Algorithms for Amplitude Evolution

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At the
ACAT 2021 Conference
Daejeon/Online | 29 November 2021
QCD cross sections

\[ d\sigma \sim L \times d\sigma_H(Q) \times PS(\mathcal{Q} \rightarrow \mu) \times MPI \times \text{Had}(\mu \rightarrow \Lambda) \times \ldots \]
Setting the scene

QCD description of collider reactions:
Complexity challenges precision.

Hard partonic scattering:
NLO QCD routinely

Jet evolution — parton branching:
NLL sometimes, mostly unclear

Multi-parton interactions
Hadronization

\[ d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \rightarrow \mu) \times MPI \times Had(\mu \rightarrow \Lambda) \times \ldots \]
(Non-)global Observables

Measure deviation from jet topology only in patch of phase space. Coherent branching breaks down, full complexity of QCD amplitudes strikes back. If non-global bit is isolated can use dipole cascades to resum.

[Dasgupta, Salam, Banfi, Marchesini, Smye, Becher et al. …]
Pressing issues in parton showers

Understand and decide on accuracy of (existing) parton shower algorithms, take as a starting point for incremental improvements.

\[ \sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l \leq 2k} c_{nkl} \alpha_s^k(Q) \ln \frac{1}{\tau} \]
Pressing issues in parton showers

Understand and decide on accuracy of (existing) parton shower algorithms, take as a starting point for incremental improvements.

[Dasgupta, Dreyer, Hamilton, Monni, Salam — PRL 125 (2020) 5]
[Forshaw, Holguin, Plätzer — JHEP 09 (2020) 014]

Dipole showers reproducing coherent branching: NLL & NLC global, LL & LC non-global

\[ \sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l \leq 2k} c_{nkl} \alpha_s^k(Q) \ln \frac{l}{\tau} \]
Cross Sections and Amplitudes
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\[ \langle f | \hat{S} | i \rangle \]
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\[ \langle f | \hat{S} | i \rangle \quad \langle i | \hat{S}^\dagger | f \rangle \]
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\[ p_i \delta(o - o_f) d\phi_f \]
Cross Sections and Amplitudes

\[ \langle f | \hat{S} | i \rangle \quad \langle i | \hat{S}^\dagger | f \rangle \quad \{ \mathcal{M} | i, f \} \quad \{ i, f | \mathcal{M} \} \]

\[ p_i \delta(o - o_f) d\phi_f \]

Correlation function of field operators.

External wave functions.
Cross Sections and Amplitudes

\[ \langle f | \hat{S} | i \rangle \quad \langle i | \hat{S}^\dagger | f \rangle \quad \{ \mathcal{M} | i, f \} \quad \{ i, f | \mathcal{M} \} \quad \{|\mathcal{M}\} \{\mathcal{M}| i, f \}\{i, f| \]

\[ p_i \delta(o - o_f) d\phi_f \]

- Correlation function of field operators.
- Cross section density operator.
- External wave functions.
- Measurement projector.
Cross Sections and Amplitudes

\[ \langle f | \hat{S} | i \rangle \quad \langle i | \hat{S}^\dagger | f \rangle \quad \{ \mathcal{M} | i, f \} \quad \{ i, f | \mathcal{M} \} \quad | \mathcal{M} \rangle \{ \mathcal{M} | i, f \} \{ i, f \} \]

\[ p_i \delta(o - o_f) d\phi_f \]

Correlation function of field operators.
External wave functions.
Cross section density operator.
Measurement projector.

Unless stated otherwise: \[ | \mathcal{M} \rangle \rightarrow | \mathcal{M} \rangle \]
Cross Sections and Amplitudes

\[ \text{Tr} \left( \begin{array}{ccc} & & \\ & & \\ & & \\ \end{array} \right) \]
\[ \sigma[u] = \sum_n \int \text{Tr} [A_n] \, u(q_1, \ldots, q_n) \, d\phi(q_1, \ldots, q_n) \]

- sum over emissions
- ‘density operator’ ~ amplitude amplitude
- observable and phase space
Cross Sections and Amplitudes

\[ A_n(q) = \int_q^Q \frac{dk}{k} \tr\left( \frac{d\kappa}{\kappa'} \Gamma(k') \right) D_n(k) A_{n-1}(k) D_n^\dagger(k) \tr\left( \frac{d\kappa}{\kappa'} \Gamma^\dagger(k') \right) \]

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
Decompose amplitudes in flow of colour charge.

$$\text{Tr} \left[ A_n \right] = \sum_{\sigma, \tau} A_{\tau \sigma} \langle \sigma | \tau \rangle$$

$$1 = \sum_{\tau} |\tau\rangle \langle \tau| = \sum_{\tau, \sigma} S^{-1}_{\tau \sigma} |\tau\rangle \langle \sigma|$$

$$S_{\tau \sigma} = \langle \tau| \sigma \rangle$$

$$N^3 \quad N^2 \quad N$$

[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
Tracking colour

Gluon emission

$$D_n(k) = \sum_i D_{ni}(k) T_i$$

Explicit suppression in $1/N$

Systematically expand around large-$N$ limit summing towers of terms enhanced by $\alpha_S N$

Gluon exchange

$$P e^{-\int_q^k \frac{dk'}{k'} \Gamma(k')} \left( \begin{array}{c} \vdots \\ \vdots \end{array} \right)$$

$$[\tau | \Gamma | \sigma \rangle = (\alpha_s N)[\tau | \Gamma^{(1)} | \sigma \rangle + (\alpha_s N)^2[\tau | \Gamma^{(2)} | \sigma \rangle + ...$$

$$[\tau | \Gamma^{(1)} | \sigma \rangle = \left( \Gamma^{(1)}_\sigma + \frac{1}{N^2} \rho^{(1)} \right) \delta_{\sigma \tau} + \frac{1}{N} \sum_{\sigma \tau}^{(1)}$$

dipole flips — implicit suppression in $1/N$

**Tracking colour**

**Gluon emission**

\[ D_n(k) = \sum_i D_{ni}(k) T_i \]

**Gluon exchange**

\[ P e^{-\int dq \frac{dk'}{k'} \Gamma(k')} \]

\[ [\tau|\Gamma|\sigma] = (\alpha_s N)[\tau|\Gamma^{(1)}|\sigma] + (\alpha_s N)^2[\tau|\Gamma^{(2)}|\sigma] + ... \]

**Explicit suppression in 1/N**

**Systematically expand around large-N limit summing towers of terms enhanced by \( \alpha_s N \)**

[Plätzer, Ruffa — JHEP 06 (2021) 007]

[Dipole flips — implicit suppression in 1/N]

Include simultaneously unresolved emissions and higher loop structures

\[ E \frac{\partial}{\partial E} A_n(E) = \Gamma_n(E) A_n(E) + A_n(E) \Gamma_n^\dagger(E) - \sum_k R_n^{(k)}(E) A_{n-k}(E) R_n^{(k),\dagger}(E) \]

combination of purely virtual and unresolved real corrections, point-by-point in phase space

resolved real emissions and virtual/unresolved corrections to emissions

Similar in origin to a fixed-order calculation with the subtraction method:

Subtract (unresolved) real emissions — cast virtual corrections into phase-space type integrals instead of integrating subtraction terms.
Beyond Leading Colour

CVolver library implements numerical evolution in colour space.

Resummation of non-global logarithms at full colour:

\[
\Sigma(\rho) = \sum_n \int d\sigma\{p_i\} \prod_i \theta_{in}(\rho - E_i)
\]

Avoid complexity which grows with colour space dimensionality:

- Monte Carlo over colour flows,
- events at intermediate steps carry complex weights.
\[ A_n(q) = \int_q^Q \frac{dk}{k} Pe^{-f_q \frac{dk}{k} \Gamma(k')} D_n(k) A_{n-1}(k) D_n^\dagger(k) Pe^{-f_q \frac{dk}{k} \Gamma^\dagger(k')} \]
Selecting Colour Flows

\[ \langle \alpha | \beta \rangle = N_c^{n - \#(\alpha, \beta)} \]

\[
\xi_{ij}^{(\tau \bar{\tau})} = \frac{\langle \tau | \tilde{\sigma} \rangle}{\langle \sigma | \tilde{\sigma} \rangle} \left[ \tau | \mathbf{T}_i \rangle \langle \sigma | \mathbf{T}_j \rangle \langle \tilde{\sigma} \right]
\]

\[
[\tau | \Gamma^{(1)} | \sigma \rangle = \left( \Gamma^{(1)}_{\sigma} + \frac{1}{N} \rho^{(1)} \right) \delta_{\sigma \tau} + \frac{1}{N} \Sigma^{(1)}_{\sigma \tau}
\]

\[
\omega_{ij} + \omega_{ik} - \omega_{jk} \quad \omega_{il} + \omega_{kj} - \omega_{kl} - \omega_{ij}
\]
Sudakov Veto Algorithms

Sudakov-type densities central to Showers

\[
\frac{dS_P(q|Q, z, x)}{dq \ dz} = \Delta_P(Q_0|Q, x)\delta(q - Q_0)\delta(z - z_0) \\
\quad + \Delta_P(q|Q, x)P(q, z, x)\theta(Q - q)\theta(q - Q_0)
\]

no emission

emission

Negative P or unknown overestimate requires weighted veto algorithm, with in principle arbitrary proposal kernel and veto probability.
Weighted Veto Algorithms & Resampling

Weighted branching algorithms exhibit prohibitive weight distributions & convergence issues.

Result without resampling

Result with resampling

Resampling algorithms can compress weight distributions at intermediate steps.

Different resampling method developed as event generator after-burner.

[Andersen, Gütschow, Maier, Prestel — EPJ C 80 (2020) 11]
Amplitude level evolution sets level of complexity to understand and design parton shower and resummation algorithms: both for incremental improvements and in its own right.

Crucial to address effects of Coulomb/Glauber phases, factorisation violation, super-leading logarithms, … — otherwise out of reach.

Investigate soft gluon effects at two loops (and related):

- Understand colour structures for many external legs and systematically expand around large-N limit.
- Design resummation of non-global observables beyond leading log and leading-N, investigate phases.
- Key ingredient for decisive statements about most flexible parton showers beyond leading order.
Thank you!