EFT validity issues in Vector Boson Scattering data analysis

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Introduction

- Vector Boson Scattering processes - probe of:
  - Higgs to VV and triple VVV couplings,
  - Quartic VVVVV couplings (most sensitive probe)

- Each of these diagrams alone is divergent. Standard Model couplings ensure finite cross sections at all energies (precise cancelation of all high energy divergences).

- Extensions of the Standard Model induce coupling modifications that can be parameterized in terms of the Effective Field Theory (EFT) approach:

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + \sum_j \frac{f_j}{\Lambda^4} \mathcal{O}_j + \ldots \]

  - Dimension-6 operators (affect triple and quartic couplings)
  - Dimension-8 operators (affect only quartic couplings)

- Traditionally, VBS analyses focus on dim-8 operators, usually on a one-by-one basis.
The unitarity problem and how we deal with it

- It is well known that any dim-8 operator causes amplitude growth which behaves asymptotically as \( \sim s^2 \) and eventually leads to unitarity violation.

**Philosophy 1. Disregard unitarity limits (CMS mainstream)**
- technically simplest,
- fair to quantify the relative precision of different measurements and the degree of agreement/disagreement with the SM,
- obtained numbers do not have direct EFT interpretation and unitarity violation usually occurs well within the measured range.

**Philosophy 2. Unitarization techniques:**
  - Amplitude saturation, e.g., K-matrix (ATLAS mainstream),
  - Form factor approach (e.g. VBFNLO)
- describe the maximum possible signal related to a given operator,
- no unique prescription,
- part of a model,
- obtained numbers not easy to interpret within the EFT.

- **Side remark:** We should never directly compare ATLAS numbers with CMS numbers!

Q: How to render experimental numbers more physical and model independent?
1. EFT validity stops at $M_{W^+W^-} = \Lambda$, the scale of new physics. $\Lambda$ can be maximally equal to the relevant unitarity limit, but it may as well be lower than that, $\Lambda \leq M^U$. The actual value of $\Lambda$ is unknown a priori and can only be deduced from the data.

2. For a given operator $\Lambda$ is one value, it applies to all affected amplitudes, even though their individual unitarity limits can be much higher. Relevant e.g. to helicity combinations.

3. $\Lambda$ must be common to different processes if they probe the same coupling (same set of higher dimension operators). For instance, the $W^+W^-$ scattering process reaches unitarity limit before $W^+W^+$ for most dim-8 operators: $O_{S1}$, $O_{T0}$, $O_{T1}$ (positive $f$), $O_{T2}$, $O_{M0}$, $O_{M1}$ and $O_{M7}$.

The same goes for $WZ \rightarrow WZ$ and $WW \rightarrow ZZ$: same WWZZ coupling, ZZ usually breaks unitarity faster.
Total $W^+W^+ \rightarrow W^+W^+$ cross section (on shell) for different dim-8 operators: S0, T2, M0, T0, M1 and M7

Measureable BSM effects are confined to a narrow energy range just before the unitarity limit (vertical lines)
Total $W^+W^+ \rightarrow W^+W^+$ cross section (on shell) for $f_{T1} = -0.1/\text{TeV}^4$

split into initial & final state helicity combinations

13 independent combinations

Unitarity limits $M^U$ (in TeV) for individual amplitudes

<table>
<thead>
<tr>
<th>Hel. $\backslash f_{T1}$</th>
<th>-0.01</th>
<th>-0.1</th>
<th>-1.0</th>
<th>-10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$--$</td>
<td>5.3</td>
<td>3.0</td>
<td>1.7</td>
<td>0.96</td>
</tr>
<tr>
<td>$--0$</td>
<td>$7.5 \times 10^7$</td>
<td>$7.5 \times 10^6$</td>
<td>$7.5 \times 10^5$</td>
<td>$7.5 \times 10^4$</td>
</tr>
<tr>
<td>$--+$</td>
<td>$1.7 \times 10^3$</td>
<td>530.</td>
<td>170.</td>
<td>53.</td>
</tr>
<tr>
<td>$-00$</td>
<td>440.</td>
<td>140.</td>
<td>44.</td>
<td>14.</td>
</tr>
<tr>
<td>$-0+$</td>
<td>74.</td>
<td>34.</td>
<td>16.</td>
<td>7.4</td>
</tr>
<tr>
<td>$--++$</td>
<td>5.5</td>
<td>3.1</td>
<td>1.7</td>
<td>0.99</td>
</tr>
<tr>
<td>$-0-0$</td>
<td>$2.5 \times 10^3$</td>
<td>800.</td>
<td>250.</td>
<td>80.</td>
</tr>
<tr>
<td>$-0-+$</td>
<td>69.</td>
<td>32.</td>
<td>15.</td>
<td>6.9</td>
</tr>
<tr>
<td>$-000$</td>
<td>$3.7 \times 10^7$</td>
<td>$3.7 \times 10^6$</td>
<td>$3.7 \times 10^5$</td>
<td>$3.7 \times 10^4$</td>
</tr>
<tr>
<td>$-0+0$</td>
<td>$2.3 \times 10^3$</td>
<td>740.</td>
<td>230.</td>
<td>74.</td>
</tr>
<tr>
<td>$-++$</td>
<td>10.</td>
<td>5.6</td>
<td>3.2</td>
<td>1.8</td>
</tr>
<tr>
<td>$0000$</td>
<td>$1.7 \times 10^3$</td>
<td>530.</td>
<td>170.</td>
<td>53.</td>
</tr>
</tbody>
</table>

Vastly different energy dependences and individual unitarity limits with aQGCs
Practical issues in data analysis

Fact: pure EFT does not provide *any* predictions for $M_{\nu\nu} > \Lambda$.

In principle, the solution is simple: Want to do EFT? Do not use this region!

What does this imply in practice?

- **For ZZ**: only potential loss of statistics – invariant mass known, so remove events above the assumed cutoff value, set limits or fit BSM accordingly.

- **For WZ and WV**: depends on resolution in invariant mass determination, crucial issues are ambiguity in neutrino kinematics reconstruction and jet pT resolution. Under study (WZ) are methods to solve the neutrino ambiguity, e.g., by taking both solutions with weights proportional to the respective cross sections.

- **For WW**: serious problem – two neutrinos = no experimental access to invariant mass. Measured signal is in general a sum of $M_{\nu\nu} < \Lambda$ and $M_{\nu\nu} > \Lambda$. The only way to correctly use EFT is to make sure the region $M_{\nu\nu} > \Lambda$ does not significantly contribute.
• The full process is \( pp \rightarrow jj \ell^+\ell^-\nu\nu \) - "gold-plated channel"

\[ D - \text{distribution of some physical observable, BSM signal: } D^{\text{BSM}} - D^{\text{SM}}, \quad \text{EFT signal: } D^{\text{EFT}} - D^{\text{SM}} \]

• What we measure - realistically modeled total signal with regularized tail:

\[
D^{\text{BSM}} = \int_{2M_W}^{\Lambda} \frac{d\sigma}{dM_{WW}} \bigg|_{\text{EFT model}} dM_{WW} + \int_{\Lambda}^{M_{\text{max}}} \frac{d\sigma}{dM_{WW}} \bigg|_{\text{regularized}} dM_{WW}
\]

• What we want - the **EFT-controlled** part of the signal:

\[
D^{\text{EFT}} = \int_{2M_W}^{\Lambda} \frac{d\sigma}{dM_{WW}} \bigg|_{\text{EFT model}} dM_{WW} + \int_{\Lambda}^{M_{\text{max}}} \frac{d\sigma}{dM_{WW}} \bigg|_{\text{SM}} dM_{WW}
\]

• \( D^{\text{BSM}} \) and \( D^{\text{EFT}} \) must be statistically indistinguishable for EFT description be successful. This is only true in a very restricted range of \( f \) (for every dim-8 operator) and \( \Lambda \).
Predicted shape of the signal depends on both $f$ and $\Lambda$ – both need to be fit to the data.

EFT description consistency requires the bulk of BSM signal originate from the EFT-controlled region. This can be verified from simulation. Only then can the obtained $f$ and $\Lambda$ values be trusted.

The inefficiency of the EFT to describe the data: requirement of BSM signal significance conflicts with requirement of EFT consistency. Only small “EFT triangles” are left (arXiv:1802.02366)

Result does not depend on the choice of EFT (SMEFT vs HEFT, arXiv:1905.03354)

Problem persists regardless of the beam energy or the accumulated statistics (arXiv:1906.10769)
EFT triangles at HL-LHC, 14 TeV (SMEFT)

- Totally empty for $O_{S1}$.

- Caution: this is a generator level study, sensitivity will degrade with full detector simulation and proper reducible background treatment.
WW case 2: setting limits on BSM

- The EFT does not predict what happens above $\Lambda$.

- Only the most conservative signal estimate gives limits guaranteed to be true. Most conservative estimate = “clip” the generated aQGC distribution: take only SM contribution above $\Lambda$. This is the practical equivalent of not using data above $\Lambda$.

- Limits on $f$ can only be determined for an assumed value of $\Lambda$ - limits are in 2 dimensions: $f$ vs. $\Lambda$.
“Clipping” technique in implementation


- 
  ssWW, WZ & combination results from the whole Run 2 (137/fb)

- Inclusive and differential cross sections still consistent with SM predictions

- First implementation of the “clipping” technique in CMS
  - Typical unitarity limits for dim-8 operators in the relevant range: ~1.5 TeV,
  - Typical fraction of generated events (aQGC samples) above unitarity limit:
    up to ~50% for WZ and up to ~80% for ssWW
“Clipping” technique in implementation

SMP-19-012: ssWW, WZ & combination results from Run 2 (137/fb)

CMS standard procedure (no unitarity preserved)

<table>
<thead>
<tr>
<th></th>
<th>Observed ($W^\pm W^\pm$) (TeV⁻⁴)</th>
<th>Expected ($W^\pm W^\pm$) (TeV⁻⁴)</th>
<th>Observed (WZ) (TeV⁻⁴)</th>
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<th>Expected (TeV⁻⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{T0}/\Lambda^4$</td>
<td>[-0.28, 0.31]</td>
<td>[-0.36, 0.39]</td>
<td>[-0.62, 0.65]</td>
<td>[-0.82, 0.85]</td>
<td>[-0.25, 0.28]</td>
<td>[-0.35, 0.37]</td>
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<tr>
<td>$f_{T1}/\Lambda^4$</td>
<td>[-0.12, 0.15]</td>
<td>[-0.16, 0.19]</td>
<td>[-0.37, 0.41]</td>
<td>[-0.49, 0.55]</td>
<td>[-0.12, 0.14]</td>
<td>[-0.16, 0.19]</td>
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<tr>
<td>$f_{T2}/\Lambda^4$</td>
<td>[-0.38, 0.50]</td>
<td>[-0.50, 0.63]</td>
<td>[-1.0, 1.3]</td>
<td>[-1.4, 1.7]</td>
<td>[-0.35, 0.48]</td>
<td>[-0.49, 0.63]</td>
</tr>
<tr>
<td>$f_{M0}/\Lambda^4$</td>
<td>[-3.0, 3.2]</td>
<td>[-3.7, 3.8]</td>
<td>[-5.8, 5.8]</td>
<td>[-7.6, 7.6]</td>
<td>[-2.7, 2.9]</td>
<td>[-3.6, 3.7]</td>
</tr>
<tr>
<td>$f_{M7}/\Lambda^4$</td>
<td>[-6.7, 7.0]</td>
<td>[-8.3, 8.1]</td>
<td>[-10, 10]</td>
<td>[-14, 14]</td>
<td>[-5.7, 6.0]</td>
<td>[-7.8, 7.6]</td>
</tr>
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</table>

Clipped at unitarity limit

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<tr>
<th></th>
<th>Observed ($W^\pm W^\pm$) (TeV⁻⁴)</th>
<th>Expected ($W^\pm W^\pm$) (TeV⁻⁴)</th>
<th>Observed (WZ) (TeV⁻⁴)</th>
<th>Expected (WZ) (TeV⁻⁴)</th>
<th>Observed (TeV⁻⁴)</th>
<th>Expected (TeV⁻⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{T0}/\Lambda^4$</td>
<td>[-1.5, 2.3]</td>
<td>[-2.1, 2.7]</td>
<td>[-1.6, 1.9]</td>
<td>[-2.0, 2.2]</td>
<td>[-1.1, 1.6]</td>
<td>[-1.6, 2.0]</td>
</tr>
<tr>
<td>$f_{T1}/\Lambda^4$</td>
<td>[-0.81, 1.2]</td>
<td>[-0.98, 1.4]</td>
<td>[-1.3, 1.5]</td>
<td>[-1.6, 1.8]</td>
<td>[-0.69, 0.97]</td>
<td>[-0.94, 1.3]</td>
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<tr>
<td>$f_{S1}/\Lambda^4$</td>
<td>[-100, 120]</td>
<td>[-100, 110]</td>
<td>[-110, 110]</td>
<td>[-120, 130]</td>
<td>[-86, 99]</td>
<td>[-91, 97]</td>
</tr>
</tbody>
</table>
Conclusions and outlook

● The usefulness of the EFT dim-8 in VBS data analysis is limited for several reasons:
  - unknown contributions of dim-6,
  - tiny BSM effects confined to narrow kinematic regions and unitarity violation within the measured range,
  - numerical results crucially depend on extra assumptions,
  - correlations between dim-8 operators: stuck at one operator at a time.

● So far, the EFT has been used mainly as a mathematical tool to quantify the relative precision of the data, but not for its physical meaning.

● If BSM physics is observed, it will be very hard to learn something about its underlying nature from the EFT approach, regardless of the accumulated statistics or beam energy.

● If agreement with the SM holds, the “clipping” technique to set limits on BSM, or equivalently removing events above a threshold if the VV invariant mass is measured, is as close as we can get to the true physical interpretation of the EFT. The machinery is being developed, first results are already available.

● Actual limits with “clipping” implemented are weaker by a factor ~4 compared to standard CMS analyses without unitarity consideration.

● Eventually we want a multi-operator analysis on combined data from different processes.
Backups
MG5 (LO) + Pythia samples (500k-1M) of the process \( pp \rightarrow jj l^+l^-\nu\nu \) @ 14 and 27 TeV for each dim-8 operator, \( f \) scan done using event reweight (including \( f=0 \) for SM).

- Tails M>\( \Lambda \) modeled by applying additional weights \((\Lambda/M)^4\)
to approximate a 1/s total cross section fall,
- Standard VBS cuts,
- Signal significances calculated from different kinematic distributions
  BSM signal significance: \( \chi^2 = \sum_i (N_i^{BSM} - N_i^{SM})^2 / N_i^{SM} \) \( (\sqrt{\chi^2} \geq 5) \)
  EFT consistency: \( \chi^2_{\text{add}} = \sum_i (N_i^{BSM} - N_i^{EFT})^2 / N_i^{BSM} \) \( (\sqrt{\chi^2_{\text{add}}} \leq 2) \)
- The most sensitive variables:
  \[ R_{p_T} \equiv p_T^{l_1} p_T^{l_2} / (p_T^{j_1} p_T^{j_2}) \] for \( O_{S0} \) and \( O_{S1} \), and
  \[ M_{o1} \equiv \sqrt{(|p_T^{l_1}| + |p_T^{l_2}| + |p_T^{\text{miss}}|)^2 - (p_T^{l_1} + p_T^{l_2} + p_T^{\text{miss}})^2} \]
  for the remaining operators
Justification of high M tail modeling

- Asymptotically, every dim-8 operator produces a divergence \( \sim s^3 \) in the total cross section.
- After regularization expected behavior \( \sim 1/s \rightarrow \) reweight like \( 1/s^4 \), i.e., \( (\Lambda/M)^4 \)

But we are mostly interested in the region just above \( \Lambda \sim M^U \)

Around unitarity limit:
- the highest power term is not dominant yet,
- the fastest growing amplitude is not dominant yet.

Hence the overall energy dependence is much less steep.

Of the simple power law scalings, \( (\Lambda/M)^4 \) fits best to the overall energy dependence around \( M^U \).

Total \( W^+W^+ \rightarrow W^+W^+ \) cross section for different \( f_{f_1} \)

Of the simple power law scalings, \( (\Lambda/M)^4 \) fits best to the overall energy dependence around \( M^U \).
### SMEFT and HEFT operators

<table>
<thead>
<tr>
<th>SMEFT</th>
<th>HEFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{S_0} = \left( (D_\mu \Phi)^\dagger D_\nu \Phi \right) \left( (D_\mu \Phi)(D_\nu \Phi)^\dagger \right)$</td>
<td>$\mathcal{P}<em>6 = \text{Tr}(\mathbf{V}</em>\mu \mathbf{V}<em>\mu)^\dagger \text{Tr}(\mathbf{V}</em>\nu \mathbf{V}_\nu)$</td>
</tr>
<tr>
<td>$O_{S_1} = \left( (D_\mu \Phi)(D_\mu \Phi)^\dagger \right) \left( (D_\nu \Phi)(D_\nu \Phi)^\dagger \right)$</td>
<td>$\mathcal{P}<em>{11} = \text{Tr}(\mathbf{V}</em>\mu \mathbf{V}<em>\nu)^\dagger \text{Tr}(\mathbf{V}</em>\mu \mathbf{V}_\nu)$</td>
</tr>
<tr>
<td>$O_{M_7} = (D_\mu \Phi)^\dagger \mathbf{W}<em>{\alpha \nu} \mathbf{W}</em>{\alpha \mu} D_\nu \Phi$</td>
<td>$\mathcal{T}<em>{42} = \text{Tr}(\mathbf{V}</em>\alpha \mathbf{W}<em>{\mu \nu})^\dagger \text{Tr}(\mathbf{V}</em>\alpha \mathbf{W}_{\mu \nu})$</td>
</tr>
<tr>
<td>$O_{M_6} = \mathbf{W}<em>{\mu \nu}^a \mathbf{W}</em>{\mu \nu}^a \left( (D_\alpha \Phi)(D_\alpha \Phi)^\dagger \right)$</td>
<td>$\mathcal{T}<em>{43} = \text{Tr}(\mathbf{V}</em>\alpha \mathbf{W}<em>{\mu \nu})^\dagger \text{Tr}(\mathbf{V}</em>\nu \mathbf{W}_{\mu \alpha})$</td>
</tr>
<tr>
<td>$O_{M_1} = \mathbf{W}<em>{\mu \nu}^a \mathbf{W}</em>{\mu \nu}^a \left( (D_\alpha \Phi)(D_\mu \Phi)^\dagger \right)$</td>
<td>$\mathcal{T}<em>{44} = \text{Tr}(\mathbf{V}</em>\nu \mathbf{W}<em>{\mu \nu})^\dagger \text{Tr}(\mathbf{V}</em>\alpha \mathbf{W}_{\mu \alpha})$</td>
</tr>
<tr>
<td>$O_{T_0} = \mathbf{W}<em>{\mu \nu}^a \mathbf{W}</em>{\mu \nu}^a \mathbf{W}<em>{\alpha \beta}^b \mathbf{W}</em>{b \alpha \beta}^b$</td>
<td>$\mathcal{T}<em>{61} = \mathbf{W}</em>{\mu \nu}^a \mathbf{W}<em>{\mu \nu}^a \text{Tr}(\mathbf{V}</em>\alpha \mathbf{V}_\alpha)$</td>
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<td>$O_{T_1} = \mathbf{W}<em>{\alpha \nu}^a \mathbf{W}</em>{\mu \nu}^a \mathbf{W}<em>{\beta \alpha \nu}^b \mathbf{W}</em>{b \alpha \nu}^b$</td>
<td>$\mathcal{T}<em>{62} = \mathbf{W}</em>{\mu \nu}^a \mathbf{W}<em>{\mu \nu}^a \text{Tr}(\mathbf{V}</em>\alpha \mathbf{V}_\alpha)$</td>
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<td>$O_{T_2} = \mathbf{W}<em>{\alpha \mu}^a \mathbf{W}</em>{\alpha \mu}^a \mathbf{W}<em>{\beta \nu}^b \mathbf{W}</em>{b \nu \alpha}^b$</td>
<td>$O_{T_0} = \mathbf{W}<em>{\mu \nu}^a \mathbf{W}</em>{\mu \nu}^a \mathbf{W}<em>{\alpha \beta}^b \mathbf{W}</em>{b \alpha \beta}^b$</td>
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<tr>
<td>$O_{T_1} = \mathbf{W}<em>{\alpha \nu}^a \mathbf{W}</em>{\alpha \nu}^a \mathbf{W}<em>{\beta \nu}^b \mathbf{W}</em>{b \nu \alpha}^b$</td>
<td>$O_{T_2} = \mathbf{W}<em>{\alpha \mu}^a \mathbf{W}</em>{\alpha \mu}^a \mathbf{W}<em>{\beta \nu}^b \mathbf{W}</em>{b \nu \alpha}^b$</td>
</tr>
</tbody>
</table>

$\mathbf{V}_\mu \equiv (D_\mu \mathbf{U})\mathbf{U}^\dagger$
HEFT vs. SMEFT

- **HEFT**: most general, non-linear realization of the Higgs sector via matrix $U = \exp(i\sigma^a \pi^a / v)$
- Expansion in $U$ derivatives, 10 operators at primary dimension $d_p = 8$.

Total $W^*W^* \rightarrow W^*W^*$ cross section (on shell) split into initial & final state helicity combinations

**Correspondence between HEFT and SMEFT operators:**
- $c_6 \mathcal{P}_6 \iff c_{(8)}^{S1} \mathcal{O}_{S1}$
- $c_{11} \mathcal{T}_{11} \iff c_{(8)}^{S0} \mathcal{O}_{S0} + c_{(8)}^{S1} \mathcal{O}_{S1}$
- $c_{61} \mathcal{T}_{61} \iff c_{(8)}^{M0} \mathcal{O}_{M0}$
- $c_{62} \mathcal{T}_{62} \iff c_{(8)}^{M1} \mathcal{O}_{M1}$

$\mathcal{T}_{42}$, $\mathcal{T}_{43}$, and $\mathcal{T}_{44}$ do not have SMEFT equivalents at or below dimension 8.

- For these 3 operators among the dominant amplitudes is --00 ($TT \rightarrow LL$)!
  - negligible in the SM,
  - higher order ($>8$) in the SMEFT,
  - LL in the final state.

- Tagging jet distributions appropriate to TT+TL, lepton distributions appropriate to LL
  - possible to disentangle based on event kinematics?