

Nonstandard neutrino interactions at COHERENT, DUNE, T2HK and LHC

In collaboration with Tao Han, Jiajun Liao, and Danny Marfatia
[arXiv:1910.03272]



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Motivation

Many new physics in neutrino sector can introduce nonstandard neutrino interactions (NSI).

The neutrino physics entered into precision era, we need to understand sub-leading effects such as NSI to determine the true value of physical parameters in neutrino sector.

NSI can be related to lepton flavor non-universality by gauge symmetry.

NSI formalism

The effects of Non-Standard neutrino interaction on low-energy observables are traditionally parametrized by an effective four-fermion Lagrangian

$$L_{\text{eff}} = -2\sqrt{2}G_F \epsilon_{\alpha\beta}^{ff'C} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P_C f')$$

$f = f'$ neutral current, matter NSI

$f \neq f'$ charged current, source NSI

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No FCNC

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$$f = f' \quad \text{neutral current, matter NSI}$$

$$\alpha = \beta$$

$$f \neq f' \quad \text{charged current, source NSI}$$

No FCNC

$$L_{\text{eff}} = -2\sqrt{2}G_F \epsilon_{\alpha\beta}^{fL} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P_L f) - 2\sqrt{2}G_F \epsilon_{\alpha\beta}^{fR} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P_R f)$$

$$= -\sqrt{2}G_F \epsilon_{\alpha\alpha}^{fV} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha) (\bar{f} \gamma_\mu f) - \sqrt{2}G_F \epsilon_{\alpha\alpha}^{fA} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha) (\bar{f} \gamma_\mu \gamma^5 f)$$

$$\epsilon_{\alpha\alpha}^{fV} \equiv \epsilon_{\alpha\alpha}^f \equiv \epsilon_{\alpha\alpha}^{fL} + \epsilon_{\alpha\alpha}^{fR}$$

$$\epsilon_{\alpha\alpha}^{fA} \equiv \epsilon_{\alpha\alpha}^{fR} - \epsilon_{\alpha\alpha}^{fL}$$

Neutrino Oscillation Experiments

The Hamiltonian for neutrino propagation in the presence of neutral current NSI is

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{pmatrix} U^\dagger + V \quad V = \sqrt{2} G_F N_e \begin{pmatrix} 1 + \epsilon_{ee} & 0 & 0 \\ 0 & \epsilon_{\mu\mu} & 0 \\ 0 & 0 & \epsilon_{\tau\tau} \end{pmatrix}$$

Effective NSI parameters $\epsilon_{\alpha\alpha} \equiv \sum_q \epsilon_{\alpha\alpha}^q \frac{N_q}{N_e}$

	Current data	DUNE+T2HK
ϵ_{ee}^u	$[-1.192, -0.802] \oplus [-0.020, +0.456]$	$[-0.407, -0.270] \oplus [-0.072, +0.064]$
$\epsilon_{\mu\mu}^u$	$[-0.130, 0.152]$	$[-0.019, +0.018]$
$\epsilon_{\tau\tau}^u$	$[-0.152, 0.130]$	$[-0.017, +0.017]$

Gauged U(1)

The effective Lagrangian formalism is non-renormalizable and not gauge invariant.

We study the non-standard interactions arising from a new flavored gauge boson Z' associated with a new $U(1)'$ symmetry.

$$X = Q'_q(B_1 + B_2 + B_3) + Q'_e L_e + Q'_\mu L_\mu + Q'_\tau L_\tau$$

With three right-handed neutrinos, we can apply anomaly-free constraints

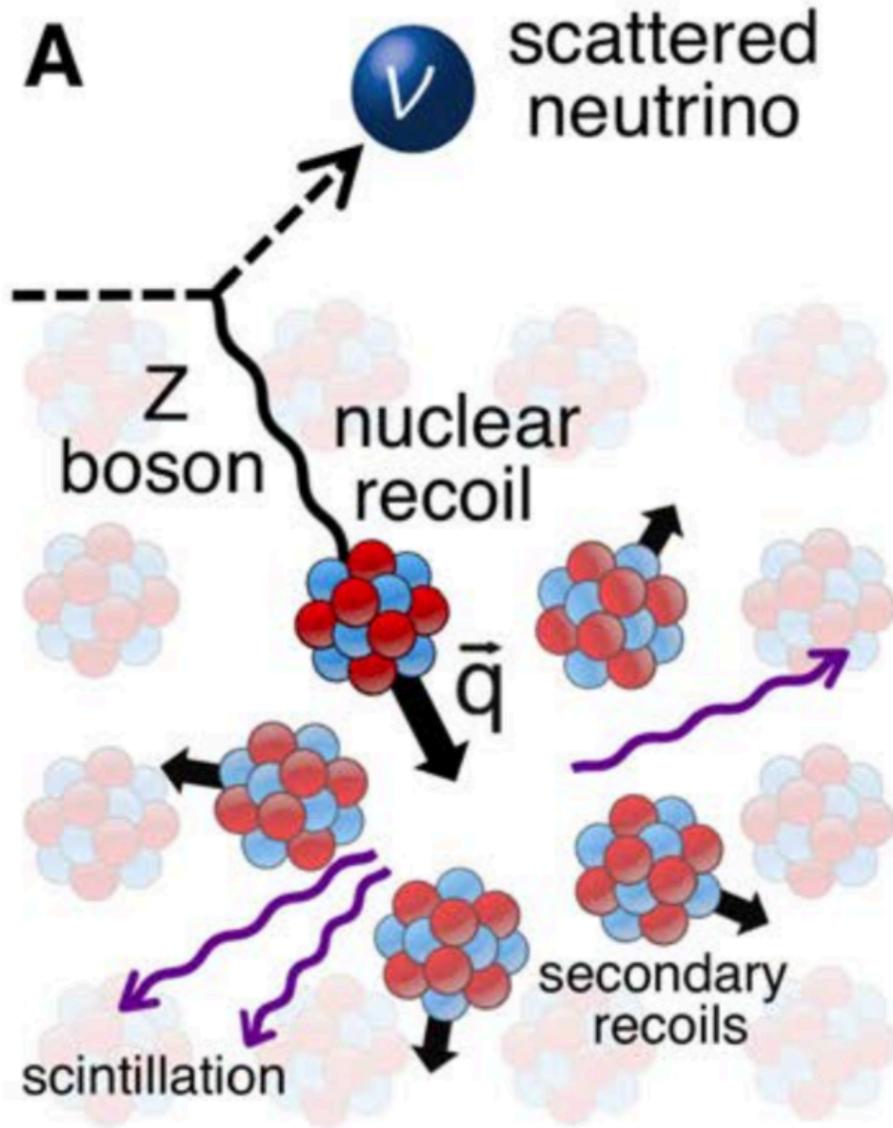
$$9 Q'_q + Q'_e + Q'_\mu + Q'_\tau = 0$$

$$\text{A) } Q'_q = 1/3, Q'_\mu = -3, Q'_e = Q'_\tau = 0$$

$$\text{B) } Q'_q = 1/3, Q'_\mu = Q'_\tau = -3/2, Q'_e = 0$$

$$\text{C) } Q'_q = 1/3, Q'_\tau = -3, Q'_e = Q'_\mu = 0$$

Coherent elastic neutrino-nucleus scattering (CEvNS)



When the momentum exchanged is significantly smaller than the inverse of the nuclear size,

$$\frac{1}{E_\nu} \sim 10^{-14} m \rightarrow E_\nu \sim 20 \text{ MeV}$$

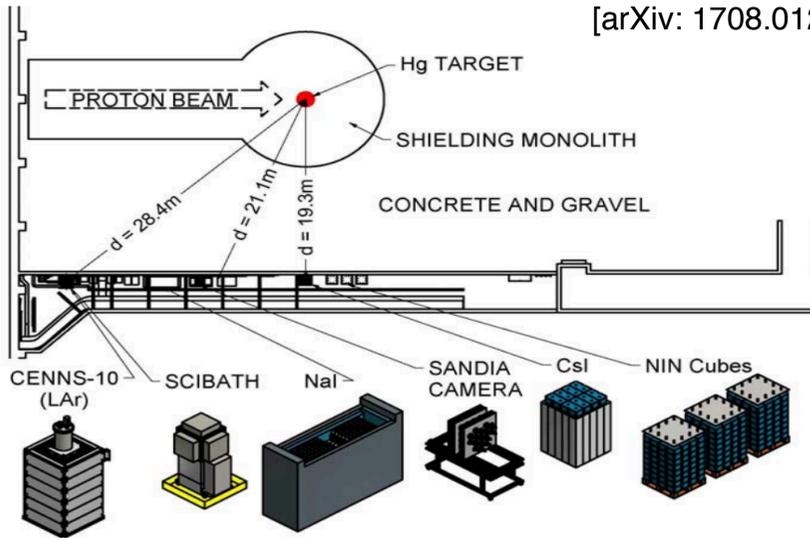
the probability of interaction would be enhanced by the square of the number of neutrons in the nucleus.

$$E_r \sim \frac{E_\nu^2}{M_{\text{LAr}}} \sim 10 \text{ KeV}$$

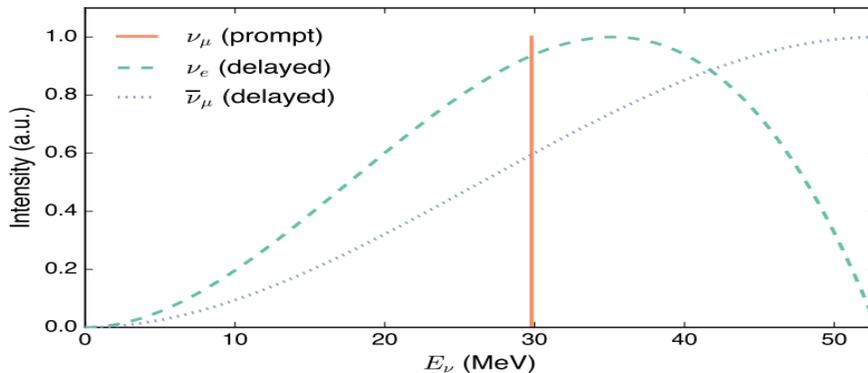
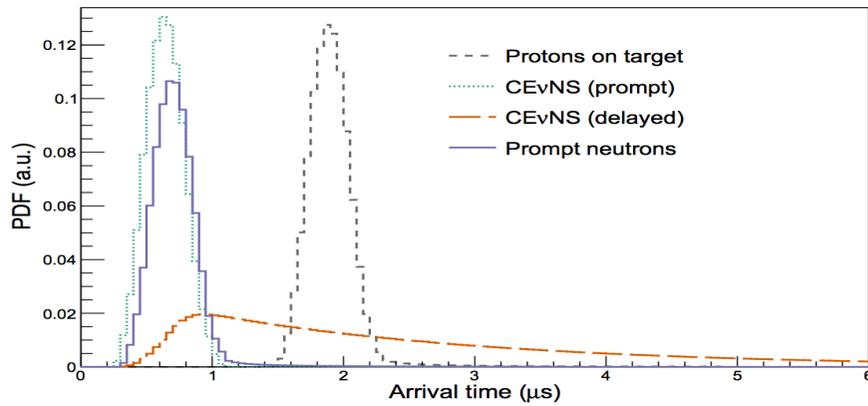
[1708.01294]

COHERENT Experiment

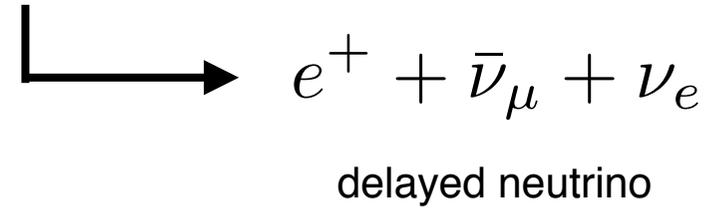
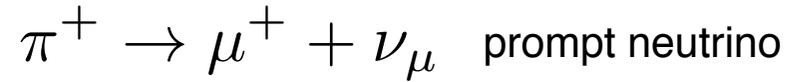
[arXiv: 1708.01294]



[arXiv: 1804.09459]



Time distribution



Energy distribution

$$\phi_{\nu_\mu}(E_\nu) = N \delta\left(E_\nu - \frac{m_\pi^2 - m_\mu^2}{2m_\pi}\right)$$

$$\phi_{\bar{\nu}_\mu}(E_\nu) = N \frac{64E_\nu^2}{m_\mu^3} \left(\frac{3}{4} - \frac{E_\nu}{m_\mu}\right)$$

$$\phi_{\nu_e}(E_\nu) = N \frac{192E_\nu^2}{m_\mu^3} \left(\frac{1}{2} - \frac{E_\nu}{m_\mu}\right)$$

Coherent elastic neutrino-nucleus scattering (CEvNS)

Differential cross section:
$$\frac{d\sigma_\alpha}{dE_r} = \frac{G_F^2}{2\pi} Q_\alpha^2 F^2(Q^2) M \left(2 - \frac{ME_r}{E_\nu^2}\right)$$

Effective charge:
$$Q_\alpha^2 = [Z(g_p^V + 2\epsilon_{\alpha\alpha}^{uV} + \epsilon_{\alpha\alpha}^{dV}) + N(g_n^V + \epsilon_{\alpha\alpha}^{uV} + 2\epsilon_{\alpha\alpha}^{dV})]^2$$

$$g_p^V = \frac{1}{2} - 2 \sin^2 \theta_W \qquad g_n^V = \frac{1}{2}$$

$$\epsilon_{ee}^{uV} = \epsilon_{ee}^{dV} = \frac{g'^2 Q'_q Q'_e}{\sqrt{2} G_F (2ME_r + M_{Z'}^2)} \qquad \epsilon_{\mu\mu}^{uV} = \epsilon_{\mu\mu}^{dV} = \frac{g'^2 Q'_q Q'_\mu}{\sqrt{2} G_F (2ME_r + M_{Z'}^2)}$$

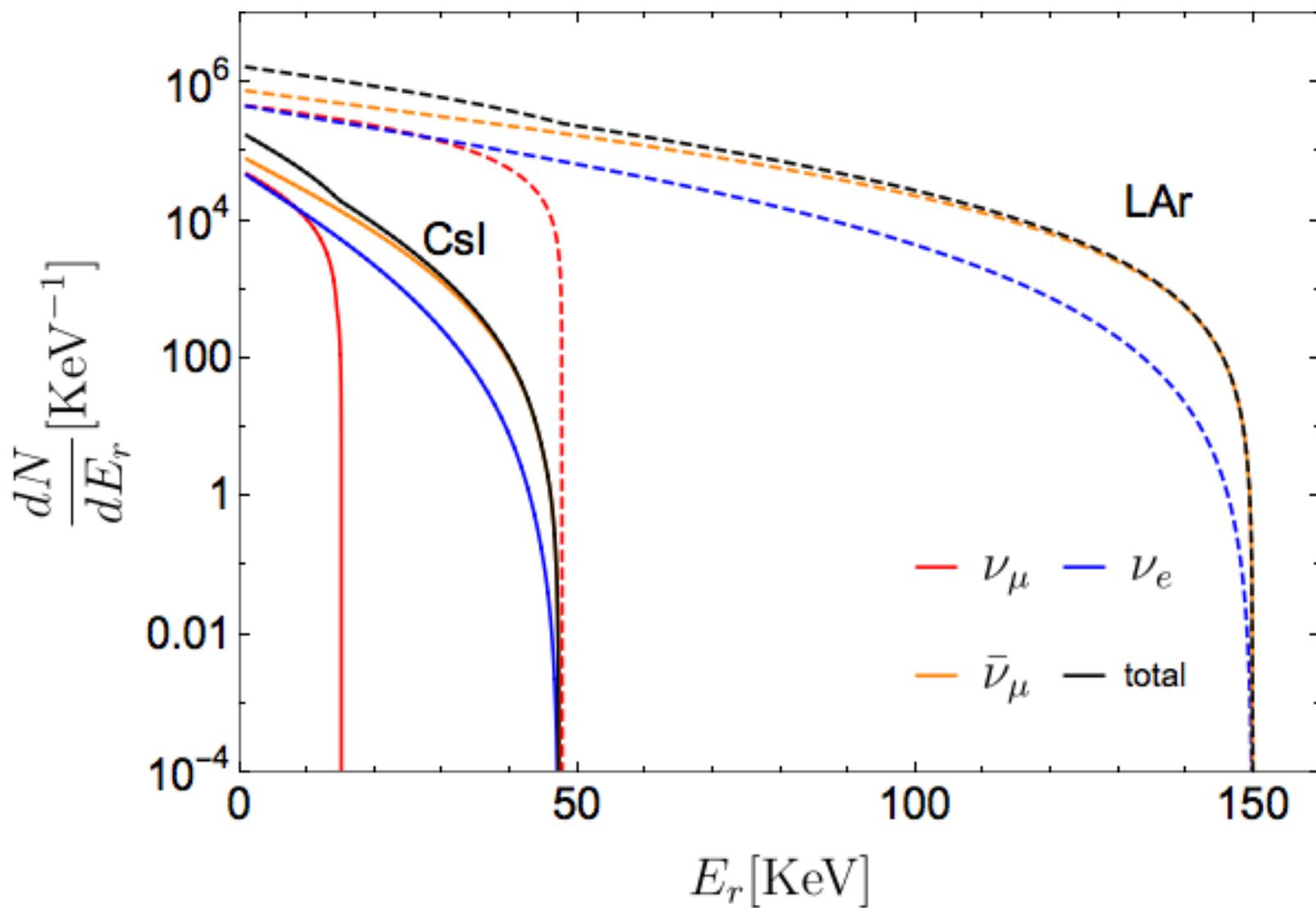
Number of events per bin

$$N_{th}(t, E_r, \epsilon) = \sum_\alpha \frac{m_{\text{det}} N_A}{M} \int_{\Delta E_r} dE_r \int_{\Delta t} dt \rho_\alpha(t) \int_{E_\nu^{\text{min}}}^{E_\nu^{\text{max}}} dE_\nu \phi_\alpha(E_\nu) \frac{d\sigma_\alpha(\epsilon)}{dE_r}$$

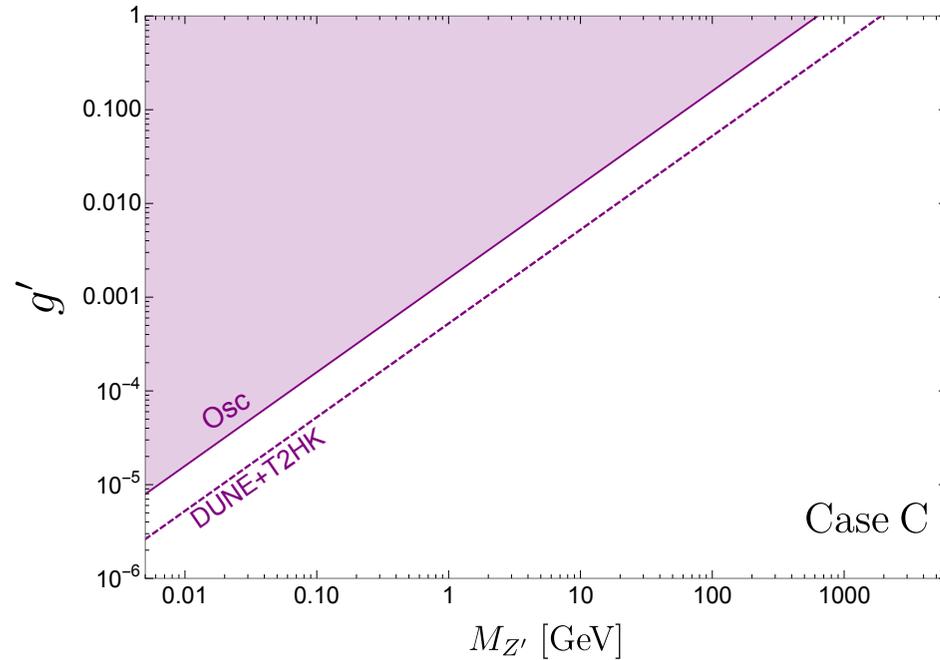
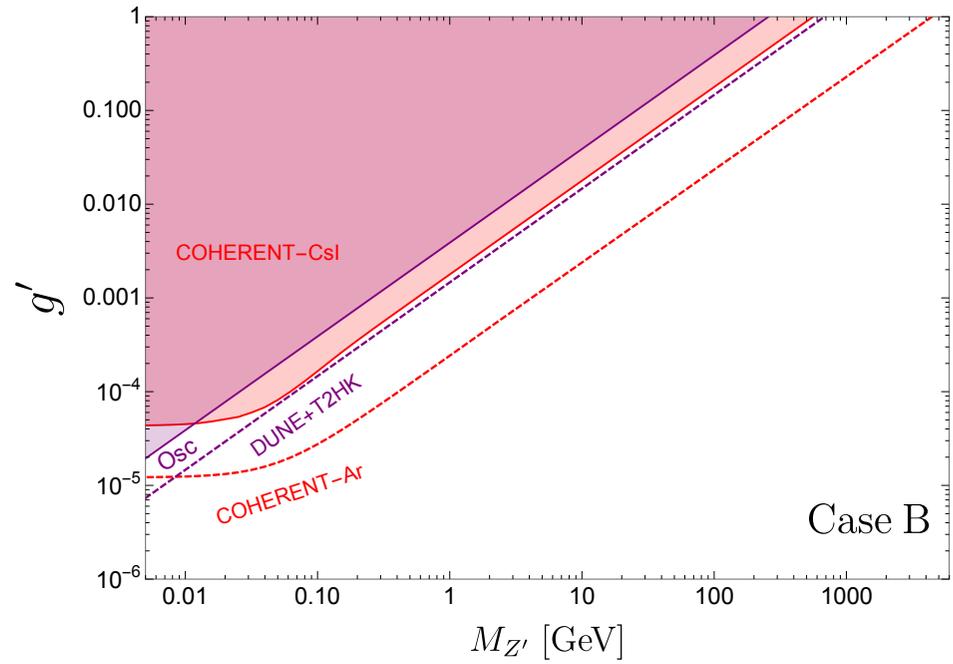
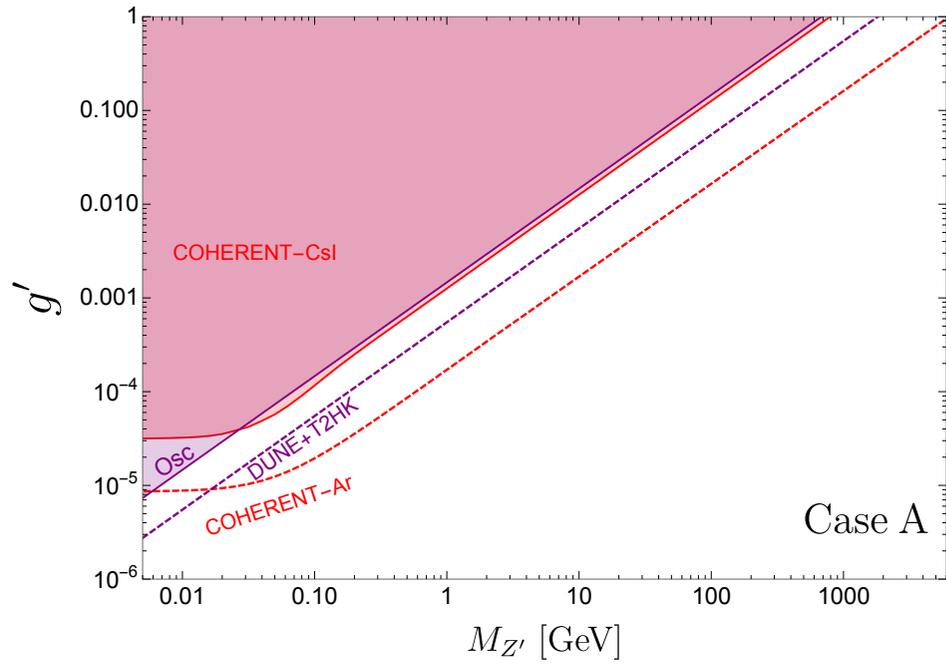
COHERENT CEvNS Detectors

	Nuclear Target	Mass (kg)	Exposure	Recoil threshold (keVr)
Current	CsI	14.6	308.1 (day)	6.5
Future	LAr	750	4 (yr)	20

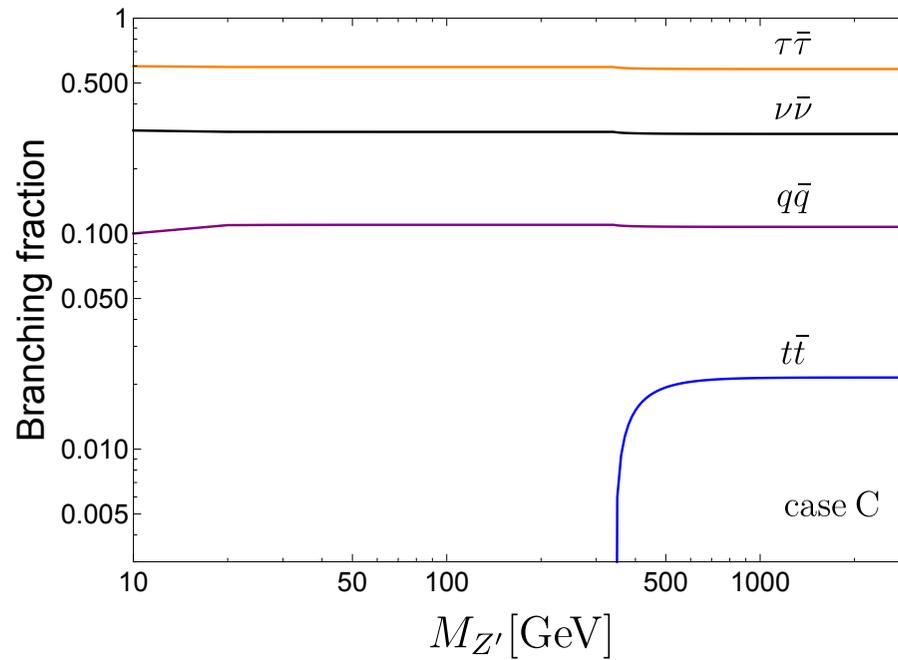
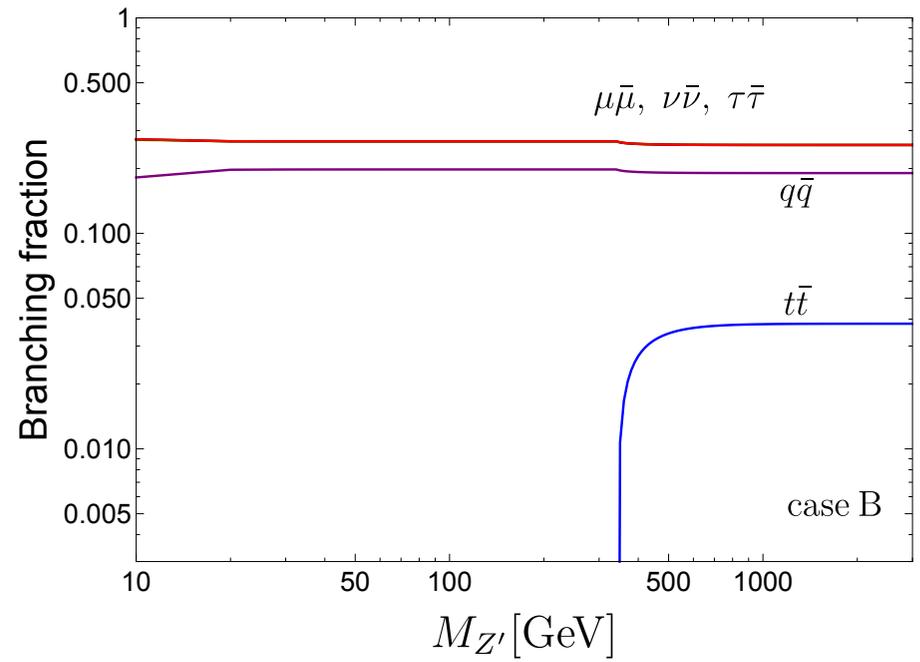
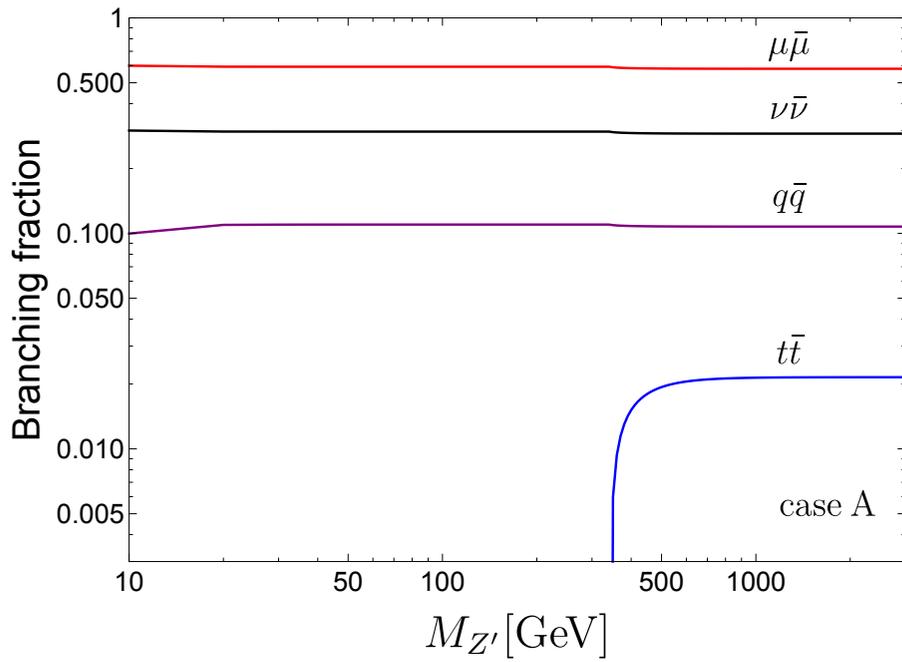
COHERENT CEvNS Detectors



DUNE+T2HK+COHERENT



LHC Searches



Collider Searches for Case A & B

$$pp \rightarrow Z' \rightarrow \ell^+ \ell^- + X$$

Case A and B : $\ell = \mu$

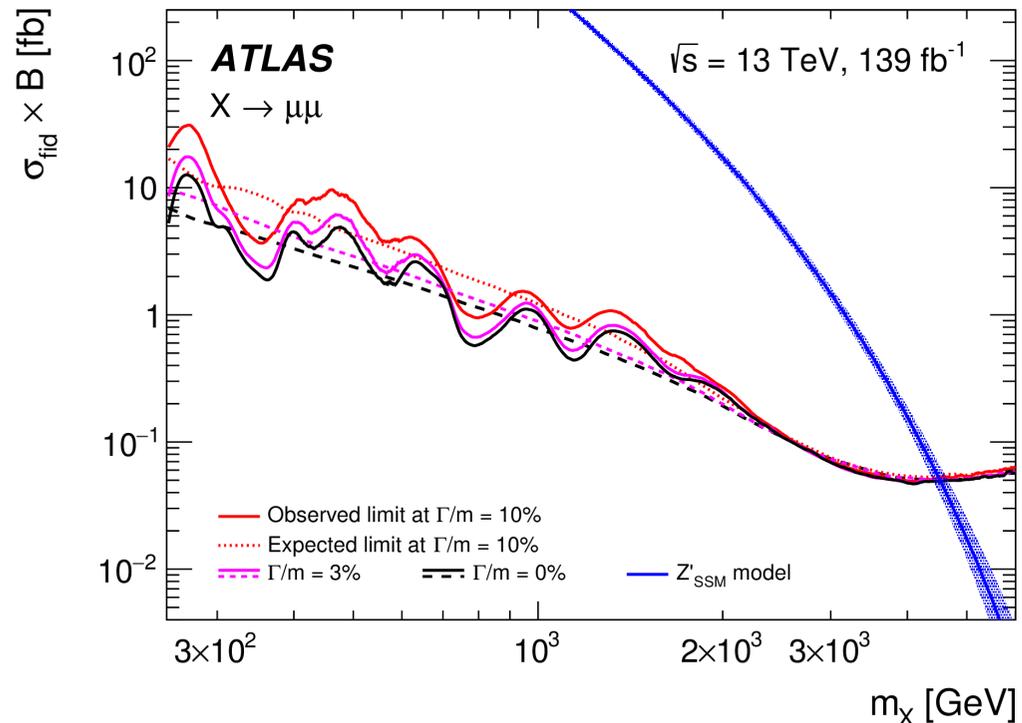
ATLAS provides di-lepton searches at $\sqrt{s} = 13$ TeV with 139 fb^{-1} integrated luminosity in the mass range (250 GeV , 6 TeV)

fiducial region

$$p_T^\mu > 30 \text{ GeV}$$

$$|\eta_\mu| < 2.5$$

$$m_{\ell\ell} > M_{Z'} - 2\Gamma_{Z'}$$



HL-LHC Projection for Case A & B

We estimate the sensitivity reach at HL-LHC with 3000 fb^{-1} integrated luminosity.

Selection rules:

$$p_T^{\mu_1} > 22 \text{ GeV} \quad p_T^{\mu_2} > 10 \text{ GeV} \quad |\eta_\mu| < 2.4$$

$$0.97 M_{Z'} < M(\ell^+ \ell^-) < 1.03 M_{Z'} \quad \text{below 3 TeV}$$

$$3 \text{ TeV} < M(\ell^+ \ell^-) < 6 \text{ TeV} \quad \text{above 3 TeV}$$

Main background : DY, ttbar, tW, VV

Collider Searches for Case C

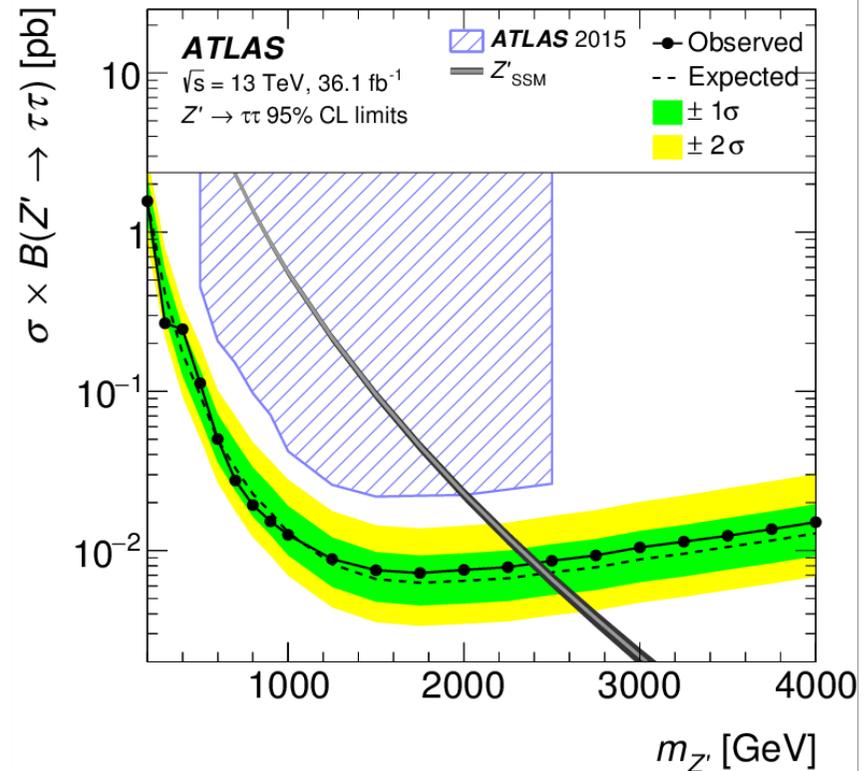
$$pp \rightarrow Z' \rightarrow \ell^+ \ell^- + X$$

Case C : $\ell = \tau$

ATLAS provides di-tau searches at $\sqrt{s} = 13$ TeV with 36.1 fb^{-1} integrated luminosity in the mass range (200 GeV , 4 TeV).

Combined three decay channels of di-tau

- semi-leptonic: $\tau_e \tau_h$ (23%), $\tau_\mu \tau_h$ (23%)
- full-hadronic: $\tau_h \tau_h$ (42%)



HL-LHC Projection for Case C

We estimate the sensitivity reach at HL-LHC with 3000 fb^{-1} integrated luminosity in the mass range (20 GeV , 6 TeV).

We combined the relatively clean full-leptonic and semi-leptonic decay modes of taus

$$\tau_e \tau_\mu (6\%) , \tau_e \tau_h (23\%) , \tau_\mu \tau_h (23\%)$$

- Main background for full-leptonic : DY, ttbar, WW
- Main background for semi-leptonic : DY, W+jets, QCD multijet (6% for $\mu+h$, 28% for $e+h$).

Apply two different selection rules SR1 and SR2 for the mass of Z' below and above the Z-pole, respectively.

Full-leptonic Channel

Both SR1 and SR2 require

- Only one muon and one oppositely charged electron with $p_T > 20$ GeV and $|\eta| < 2.4$,
- veto b-tagged jets,
- $0.2M_{Z'} < M_{\tau_1\tau_2} < 0.8M_{Z'}$,
- $M_T^\mu < 40$ GeV,

$$M_T^\mu = \sqrt{2P_T^\mu \cdot E_T^{\text{miss}}(1 - \cos \Delta\phi(\mu, \vec{E}_T^{\text{miss}}))}$$

SR1 further requires

- $\Delta R(\tau_1, \tau_2) < \Delta R_{\text{cut}}$

ΔR_{cut} and $E_{T,\text{cut}}^{\text{miss}}$ are varied with $M_{Z'}$

SR2 further requires

to optimize the significance

- $\cos \Delta\phi(\tau_1, \tau_2) < -0.95$,
- $\cos \Delta\phi(\tau_1, \vec{E}_T) + \cos \Delta\phi(\tau_2, \vec{E}_T) > -0.1$,
- $E_T^{\text{miss}} > E_{T,\text{cut}}^{\text{miss}}$

Semi-leptonic Channel

Both SR1 and SR2 require

- Only one muon and at least one opposite-sign tau-tagged jet with $p_T > 20$ GeV and $|\eta| < 2.4$,
- veto b-tagged jets,
- $0.3M_{Z'} < M_{\tau_1\tau_2} < 0.9M_{Z'}$,
- $M_T^\ell < 40$ GeV,

SR1 further requires

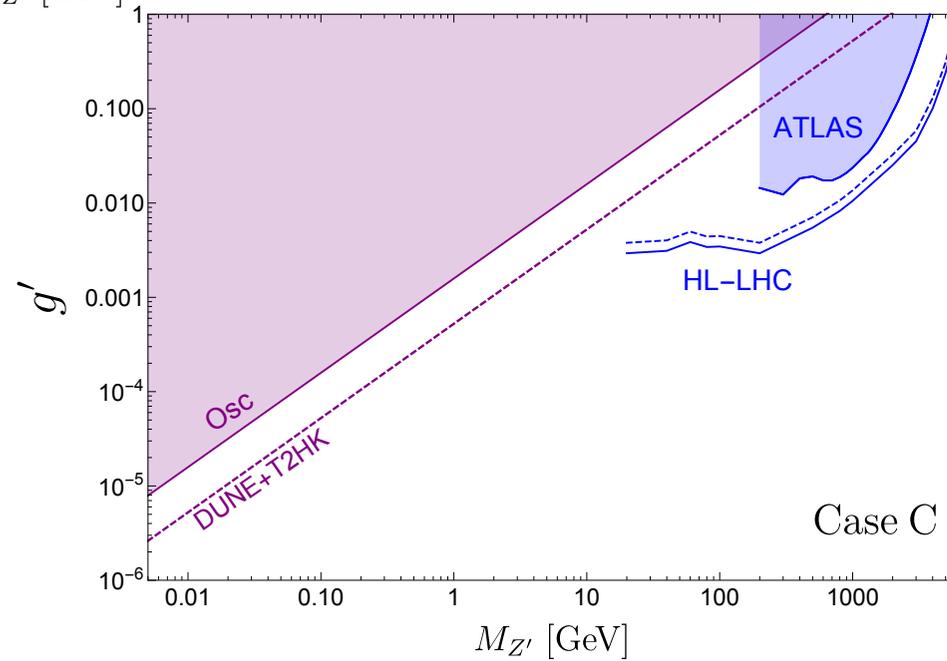
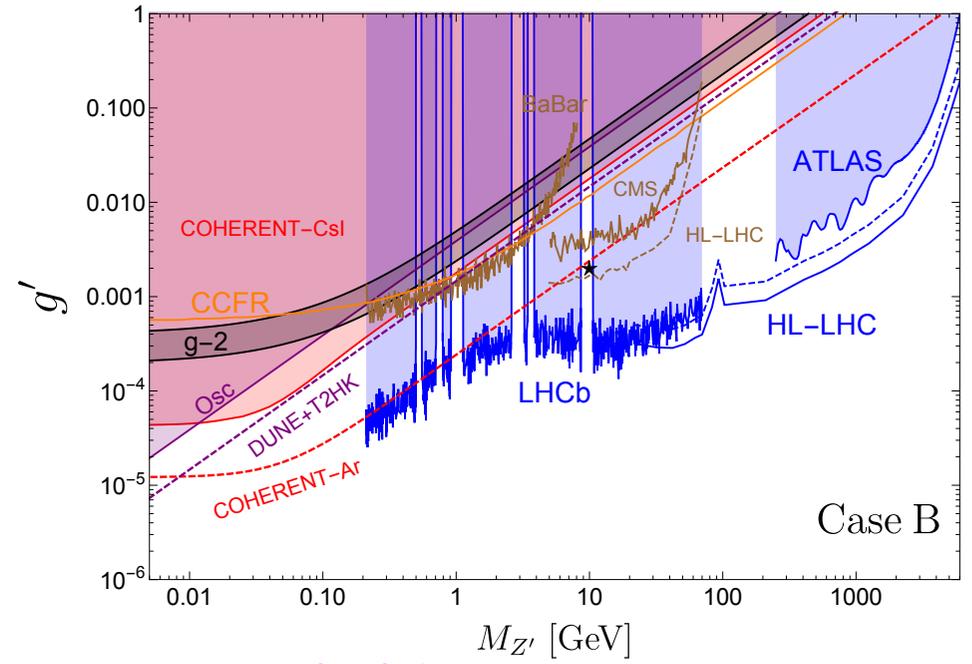
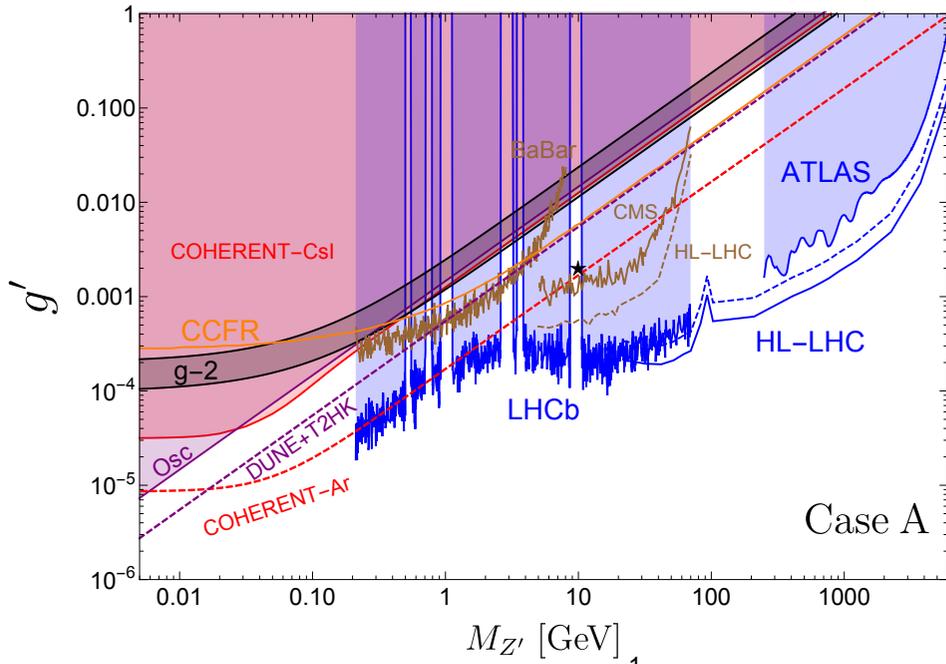
- $\Delta R(\tau_1, \tau_2) < \Delta R_{\text{cut}}$

ΔR_{cut} and $E_{T,\text{cut}}^{\text{miss}}$ are varied with $M_{Z'}$
to optimize the significance

SR2 further requires

- $\cos \Delta\phi(\tau_1, \tau_2) < -0.95$,
- $\cos \Delta\phi(\tau_1, \vec{E}_T) + \cos \Delta\phi(\tau_2, \vec{E}_T) > -0.1$,
- $E_T^{\text{miss}} > E_{T,\text{cut}}^{\text{miss}}$

Results



Conclusion

Constraints from current data

- In Case A and B, neutrino oscillation and CEvNS experiments give the most stringent bounds for masses below the di-muon threshold.
- In Case A and B, ATLAS di-muon searches give the strongest bounds in the mass range, $250 \text{ GeV} \leq M_{Z'} \leq 6 \text{ TeV}$.
- The $(g - 2)_\mu$ favored region is excluded by a combination of the experiments in the mass range considered.
- In Case C, neutrino oscillation experiments set the strongest constraints up to 200 GeV. The LHC gives the strongest constraints for $200 \text{ GeV} \leq M_{Z'} \leq 4 \text{ TeV}$

Future projections

- In Case A and B, neutrino oscillation and CEvNS experiments give the most stringent bounds for masses below the di-muon threshold.
- We estimated the sensitivity of the high luminosity LHC with an integrated luminosity of 3 ab^{-1} and find that the reach of the $Z' \rightarrow \ell\ell$ channel is significantly improved in all of three scenarios.
- If the new gauge boson couples to first and second generation leptons, future CEvNS data can set stronger bounds than next-generation neutrino oscillation experiments in almost the entire mass range.

Thanks!

Back-up slides

Current Oscillation Constraints

OSC			+COHERENT		
	LMA	LMA \oplus LMA-D		LMA	LMA \oplus LMA-D
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	$[-0.020, +0.456]$	$\oplus[-1.192, -0.802]$	ε_{ee}^u	$[-0.008, +0.618]$	$[-0.008, +0.618]$
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	$[-0.005, +0.130]$	$[-0.152, +0.130]$	$\varepsilon_{\mu\mu}^u$	$[-0.111, +0.402]$	$[-0.111, +0.402]$
$\varepsilon_{e\mu}^u$	$[-0.060, +0.049]$	$[-0.060, +0.067]$	$\varepsilon_{\tau\tau}^u$	$[-0.110, +0.404]$	$[-0.110, +0.404]$
$\varepsilon_{e\tau}^u$	$[-0.292, +0.119]$	$[-0.292, +0.336]$	$\varepsilon_{e\mu}^u$	$[-0.060, +0.049]$	$[-0.060, +0.049]$
$\varepsilon_{\mu\tau}^u$	$[-0.013, +0.010]$	$[-0.013, +0.014]$	$\varepsilon_{e\tau}^u$	$[-0.248, +0.116]$	$[-0.248, +0.116]$
$\varepsilon_{ee}^d - \varepsilon_{\mu\mu}^d$	$[-0.027, +0.474]$	$\oplus[-1.232, -1.111]$	$\varepsilon_{\mu\tau}^u$	$[-0.012, +0.009]$	$[-0.012, +0.009]$
$\varepsilon_{\tau\tau}^d - \varepsilon_{\mu\mu}^d$	$[-0.005, +0.095]$	$[-0.013, +0.095]$	ε_{ee}^d	$[-0.012, +0.565]$	$[-0.012, +0.565]$
$\varepsilon_{e\mu}^d$	$[-0.061, +0.049]$	$[-0.061, +0.073]$	$\varepsilon_{\mu\mu}^d$	$[-0.103, +0.361]$	$[-0.103, +0.361]$
$\varepsilon_{e\tau}^d$	$[-0.247, +0.119]$	$[-0.247, +0.119]$	$\varepsilon_{\tau\tau}^d$	$[-0.102, +0.361]$	$[-0.102, +0.361]$
$\varepsilon_{\mu\tau}^d$	$[-0.012, +0.009]$	$[-0.012, +0.009]$	$\varepsilon_{e\mu}^d$	$[-0.058, +0.049]$	$[-0.058, +0.049]$
$\varepsilon_{ee}^p - \varepsilon_{\mu\mu}^p$	$[-0.041, +1.312]$	$\oplus[-3.327, -1.958]$	$\varepsilon_{e\tau}^d$	$[-0.206, +0.110]$	$[-0.206, +0.110]$
$\varepsilon_{\tau\tau}^p - \varepsilon_{\mu\mu}^p$	$[-0.015, +0.426]$	$[-0.424, +0.426]$	$\varepsilon_{\mu\tau}^d$	$[-0.011, +0.009]$	$[-0.011, +0.009]$
$\varepsilon_{e\mu}^p$	$[-0.178, +0.147]$	$[-0.178, +0.178]$	ε_{ee}^p	$[-0.010, +2.039]$	$[-0.010, +2.039]$
$\varepsilon_{e\tau}^p$	$[-0.954, +0.356]$	$[-0.954, +0.949]$	$\varepsilon_{\mu\mu}^p$	$[-0.364, +1.387]$	$[-0.364, +1.387]$
$\varepsilon_{\mu\tau}^p$	$[-0.035, +0.027]$	$[-0.035, +0.035]$	$\varepsilon_{\tau\tau}^p$	$[-0.350, +1.400]$	$[-0.350, +1.400]$
			$\varepsilon_{e\mu}^p$	$[-0.179, +0.146]$	$[-0.179, +0.146]$
			$\varepsilon_{e\tau}^p$	$[-0.860, +0.350]$	$[-0.860, +0.350]$
			$\varepsilon_{\mu\tau}^p$	$[-0.035, +0.028]$	$[-0.035, +0.028]$

[arXiv:1805.04530]

DUNE+T2HK

Table 1: Comparison of the experiments considered in this work.

Experiment	$\frac{L(\text{km})}{E_{\text{peak}}(\text{GeV})}$	$\nu + \bar{\nu}$ Exposure (kt·MW·10 ⁷ s)	Signal norm. uncertainty	Background norm. uncertainty
DUNE (LAr)	$\frac{1300}{3.0}$	264 + 264 (80 GeV protons, 1.07 MW power, 1.47×10 ²¹ POT/yr, 40 kt fiducial mass, 3.5+3.5 yr)	app: 2.0% dis: 5.0%	app: 5-20% dis: 5-20%
T2HK (WC)	$\frac{295}{0.6}$	864.5 + 2593.5 (30 GeV protons, 1.3 MW power, 2.7×10 ²¹ POT/yr, 0.19 Mt each tank, 1.5+4.5 yr with 1 tank, 1+3 yr with 2 tanks)	app: 2.5% dis: 2.5%	app: 5% dis: 20%
T2HKK-1.5 (WC)	$\frac{295}{0.6} + \frac{1100}{0.8}$	1235 + 3705 (30 GeV protons, 1.3 MW power, 2.7×10 ²¹ POT/yr, 0.19 Mt each tank, 2.5+7.5 yr with 1 tank at KD and HK)	app: 2.5% dis: 2.5%	app: 5% dis: 20%
T2HKK-2.5 (WC)	$\frac{295}{0.6} + \frac{1100}{0.6}$			

[arXiv:1612.01443]

For DUNE, 1 yr = 1.76 × 10⁷s; for HyperK, 1 yr = 1.0 × 10⁷s.

appearance

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= x^2 f^2 + 2xyfg \cos(\Delta + \delta) + y^2 g^2 \\
 &+ 4\hat{A}\epsilon_{e\mu} \left\{ xf[s_{23}^2 f \cos(\phi_{e\mu} + \delta) + c_{23}^2 g \cos(\Delta + \delta + \phi_{e\mu})] \right. \\
 &\quad \left. + yg[c_{23}^2 g \cos \phi_{e\mu} + s_{23}^2 f \cos(\Delta - \phi_{e\mu})] \right\} \\
 &+ 4\hat{A}\epsilon_{e\tau} s_{23} c_{23} \left\{ xf[f \cos(\phi_{e\tau} + \delta) - g \cos(\Delta + \delta + \phi_{e\tau})] \right. \\
 &\quad \left. - yg[g \cos \phi_{e\tau} - f \cos(\Delta - \phi_{e\tau})] \right\} \\
 &+ 4\hat{A}^2 (g^2 c_{23}^2 |c_{23}\epsilon_{e\mu} - s_{23}\epsilon_{e\tau}|^2 + f^2 s_{23}^2 |s_{23}\epsilon_{e\mu} + c_{23}\epsilon_{e\tau}|^2) \\
 &+ 8\hat{A}^2 fg s_{23} c_{23} \left\{ c_{23} \cos \Delta [s_{23}(\epsilon_{e\mu}^2 - \epsilon_{e\tau}^2) + 2c_{23}\epsilon_{e\mu}\epsilon_{e\tau} \cos(\phi_{e\mu} - \phi_{e\tau})] \right. \\
 &\quad \left. - \epsilon_{e\mu}\epsilon_{e\tau} \cos(\Delta - \phi_{e\mu} + \phi_{e\tau}) \right\} + \mathcal{O}(s_{13}^2 \epsilon, s_{13}\epsilon^2, \epsilon^3),
 \end{aligned}$$

where following Ref. [27],

$$\begin{aligned}
 x &\equiv 2s_{13}s_{23}, \quad y \equiv 2rs_{12}c_{12}c_{23}, \quad r = |\delta m_{21}^2/\delta m_{31}^2|, \\
 f, \bar{f} &\equiv \frac{\sin[\Delta(1 \mp \hat{A}(1 + \epsilon_{ee}))]}{(1 \mp \hat{A}(1 + \epsilon_{ee}))}, \quad g \equiv \frac{\sin(\hat{A}(1 + \epsilon_{ee})\Delta)}{\hat{A}(1 + \epsilon_{ee})}, \\
 \Delta &\equiv \left| \frac{\delta m_{31}^2 L}{4E} \right|, \quad \hat{A} \equiv \left| \frac{A}{\delta m_{31}^2} \right|.
 \end{aligned}$$

disappearance

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_\mu) &= 1 - \sin^2 2\theta_{23} \sin^2 \Delta + 4rc_{12}^2 c_{23}^2 s_{23}^2 \Delta \sin 2\Delta - \frac{4s_{23}^4 s_{13}^2 \sin^2(1 - \hat{A})\Delta}{(1 - \hat{A})^2} \\
 &- \frac{\sin^2 2\theta_{23} s_{13}^2}{(1 - \hat{A})^2} \left[\hat{A}(1 - \hat{A})\Delta \sin 2\Delta + \sin(1 - \hat{A})\Delta \sin(1 + \hat{A})\Delta \right] \\
 &- 2\hat{A}\epsilon_{\mu\tau} \cos \phi_{\mu\tau} (\sin^3 2\theta_{23} \Delta \sin 2\Delta + 2 \sin 2\theta_{23} \cos^2 2\theta_{23} \sin^2 \Delta) \\
 &+ \hat{A}(\epsilon_{\mu\mu} - \epsilon_{\tau\tau}) \sin^2 2\theta_{23} \cos 2\theta_{23} (\Delta \sin 2\Delta - 2 \sin^2 \Delta) \\
 &- 2\hat{A}^2 \sin^2 2\theta_{23} \epsilon_{\mu\tau}^2 (2 \sin^2 2\theta_{23} \cos^2 \phi_{\mu\tau} \Delta^2 \cos 2\Delta + \sin^2 \phi_{\mu\tau} \Delta \sin 2\Delta) \\
 &- \hat{A}^2 \sin^4 2\theta_{23} (\epsilon_{\mu\mu} - \epsilon_{\tau\tau})^2 \left(\frac{1}{2} \Delta \sin 2\Delta - \sin^2 \Delta \right) \\
 &+ \mathcal{O}(s_{13}^2 \epsilon, r\epsilon, s_{13}\epsilon^2, \cos 2\theta_{23}\epsilon^2, \epsilon^3).
 \end{aligned}$$

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