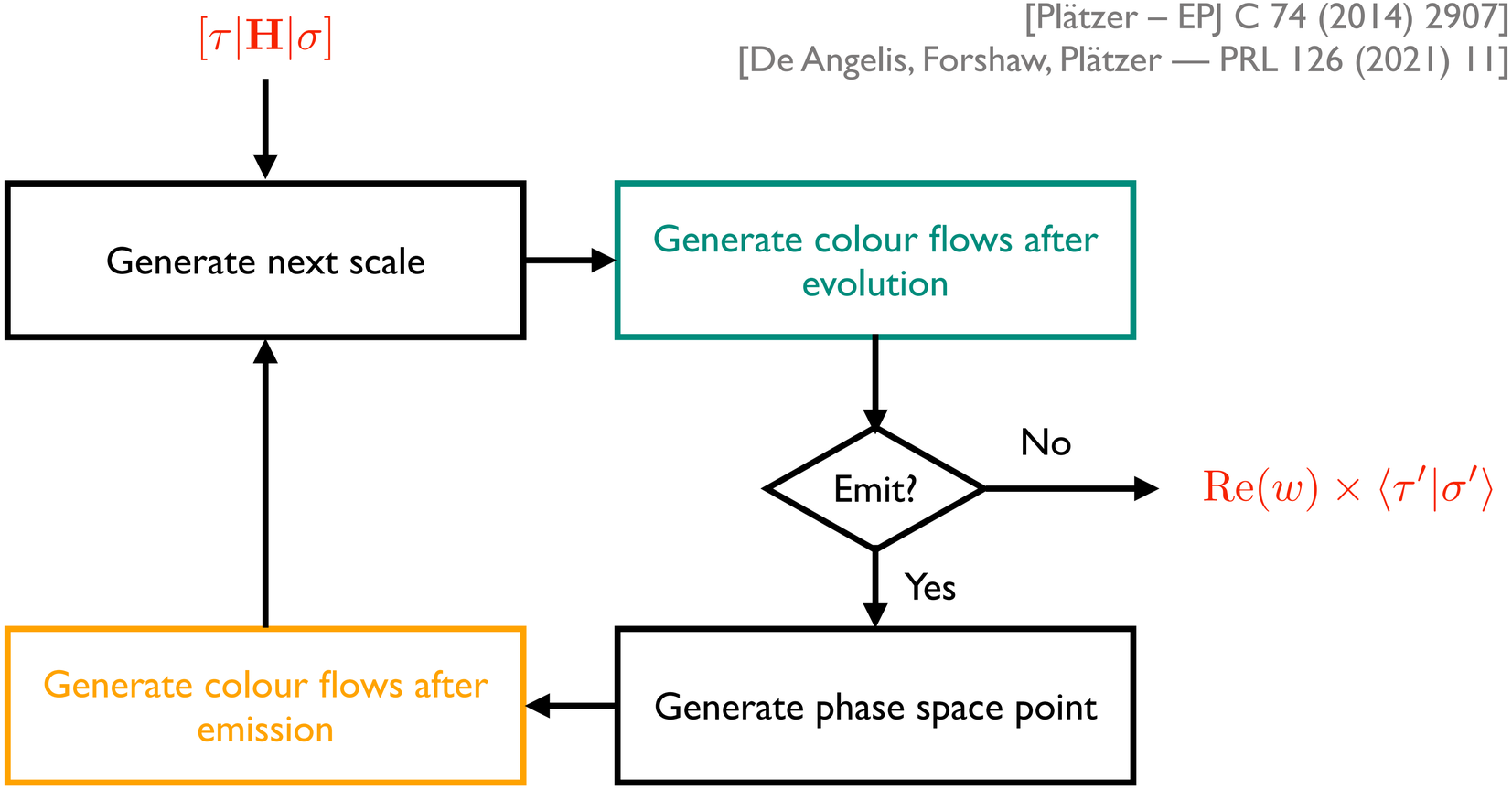
- Monte Carlo over colour flows,
- events at intermediate steps carry complex weights.

Structure of CVolver

[Figure by De Angelis]



[Plätzer – EPJ C 74 (2014) 2907]
[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Selecting Colour Flows

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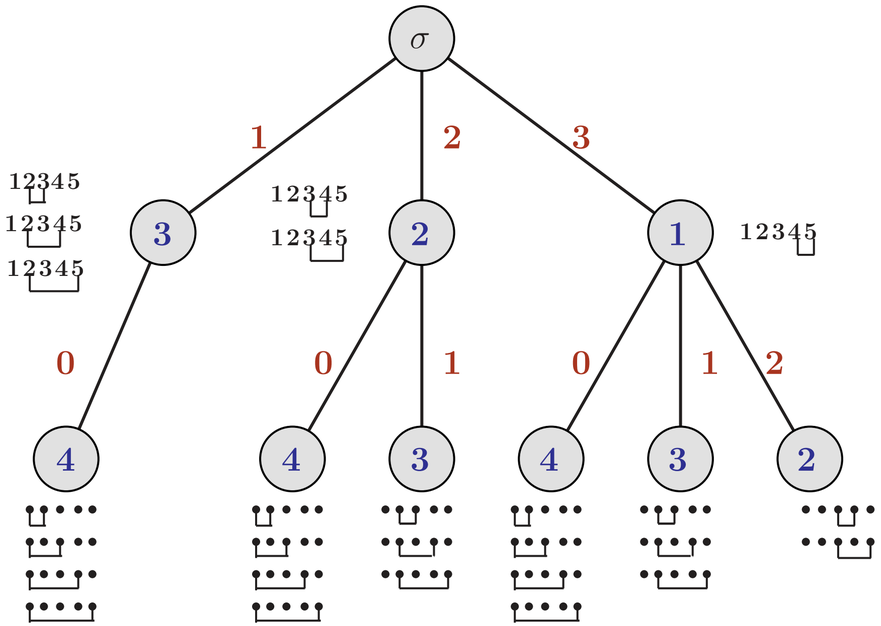
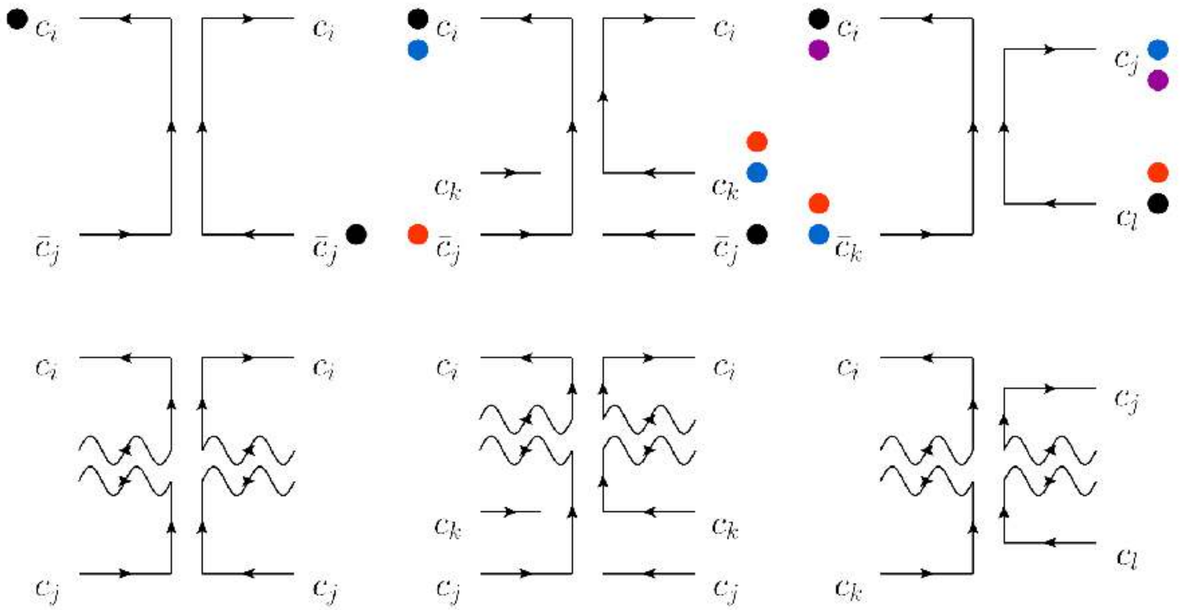
more to [De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]
[Holguin, Forshaw, Plätzer — arXiv:soon]

$$\langle\alpha|\beta\rangle = N_c^{n-\#(\alpha,\beta)}$$

$$\xi_{ij}^{(\tau\sigma\bar{\sigma}\bar{\tau})} = \frac{\langle\tau|\bar{\tau}\rangle}{\langle\sigma|\bar{\sigma}\rangle}[\tau|\mathbf{T}_i|\sigma\rangle\langle\bar{\sigma}|\mathbf{T}_j|\bar{\tau}]$$





$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

ω_{ij} $\qquad$ $\omega_{ij} + \omega_{ik} - \omega_{jk}$ $\qquad$ $\omega_{il} + \omega_{kj} - \omega_{kl} - \omega_{ij}$

Sudakov Veto Algorithms

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Sudakov-type densities central to Showers

[Olsson, Plätzer, Sjödahl — EPJC 80 (2020) 10]
[Plätzer, Sjödahl — EPJ Plus 127 (2012) 26]
Also cf. shower variations e.g.
[Bellm, Plätzer, et al. — Phys.Rev.D 94 (2016) 3, 034028]

no emission

$$\frac{\mathrm{d}S_P(q|Q,z,x)}{\mathrm{d}q\ \mathrm{d}z} = \Delta_P(Q_0|Q,x)\delta(q-Q_0)\delta(z-z_0) + \Delta_P(q|Q,x)P(q,z,x)\theta(Q-q)\theta(q-Q_0)$$

emission

Negative P or unknown overestimate requires weighted veto algorithm, with in principle arbitrary proposal kernel and veto probability.

$Q' \leftarrow Q, w \leftarrow w_0$
loop
A trial splitting scale and variables, q, z, are generated according to $S_R(q|Q', z, x)$, for example using Alg. 1.
if $q = Q_0$ **then**
There is no emission and the cut-off scale Q_0 is returned while the event weight is kept at w.
else
if rnd$\leq \epsilon$ **then**
The trial splitting variables q, z are accepted, and

$$w \leftarrow w \times \frac{1}{\epsilon} \times \frac{P(Q', z, x)}{R(Q', z, x)}. \tag{3}$$

else
The emission is rejected, and the algorithm continues with

$$w \leftarrow w \times \frac{1}{1-\epsilon} \times \left(1 - \frac{P(q, z, x)}{R(q, z, x)}\right)$$
$$Q' \leftarrow q. \tag{4}$$

end if
end if
end loop

Weighted Veto Algorithms & Resampling

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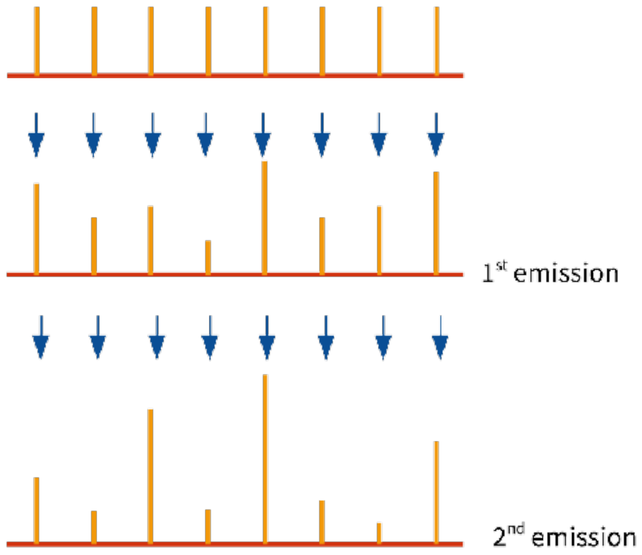
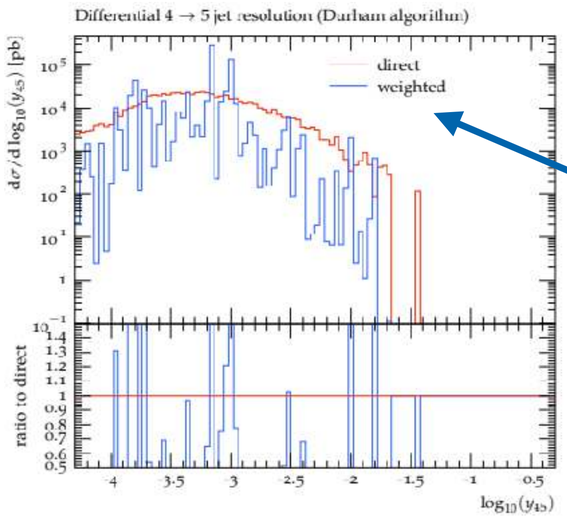
[Olsson, Plätzer, Sjödahl — EPJ C80 (2020) 10, 934]

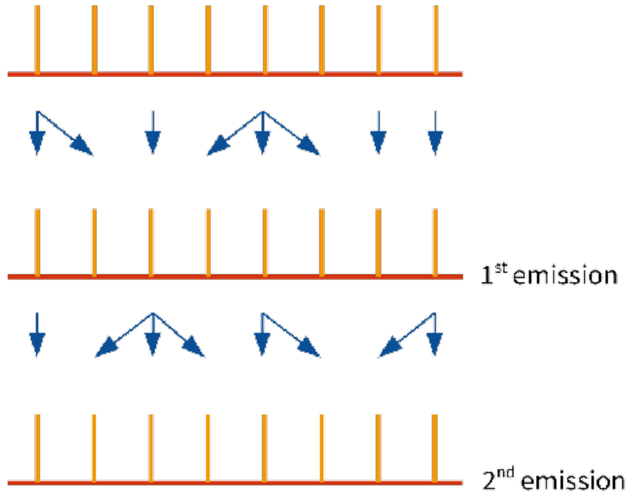


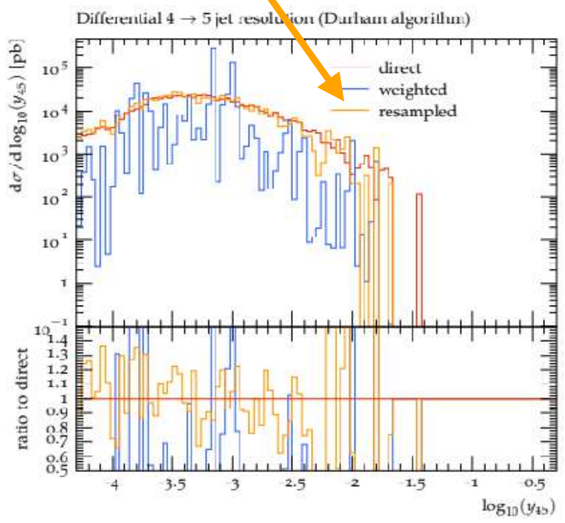


Weighted branching algorithms exhibit prohibitive weight distributions & convergence issues.

Result without resampling

Result with resampling





Resampling algorithms can compress weight distributions at intermediate steps.

Different resampling method developed as event generator after-burner.

[Andersen, Gütschow, Maier, Prestel — EPJ C 80 (2020) 11]

Summary & Outlook

Amplitude level evolution sets level of complexity to understand and design parton shower and resummation algorithms: both for incremental improvements and in its own right.

Crucial to address effects of Coulomb/Glauber phases, factorisation violation, super-leading logarithms, … — otherwise out of reach.

Investigate soft gluon effects at two loops (and related):

- Understand colour structures for many external legs and systematically expand around large-N limit.
- Design resummation of non-global observables beyond leading log and leading-N, investigate phases.
- Key ingredient for decisive statements about most flexible parton showers beyond leading order.

Thank you!

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