

Mixed QCDxEW corrections for Drell-Yan processes

Zoom personal room for discussions

ID: 872 1692 2022

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Outline

Mixed QCDxEW corrections for Drell-Yan processes

complexities and approximations

for motivations see
A. Vicini's talk



Mixed QCDxEW corrections in the resonance region: the Z case

motivations and simplifications

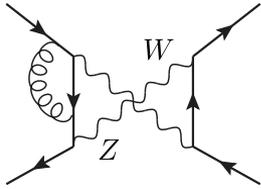
recent progress: inclusive cross section and differential distributions

Conclusions and outlook

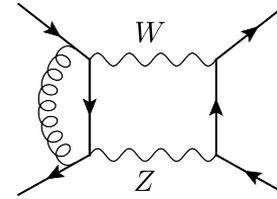
Mixed QCDxEW to Drell-Yan processes: why it is hard

The complete calculation of the mixed QCDxEW corrections for a fully off-shell dilepton system involve several technical complexities:

Amplitudes/Integrals



Two-loop integrals involving several energy scales

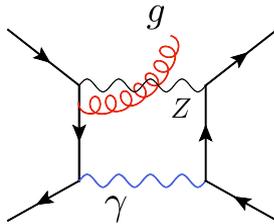


Scattering amplitudes not available yet

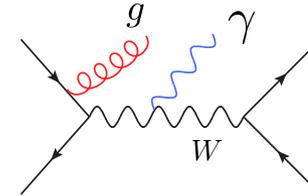
Recent developments:

[Bonciani, Di Vita, Mastrolia, Schubert 1604.08581] [Heller, von Manteuffel, Schabinger 1907.00491] [Mehedi Hasan, Schubert 2004.14908]

Subtraction of IR singularities



Complex infrared structure of the amplitude



Fully off-shell case with charged leptons not available yet

Recent developments:

[Delto, Jaquier, Melnikov, Rötsch 1909.08428] [Cieri, De Florian, Der, Mazzitelli 2005.01315]

[Buonocore, Grazzini, Tramontano 1911.10166]

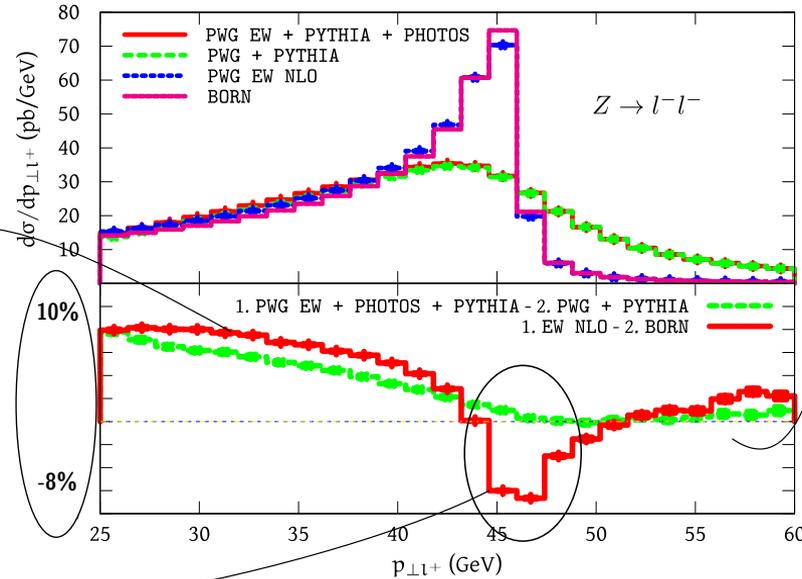
Fully off-shell NNLO QCDxQED for

$Z \rightarrow \nu\bar{\nu}$

Available approximations to full NNLO QCDxEW: PS

NLOEW and QED multiple photon corrections within native NLO and QCD Parton Shower in POWHEG BOX [Balossini et al. 0907.0276, Barze' et al. 1302.4606]

NLO QCD and EW + PS included into a single generator. Made available both for NCDY (Z) and CCDY (W)

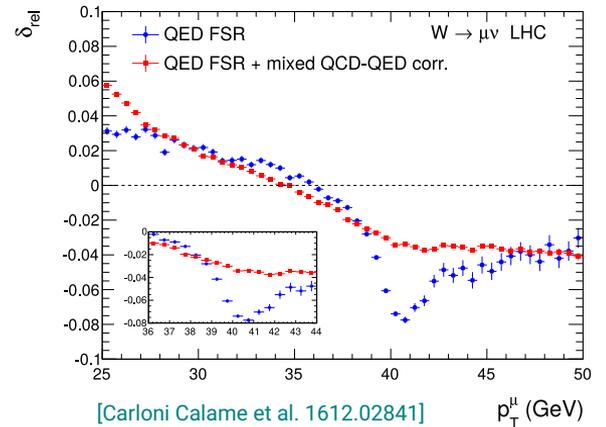


Pure NLO EW effects

Red vs Green: impact of mixed corrections: up to several %

Combination of NLO QCD x NLO EW + PS (QCD & QED) vs NLO QCD+PS

Detailed phenomenological study indicate that impact of mixed QCDxEW correction on W mass extraction $\Delta M_W \sim -16$ MeV



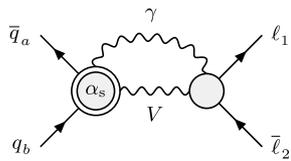
Drell-Yan at the resonance region: can we simplify it?

Resonance region relevant for EW precision studies at the LHC.

Moreover: remarkable technical simplification wrt fully off-shell case. One can work in the narrow width approximation [Fadin, Khoze, Martin hep-ph/9309234]

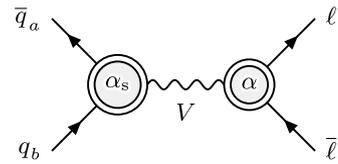
$$\sigma = \text{prod} \times \text{dec} + \mathcal{O}\left(\frac{\Gamma}{M}\right)$$

The pole approximation [Stuart '91] well suited for describing (mixed qcd-) electroweak effects near the resonance region



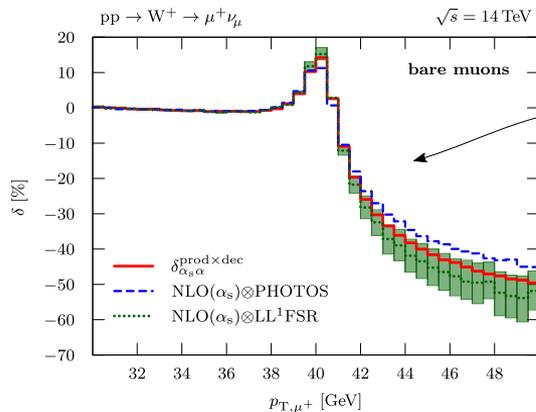
[Dittmaier, Huss, Schwinn. 1403.3216]

IS-FS non-factorizable contributions
subdominant in the resonance region.
Phenomenologically negligible



[Dittmaier, Huss, Schwinn. 1511.08016]

Dominant effects from IS-FS factorizable contributions. Authors consider initial-final and final-final.



Comparison of IS-FS correction with NLO QCD (IS) x QED PS (FS). Generally good agreement

(one of the) **Outcome** of the study:
Mixed QCDxEW corrections can have an impact as **14 MeV** in the extraction of the **W mass**.

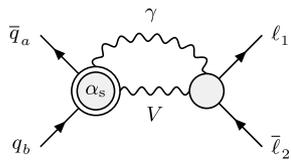
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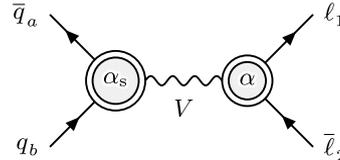
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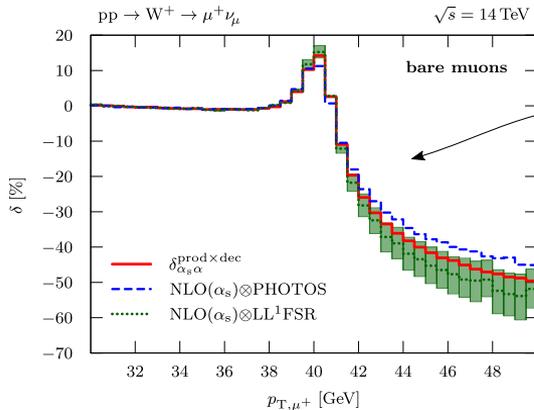
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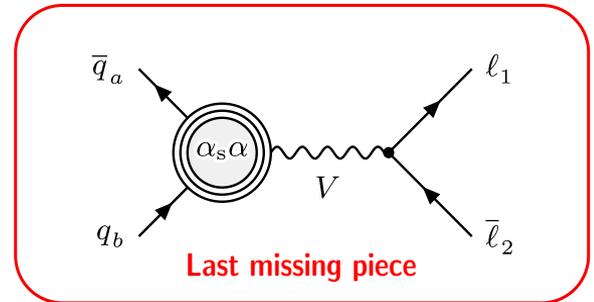
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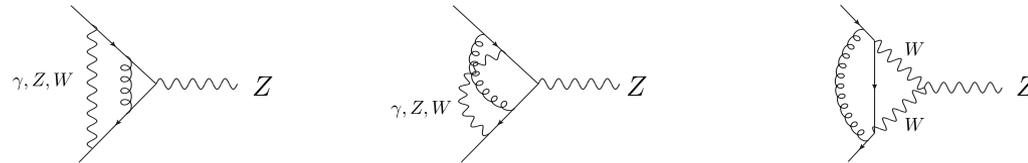
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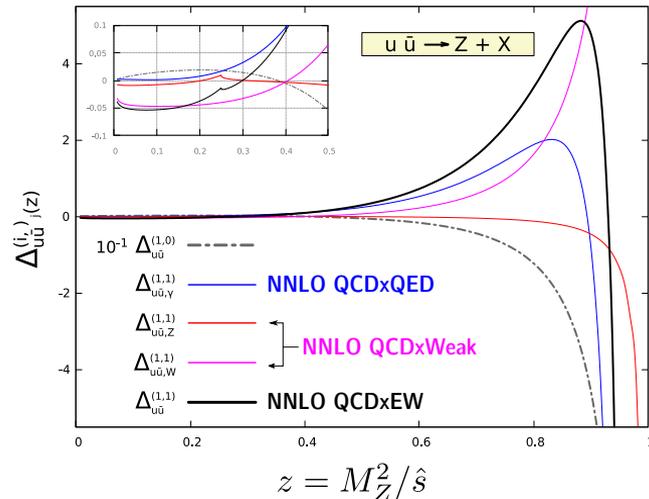
Complete QCDxEW corrections to on-shell Z: fully inclusive XS

NNLO QCDxQED corrections to inclusive Z obtained through "abelianisation" [De Florian, Der, Fabre 1805.12214] of NNLO QCD [Hamberg, Matsuura, van Neerven '90]

NNLO QCDxEW corrections to inclusive Z production: $q\bar{q} \rightarrow Z$ channel [Bonciani, F.B, Rana, Triscari, Vicini 1911.06200]



Fully analytic computation of the amplitudes and of the required loop and phase-space integrals. Important benchmark for Monte Carlo calculations



QCDxEW renormalization [Djoaudi, Gambino hep-ph/9309298] + mass factorization [De Florian, Rodrigo, Sborlini 1512.00612]

$$\alpha_s \alpha \sigma_{q\bar{q}}^{(1,1)} = \sigma_{q\bar{q}}^{(0)} \left(\Delta_{q\bar{q},\gamma}^{(1,1)} + \Delta_{q\bar{q},Z}^{(1,1)} + \Delta_{q\bar{q},W}^{(1,1)} \right)$$

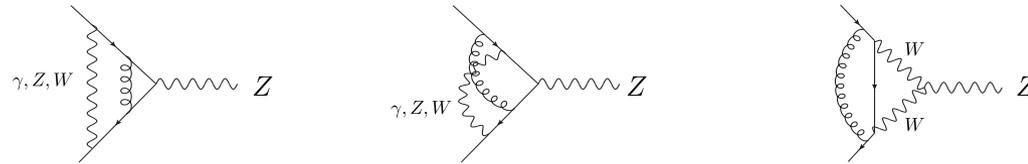
Purely weak contribution are large wrt to pure QED effects already at the partonic level

For a realistic assessment of impact of QCDxEW corrections and investigation of theory uncertainty fully hadronic XS is needed.

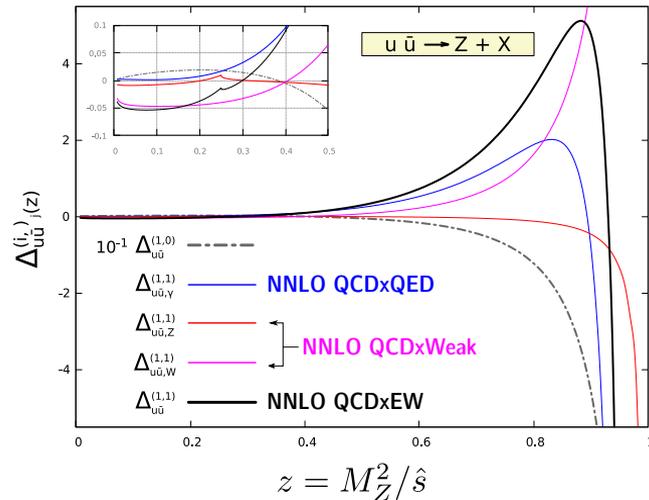
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Coming soon! (stay tuned)

Mixed QCDxEW to on-shell Z production: going differential

Complete calculation of mixed QCDxEW corrections to on-shell Z boson production at the LHC [F.B., Caola, Delto, Jaquier, Melnikov, Rötsch 2005.10221]

$$d\sigma_{pp \rightarrow e^+e^-} = \text{Br}(Z \rightarrow e^+e^-) d\sigma_{pp \rightarrow Z} \frac{d\Gamma_{Z \rightarrow e^+e^-}}{\Gamma_{Z \rightarrow e^+e^-}}$$

In perturbation theory

$$d\sigma = \sum_{i,j} \frac{\alpha_s^i}{2\pi} \frac{\alpha^j}{2\pi} \delta\sigma^{(i,j)} = \sum_{i,j} d\sigma^{(i,j)}$$

$d\sigma^{(2,0)}$ **NNLO QCD**
 $d\sigma^{(1,1)}$ **NNLO QCDxEW**

Main aspects of the calculation:

- Independent calculation of **2-loop form factors**: agreement with results available from the literature [Kotikov, Kuhn, Veretin hep-ph/0703013]
MIs known in the literature [Aglietti, Bonciani hep-ph/0304028] [Aglietti, Bonciani hep-ph/0401193]
- One-loop **real-virtual integrals** from OpenLoops2 [F.B., Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 1907.13071]
It guarantees numerical stability in unresolved IR regions.
- **QCDxEW renormalization** [Djoaudi, Gambino hep-ph/9309298]
- **Subtraction of IR singularities** performed within the nested soft collinear subtraction formalism [Caola, Melnikov, Rötsch 1702.01352]
adapted to QCDxQED corrections [Delto, Jaquier, Melnikov, Rötsch 1909.08428]

Corrections to the integrated cross sections

Electroweak corrections can be further split as $d\sigma^{(i,1)} = d\sigma_{\text{QED}}^{(i,1)} + d\sigma_{\text{weak}}^{(i,1)}$

bin-by-bin in differential distributions

Ratio to NLO QCD

$$d\Delta^{(i,j)} = \frac{d\sigma^{(i,j)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

the $Z \rightarrow e^+e^-$ branching ratio drops in the ratio

We present **results for**:

- 13 TeV LHC
- G_μ scheme: (G_μ , M_W , M_Z) as input
- $\mu_F = \mu_R = M_Z/2$

Standard selection criteria (cuts):

- $p_{T,l_1} > 24$ GeV (harder lepton)
- $p_{T,l_2} > 16$ GeV (softer lepton)
- $-2.4 < y_l < 2.4$
- $m_{ll} > 50$ GeV
- e- γ recombination for $R_{e\gamma} < 0.1$

$$R_{e\gamma} = \sqrt{(y_e - y_\gamma)^2 + (\varphi_e - \varphi_\gamma)^2}$$

	Inclusive	Cuts	Cuts (production)
$\Delta_{\text{QED}}^{(0,1)}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta_{\text{weak}}^{(0,1)}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
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$\Delta_{\text{QED}}^{(1,1)}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
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Tiny EW corrections due to G_μ scheme

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(QCDx)QED corrections strongly sensitive to selection cuts

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NNLO QCD unnaturally (and accidentally) small. QCDxEW compete (eventually larger)

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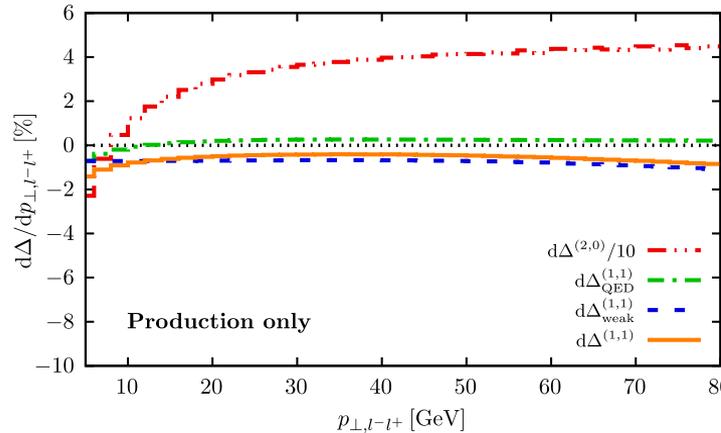
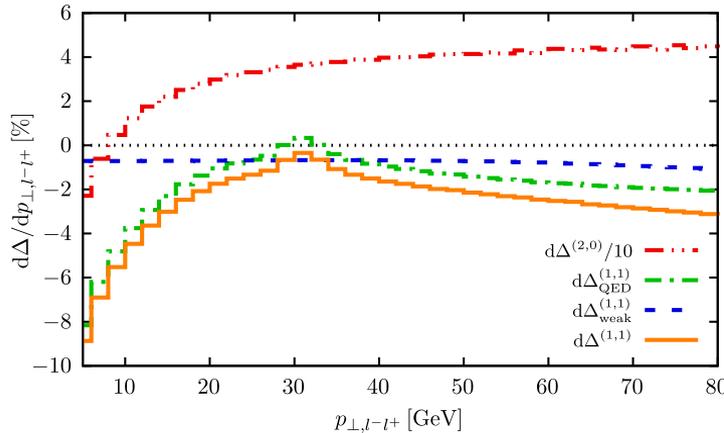
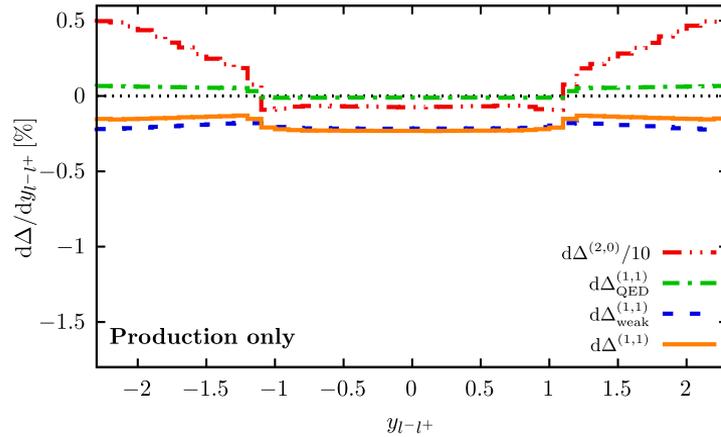
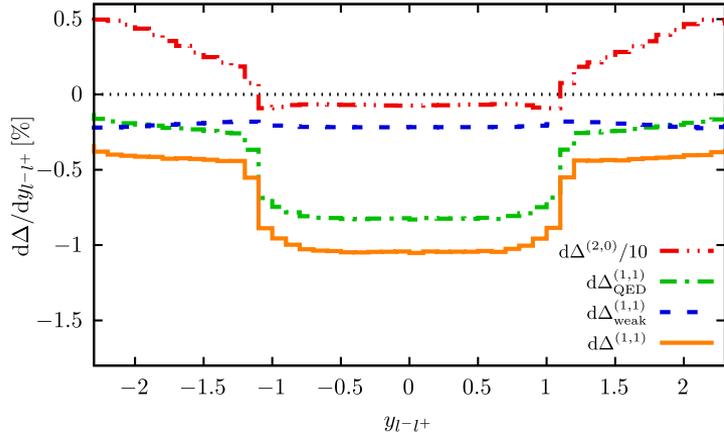
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Production process.

It removes (strong) dependence on lepton cuts:
weak corrections dominant over QED

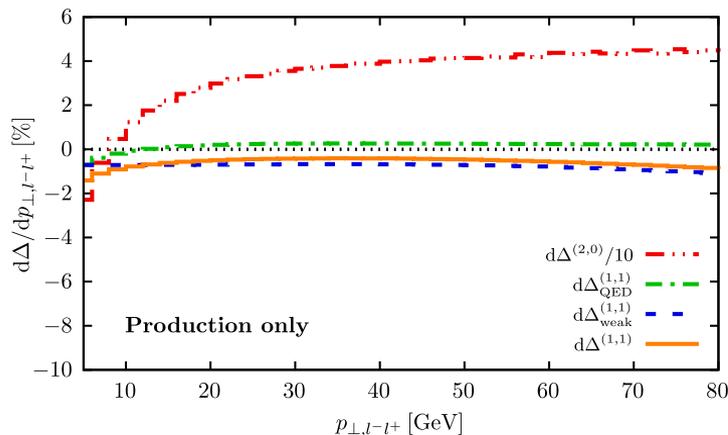
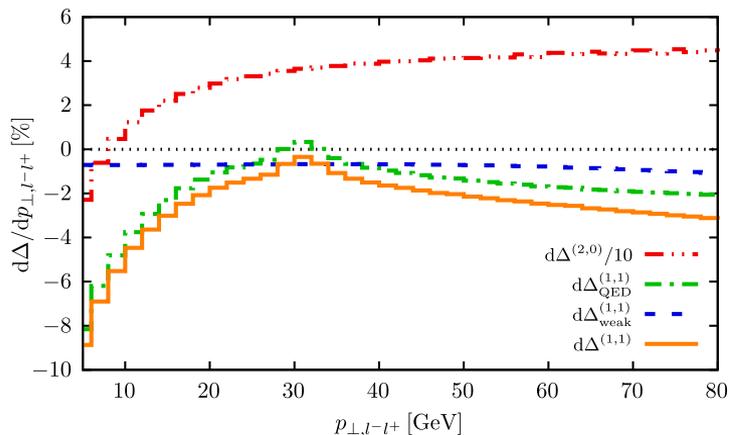
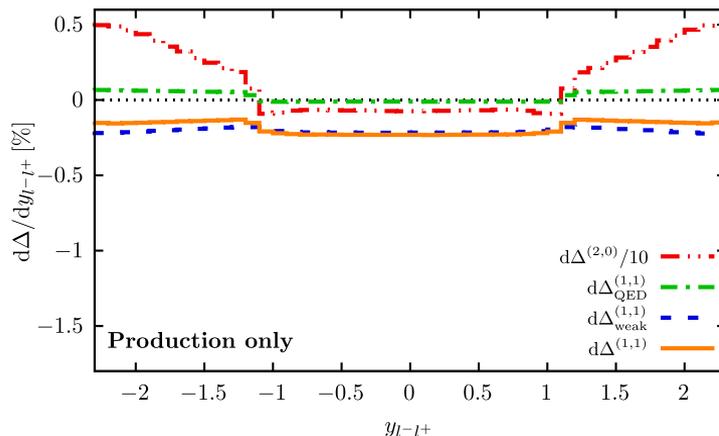
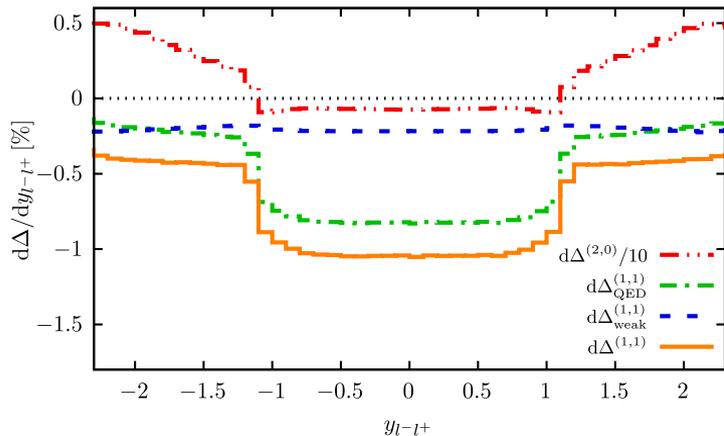
Differential distributions



As for the integrated XS:

- QCDxWeak corrections dominate over QCDxQED in the production mechanism
- In certain kinematic regions QCDxEW effects comparable to NNLO QCD ones, e.g. central y_{ll} and low $p_{T,ll}$
- The impact of mixed QCDxEW, as well as the QED-Weak interplay is observable dependent

Differential distributions

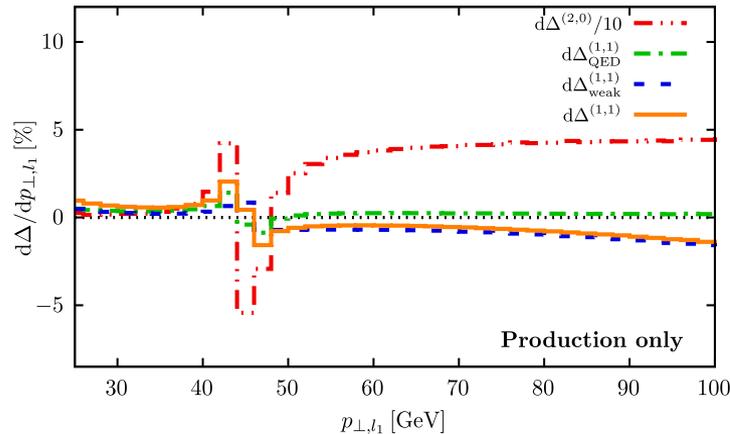
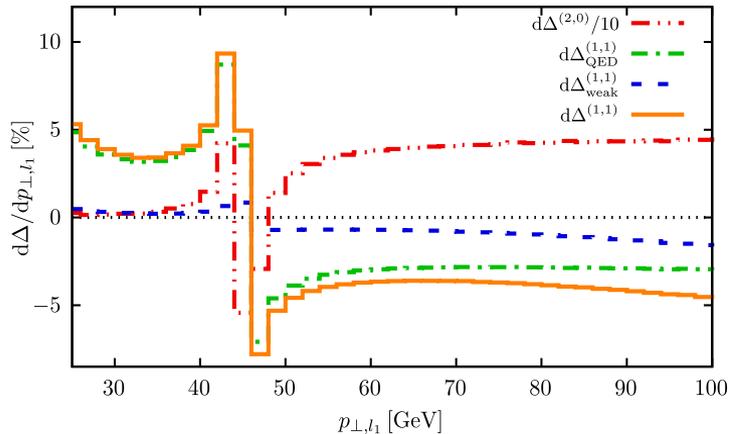
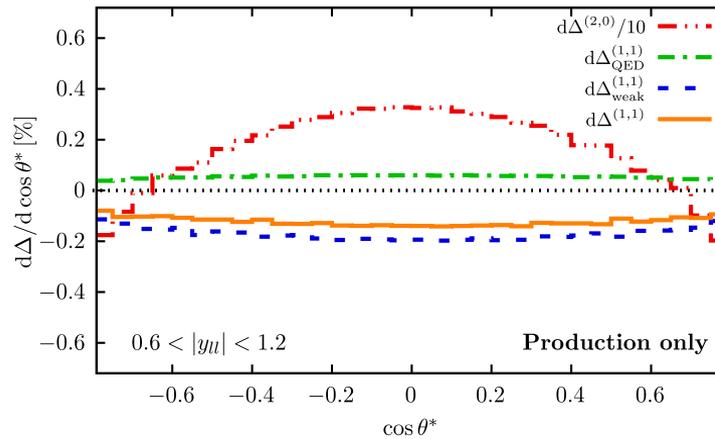
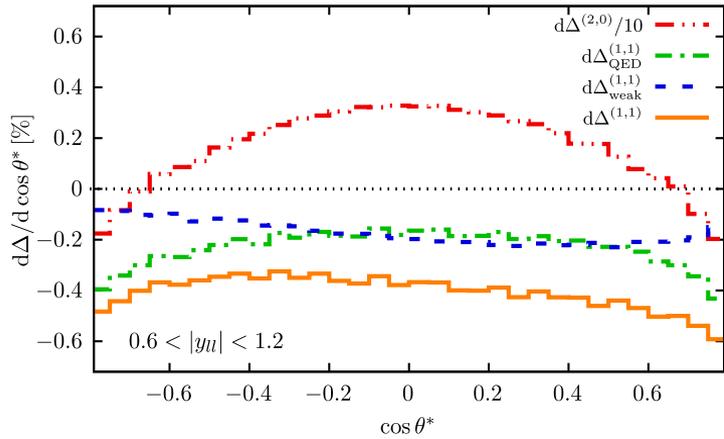


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both EW and QED effects must be considered for the required $O(\alpha_s)$ accuracy

Differential distributions



Collins-Soper angle ϑ^*

$$\cos \theta^* = \text{sgn}(p_{z,u}) \frac{P_{l^-}^+ P_{l^+}^- - P_{l^-}^- P_{l^+}^+}{\sqrt{m_u^2 (m_u^2 + p_{\perp,u}^2)}}$$

$\cos \theta^*$ allows for a precise determination of the weak mixing angle at the LHC

- for $\cos \theta^*$ QED and weak effects have a similar impact even when fs corrections are included
- LO kinematic boundary $p_{\perp,l1} < M_Z/2$. Effects more pronounced when FS QED corrections are included. Sensitivity to selection cuts

Comparison against factorised approximation

Preliminary

Exact: $d\sigma = d\sigma^{\text{LO}} (1 + \delta_{\text{QCD}} + \delta_{\text{EW}} + \delta_{\text{QCD}\times\text{EW}})$ $\delta_{\text{QCD}} \sim \text{NLO+NNLO QCD}$

Multiplicative approximation:

$$d\sigma = d\sigma^{\text{LO}} (1 + \delta_{\text{QCD}}) (1 + \delta_{\text{EW}})$$

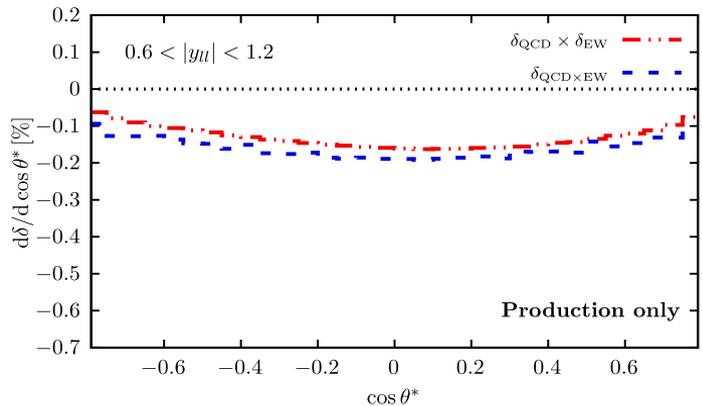
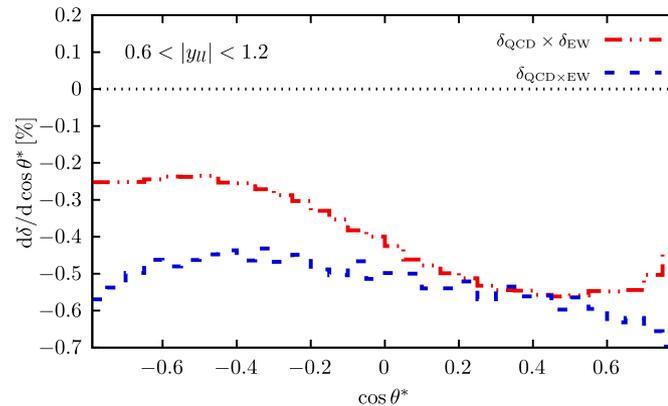
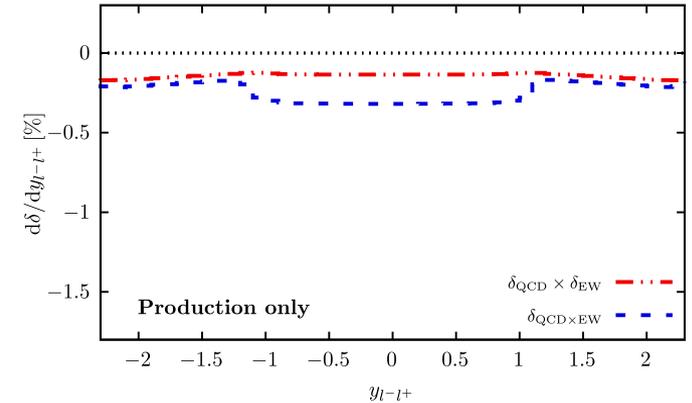
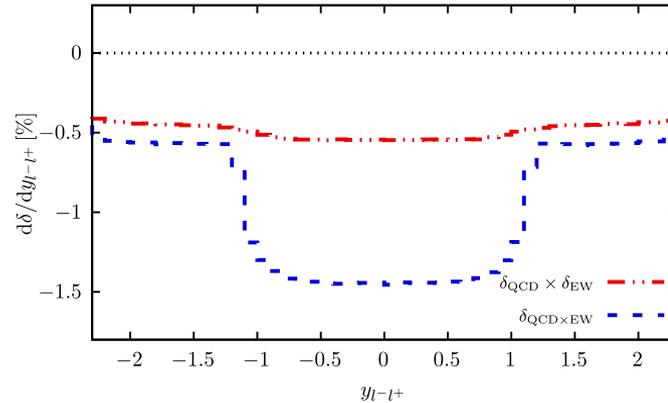
$$\delta_{\text{QCD}} \times \delta_{\text{EW}}$$

widely adopted method to estimate missing mixed QCDxEW effects.

NB: δ defined wrt LO

At the integrated cross section level:

	$\delta_{\text{QCD}} \times \delta_{\text{EW}}$	$\delta_{\text{QCD}\times\text{EW}}$
Inclusive	-1.17×10^{-3}	-1.40×10^{-3}
Cuts	-0.51×10^{-2}	-1.09×10^{-2}
Cuts (production)	-1.38×10^{-3}	-2.65×10^{-3}



Similar studies in [\[Dittmaier, Huss, Schwinn. 1511.08016\]](#)

Conclusions and outlook

The Drell-Yan process is a paradigm of precision physics at hadron colliders

For the required level of accuracy and for the upcoming HL-LHC phase, NNLO QCDxEW corrections need to be taken into account.

This is an active and lively topic of research and we are witnessing fast and constant progress from the side of theoretical calculations.

Mixed QCDxEW corrections to Z production at the resonance:

- mixed QCDxEW corrections are generally small: $O(10^{-3})$, **however...**
- significant **interplay between QCDxQED and QCDxWeak**
- effects **vary with the observable** of interest and **depend strongly on the selection cuts**
- proper assessment of the reduction of theory uncertainty **needs a complete N3LO QCD calculation**

complete mixed QCDxEW corrections needed (where possible)
if accuracy target is $< 1\%$

Outlook:

Complete QCDxEW corrections at the W resonance:

- W boson mass extraction from LHC data with target accuracy of $O(10 \text{ MeV})$. **Mixed corrections need to be taken into account.**

Mixed QCDxEW corrections away from the resonance

- Crucial for studies in high-invariant mass regions
- Assessment of validity of most used approximations
- Many interesting formal aspects of perturbative calculations

Backup

Corrections to the integrated cross sections. $\mu_F = \mu_R = M_Z$

$$d\Delta^{(i,j)} = \frac{d\sigma^{(i,j)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

13 TeV LHC
 G_μ scheme: (G_μ , M_W , M_Z) as input
 $\mu_F = \mu_R = M_Z$

$p_{T,l_1} > 24$ GeV (harder lepton)
 $p_{T,l_2} > 16$ GeV (softer lepton)
 $-2.4 < y_l < 2.4$
 $m_{ll} > 50$ GeV
e- γ recombination for $R_{e\gamma} < 0.1$

Ratio to NLO QCD

$\mu_F = \mu_R = M_Z/2$

	Inclusive	Cuts	Cuts (production)
$\Delta_{\text{QED}}^{(0,1)}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta_{\text{weak}}^{(0,1)}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
$\Delta^{(0,1)}$	-3.2×10^{-3}	-1.0×10^{-2}	-2.8×10^{-3}
$\Delta^{(2,0)}$	$+1.3 \times 10^{-2}$	$+5.8 \times 10^{-3}$	$+5.8 \times 10^{-3}$
$\Delta_{\text{QED}}^{(1,1)}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
$\Delta_{\text{weak}}^{(1,1)}$	-1.6×10^{-3}	-2.1×10^{-3}	-2.1×10^{-3}
$\Delta^{(1,1)}$	-1.1×10^{-3}	-8.0×10^{-3}	-2.0×10^{-3}

$\mu_F = \mu_R = M_Z$

	Inclusive	Cuts	Cuts (production)
$\Delta_{\text{QED}}^{(0,1)}$	$+3.1 \times 10^{-3}$	-5.5×10^{-3}	$+3.0 \times 10^{-3}$
$\Delta_{\text{weak}}^{(0,1)}$	-6.2×10^{-3}	-5.8×10^{-3}	-5.8×10^{-3}
$\Delta^{(0,1)}$	-3.1×10^{-3}	-1.1×10^{-2}	-2.9×10^{-3}
$\Delta^{(2,0)}$	-6.3×10^{-3}	-1.2×10^{-2}	-1.2×10^{-2}
$\Delta_{\text{QED}}^{(1,1)}$	$+2.9 \times 10^{-4}$	-5.2×10^{-3}	-1.5×10^{-4}
$\Delta_{\text{weak}}^{(1,1)}$	-9.2×10^{-4}	-1.3×10^{-3}	-1.3×10^{-3}
$\Delta^{(1,1)}$	-6.4×10^{-4}	-6.5×10^{-3}	-1.5×10^{-3}

On-shell Z production in the narrow-width approximation

The production of a Z boson is computed in the narrow width approximation

$$d\sigma_{pp \rightarrow e^+e^-} = \text{Br}(Z \rightarrow e^+e^-) d\sigma_{pp \rightarrow Z} \frac{d\Gamma_{Z \rightarrow e^+e^-}}{\Gamma_{Z \rightarrow e^+e^-}}$$

The cross section for the process $pp \rightarrow Z \rightarrow e^+e^-$ can be expanded in power of α_s and α

$$d\sigma = \sum_{i,j} \frac{\alpha_s^i}{2\pi} \frac{\alpha^j}{2\pi} \delta\sigma^{(i,j)} = \sum_{i,j} d\sigma^{(i,j)}$$

The partial decay width is expanded perturbatively, so is the ratio $d\Gamma/\Gamma$:

$$\Gamma_{Z \rightarrow e^+e^-} = \Gamma^0 \times \left(1 + \alpha\delta_{\text{dec}}^{(0,1)} + \alpha\alpha_s\delta_{\text{dec}}^{(1,1)}\right) + \mathcal{O}(\alpha^2, \alpha_s^2) \quad d\Gamma_{Z \rightarrow e^+e^-} = d\Gamma^{(0,0)} + d\Gamma^{(0,1)} + d\Gamma^{(1,1)} + \mathcal{O}(\alpha^2, \alpha_s^2)$$

The mixed QCDxEW corrections to the cross sections thus read

$$d\sigma^{(1,1)} = \text{Br}(Z \rightarrow e^+e^-) \times \left[\underbrace{d\sigma_{pp \rightarrow Z}^{(1,1)} \times \frac{d\Gamma^{(0,0)}}{\Gamma^0}}_{\text{"production only"}} + \underbrace{d\sigma_{pp \rightarrow Z}^{(1,0)} \times \left(\frac{d\Gamma^{(0,1)}}{\Gamma^0} - \alpha \frac{d\Gamma^{(0,0)}}{\Gamma^0} \delta_{\text{dec}}^{(0,1)} \right) + d\sigma_{pp \rightarrow Z}^{(0,0)} \times \left(\frac{d\Gamma^{(1,1)}}{\Gamma^0} - \alpha\alpha_s \frac{d\Gamma^{(0,0)}}{\Gamma^0} \delta_{\text{dec}}^{(1,1)} \right)}_{\text{final-state effects}} \right]$$