

Verifying mathematical proofs with computers.

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What's this talk about?

For decades, computers have been used by scientists to *compute*.

Example of a computation: “compute all prime numbers less than 1000”.

Computers can also be used to *reason* – i.e., to prove theorems.

Example of reasoning: “prove that there are infinitely many prime numbers.”

Live demo

A Proof Assistant is an app which knows the axioms of mathematics and the rules of logic.

Lean is a free and open source proof assistant designed principally by Leonardo de Moura at Microsoft Research.

Let's see it in action.

What did we just learn?

If you want to compute in a proof assistant, you *can do it*.
However for every algorithm you write, you must *prove it works*.

Does it scale?

It has taken decades to make it scale.

Computer scientists have spent decades developing these systems.

Most modern systems now have tactics like `ring`, enabling mathematicians to work in a more “normal” (for them) way.

Mathematicians have to be trained how to use the software.

Where are the benefits of using it?

Recent breakthroughs: research mathematics

Lean's mathematics library was used to power the verification of a theorem from 2020 by Clausen and Scholze.

Scholze was a 2018 Fields Medallist.

The formalisation was led by Johan Commelin.

We got a write-up in Nature.

The future: better mathematical documents

Patrick Massot has emphasized that proving theorems using these tools. . .

. . . can result in a new kind of mathematical document.

The future: AI solving hard math problems?

Computers are now better than humans at board games.

AI researchers have pointed out that mathematics is a natural next step.

Maybe computers can start proving theorems by themselves!

Unfortunately, mathematics is infinite $(1,2,3,4,\dots)$.

I think we're nowhere near this right now, for research level theorems.

What about Olympiad questions?

The International Mathematical Olympiad consists of extremely hard problems which can be solved using only pre-university mathematics.

I have seen an AI solve a problem at International Mathematical Olympiad level.

So it's coming.

Thank you for your attention.