More Axions from Strings

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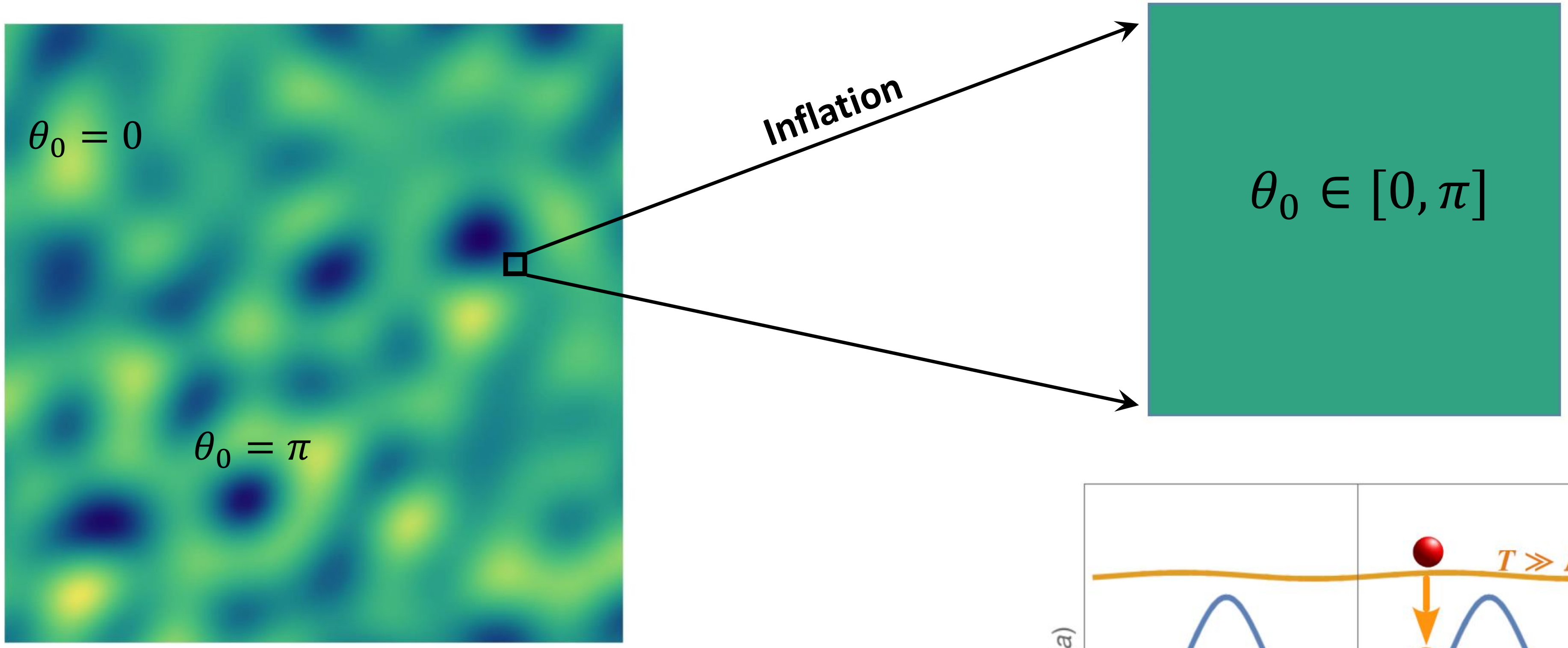
[1806.04677 + 2007.04990]

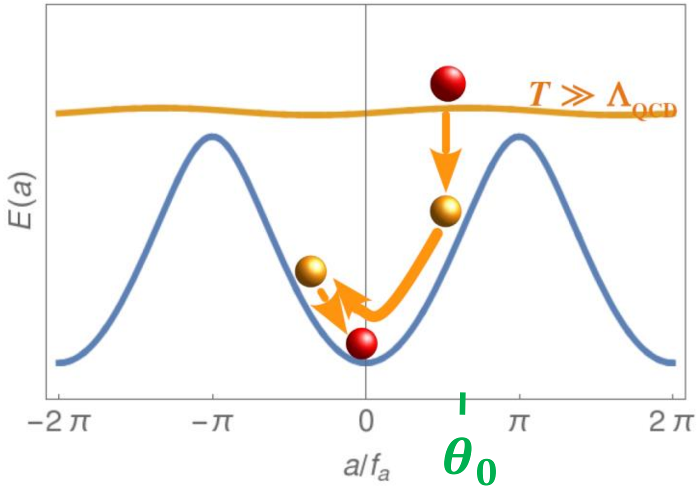
Axion Cosmology

Pre-inflationary:

$f_a \gtrsim H_I, T_R$



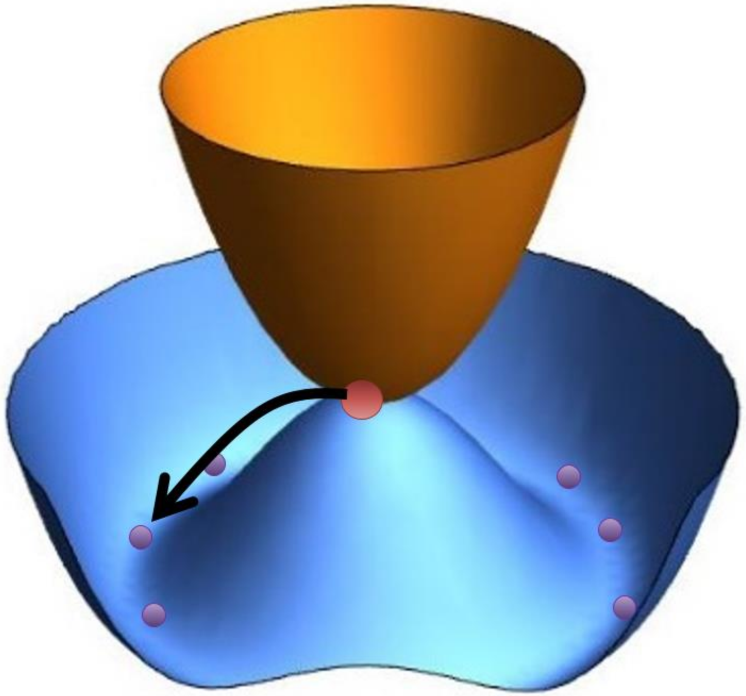
$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\mathrm{GeV}}\right]^{1+\epsilon}$$



The PQ Phase Transition

$$T \gtrsim f_a$$

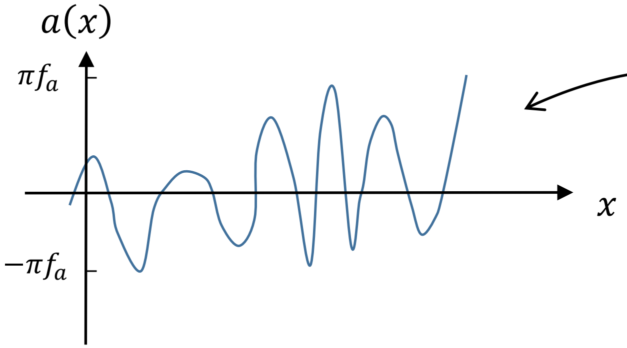
$$T \lesssim f_a$$



$$\phi = |\phi| e^{i\frac{a}{f_a}}$$

Similarly if:

$$H_I \gtrsim f_a$$

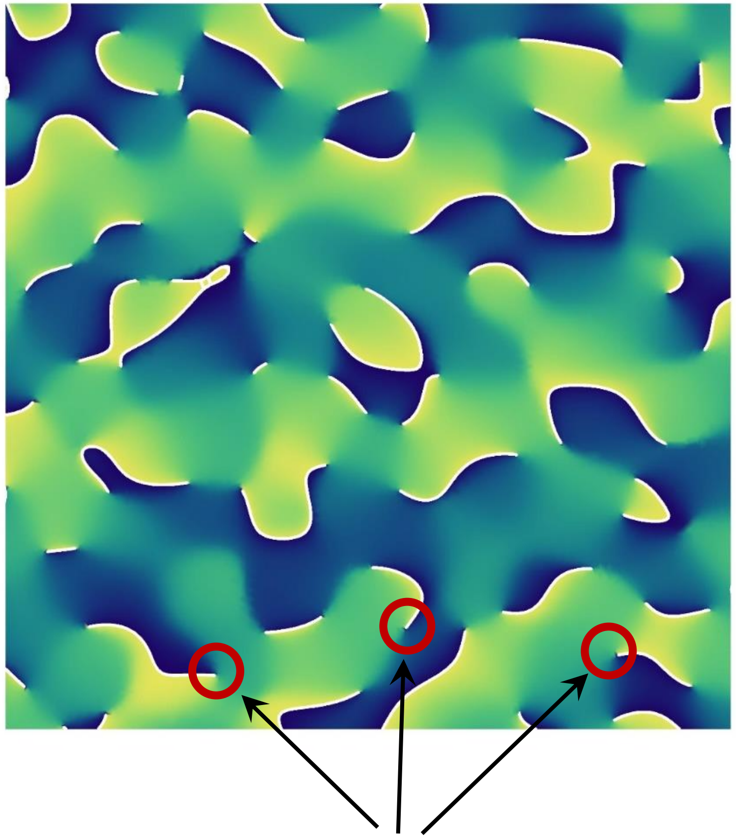


after PQ breaking axion field has random fluctuations over the observable universe

The Post-Inflationary scenario

Post-inflationary:

$$f_a \lesssim H_I, T_R$$



Axion Strings

⬇

non-linear evolution

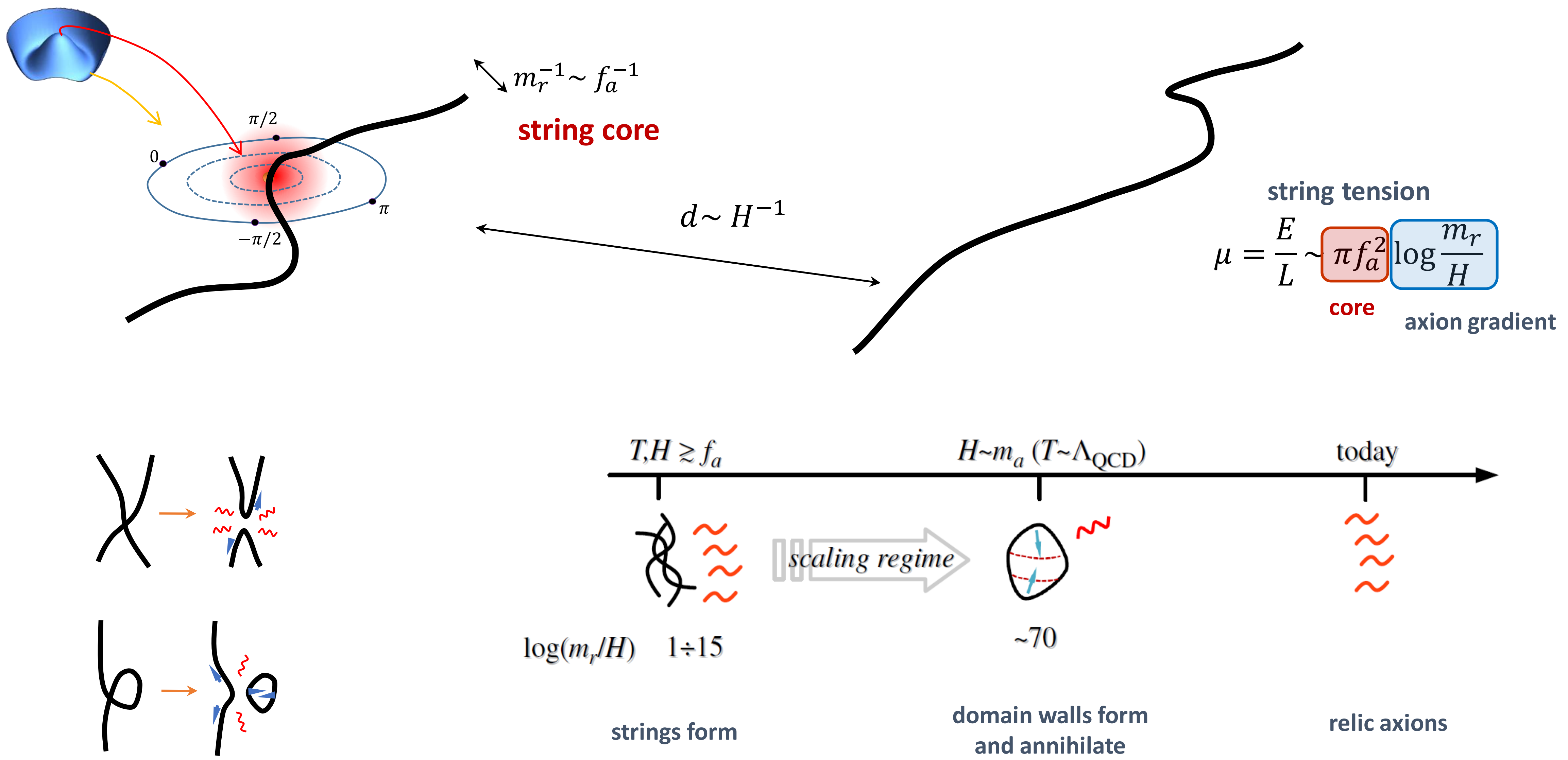
no free parameters in the initial conditions

⬇

$$\Omega_a = \Omega_a(f_a) \stackrel{!}{=} \Omega_{DM}$$

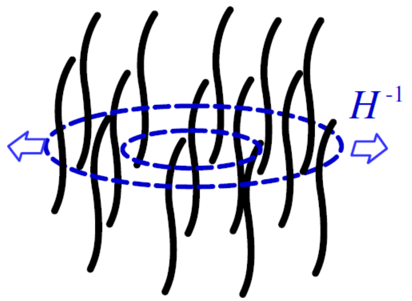
prediction for f_a!

Axion Strings



The Scaling Solution

free strings



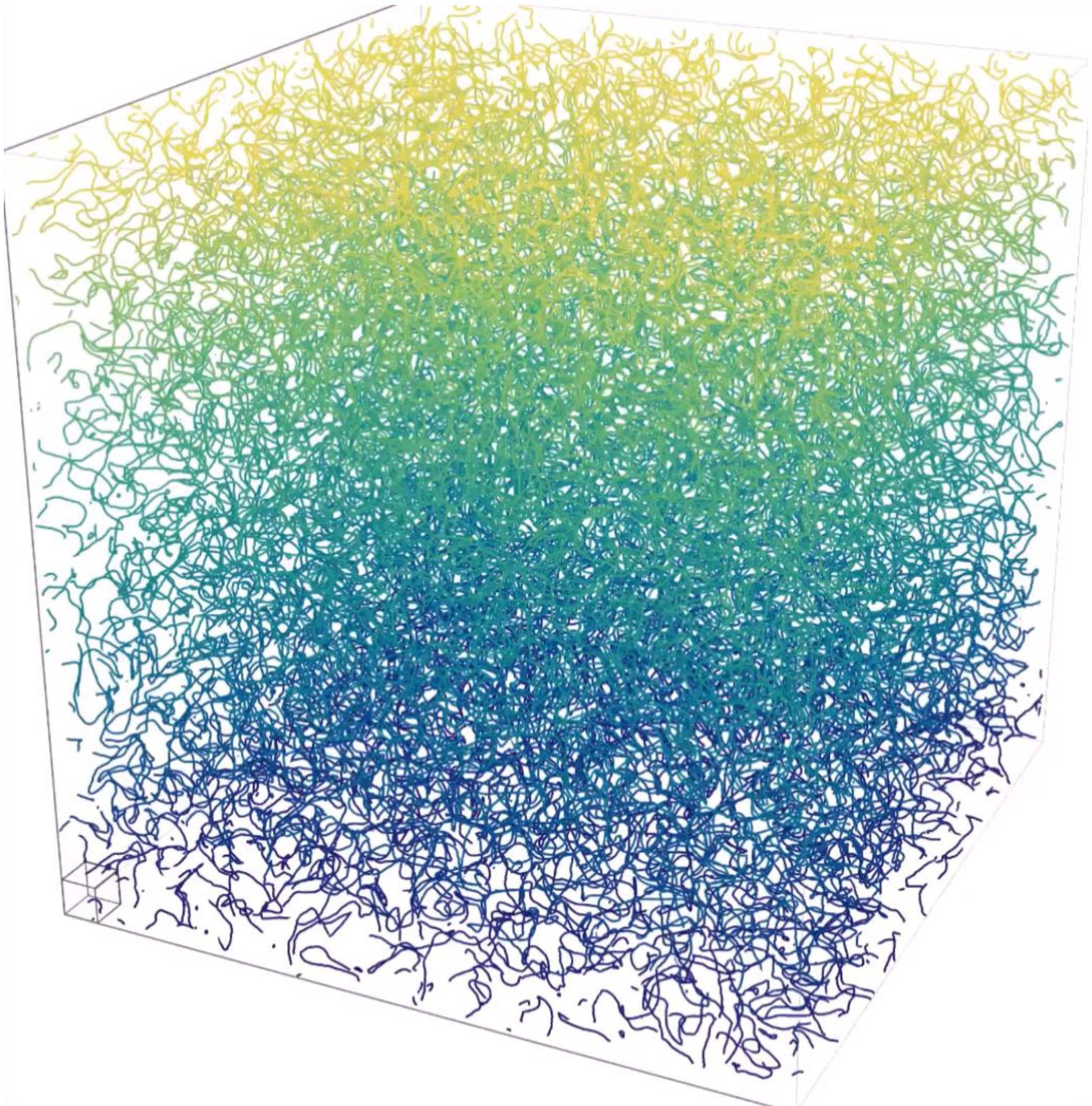
$$\rho_{\rm free} \propto \frac{1}{R^2(t)} = \frac{1}{t}$$

string recombination

scaling solution

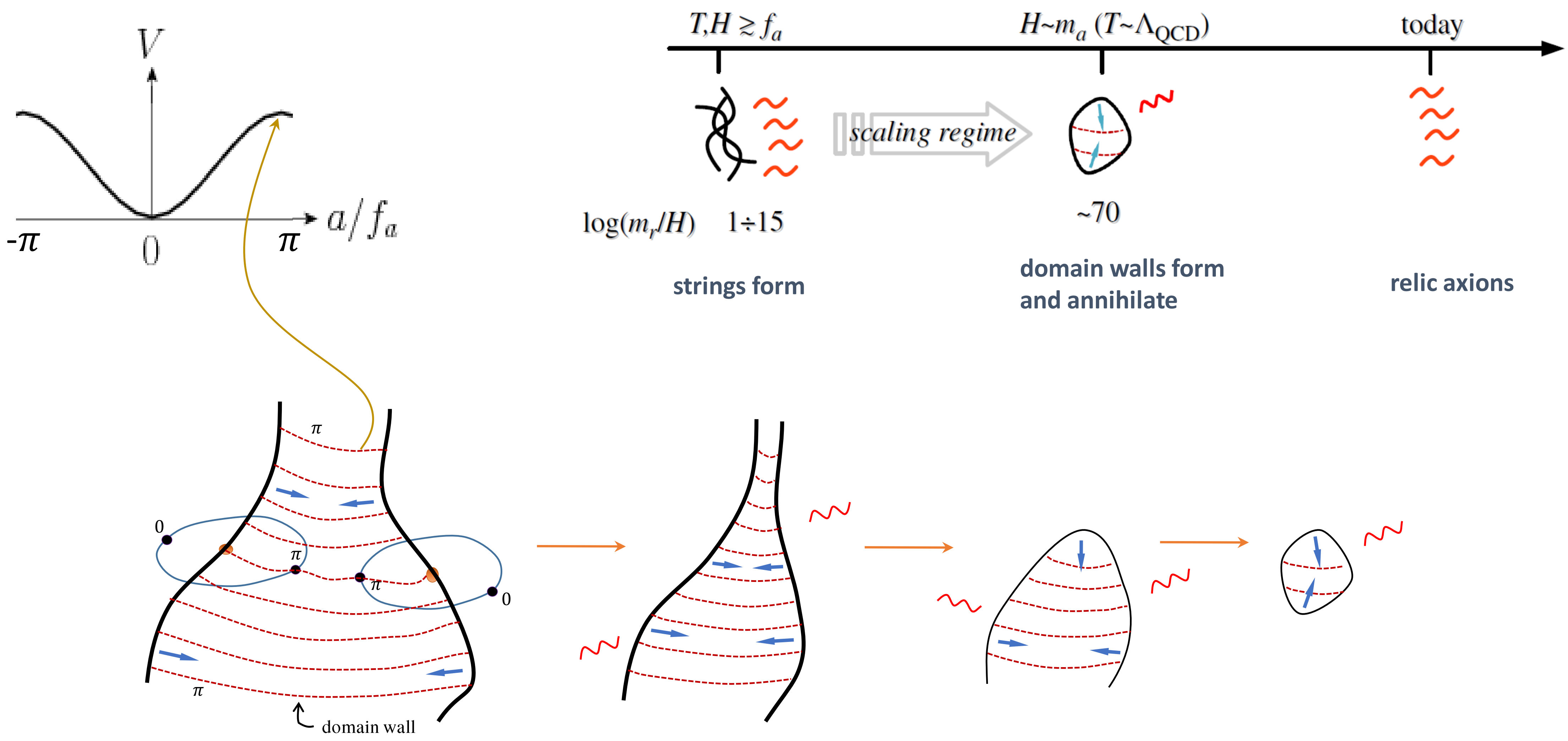
$$\rho_s = \xi \frac{\mu}{t^2}$$

ξ = (# strings) / (Hubble Patch)

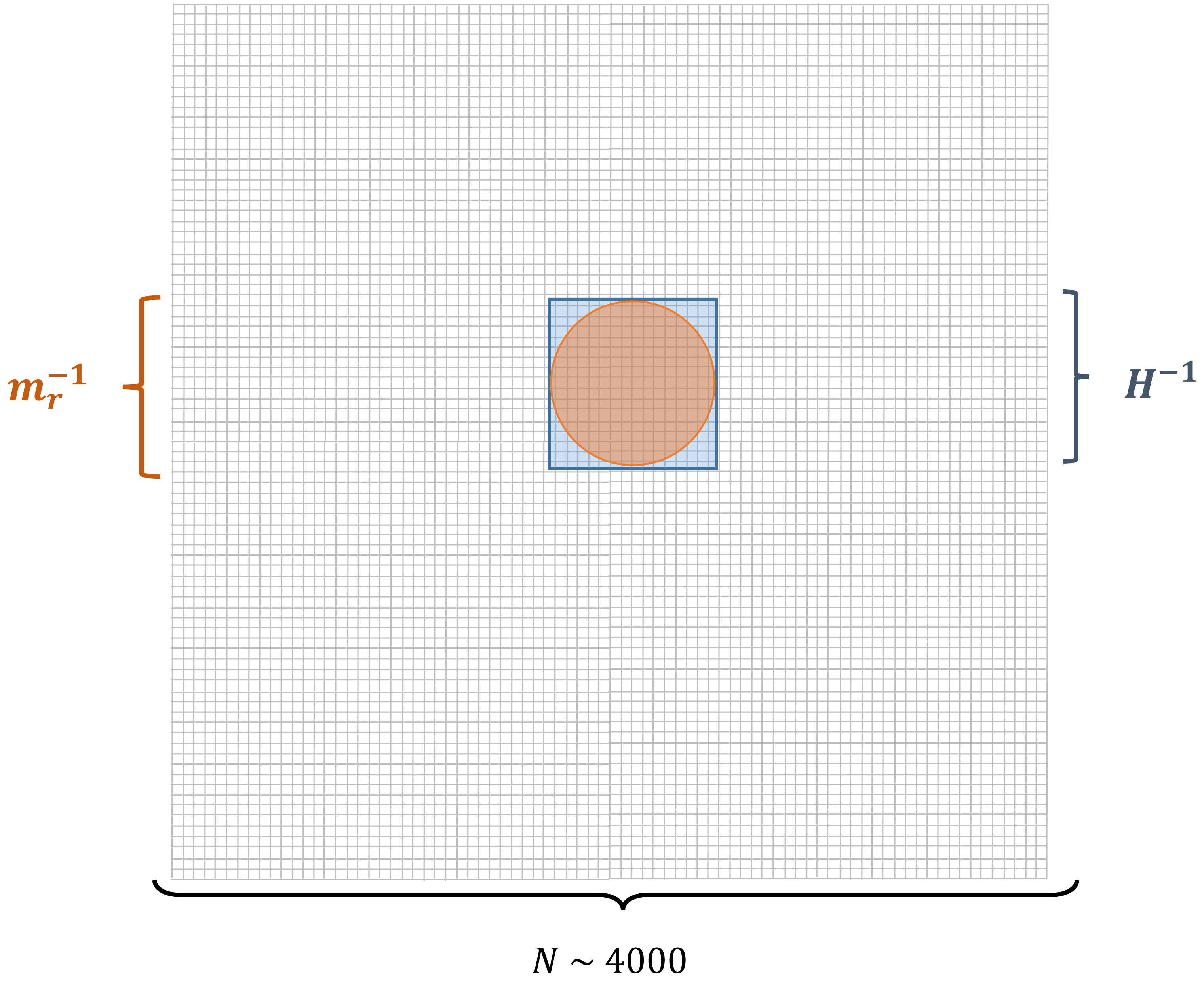


Axion Domain Walls

@ $H \sim m_a$ ($T \sim \Lambda_{QCD}$)

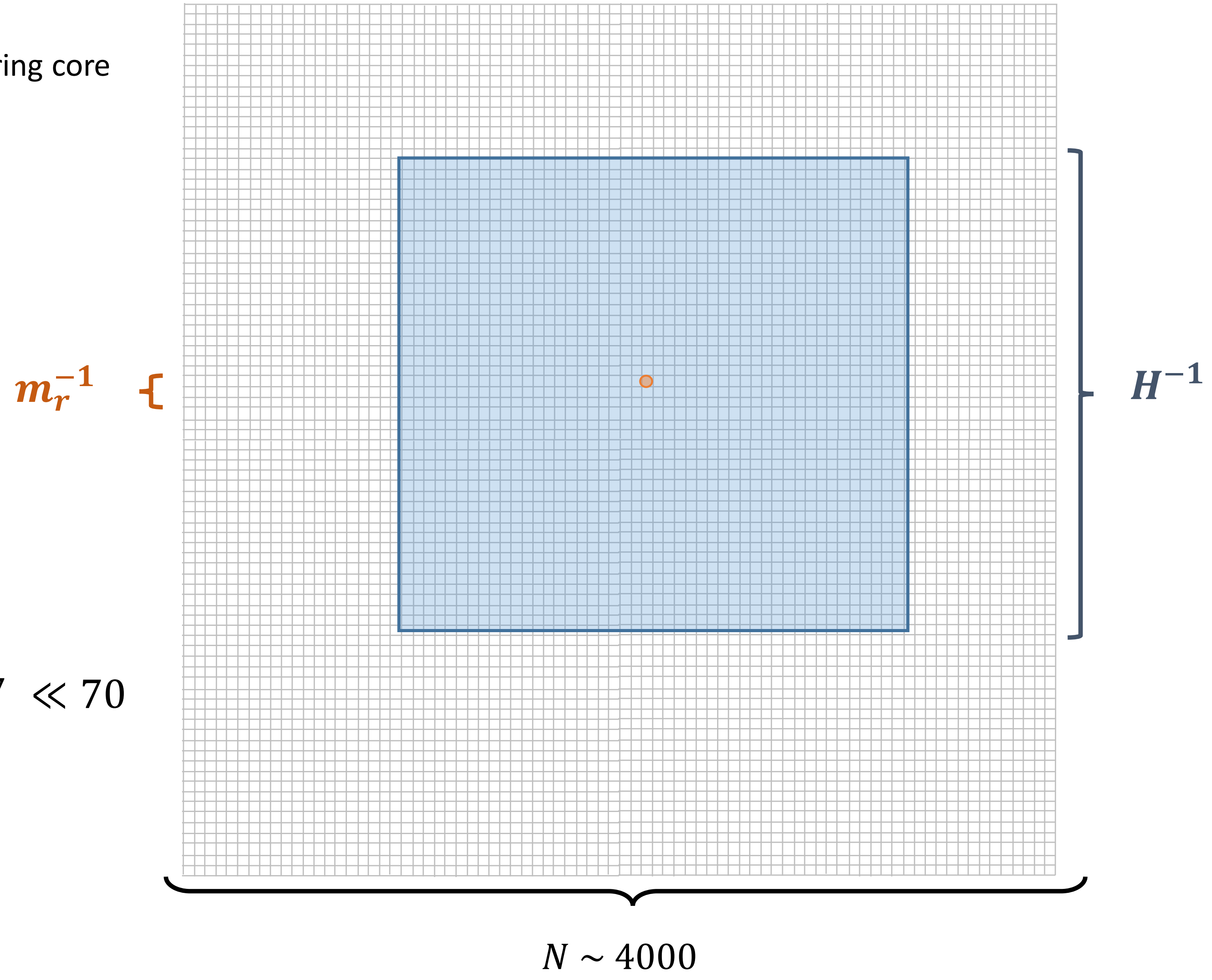


The Bottle Neck



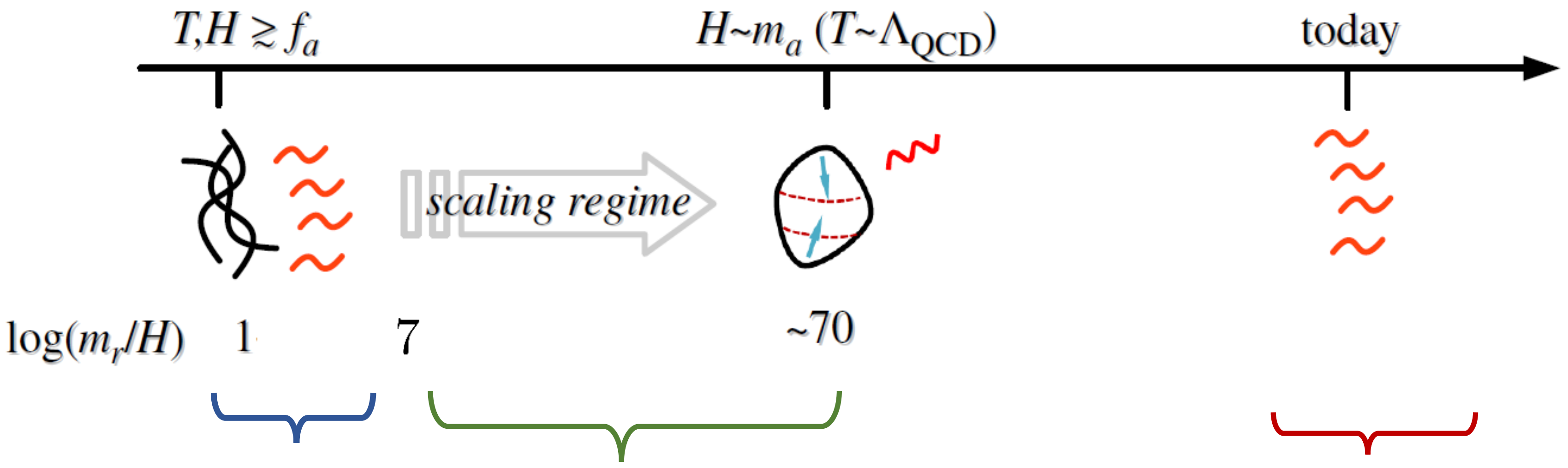
The Bottle Neck

- a few lattice points per string core
- a few Hubble patches



$$\log\frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 7 \ll 70$$

A Less Ambitious Goal: a Lower Bound



simulate and extract the attractor properties

reconstruct the attractor

lower bound on axions from the scaling regime

1) Energy in axions:

number of strings

$$\frac{d\rho_a}{dt} \simeq \frac{\boxed{\xi}\,\mu}{t^3}$$

2) Axion number density:

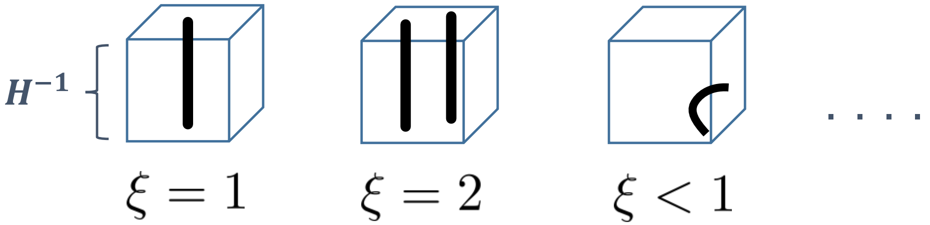
$$n_a = \int \frac{dk}{\omega_k} \boxed{\frac{\partial \rho_a}{\partial k}}$$

axion energy density spectrum

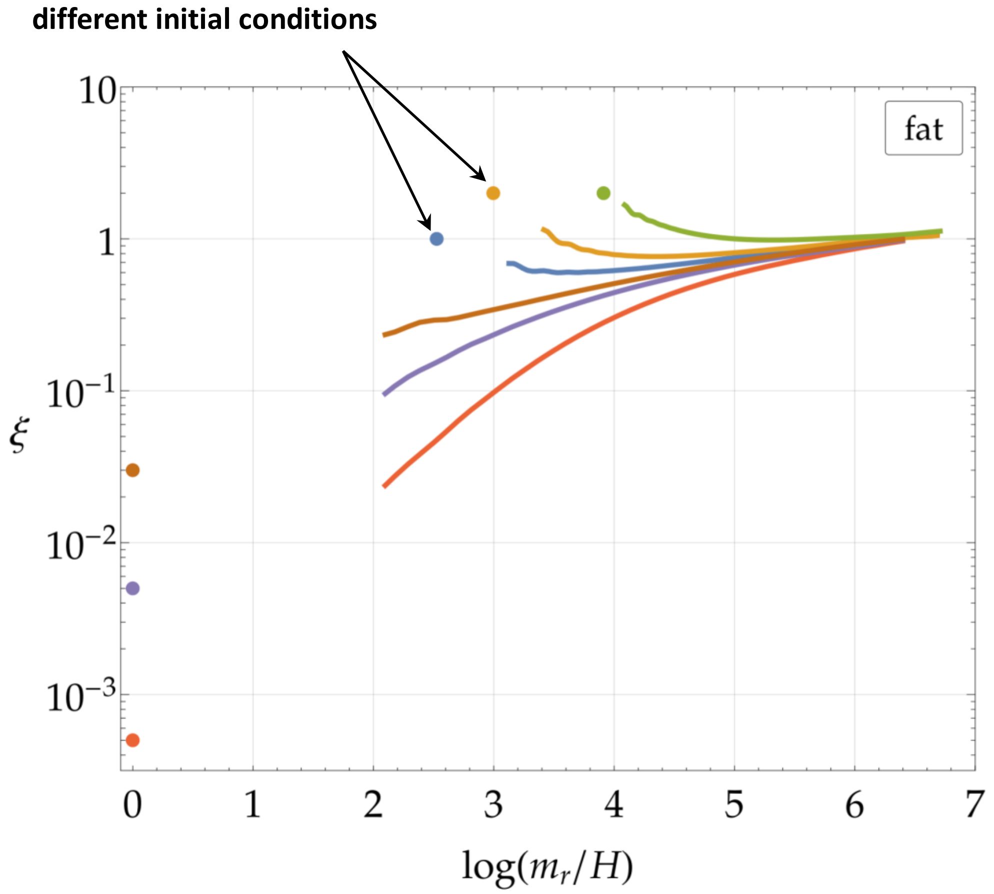
3) Axion number density through the axion potential turn on

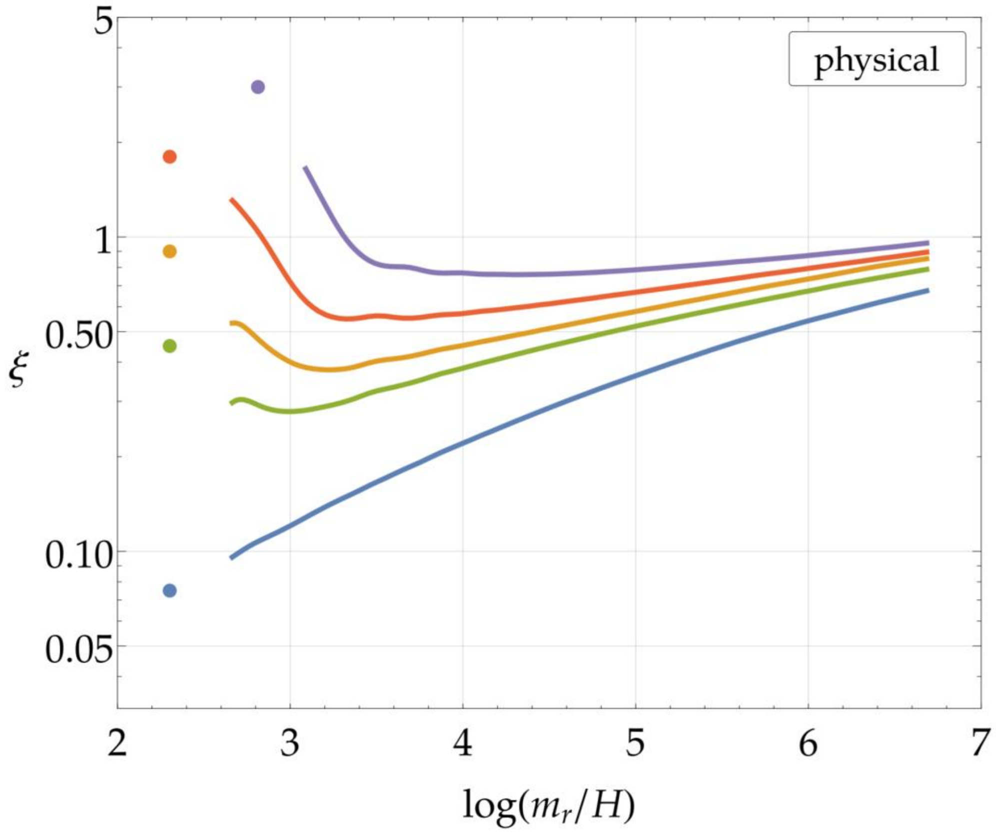
1) The Number of Strings per Hubble Volume

$$\xi \equiv \frac{N_{\text{strings}}}{H^{-3}}$$

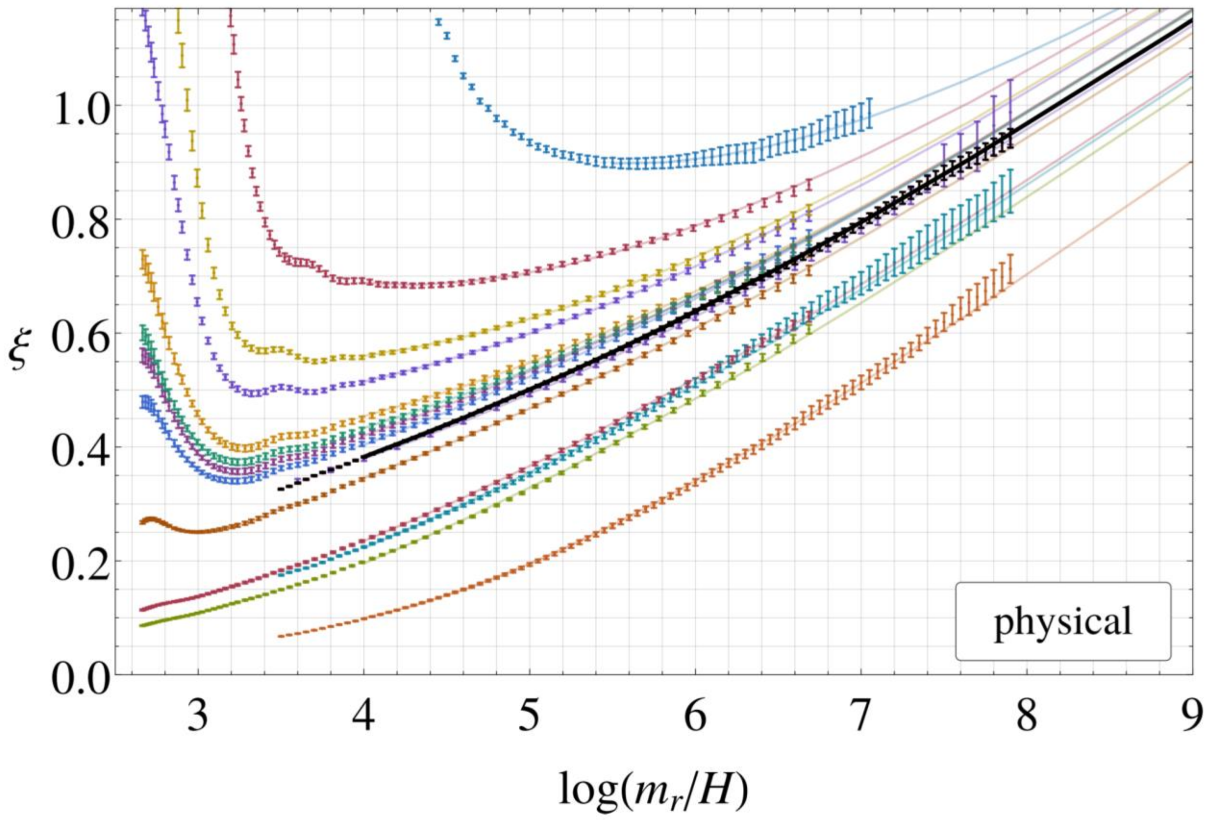


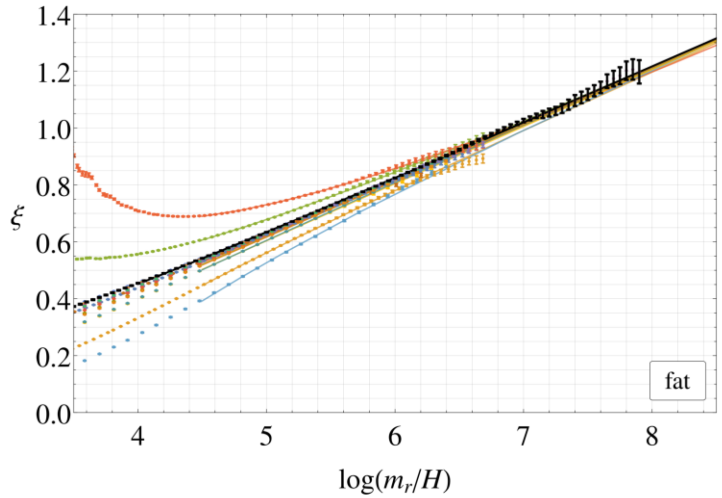
string length in one Hubble volume in units of H^{-1}





1) The Number of Strings per Hubble Volume





Scaling Violation

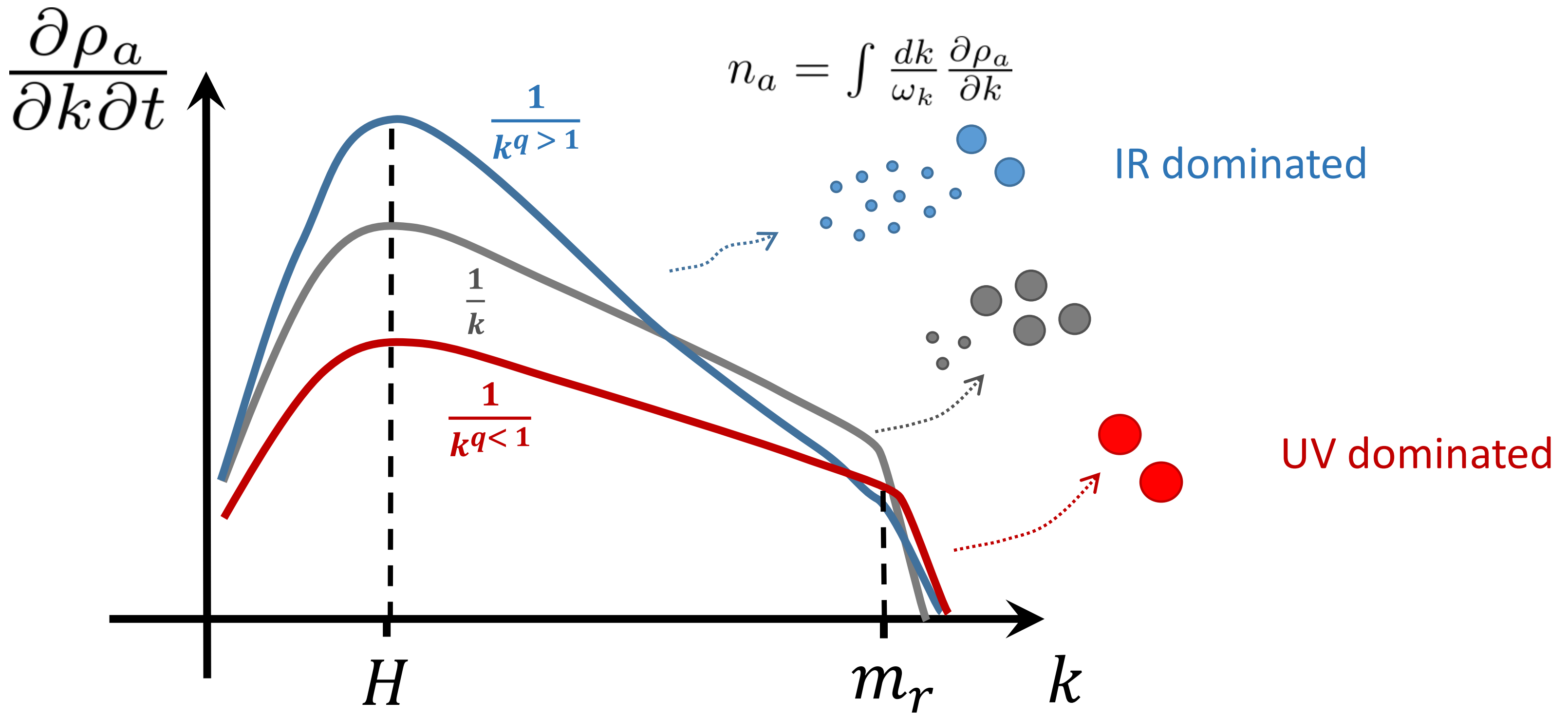
$$\boxed{\xi = c_1 \log} + c_0 + \frac{c_{-1}}{\log} + \frac{c_{-2}}{\log^2} \quad \rightarrow \quad \boxed{\xi \xrightarrow{\log \to 70} 15(2)}$$

2) The Axion Spectrum

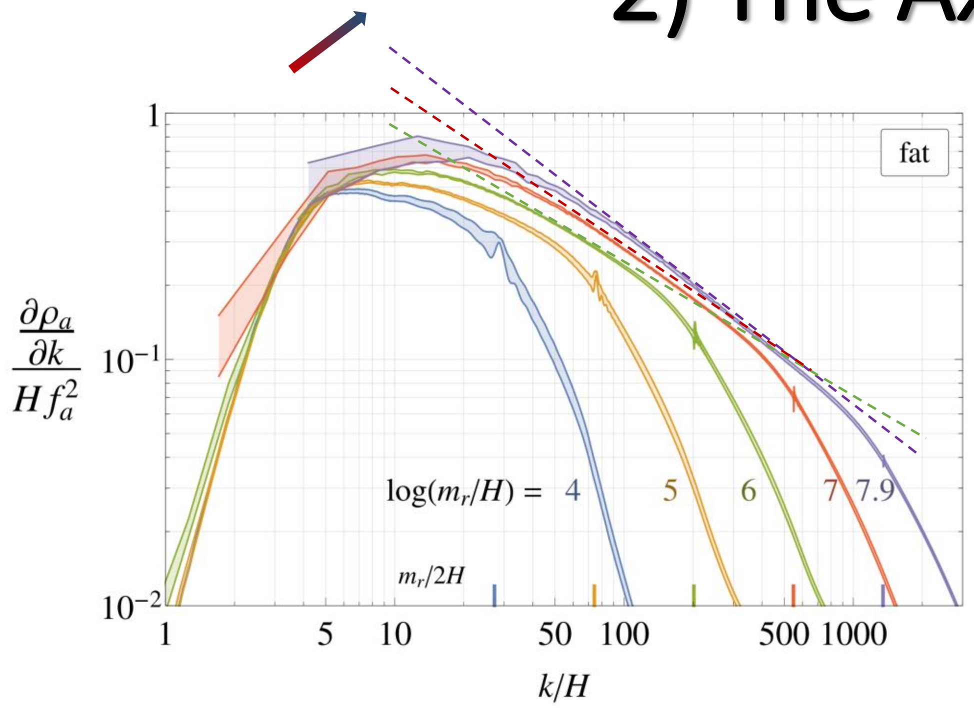
$$\frac{\partial \rho_a}{\partial k \partial t} \equiv \text{energy spectrum of axions emitted}$$

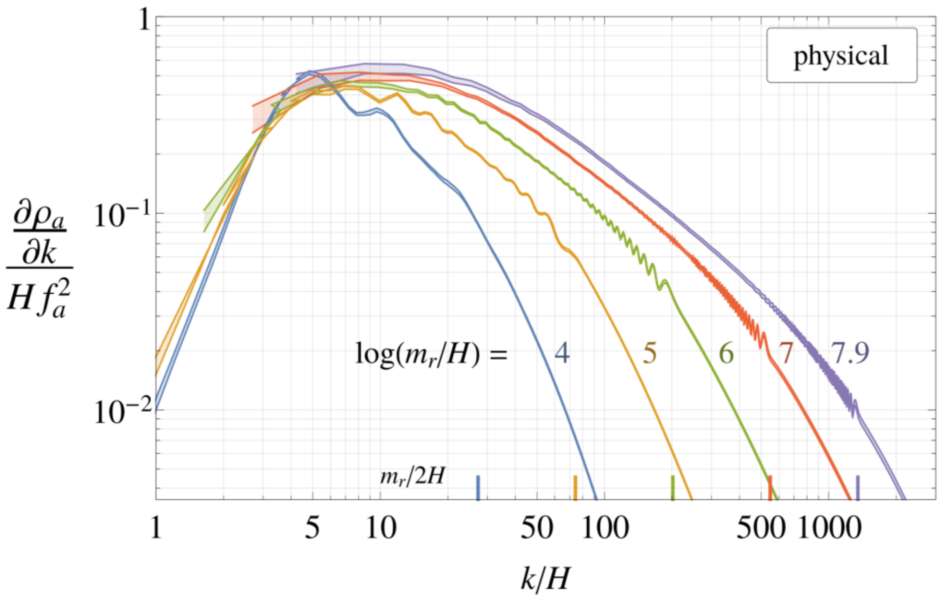
Theoretical expectation

- natural cut-offs at H and m_r
- peak at H because strings have curvature of $O(H)$
- in between an approximate power law: $\boxed{\frac{\partial \rho_a}{\partial k \partial t} \propto \frac{1}{k^q}}$
- in principle q could be time-dependent, $q = q(\log)$



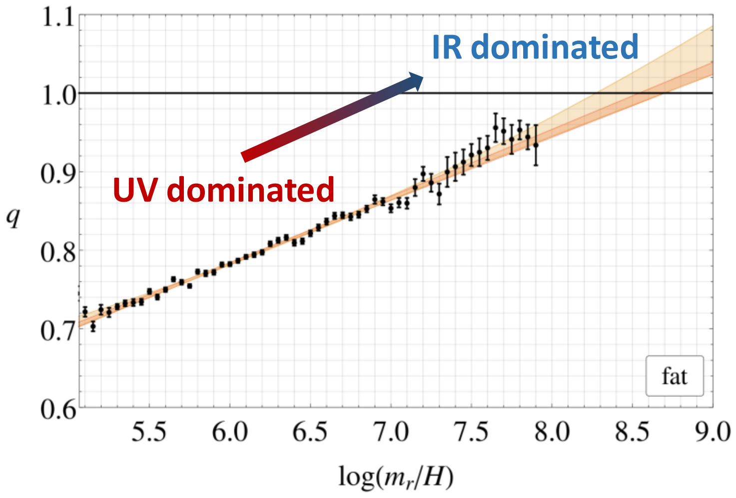
2) The Axion Spectrum

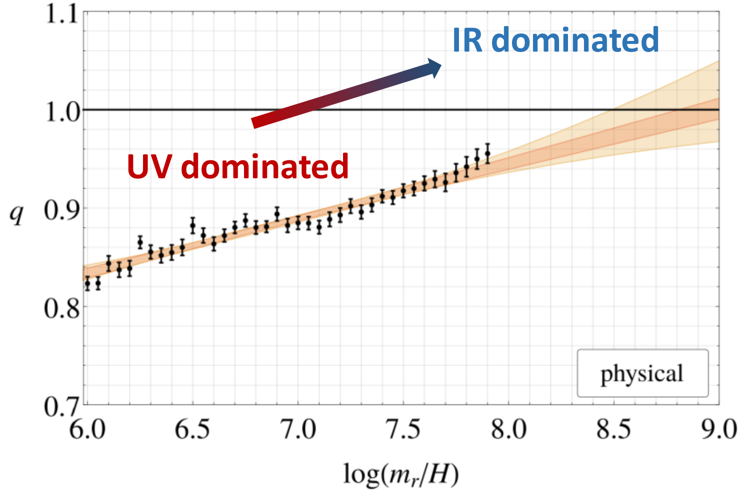




Running of q

$$\overset{\log \to 70}{q \;>\; 1}$$





3) Axion waves through the nonlinear regime

$$\boxed{@\ \ H = m_a \equiv H_\star \ \longleftrightarrow\ \log \sim 70}$$

$$\rho_a \sim \frac{\xi\mu}{t^2} \sim 10^3 \left[\frac{\overset{15}{\uparrow}\hspace{-0.5em}\xi\ \overset{70}{\uparrow}\hspace{-0.5em}\log}{10^3}\right] H^2 f_a^2 \quad \gg m_a^2 f_a^2 \sim \rho_{\rm mis} \sim V(a) \; = m_a^2 f_a^2 (1-\cos(\tfrac{a}{f_a}))$$

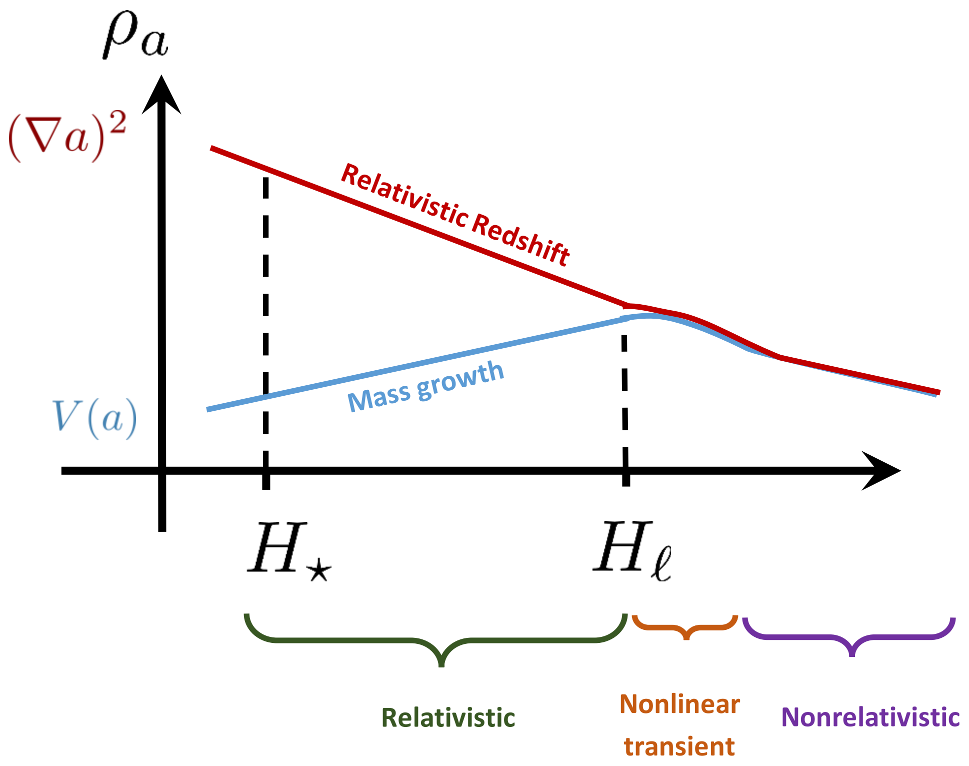
$$\rho_a \sim (\nabla a)^2 + \cancel{V(a)}$$

neglibile @ H_*

The potential becomes relevant when:

$$\xi \log H^2 f_a^2 \sim m_a^2 f_a^2 \ \Rightarrow\ m_a \sim \sqrt{\xi \log} H$$

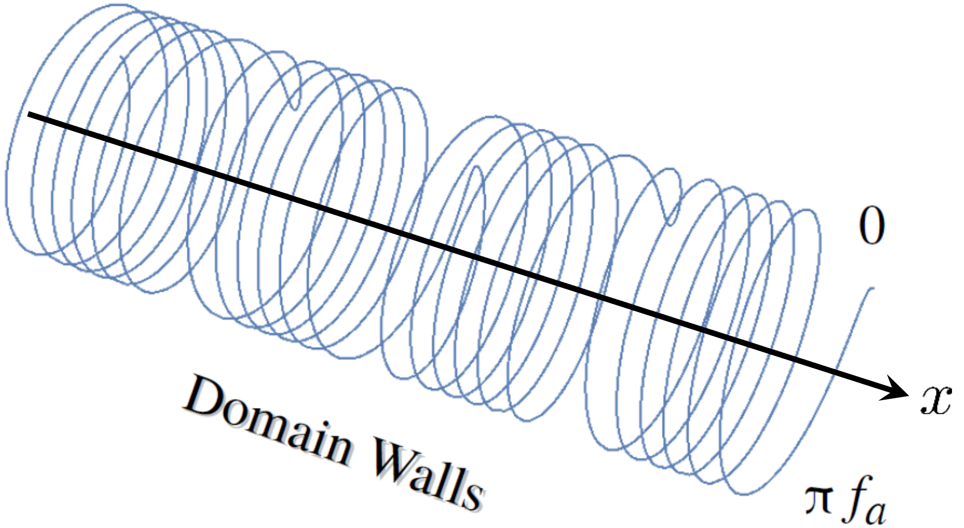
$$n_a \sim \frac{\rho_a}{m_a} \sim \boxed{\sqrt{\xi \log} H f_a^2} \sim \sqrt{\xi \log}\, n_a^{\rm mis}$$

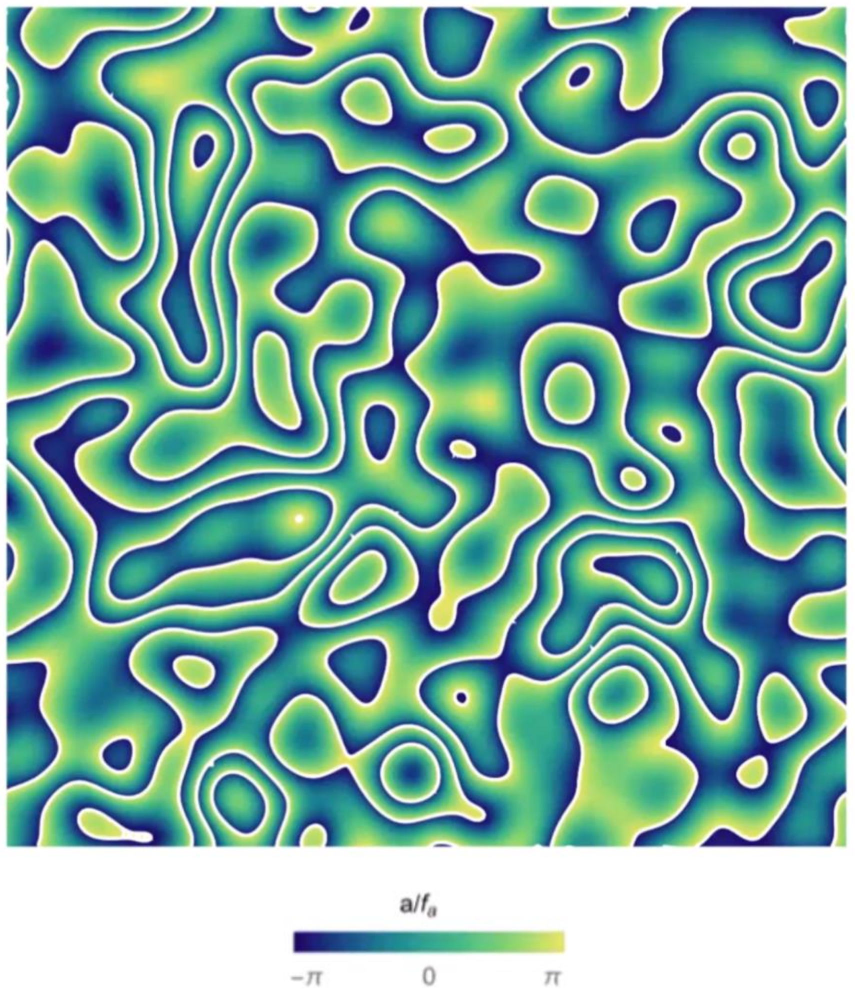


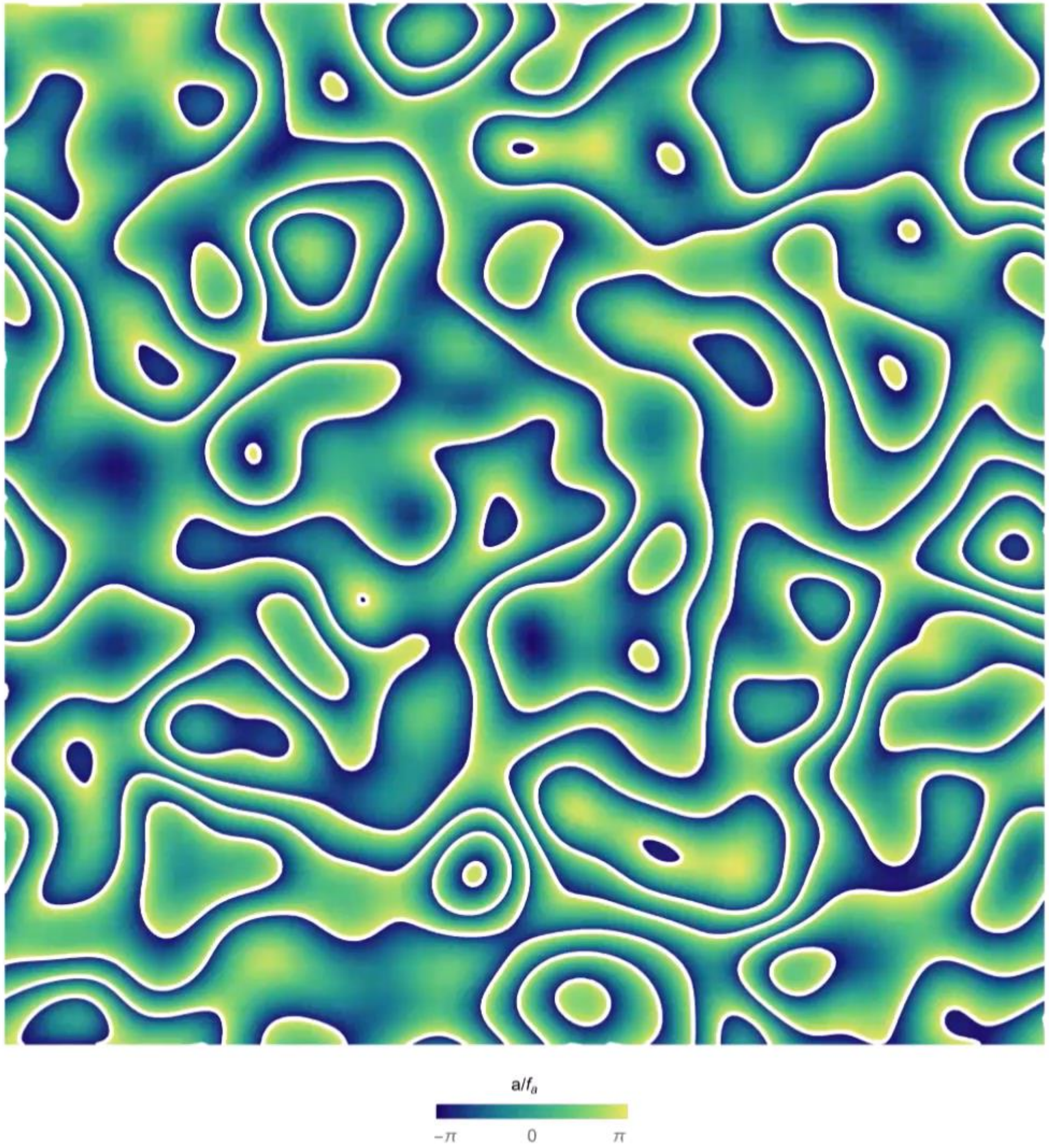
3) Axion waves through the nonlinear regime

$@\ H_\star$

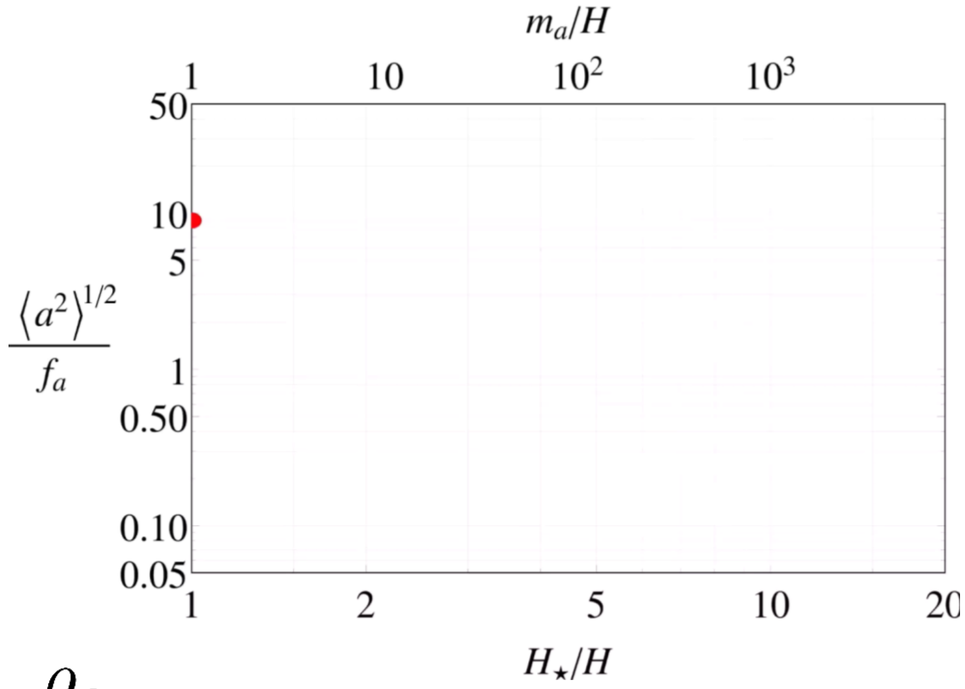
$$\rho_a \sim (\nabla a)^2 \sim H^2 a^2 \ \sim \xi \log H^2 f_a^2 \quad \Rightarrow \quad \frac{a}{2\pi f_a} \sim \sqrt{\xi \log} = O(10)$$



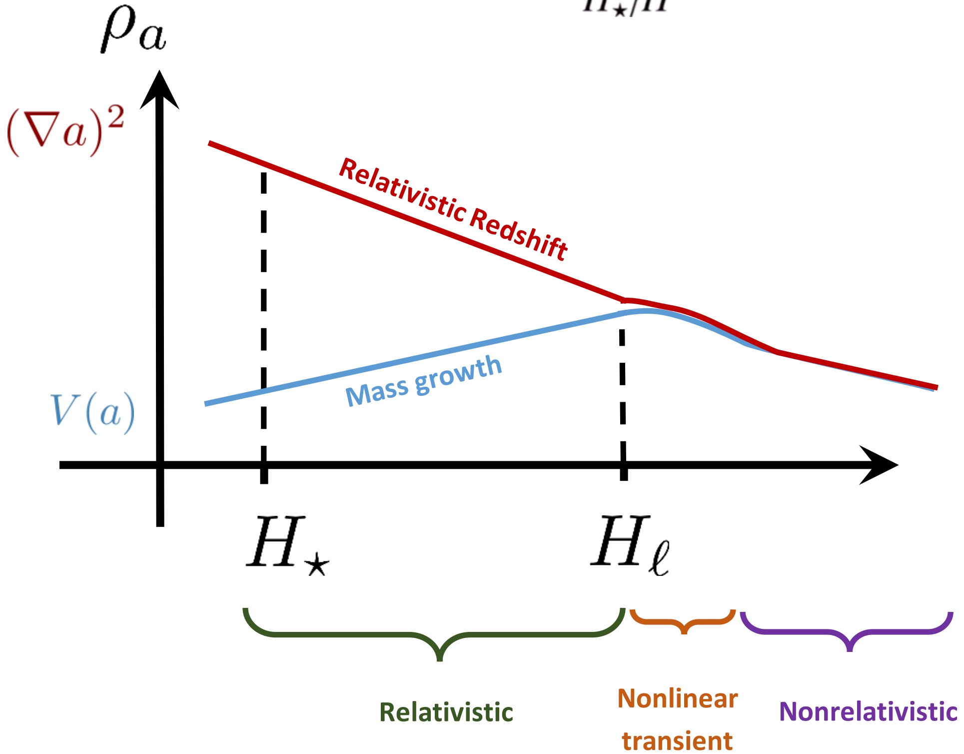




a/f_a
−π
0
π

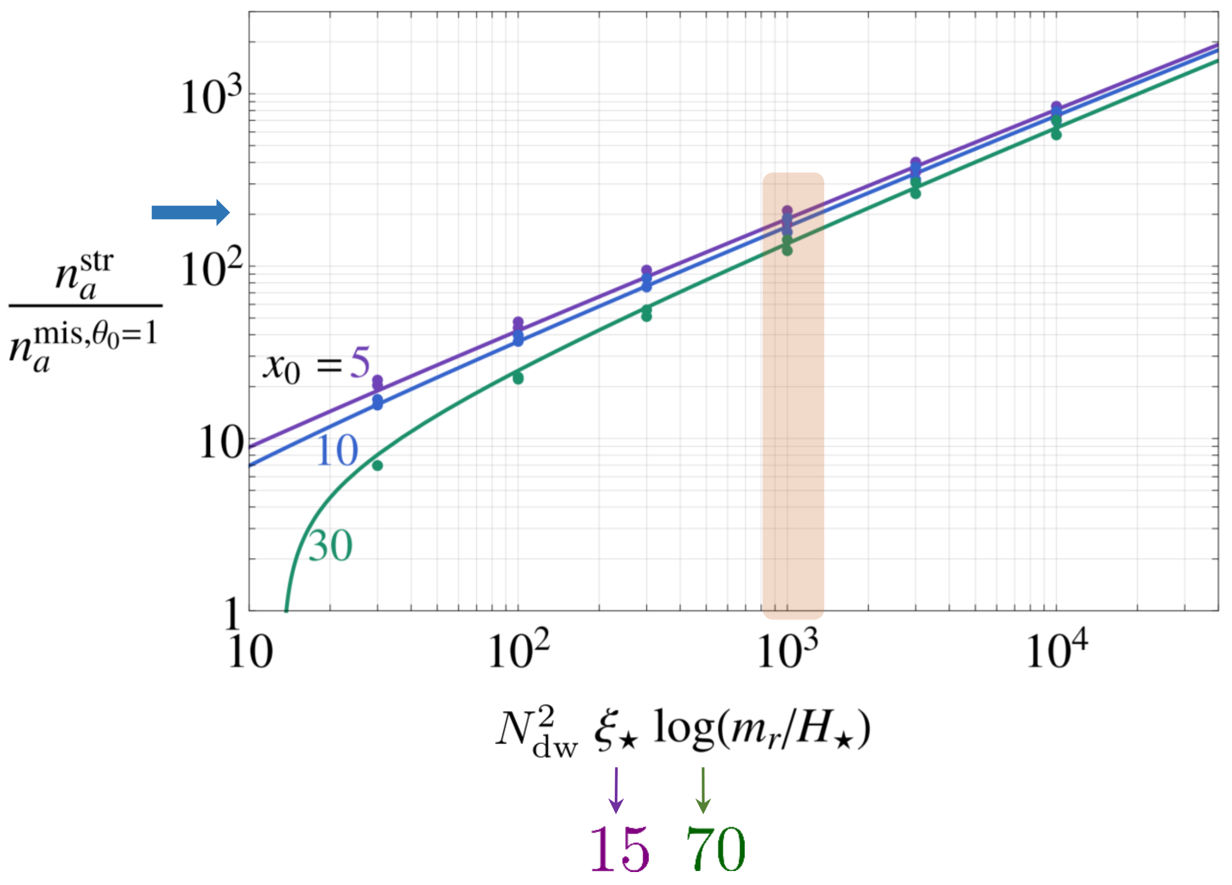


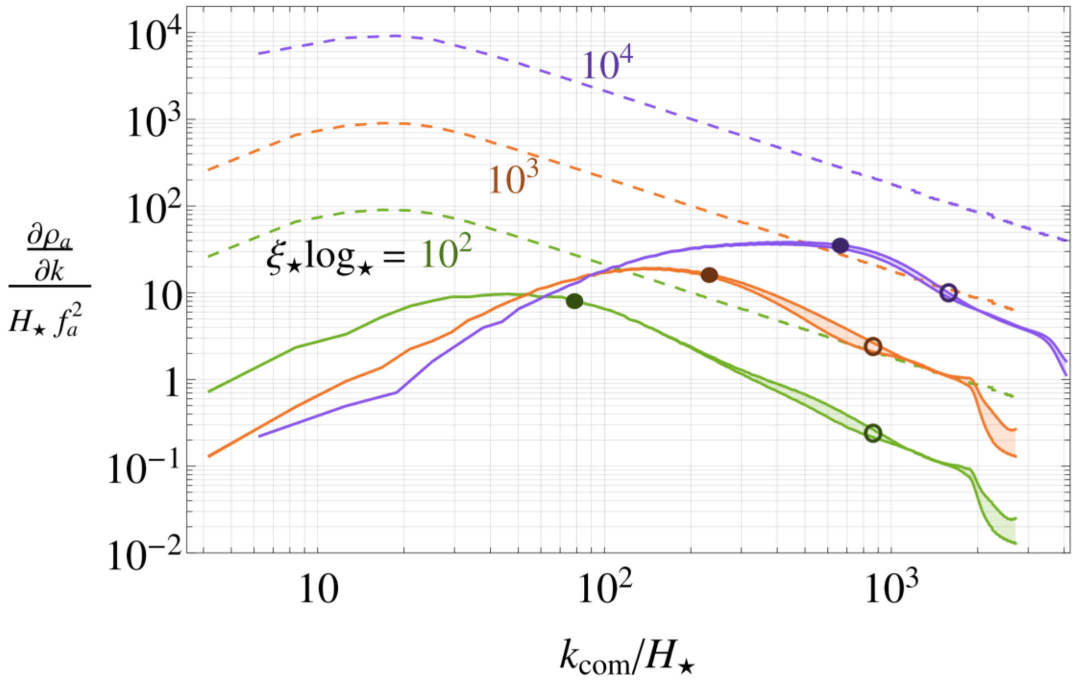
m_a/H
1
10
10^2
10^3
50
10
5
1
0.50
0.10
0.05
$\frac{\langle a^2 \rangle^{1/2}}{f_a}$
1
2
5
10
20
$H_\star/H$



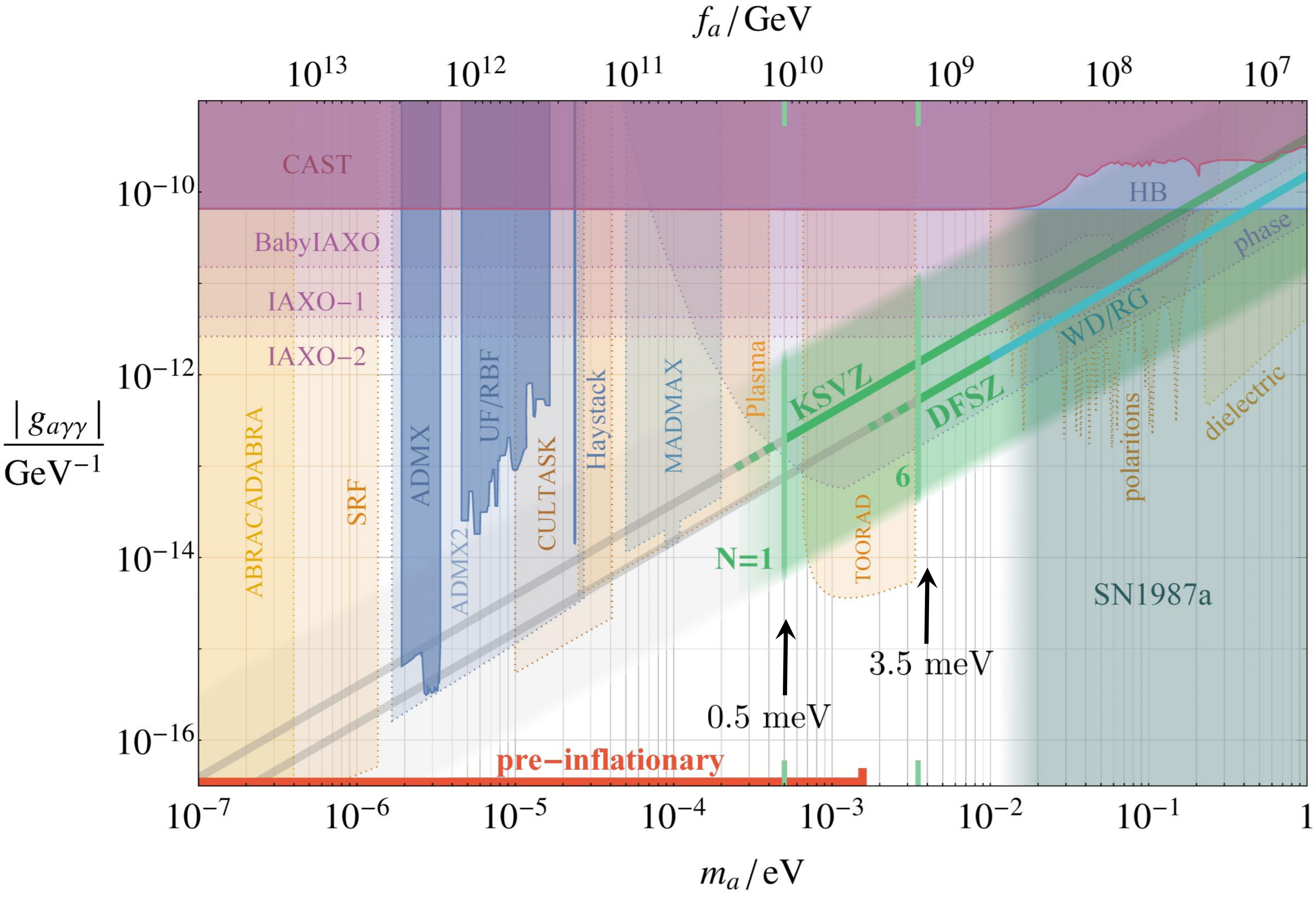
ρ_a
$(\nabla a)^2$
Relativistic Redshift
Mass growth
$V(a)$
$H_\star$
H_ℓ
Relativistic
Nonlinear transient
Nonrelativistic

3) Axion waves through the nonlinear regime





A Lower Bound on the Axion Mass



Conclusions

1) The system of axion strings is driven towards an attractor solution

- evidence of logarithmic violations in ξ and q
- most conservative extrapolation implies More Axions from Strings

2) The Axions from Strings experience nonlinear evolution at the QCD transition

- a period of relativistic redshfit:

  A) partially reduces the number density

  B) makes the spectrum more UV

$$\longrightarrow \quad m_a \approx 0.5 \div 20 \ \mathrm{meV}$$

Thank you

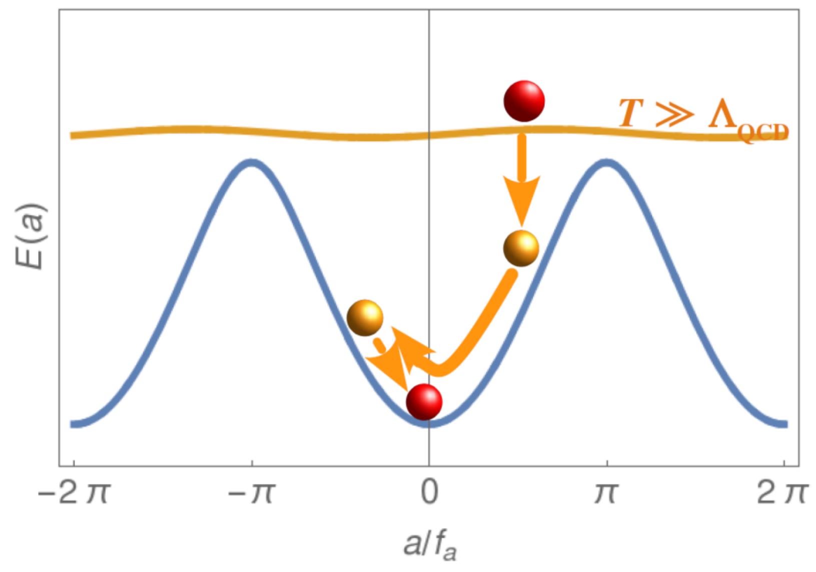
Backup

Scenario #1: $(T, H) < f_a$

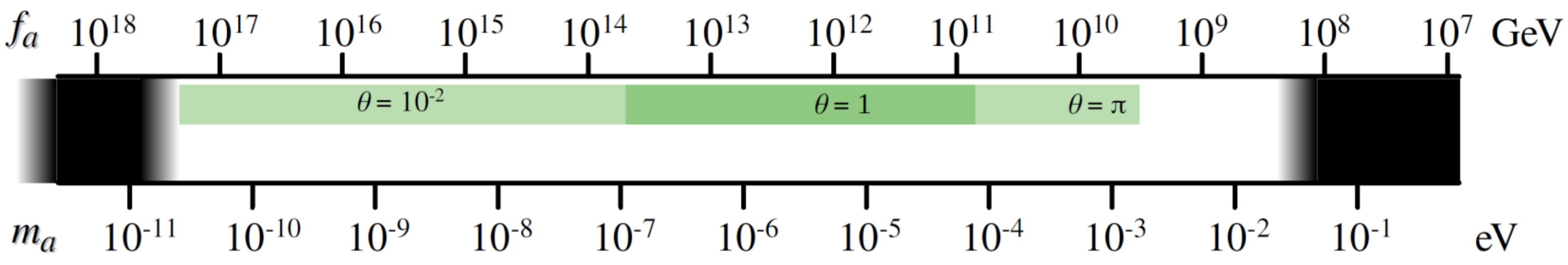
$\theta = 0 \div \pi$

$$a(t_0) = \text{const} \equiv \theta_0 f_a$$

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

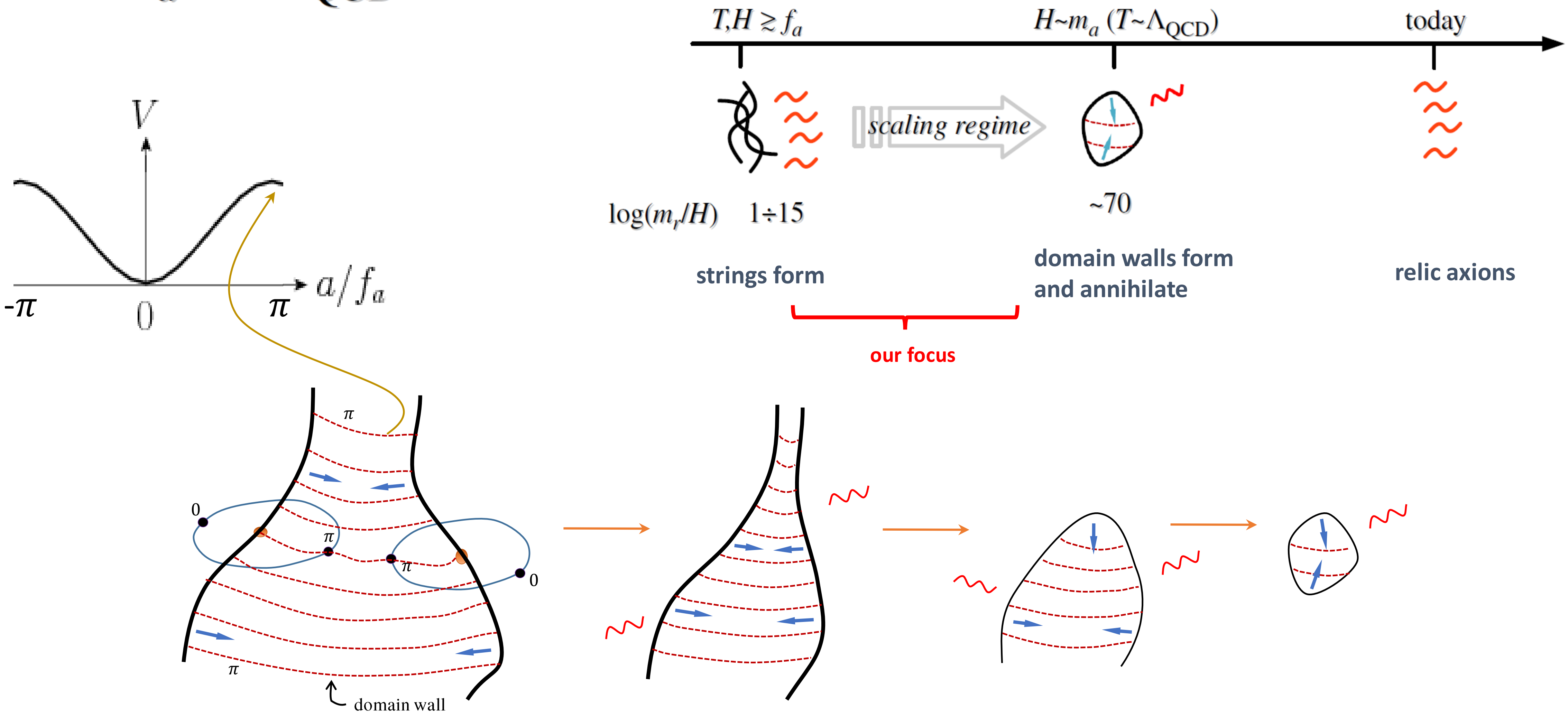


$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\text{GeV}}\right]^{1+\epsilon} \quad = \Omega_{DM}$$

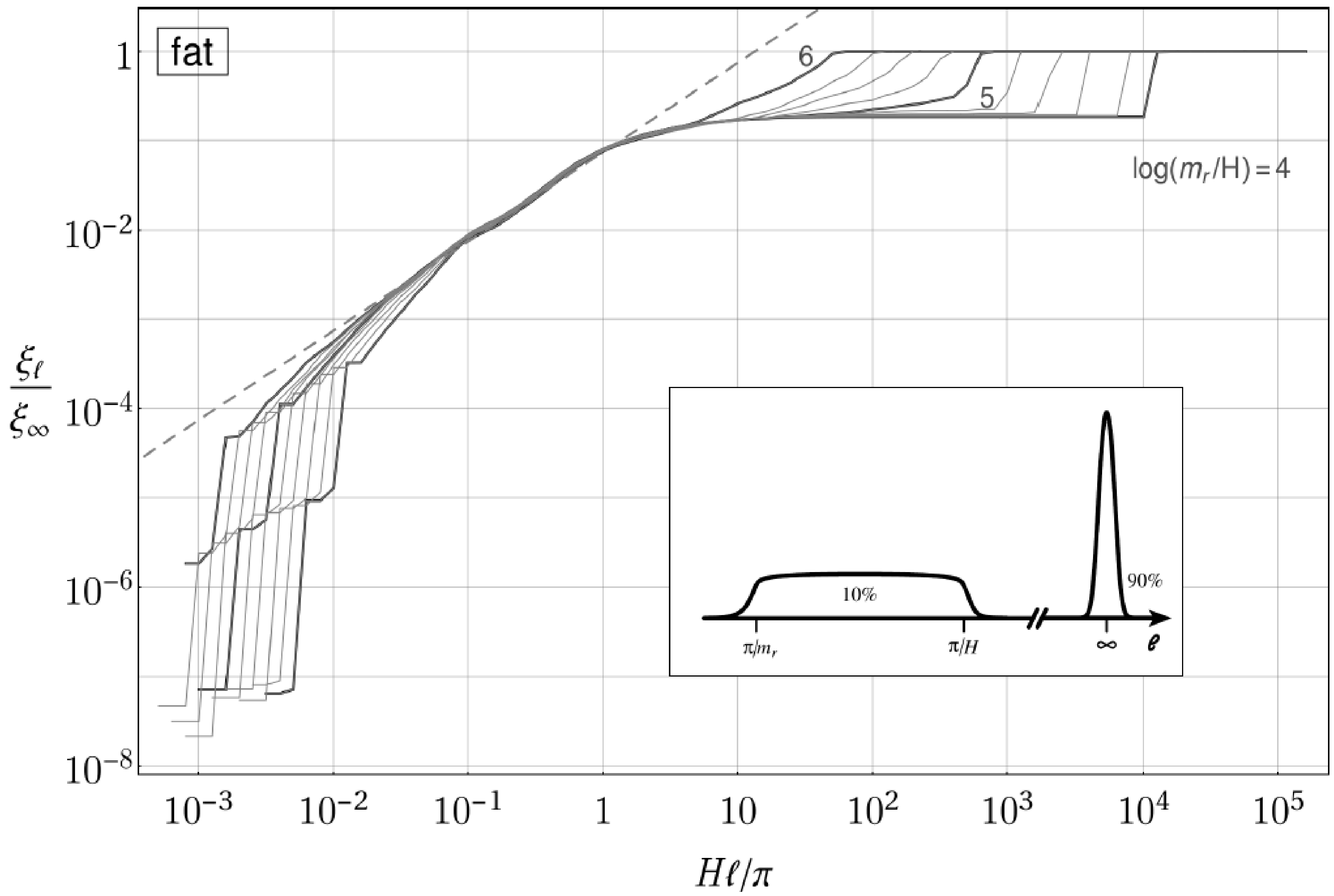


Domain Walls

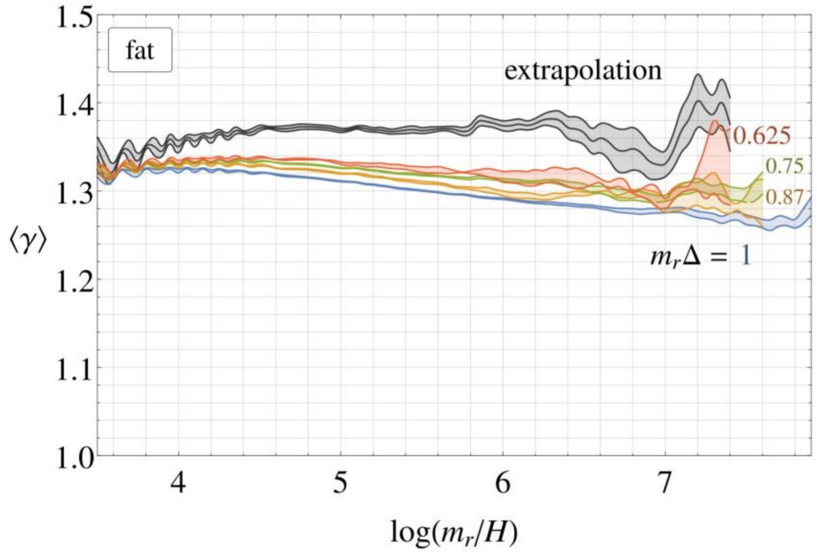
@ $H \sim m_a$ ($T \sim \Lambda_{QCD}$)

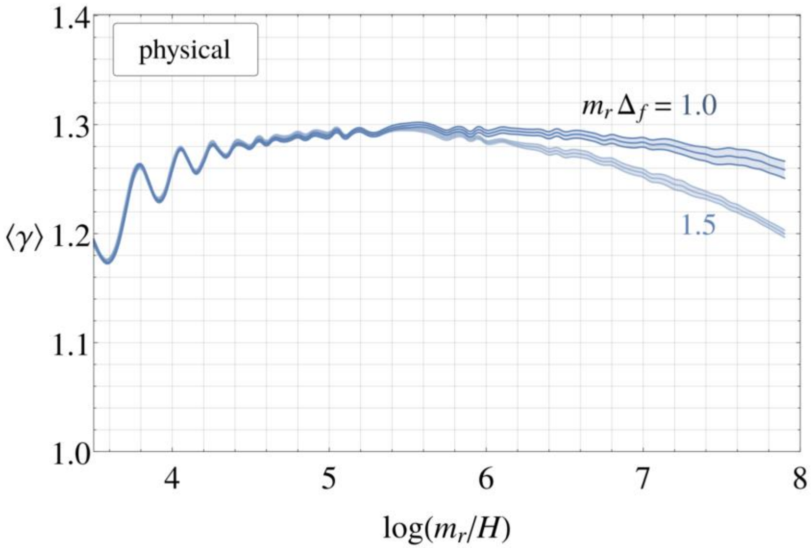


Loop Distribution

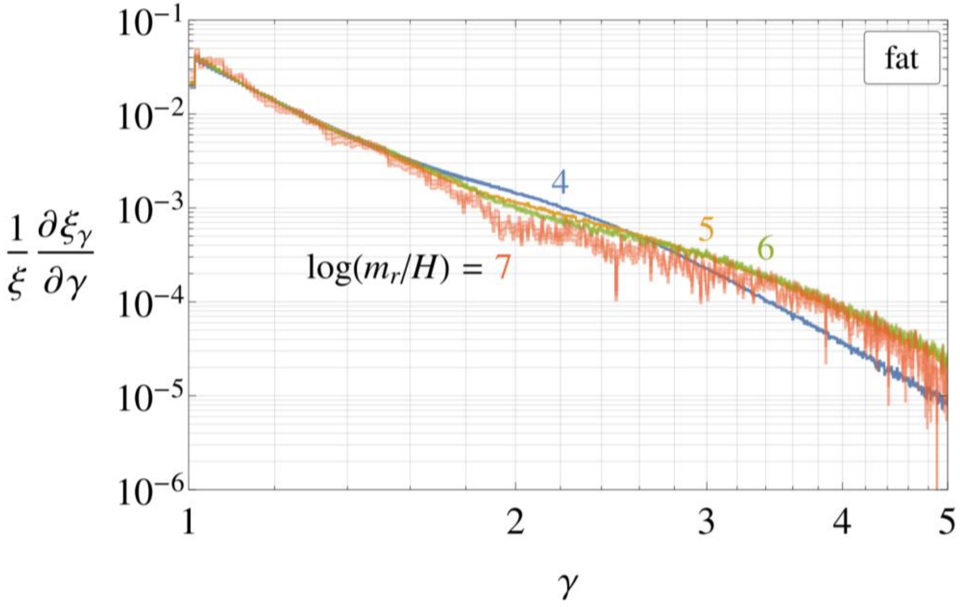


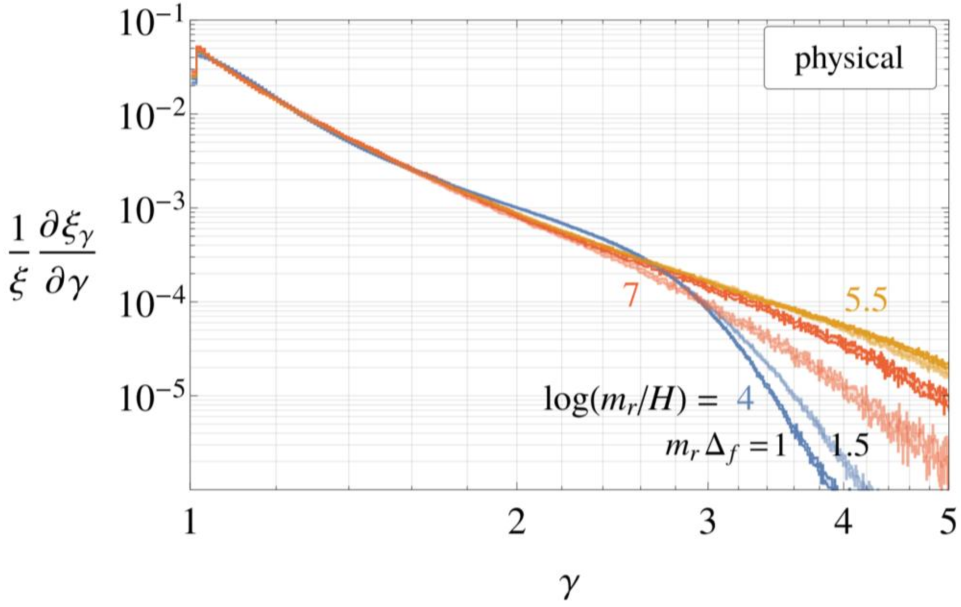
Boost Factors



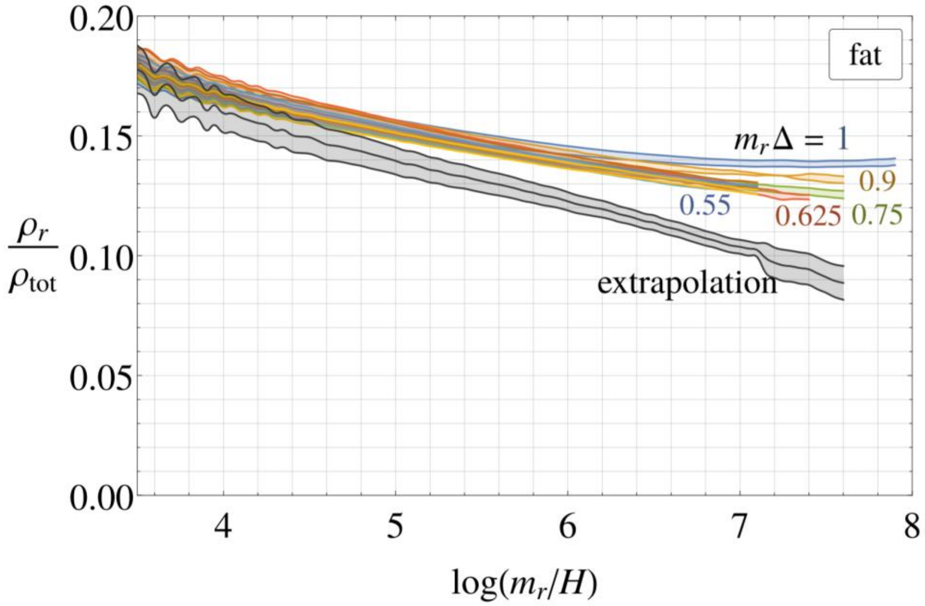


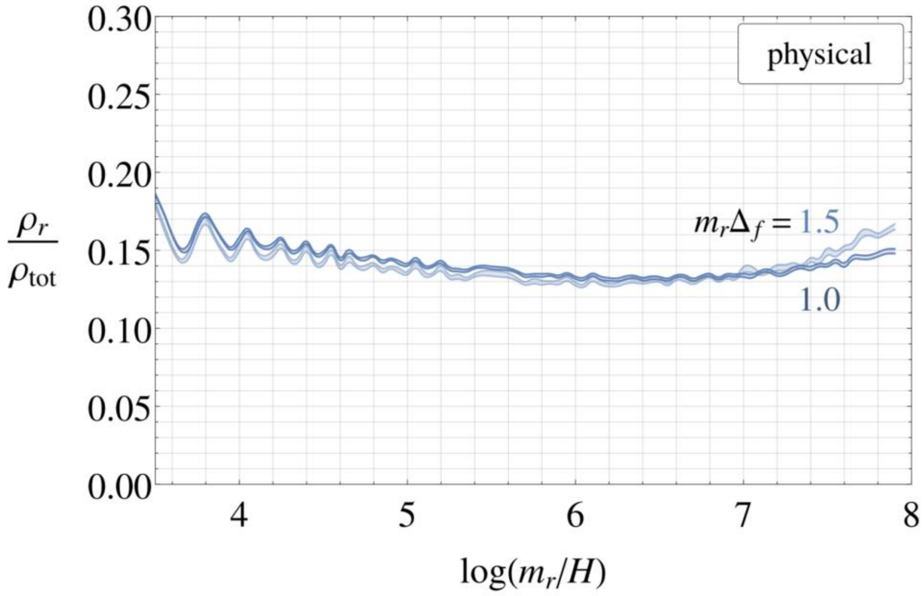
Increase in Higher Boosts:



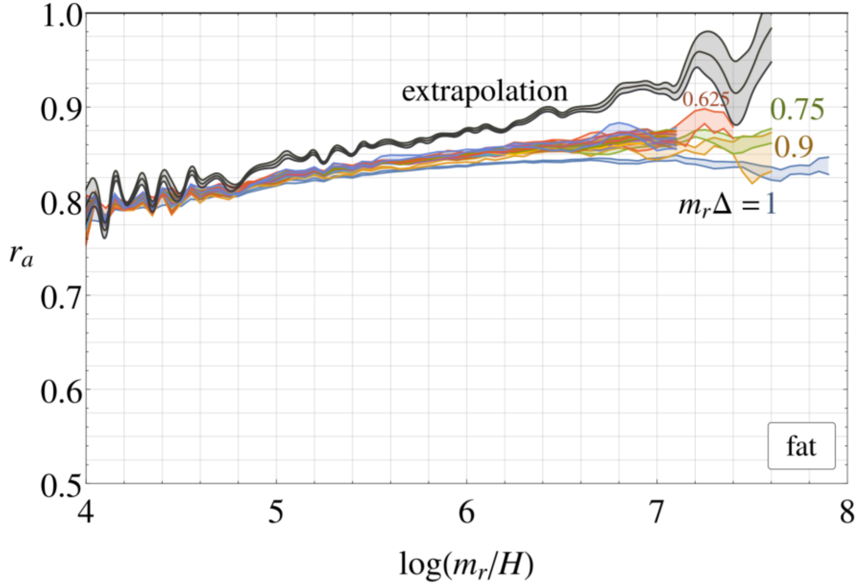


Radial Energy and Axion Emission



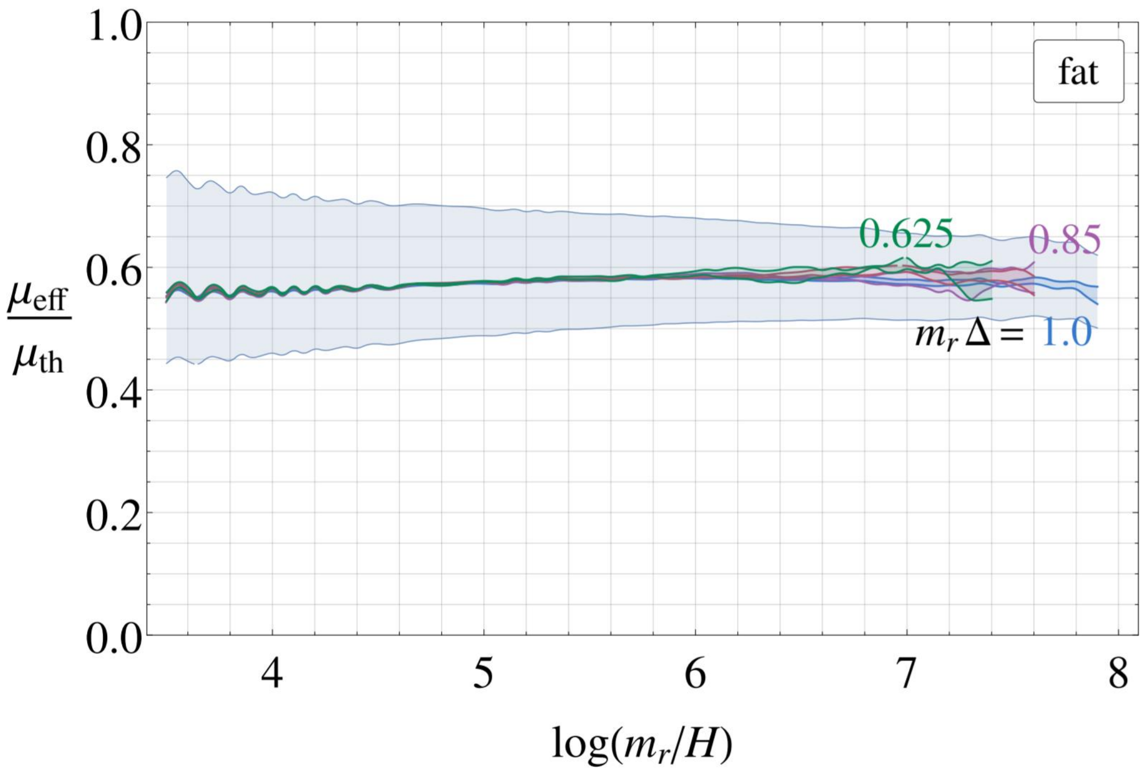


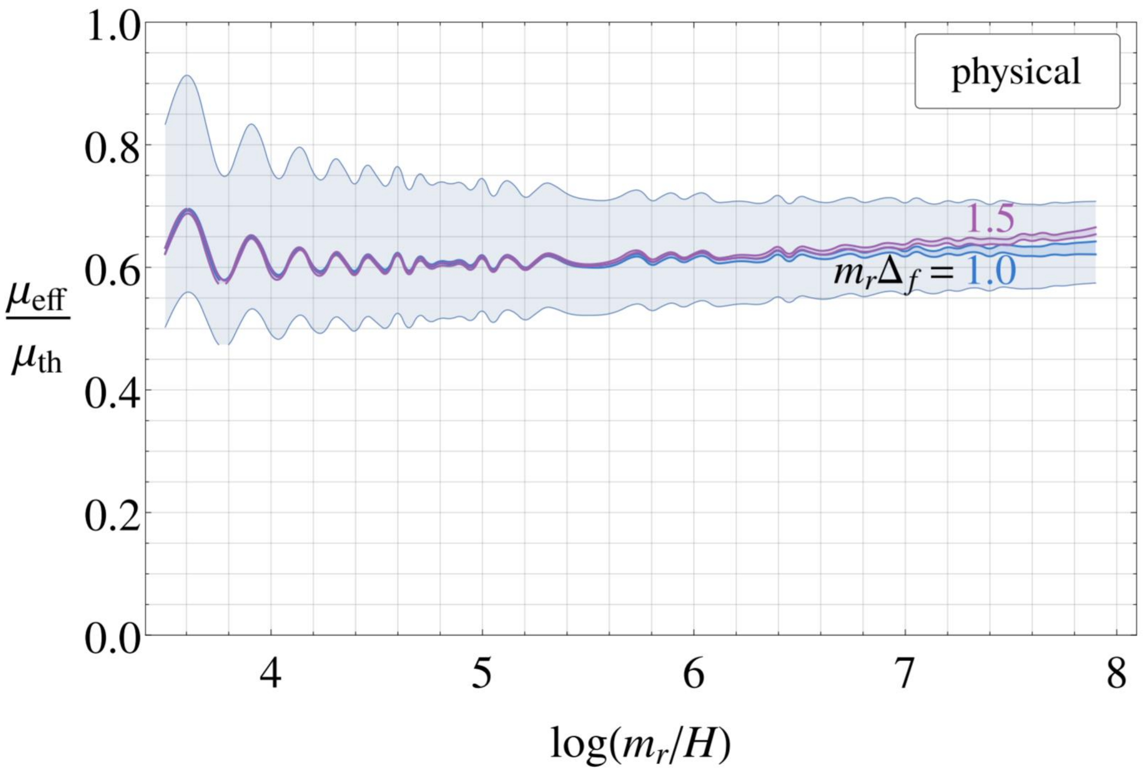
ρ_r/ρ_{tot} : **decreases**
(from ~0.2 to ~ 0.1)
[lattice spacing effects]



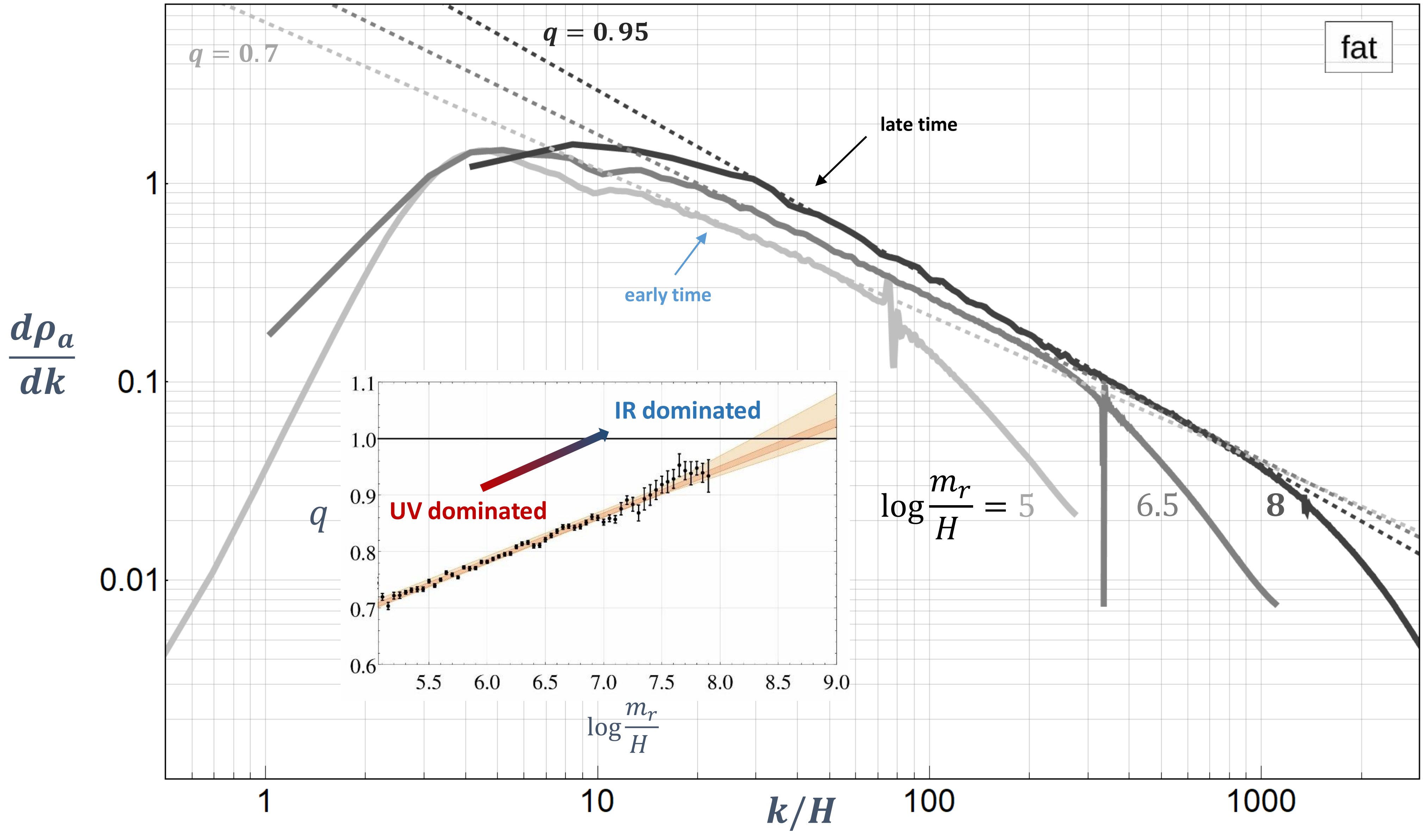
Effective String Tension

$$\mu_{\rm th} = \langle\gamma\rangle\pi f_a^2 \log\left(\frac{m_r\,\eta}{H\sqrt{\xi}}\right)$$

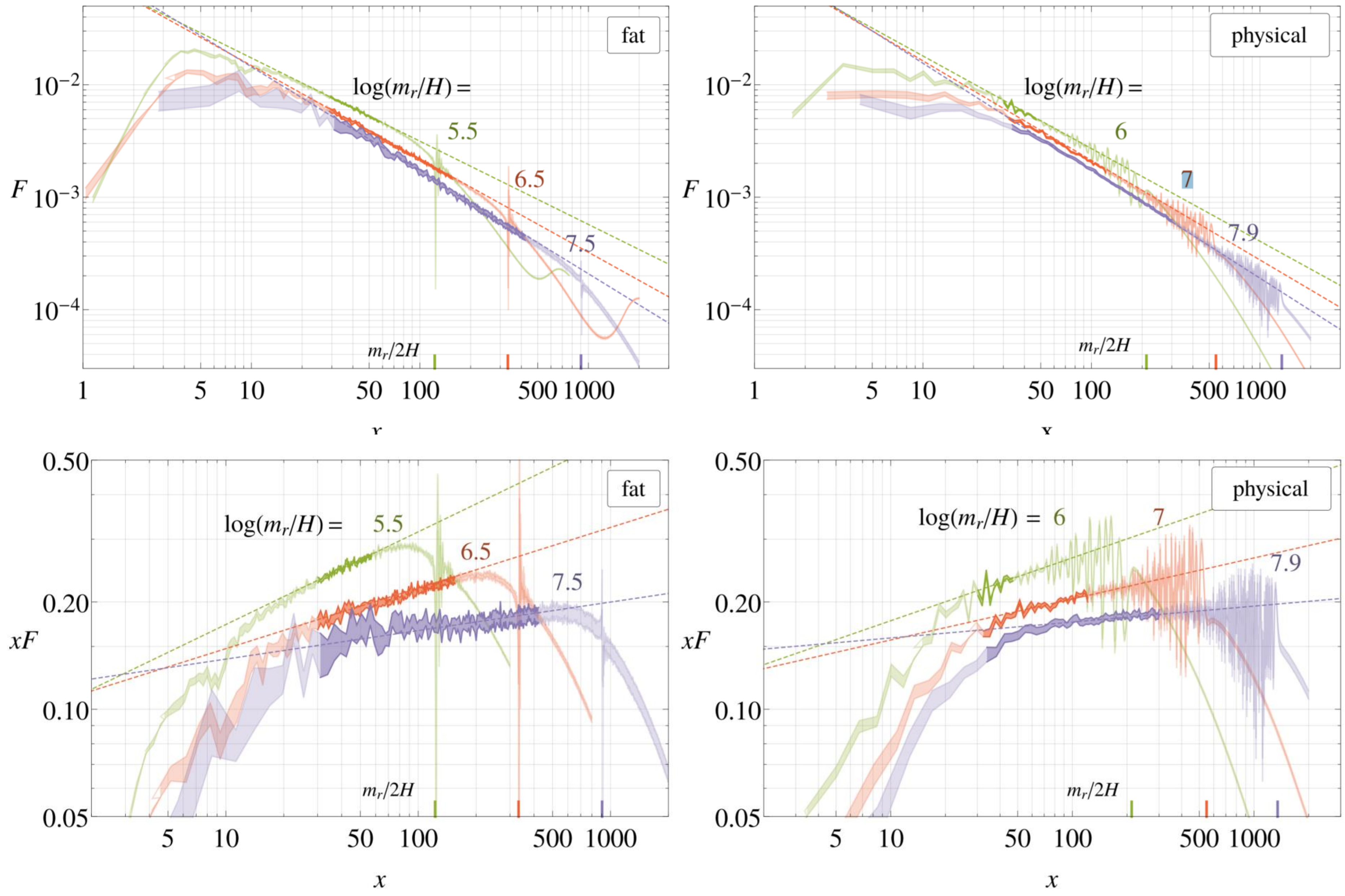




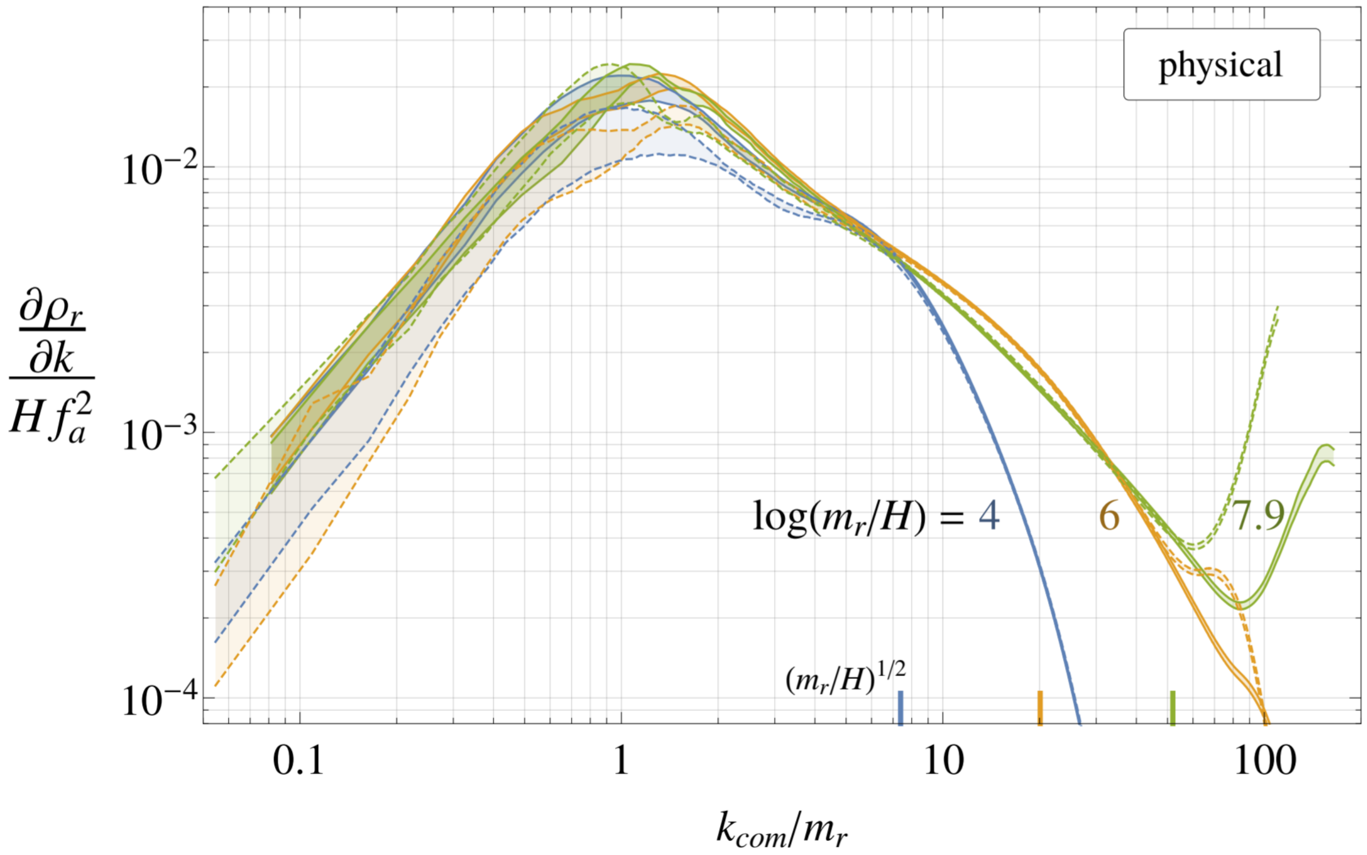
Instantaneous Spectrum (1)



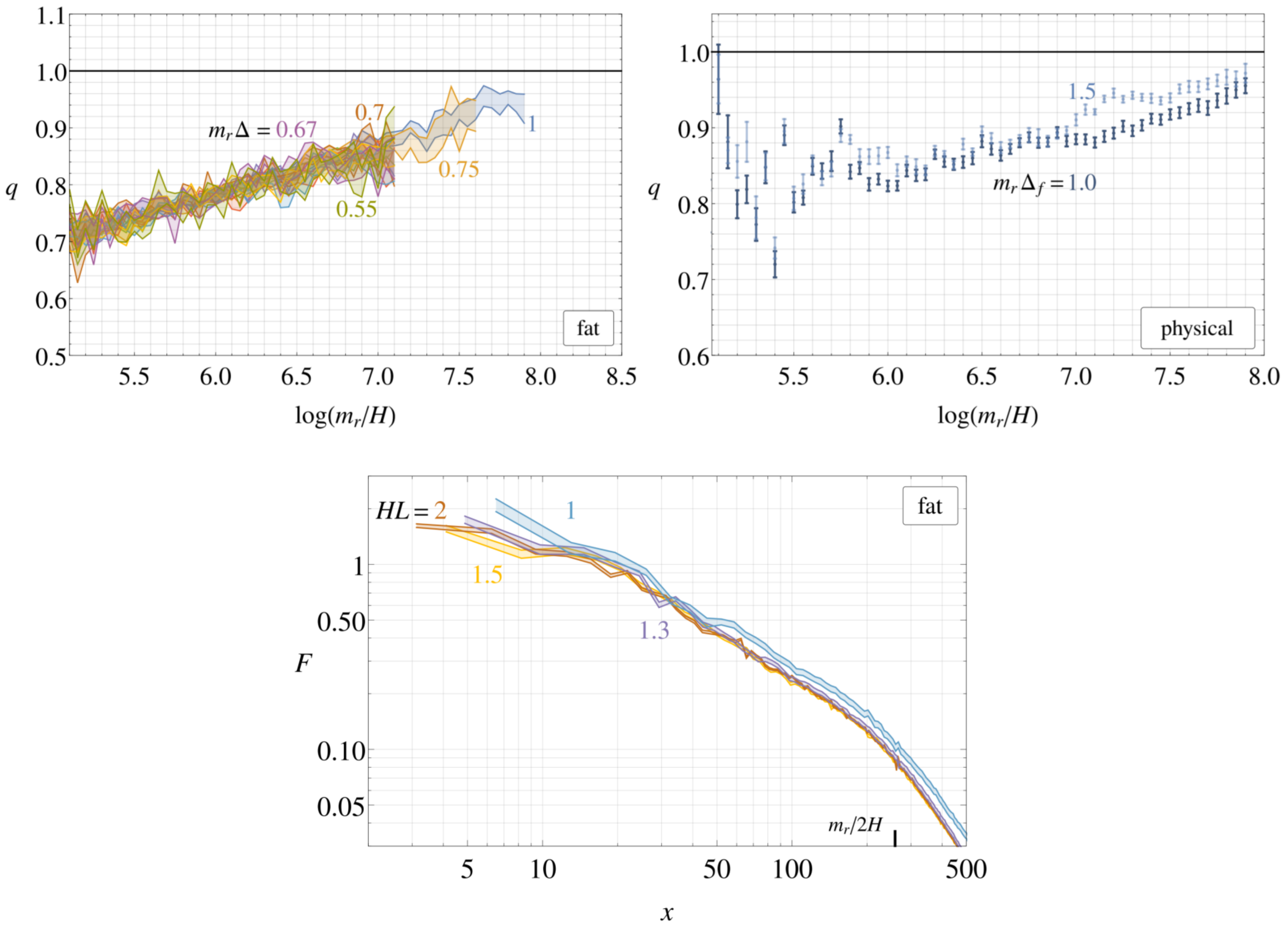
Instantaneous Spectrum (2)



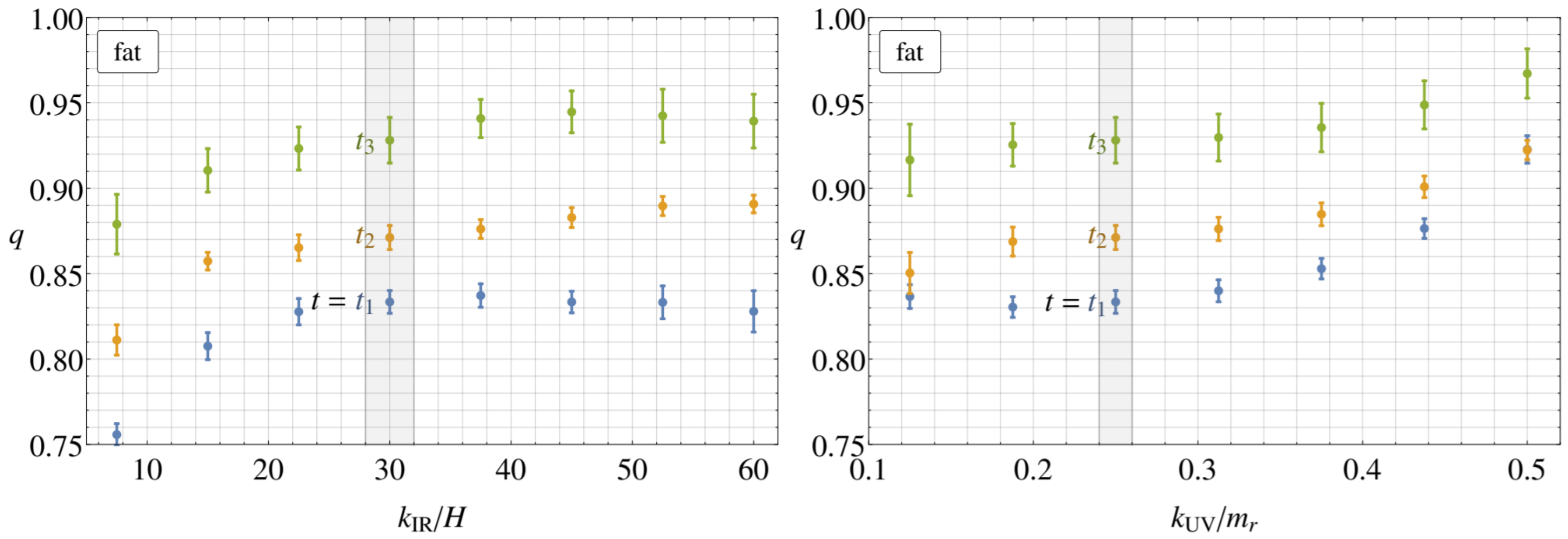
Radial Spectrum

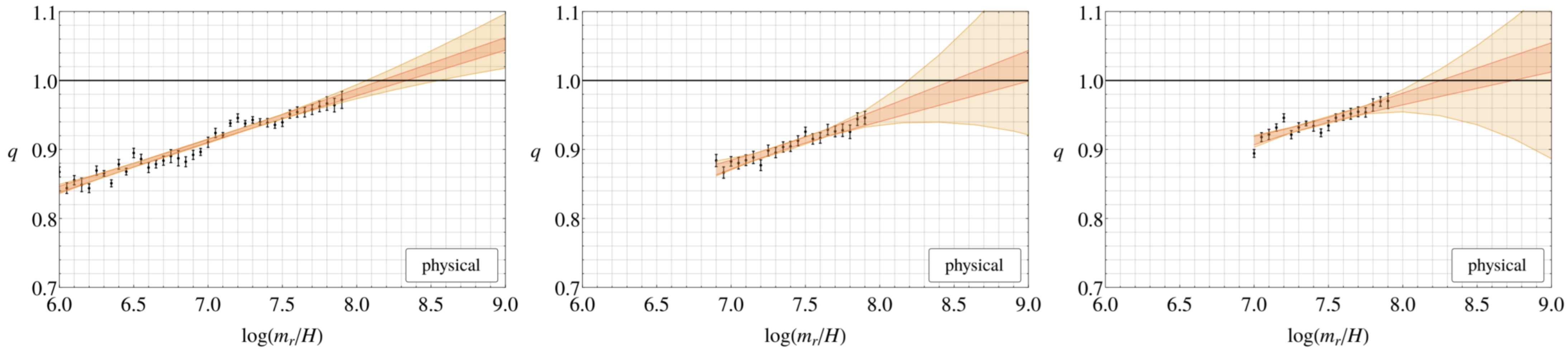


Lattice Spacing and Finite Volume Effects on q

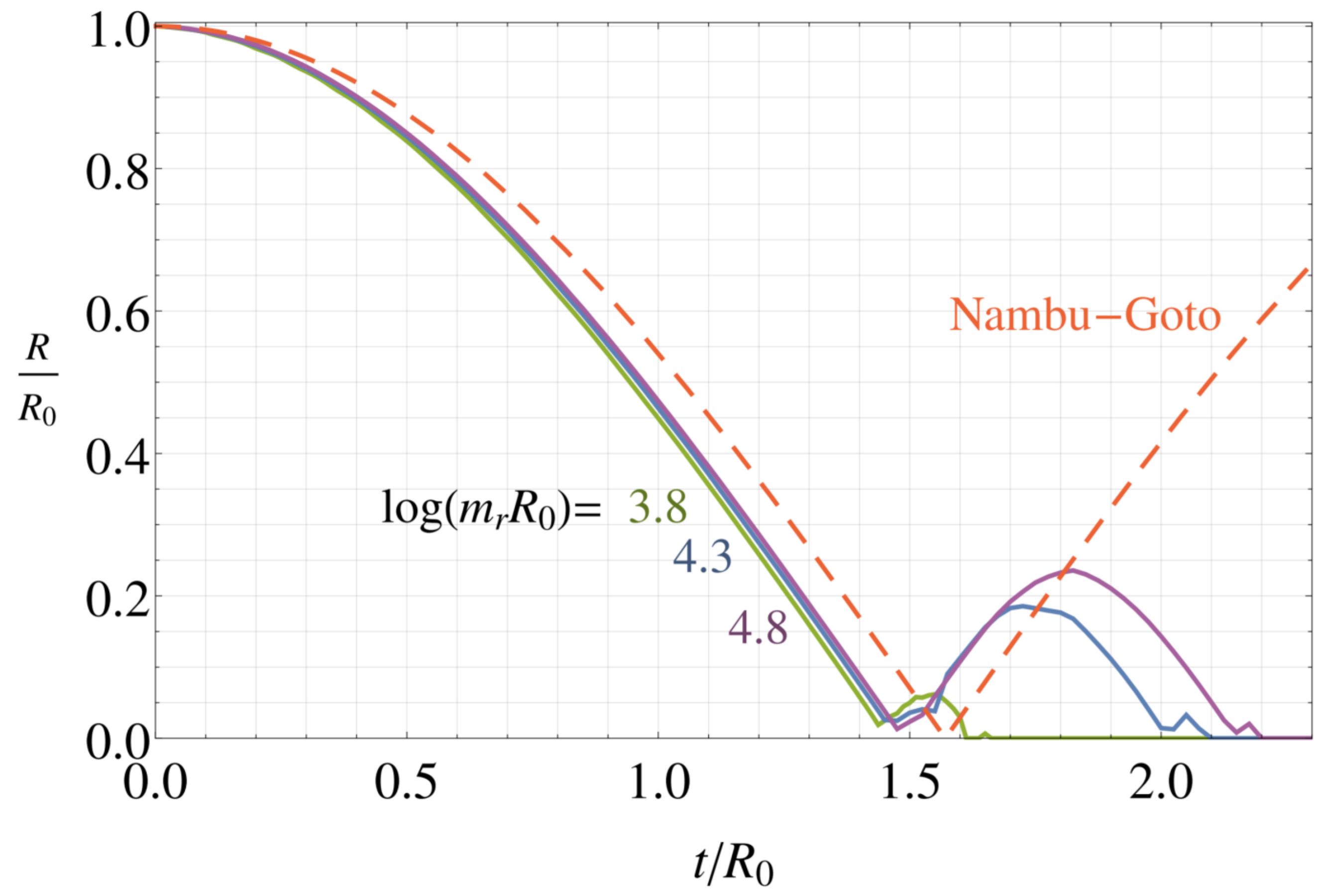


Range of fitted momenta for q

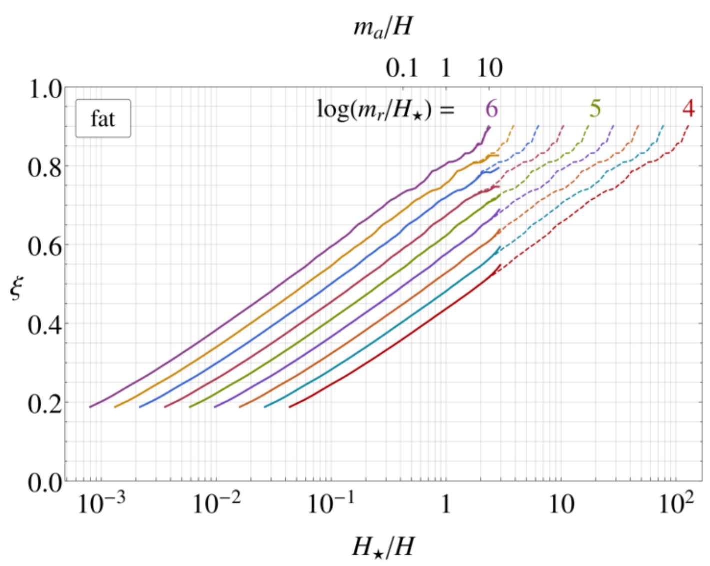


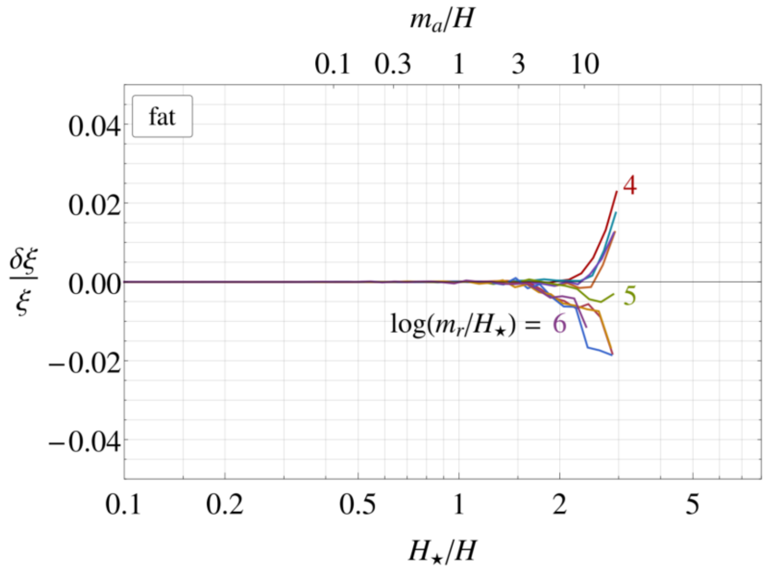


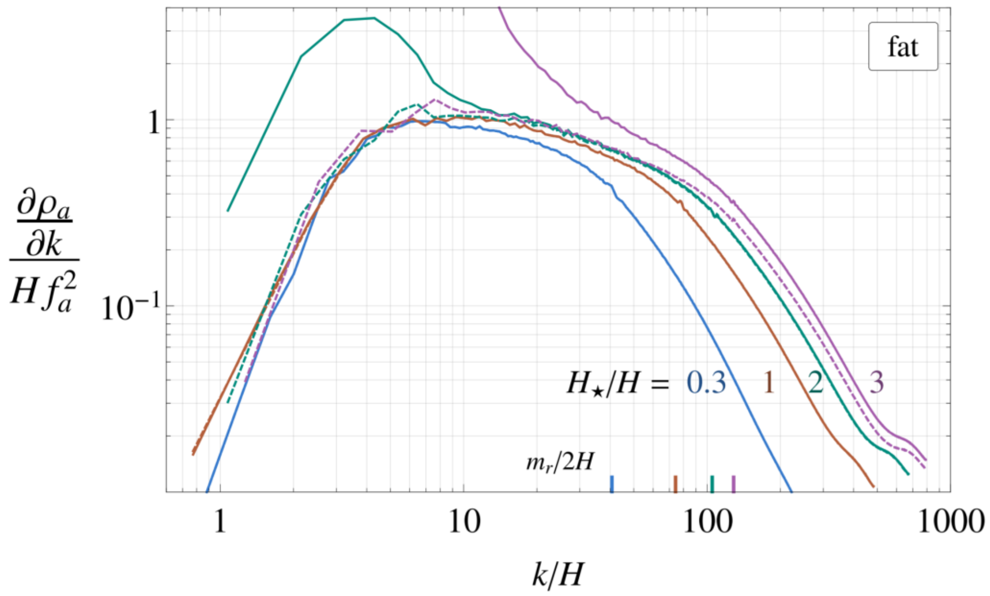
Circular Loops Bounce More



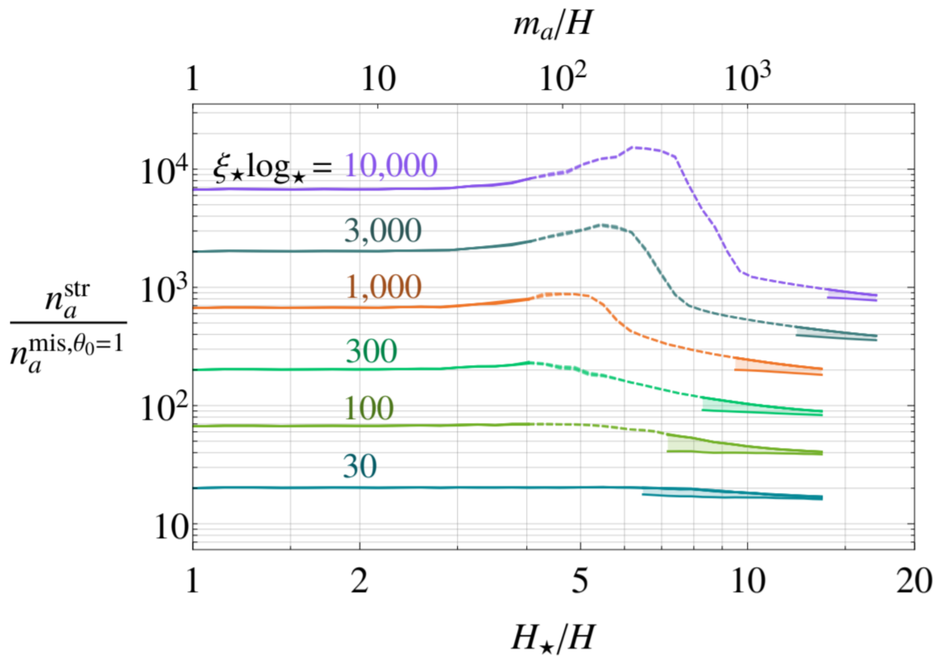
End of the Scaling regime: $H = m_a \equiv H_*$

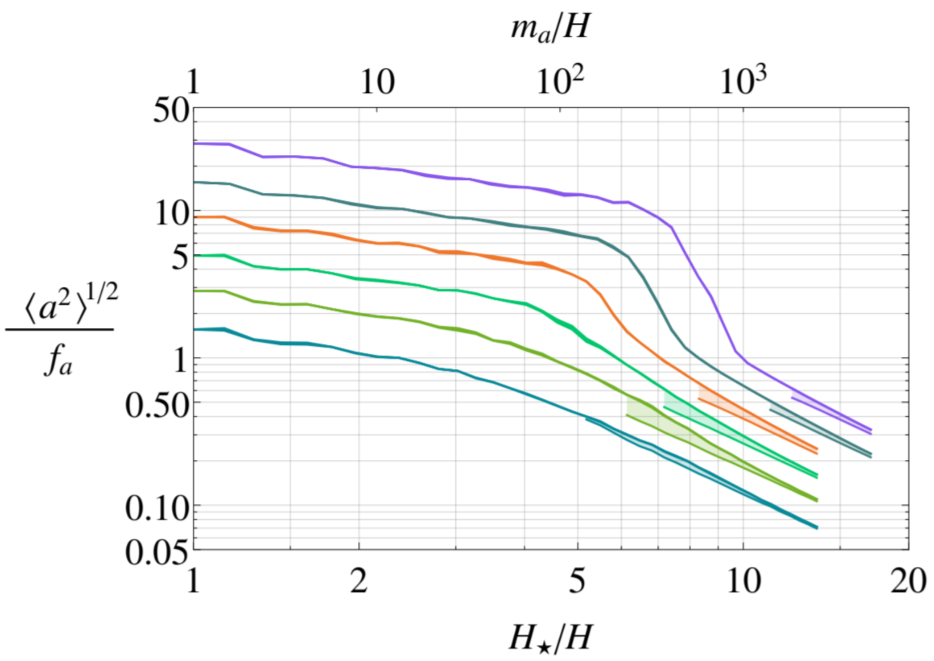


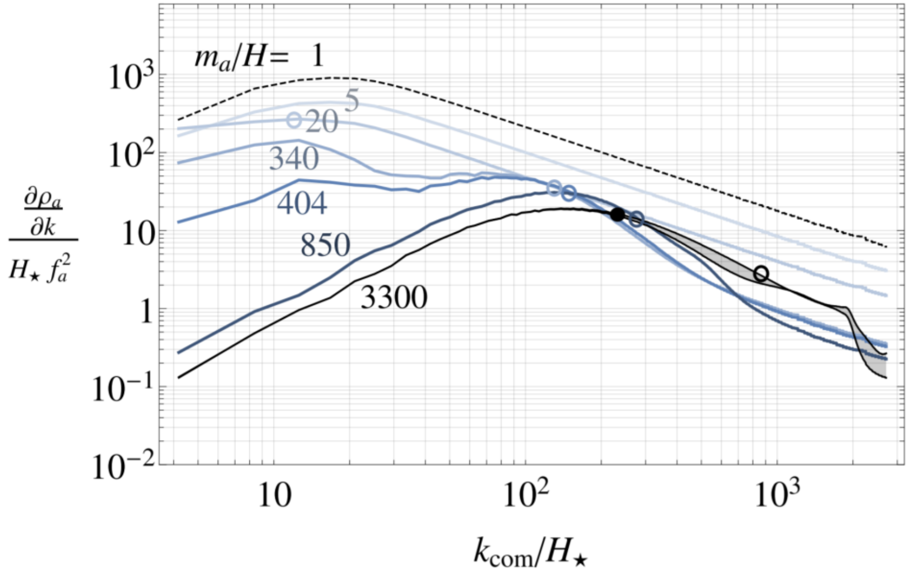




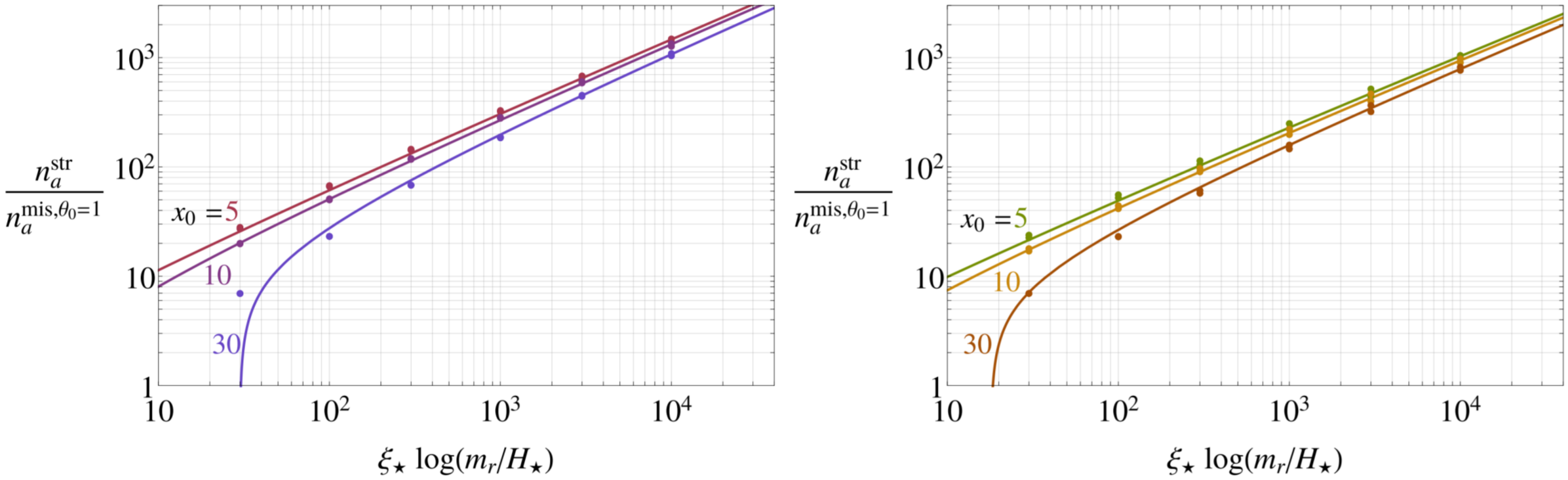
Relativistic Regime and Nonlinear Transient





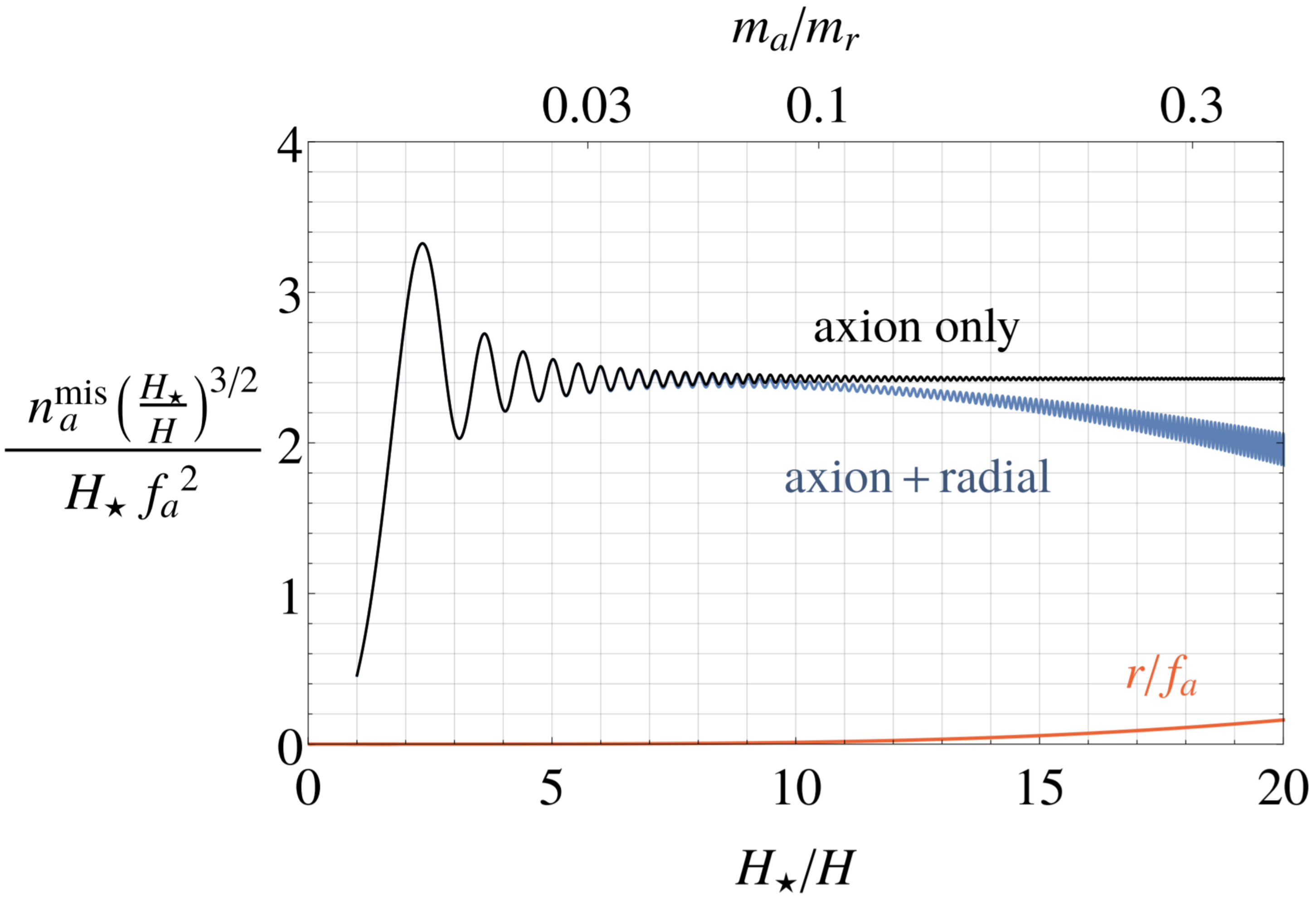


Axion Number Density after the transient

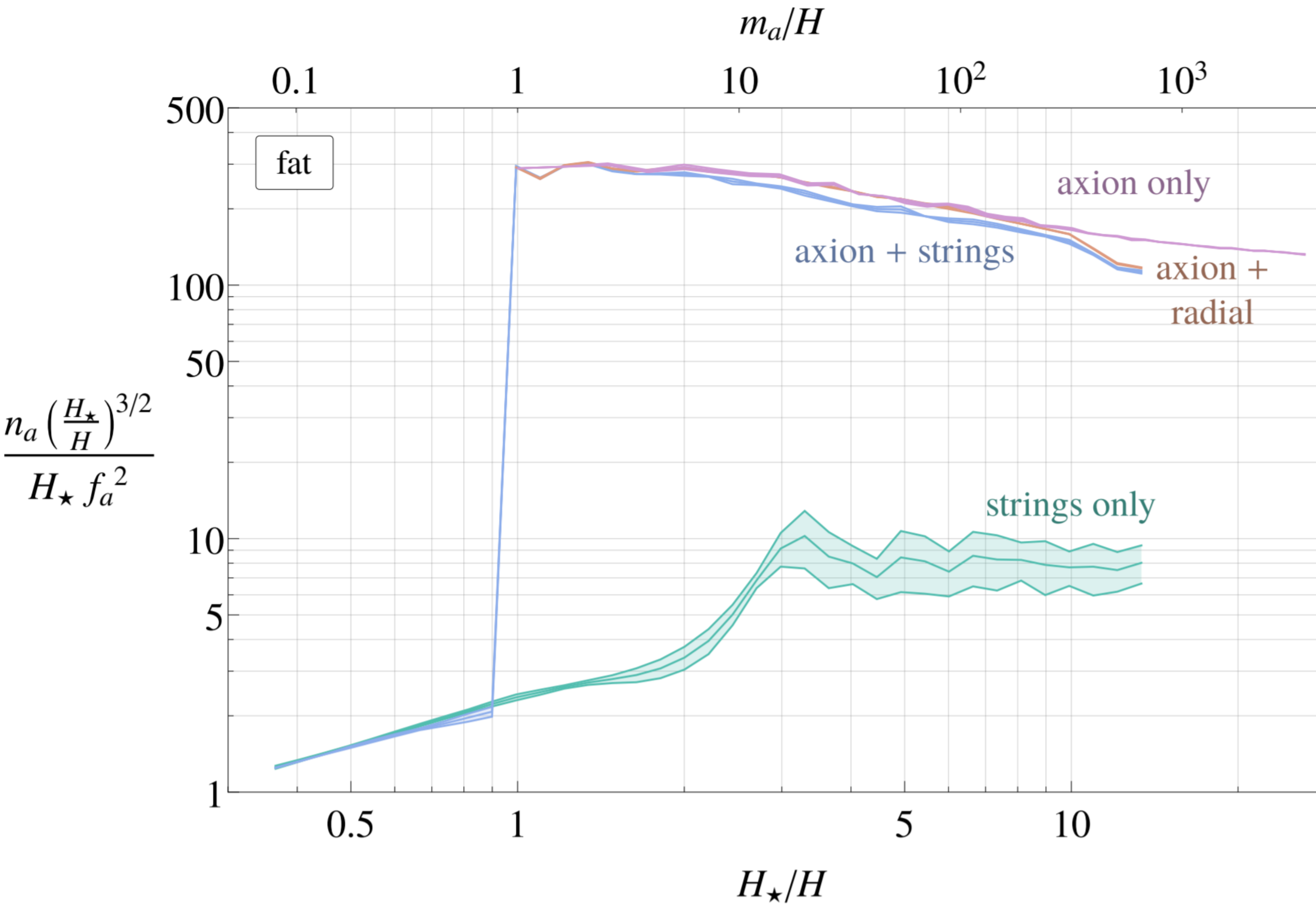


$$
\begin{aligned}
Q(t_\ell) &\equiv \frac{n_a^{\rm str}(t_\ell)}{n_a^{{\rm mis},\theta_0=1}(t_\ell)} \\
&= \frac{c_n}{c_n'} c_V \left[\frac{W_{-1}\left(-\frac{c_V\left(1+\frac{2}{\alpha+2}\right)}{4\pi\xi_\star \log_\star} \left(\frac{x_0}{c_m}\right)^{2\left(1+\frac{2}{\alpha+2}\right)} \right)}{-\frac{c_V\left(1+\frac{2}{\alpha+2}\right)}{4\pi\xi_\star \log_\star}} \right]^{\frac{1}{2}\left(1+\frac{2}{\alpha+4}\right)} \\
&= \frac{c_n}{c_n'} c_V \left[\frac{4\pi\xi_\star \log_\star}{c_V} \left[1 - \frac{2}{\alpha+4}\right] \log\left(\frac{4\pi\xi_\star \log_\star}{c_V} \left[1 - \frac{2}{\alpha+4}\right] \left[\frac{c_m}{x_0}\right]^{2\left(1+\frac{2}{\alpha+2}\right)} \log(\ldots) \right) \right]^{\frac{1}{2}\left(1+\frac{2}{\alpha+4}\right)} .
\end{aligned}
\tag{36}
$$

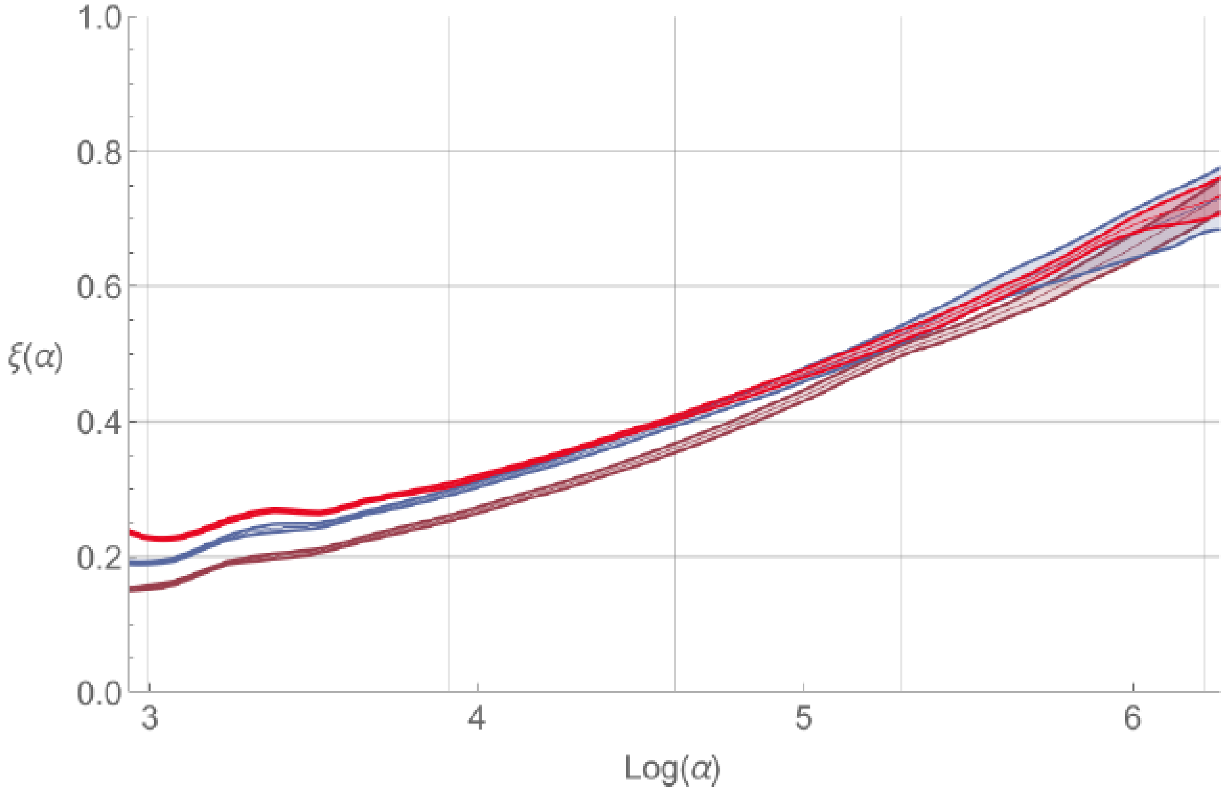
Radial Mode Decoupling



Axions Waves in a String Background



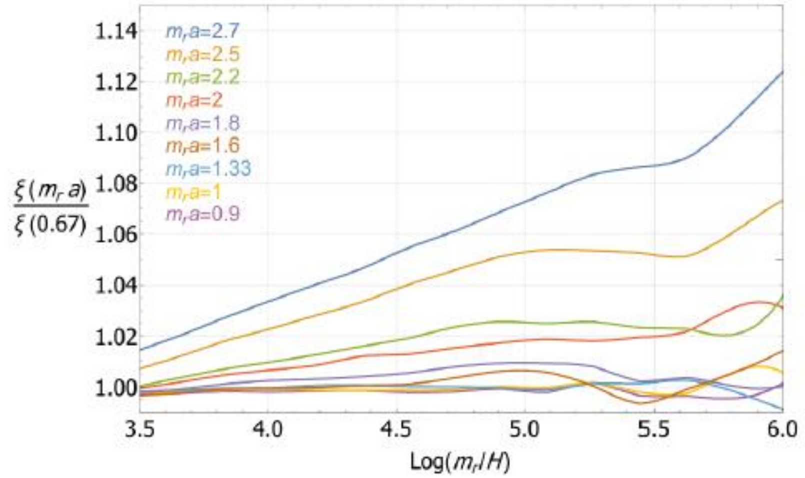
Local Strings

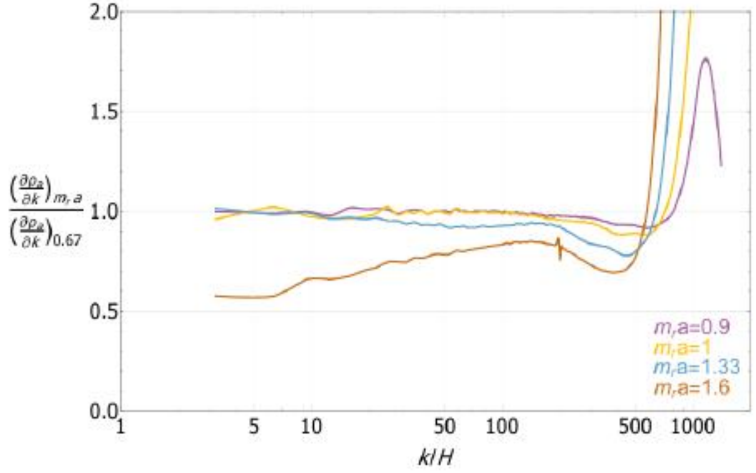


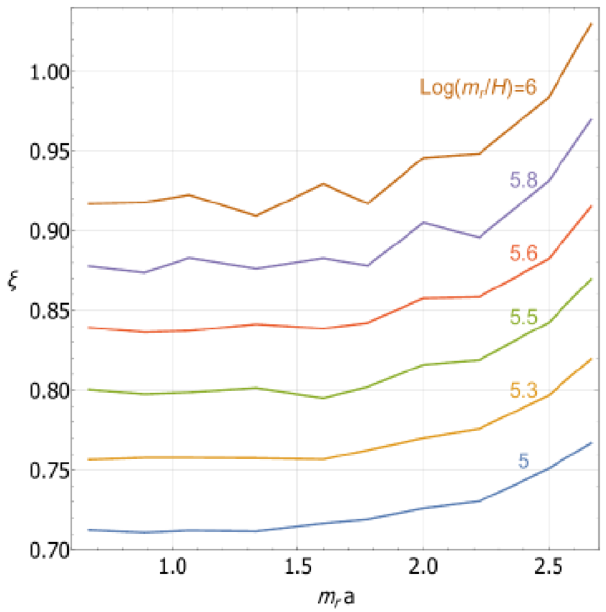
These also seem to have a log increase in $\xi\,(t)$ even though tension is constant

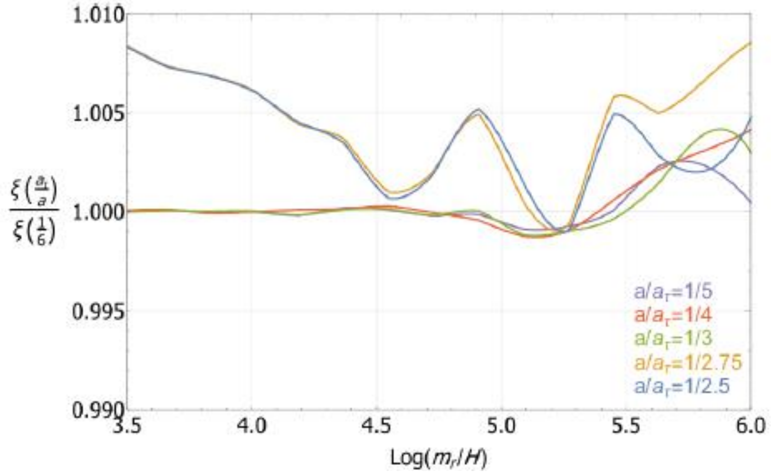
Emission to heavy modes not so suppressed, but mysterious where log is coming from?!

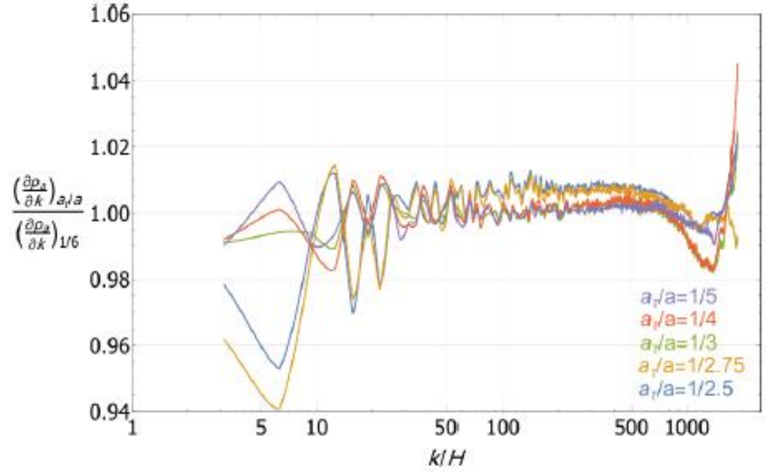
Lattice Spacing



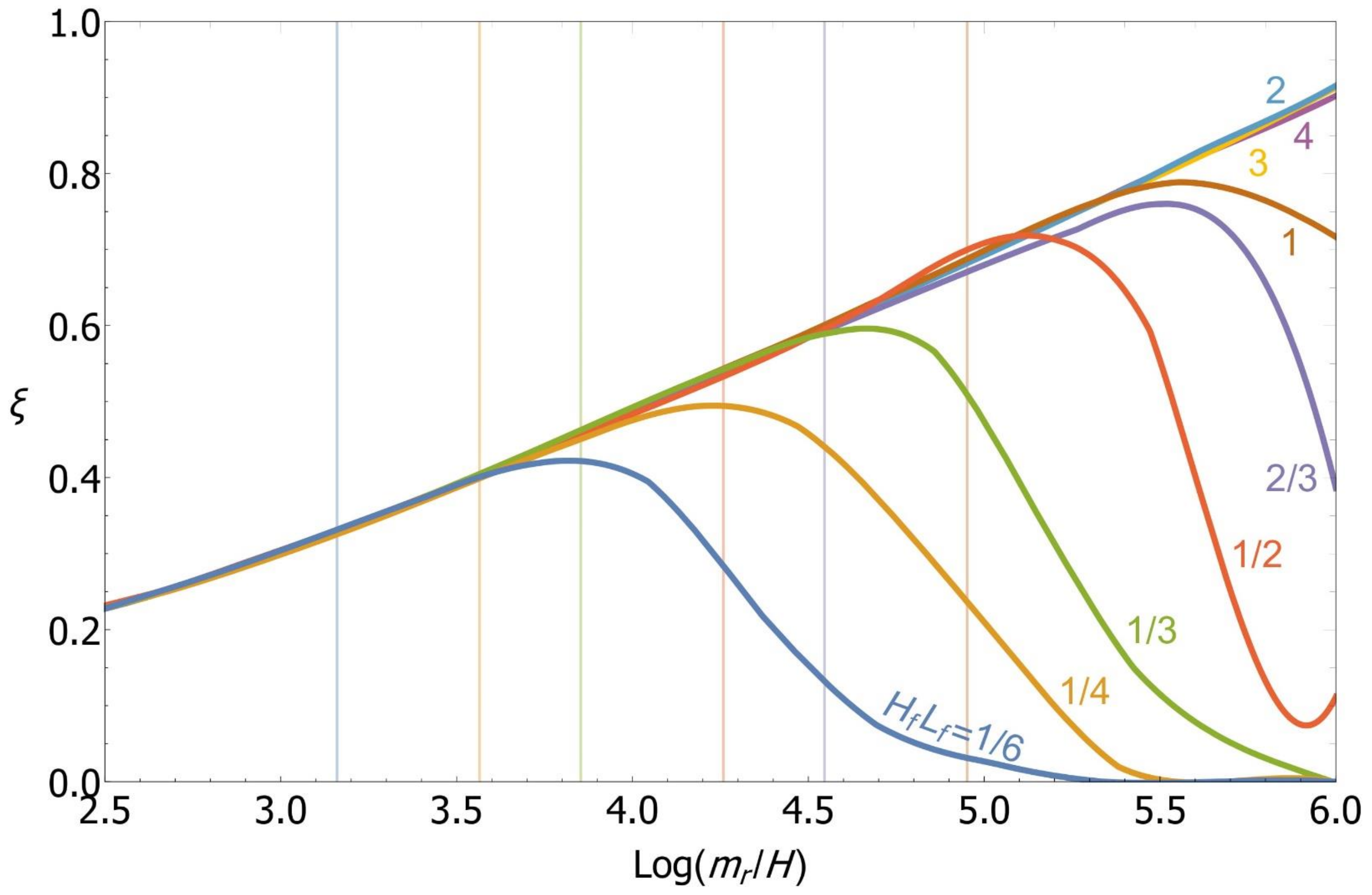




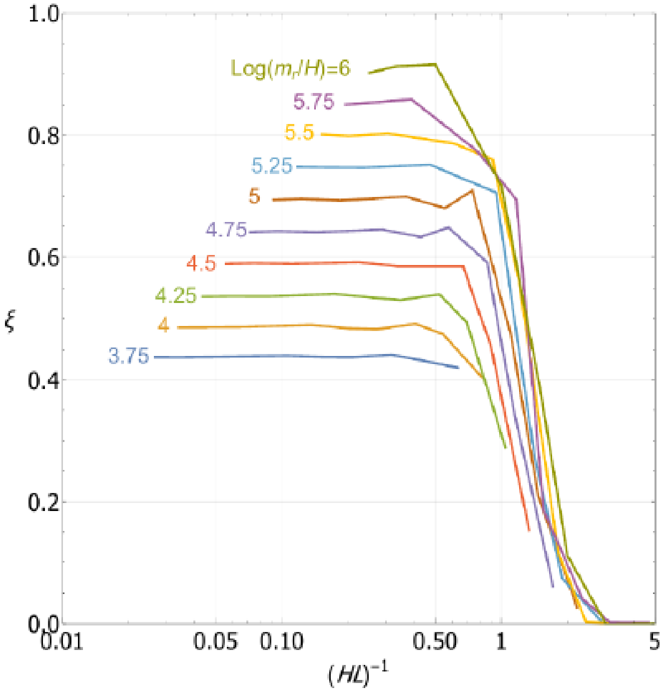


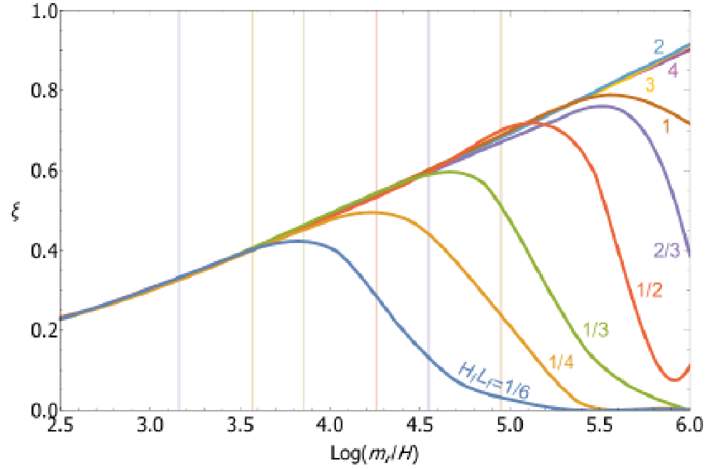


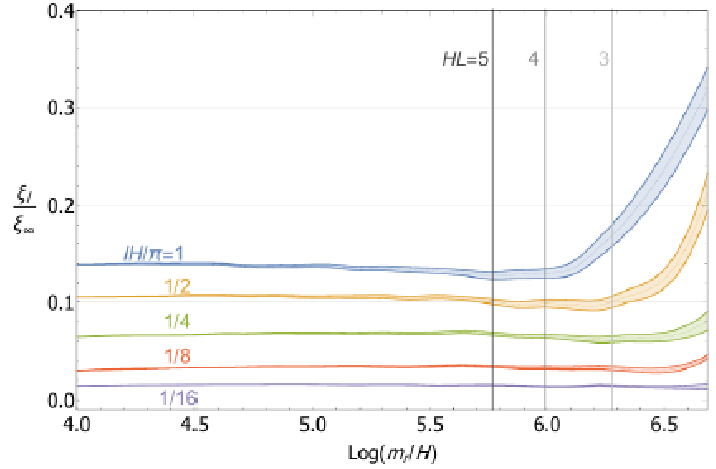
Finite Volume

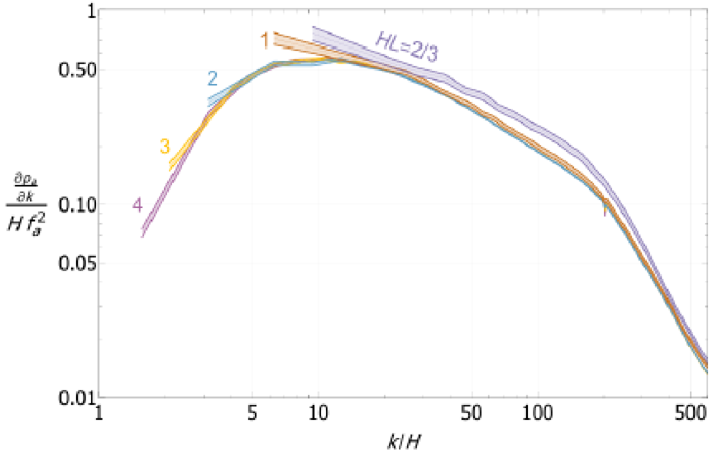


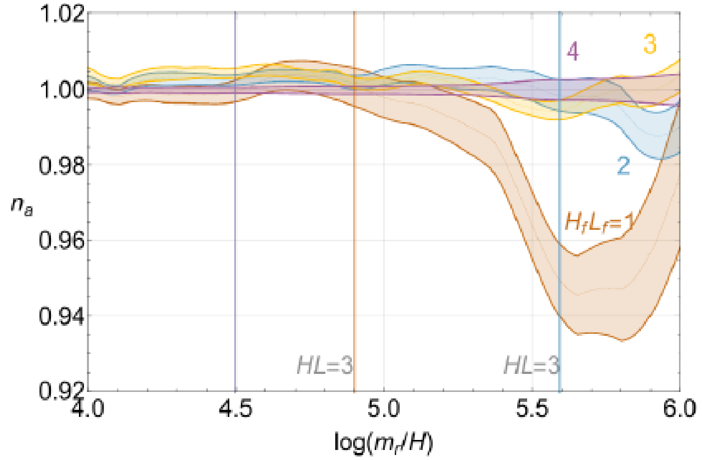
Finite Volume



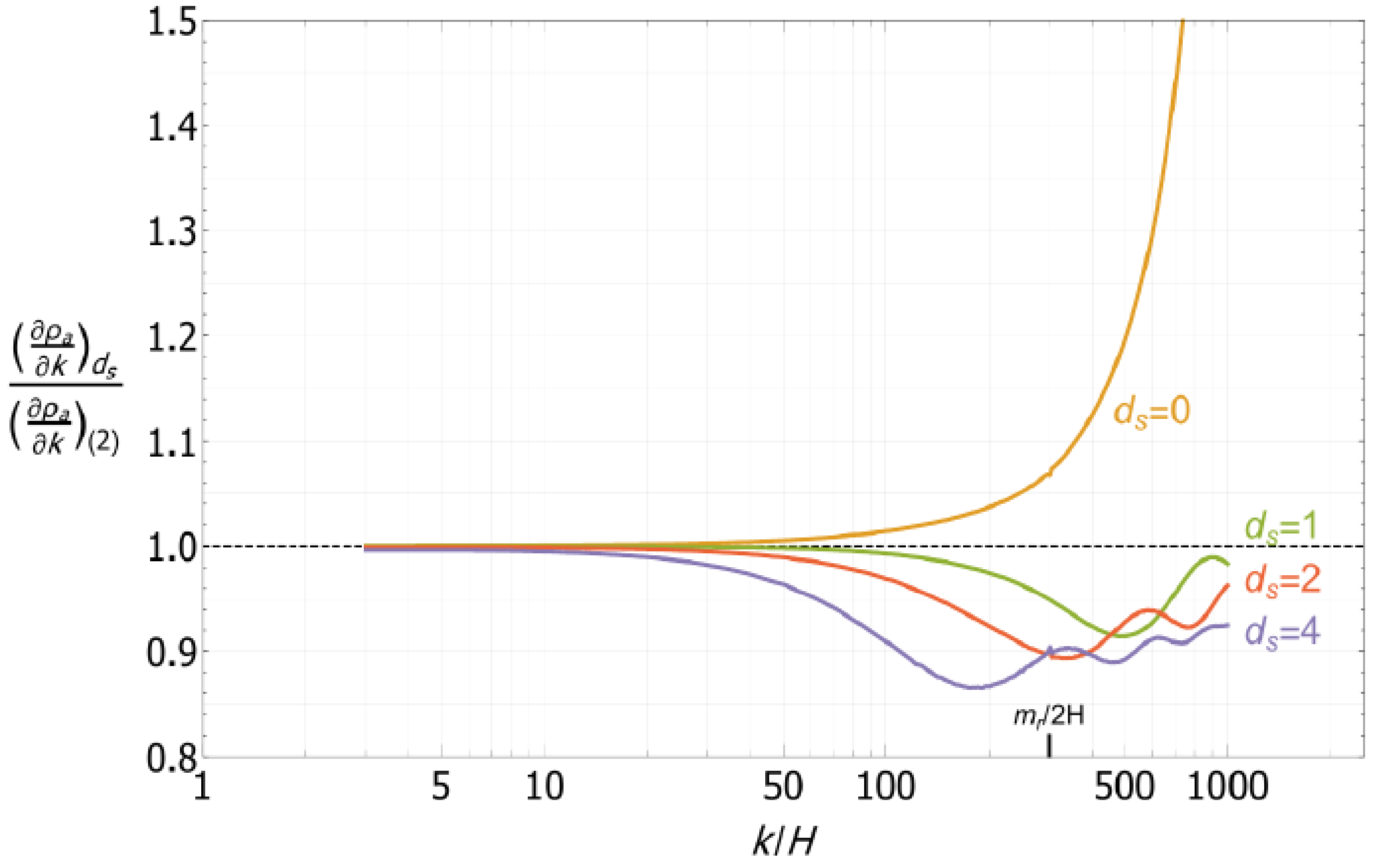






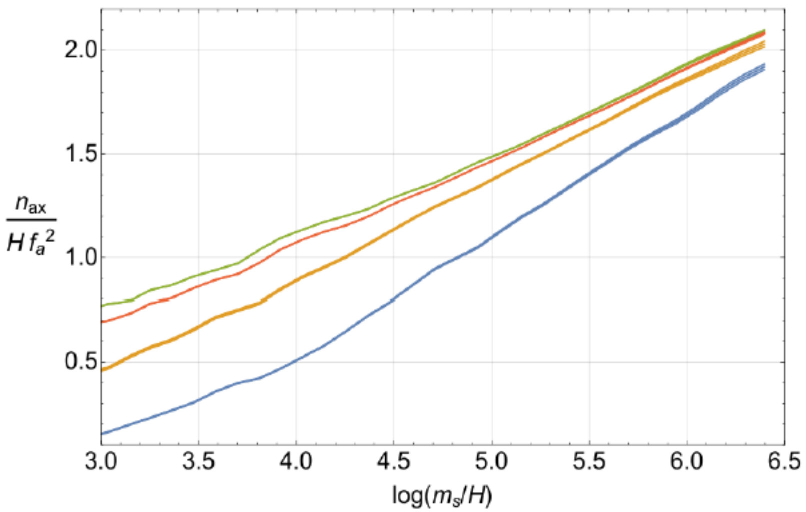


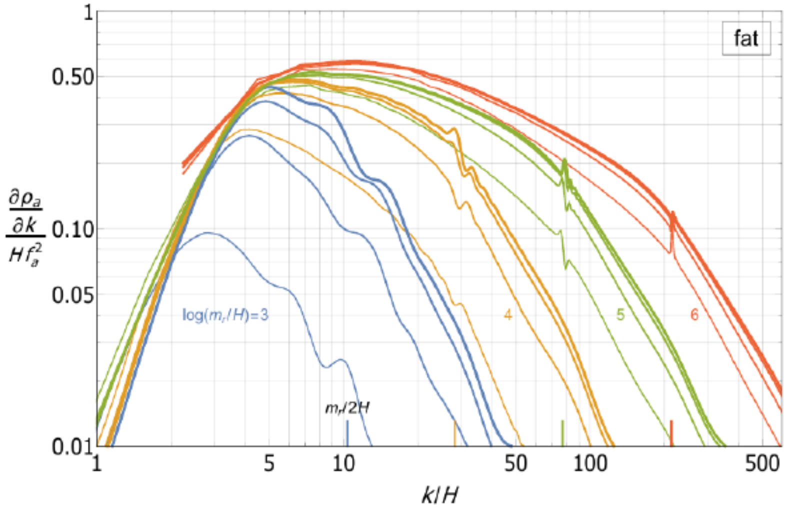
Strings Screening

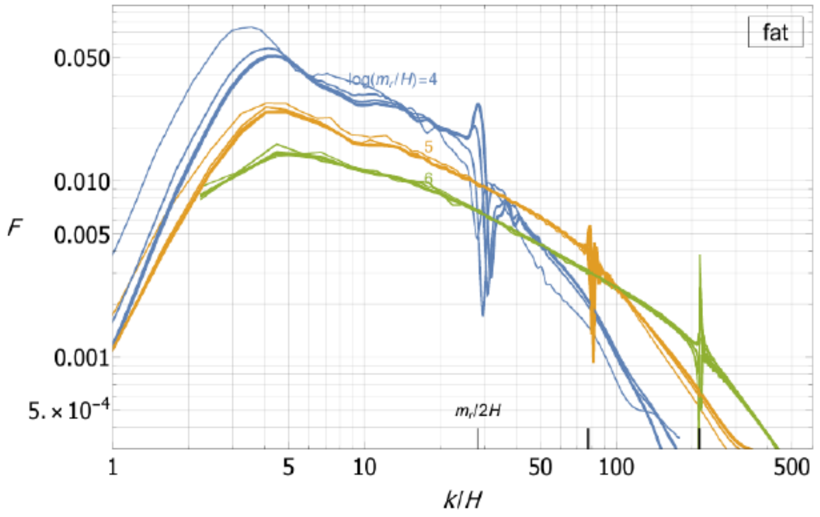


Dependence on the Initial Conditions

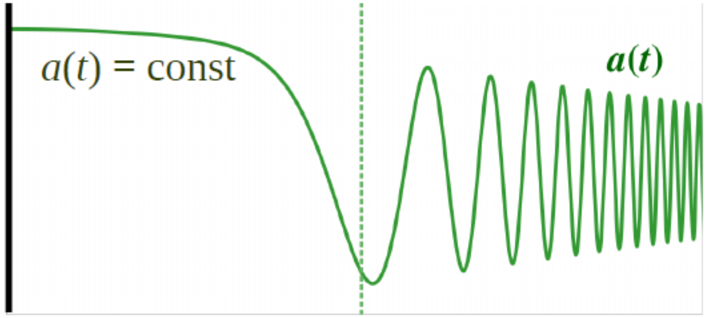
Scaling Solution *vs* Initial Conditions







Misalignment Relic Density



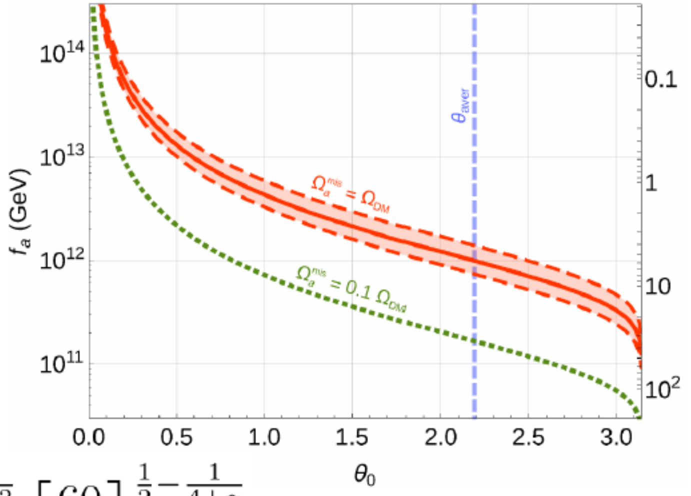
$$\frac{\langle\rho\rangle R^3}{m_a} = \text{const}$$

$$m_a(T_\star) = H(T_\star)$$

$$m_a(T) = \frac{\Lambda^2}{f_a}\left(\frac{\Lambda}{T}\right)^{\alpha/2} \Rightarrow \frac{T_\star}{\Lambda} = \left[\frac{M_P}{f_a}\frac{3\sqrt{10}}{\pi g_\star^{1/2}}\right]^{\frac{2}{4+\alpha}}$$

$$\Omega_a = \frac{\rho_a^{\text{today}}}{\rho_c} = \Omega_\gamma \frac{m_a}{T_\gamma}\frac{86}{33}\frac{n_{a\star}}{s_\star}$$

$$\frac{n_{a\star}}{s_\star} = \frac{\langle m_a(T_\star)a_0^2(t_\star)\rangle}{\frac{2\pi^2}{45}g_\star T_\star^3} = k\frac{\theta_0^2 f_a^2}{\sqrt{g_\star}M_P T_\star}$$



$$\Omega_a = 0.1\ k_\alpha \left[\frac{\theta_0}{2.15}\right]^2 \left[\frac{f_a}{10^{12}\ \text{GeV}}\right]^{1+\frac{1}{2+\alpha/2}} \left[\frac{60}{g_\star}\right]^{\frac{1}{2}-\frac{1}{4+\alpha}}$$