Higgs Boson Pair Production via Gluon Fusion: NLO QCD Corrections

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Based on arxiv: 2003.03227 and 1811.05692
Motivation

- Detection of a Higgs boson with a mass $\sim 125$ GeV
- Higgs mass, coupling strengths, spin and CP already determined
- Self-coupling strength still unknown

\[ V(\phi) = \frac{\lambda}{2} \left( |\phi|^2 - \frac{v^2}{2} \right)^2 \]

\[ \lambda_{h^3} = 3 \frac{m_h^2}{v} \]

\[ \lambda_{h^4} = 3 \frac{m_h^2}{v^2} \]
Motivation

Higgs boson pair production

Production channels

Cross sections

Higgs boson pair production channel diagrams and cross section plots.

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Motivation

Uncertainties:

\[
\sigma(pp \rightarrow HH + X) \ [fb] \\
\sqrt{s} = 14 \text{ TeV}, M_H = 125 \text{ GeV}
\]

\[\begin{align*}
gg & \rightarrow HH \\
qq' & \rightarrow HHqq' \\
qq' & \rightarrow WHH \\
qq & \rightarrow ZHH
\end{align*}\]

\[
gg \rightarrow HH : \frac{\Delta \sigma}{\sigma} \sim -\frac{\Delta \lambda}{\lambda}
\]

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira
Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100%
- Large top mass expansion: ~ ±10%
- NLO mass effects of the real NLO correction alone ~ -10 %
- NLO QCD corrections including the full top mass dependence: - 15 % NLO mass effects
- New expansion/extrapolation methods:
  - $1/m_t^2$ expansion & conformal mapping & Padé approximants
  - $p_T^2$ expansion
  - high-energy

Dawson, Dittmaier, Spira
Grigo, Melnikov, Steinhauser
de Florian, Mazzitelli
Grigo, Hoff, Melnikov, Steinhauser

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

Baglio, Campanario, SG, Mühleitner, Spira, Streicher

Gröber, Maier, Rauh
Bonciani, Degrassi, Giardino, Gröber
Davies, Mishima, Steinhauser, Wellmann
$\sigma_{NLO}(pp \rightarrow HH + X) = \sigma_{LO} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$

$\sigma_{LO}$:

$\Delta\sigma_{\text{virt}}$:

$\Delta\sigma_{ij}$:
Virtual Corrections

Triangular diagrams
- Use existing results of single Higgs calculation

One-particle reducible diagrams
- Use existing results of $H \rightarrow Z\gamma$

Box diagrams
- Treat every diagram individually (no reduction to master integrals)
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions
- Extract the infrared and collinear divergences using a ‘proper’ subtraction of the integrand
- Integration by parts to cope with numerical instabilities above the thresholds where

$$m_{HH}^2 > 0, m_{HH}^2 > 4m_t^2 \Rightarrow m_t^2 \rightarrow (1 - i\bar{\epsilon}) \text{ with } \bar{\epsilon} \ll 1$$
Virtual Corrections

Total virtual corrections

- Numerical evaluation using Vegas

\[ Q^2 \frac{d\Delta \sigma_{\text{virt}}}{dQ^2} = \tau \frac{dL_{gg}}{d\tau} \hat{\sigma}_{\text{virt}}(Q^2) \bigg|_{\tau = \frac{Q^2}{s}} \]

(P. Lepage)

\[ \frac{dL_{gg}}{d\tau} = \text{gluon luminosity} \]

\[ \hat{\sigma}_{\text{virt}} = \text{virtual part of the partonic cross section} \]

\[ (Q^2 = m_{HH}^2) \]

- Renormalization: \( \alpha_s \) in \( \overline{MS} \) with \( N_F = 5 \) and \( m_t \) on shell (central value)

- Subtraction of Born-improved HTL \( \rightarrow \) IR finite top mass effects

- Numerical instabilities due to the small imaginary parts of the top mass:

  \( Richardson \ extrap \)olation to get the narrow-width limit for \( m_t \)

Real corrections

- Full matrix elements generated with FeynArts and FormCalc

- Matrix element in the HTL (massive LO) subtracted \( \rightarrow \) IR finite top mass effects
Differential cross section

\[ \frac{d\sigma}{dm_{HH}} [\text{fb/GeV}] \]

\[ K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}} \]

Total hadronic cross section

<table>
<thead>
<tr>
<th>Energy</th>
<th>( m_t = 172.5 \text{ GeV} )</th>
</tr>
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<tbody>
<tr>
<td>13 TeV</td>
<td>( 27.73(7)^{+13.8%}_{-12.8%} \text{ fb} )</td>
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<tr>
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using PDF4LHC PDFs
Top scale and scheme uncertainty

Uncertainty due to $m_t$: differential cross section

→ uncertainty related to the scheme and scale choice of the top mass
→ calculated the total NLO results from the differential cross section for the $\overline{MS}$ top mass at different scale choices
→ $\overline{MS}$ top mass scale in the range [$Q/4, Q$], $m_t$

\[
\begin{align*}
\left. \frac{d\sigma(gg \to HH)}{dQ} \right|_{Q=300 \text{ GeV}} &= 0.02978(7)^{+6%}_{-34%} \text{ fb/GeV} \\
\left. \frac{d\sigma(gg \to HH)}{dQ} \right|_{Q=400 \text{ GeV}} &= 0.1609(4)^{+0%}_{-13%} \text{ fb/GeV} \\
\left. \frac{d\sigma(gg \to HH)}{dQ} \right|_{Q=600 \text{ GeV}} &= 0.03204(9)^{+0%}_{-30%} \text{ fb/GeV} \\
\left. \frac{d\sigma(gg \to HH)}{dQ} \right|_{Q=1200 \text{ GeV}} &= 0.000435(4)^{+0%}_{-35%} \text{ fb/GeV}
\end{align*}
\]  
($\sqrt{s} = 14$ TeV)
Top scale and scheme uncertainty

Uncertainty due to $m_t$: total hadronic cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma(gg \rightarrow HH) = 32.81(7)^{+4\%}_{-18\%} \text{ fb}$$

with PDF4LHC15
Variation of the cross section with $\lambda_{H^3}$

**Diagram Description:**
- **Left Diagram:**
  - Plot of $\sigma(gg \rightarrow HH)$ at NLO QCD with $\sqrt{s} = 14$ TeV and PDF4LHC15.
  - Graphs for HTL, HTL + full reals, HTL + full virtuals, and Full NLO.
  - Ratio to HTL shown.
- **Right Diagram:**
  - Plot of $gg \rightarrow HH$ K-factor for $m_t = 172.5$ GeV and MMHT2014.
  - Graphs for $\sqrt{s} = 13$ TeV, $\sqrt{s} = 14$ TeV, $\sqrt{s} = 27$ TeV, and $\sqrt{s} = 100$ TeV.
  - HTL and full NLO are also shown.
Conclusion

- Calculation of two-loop integrals with three free parameter ratios (without reduction to master integrals)
- NLO top mass effects of ~ -15% compared to Born-improved HTL result
- Factorisation / renormalisation scale dependence: ~ 15% uncertainties
- Top mass scheme and scale uncertainties: ≲ 30% (differential)

Total cross section at 14 TeV: \( \sigma(gg \rightarrow HH) = 32.81(7)^{+4\%}_{-18\%} \text{ fb} \)

- Lambda variation leads to a shift of the minimum of the cross section

Outlook

- Extension to the 2HDM
- Bottom loops
BACK-UP
Scale dependence

Factorisation / renormalisation scale dependence

varying both scales by a factor of two around central value of $\mu_F = \mu_R = m_{hh}/2$

Differential cross section:

\[
\begin{align*}
\frac{d\sigma}{dQ}(gg \rightarrow HH) \bigg|_{Q=300 \text{ GeV}} &= 0.02978(8)^{+15.3\%}_{-13.0\%} \text{ fb/GeV} \\
\frac{d\sigma}{dQ}(gg \rightarrow HH) \bigg|_{Q=400 \text{ GeV}} &= 0.1609(4)^{+14.4\%}_{-12.8\%} \text{ fb/GeV} \\
\frac{d\sigma}{dQ}(gg \rightarrow HH) \bigg|_{Q=600 \text{ GeV}} &= 0.03204(9)^{+10.9\%}_{-11.5\%} \text{ fb/GeV} \\
\frac{d\sigma}{dQ}(gg \rightarrow HH) \bigg|_{Q=1200 \text{ GeV}} &= 0.000435(4)^{+7.1\%}_{-10.6\%} \text{ fb/GeV}
\end{align*}
\]

Total cross section:

\[
\sigma(gg \rightarrow HH) = 32.81(7)^{+13.5\%}_{-12.5\%} \text{ fb} \quad (\text{PDF4LHC15})
\]

($\sqrt{s} = 14 \text{ TeV}$)

Baglio, Campanario, G, Mühleitner, Ronca, Spira, Streicher: 2003.03227
### Total hadronic cross section

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HH White Paper
Results

Total hadronic cross section

Range 275-300 GeV:

- extension of Boole’s rule \((h \text{ step size of } 5 \text{ GeV})\)

\[
\int_{x_0}^{x_5} f(x) \, dx \approx \frac{5h}{288} [19f(x_0) + 75f(x_1) + 50f(x_2) + 50f(x_3) + 75f(x_4) + 19f(x_5)]
\]

error: \(-\frac{275}{12096} h^7 f(x_6) \) \((x_0 < x_6 < x_5)\)

Range 300-1500 GeV:

- Richardson extrapolation for differential cross sections with bin sizes of 50, 100, 200 and 400 GeV
- Trapezoidal rule

\[
\int_{a}^{b} f(x) \, dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]
\]

error: \(-\frac{nh^3}{12} f''(\zeta) \) \((a < \zeta < b)\)
Top mass uncertainty

NNLO

Spira, Djouadi, Graudenz, Zerwas
NLO Corrections

\[ \sigma_{NLO}(pp \rightarrow HH + X) = \sigma_{LO} + \Delta \sigma_{\text{virt}} + \Delta \sigma_{gg} + \Delta \sigma_{gq} + \Delta \sigma_{q\bar{q}}, \]

\[ \sigma_{LO} = \int_{\tau_0}^{1} d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{LO}(Q^2 = \tau s) \]

\[ \Delta \sigma_{\text{virt}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{LO}(Q^2 = \tau s) C \]

\[ \Delta \sigma_{gg} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \int_{\tau_0/\tau}^{1} \frac{dz}{z} \hat{\sigma}_{LO}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log \frac{M^2}{\tau s} \right. \]

\[ + d_{gg}(z) + 6[1 + z^4 + (1 - z)^4] \left( \frac{\log(1 - z)}{1 - z} \right) \right\} \]

\[ \Delta \sigma_{gq} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \hat{\sigma}_{LO}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1 - z)^2} \right. \]

\[ + d_{gq}(z) \right\} \]

\[ \Delta \sigma_{q\bar{q}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \hat{\sigma}_{LO}(Q^2 = z\tau s) d_{q\bar{q}}(z) \]

\[ C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta \Delta}, \quad d_{gg} \rightarrow -\frac{11}{2} (1 - z)^3, \quad d_{gq} \rightarrow \frac{2}{3} z^2 - (1 - z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27} (1 - z)^3 \]
Virtual Corrections

47 two-loop box diagrams + 8 triangular diagrams + 2 one-particle reducible diagrams

Triangular Diagrams

single Higgs case

One-particle reducible diagrams

analytical results for $C_{\Delta\Delta}$

$(H \rightarrow Z\gamma)$

see e.g. Degrassi, Giardino, Gröber
Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → Reduce, Mathematica, Form)

- Perform Feynman parametrisation → additional 6-dimensional integrals

- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

\[
\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)
\]

- Extract the infrared and collinear divergences using a ‘proper’ subtraction of the integrand (based on HTL calculation)

- Integration by parts due to numerical instabilities at the thresholds

\[
m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2 (1 - i\bar{\epsilon}) \quad \text{with} \quad \bar{\epsilon} \ll 1
\]

\[
\int_0^1 dx \frac{f(x)}{(a + bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a + b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a + bx)^2}
\]
Virtual Corrections

Differential cross section

\[ Q^2 \frac{d\Delta \sigma_{\text{virt}}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{virt}}(Q^2) \bigg|_{\tau = \frac{Q^2}{s}} \]  \hspace{1cm} (Q^2 = m^2_{HH})

\[ \frac{d\mathcal{L}^{gg}}{d\tau} = \text{gluon luminosity} \]

\[ \hat{\sigma}_{\text{virt}} = \text{virtual part of the partonic cross section} \]

\[ \rightarrow \] 7 dimensional integrals (6 Feynman and one phase space integration)

\[ \rightarrow \] use Vegas for numerical integration \hspace{1cm} (P. Lepage)

\[ \rightarrow \] numerical instabilities due to the small imaginary parts of the top mass above the thresholds: Richardson extrapolation
Virtual Corrections

Renormalisation

$\alpha_s$ and $m_t$ need to be renormalised

$\rightarrow \quad \alpha_s$ in $\overline{MS}$ with $N_F = 5$

$\rightarrow \quad m_t$ on shell (central prediction)

$$\delta \sigma = \delta \alpha_s \frac{\delta \sigma_{LO}}{\delta \alpha_s} + \delta m_t \frac{\delta \sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit $\rightarrow$ virtual mass effects only (infrared finite)

$$\Delta C_{\text{mass}} = C^0 - C_{HTL}^0$$

Adding back the results in the heavy-top limit (HPAIR)

$$C = C_{HTL} + \Delta C_{\text{mass}}$$
Virtual Corrections

Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand $\rightarrow$ Reduce, Mathematica)

- Use dimensional regularisation: $D = 4 - 2\epsilon$

- Perform Feynman parametrisation $\rightarrow$ additional 6-dimensional integrals

\[
\frac{1}{A_1^{\alpha_1} \cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1 + \cdots + \alpha_n)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_n)} \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-\cdots-u_{n-2}} du_{n-1} \frac{u_1^{\alpha_1-1} \cdots u_{n-1}^{\alpha_{n-1}-1}(1 - u_1 - \cdots - u_{n-1})^{\alpha_n-1}}{u_1 A_1 + \cdots + u_{n-1} A_{n-1} + (1 - u_1 - \cdots - u_{n-1}) A_n} \big|_{\alpha_1 + \cdots + \alpha_n}.
\]

- Substitution to obtain integrals from 0 to 1

- Evaluating momentum integrals using:

\[
\int \frac{d^D k}{(2\pi)^D (k^2 - M^2 + i\epsilon)^N} = \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(N - \frac{D}{2})}{\Gamma(N)} \frac{1}{(M^2 - i\epsilon)^{N - \frac{D}{2}}}
\]

\[
\int \frac{d^D k}{(2\pi)^D (k^2 - M^2 + i\epsilon)^N} = \frac{i}{2} \frac{(-1)^{N-1} \Gamma(N - 1 - \frac{D}{2})}{(4\pi)^{D/2}} \frac{g_{\mu\nu}}{(M^2 - i\epsilon)^{N-1 - \frac{D}{2}}}
\]

etc.
Virtual Corrections

Divergences

- Integration by parts due to numerical instabilities at the thresholds

\[ m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \rightarrow m_t^2 (1 - i\bar{\epsilon}) \text{ with } \bar{\epsilon} \ll 1 \]

\[
\int_0^1 dx \frac{f(x)}{(a + bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a + b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a + bx)^2}
\]

(more involved for second order polynomials)

Further integration by parts not successful since new divergences are created (investigated further)
Divergences

- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

\[
\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-\epsilon}}
\]

\[
= \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)
\]

- Extract the infrared and collinear divergences using a proper subtraction of the integrand

**denominator:**

\[
N = ar^2 + br + c
\]

\[
N_0 = br + c
\]

\[
a = \mathcal{O}(\rho)
\]

\[
b = 1 + \mathcal{O}(\rho)
\]

\[
c = -\rho_s x (1-x) (1-s) t
\]

\[
\int_0^1 d\vec{x} dr \frac{r H(\vec{x}, r)}{N^{3+2\epsilon}} = \int_0^1 d\vec{x} dr \left\{ \left( \frac{r H(\vec{x}, r)}{N^{3+2\epsilon}} - \frac{r H(\vec{x}, 0)}{N_0^{3+2\epsilon}} \right) + \frac{r H(\vec{x}, 0)}{N_0^{3+2\epsilon}} \right\}
\]

Taylor expansion in \( \epsilon \)

analytical r-integration

Virtual Corrections
Richardson extrapolation

→ sequence acceleration method to obtain a better convergence behaviour

Approximation polynomial

\[ M_{i+1}(h) = \frac{t^{k_i} M_i \left( \frac{h}{t} \right) - M_i(h)}{t^{k_i} - 1} \]

\( h \) and \( h/t \) the two step sizes and \( k_i \) the truncation error

\[
M_2[f(h), f(2h)] = 2f(h) - f(2h) = f(0) + O(h^2)
\]

\[
M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 = f(0) + O(h^3)
\]

\[
M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 = f(0) + O(h^4)
\]

In our case \( \bar{h} = \bar{\varepsilon} \) and \( \bar{\varepsilon}_n = 0.05 \times 2^n \quad n = 0 \ldots 9 \)

Theoretical error from Richardson extrapolation estimated by the difference of the fifth and the
Numerical Instabilities

1. due to phase-space integration over Mandelstam variable $t$
   - cut-off at $t = 10^{-8}$ for individual diagrams (total sum finite)
   - logarithmic substitution with $y = \log \frac{t - t_\text{-cut}}{m_t^2}$

2. due to the small imaginary parts $\bar{\epsilon}$ of the top mass above the thresholds, where
   $m_t^2 \to m_t^2 (1 - i\bar{\epsilon})$
   need value in narrow width approximation where $\bar{\epsilon} \to 0$
   calculate partonic cross section for different $\bar{\epsilon} \to \text{Richardson extrapolation}$
Results

K-factor distribution

\[ K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}} \]

Triangular contributions

Box contributions
Results

Triangular contributions

Box contributions

\[ gg \rightarrow HH \text{ at NLO QCD} \mid \sqrt{s} = 14 \text{ TeV} \mid \text{MMHT2014} \]

- K-fac\^\text{continuum}/K-fac\text{full} for different cases:
  - HTL
  - HTL + full reals
  - HTL + full virtuals
  - Full

\[ \mu_R = \mu_F = \frac{m_{hh}}{2} \]

\[ m_{hh} \text{ [GeV]} \]

\[ \mu_R = \mu_F = \frac{m_{hh}}{2} \]

\[ m_{hh} \text{ [GeV]} \]
Results

Uncertainty due to $m_t$ for single Higgs

→ $\overline{MS}$ top mass in the range $[Q/4, Q]$  

\[
\begin{align*}
\sigma(gg \rightarrow H) \bigg|_{m_H=125 \text{ GeV}} &= 42.17^{+0.4%}_{-0.5%} \text{ pb} \\
\sigma(gg \rightarrow H) \bigg|_{m_H=300 \text{ GeV}} &= 9.85^{+7.5%}_{-0.3%} \text{ pb} \\
\sigma(gg \rightarrow H) \bigg|_{m_H=400 \text{ GeV}} &= 9.43^{+0.1%}_{-0.9%} \text{ pb} \\
\sigma(gg \rightarrow H) \bigg|_{m_H=600 \text{ GeV}} &= 1.97^{+0.0%}_{-15.9%} \text{ pb} \\
\sigma(gg \rightarrow H) \bigg|_{m_H=900 \text{ GeV}} &= 0.230^{+0.0%}_{-22.3%} \text{ pb} \\
\sigma(gg \rightarrow H) \bigg|_{m_H=1200 \text{ GeV}} &= 0.0402^{+0.0%}_{-26.0%} \text{ pb}
\end{align*}
\]
triangles for $Q^2 \gg 4M_t^2$:

$$C \to \frac{C_A - C_F}{12} \left[ \log \frac{Q^2}{m_t^2} - i\pi \right]^2 - C_F \left[ \log \frac{Q^2}{m_t^2} - i\pi \right] + 3C_F \log \frac{\mu^2}{m_t^2} + \mathcal{O}(1)$$
resummation for large $Q$

- Abelian logs ($C_F$): $H \to \gamma \gamma$

  \[
  \begin{align*}
  H & \xrightarrow{Q} H \\
  & \text{Sudakov FF}
  \end{align*}
  \]

  Kotsky, Yakovlev, PLB 418 (1998) 335 (LL)
  Akhoury, Wang, Yakovlev, PRD 64 (2001) 113008 (NLL)

- non-Abelian logs ($C_A$): LL related to IR singularities $\to$ exponentiate


- non-Abelian NLL?

- remainder (NNLL)?

- boxes? (more scales)
- threshold: $p$-wave QCD potential $\rightarrow$ Coulomb singularities

- matrix element $\propto \beta^2$, phase space $\propto \beta$
  imaginary part $\propto \beta^3 \ @ \ LO$

- Coulomb singularity at each order in imaginary part:
  
  $C_{Coul} = \frac{Z}{1 - e^{-Z}} = 1 + \frac{Z}{2} + \cdots$ with $Z = C_F \pi \alpha_s \frac{\beta}{\beta}$

  $\Rightarrow$ step in imaginary part $\ @ \ N_3^{3LO}$
  $\Rightarrow$ log. sing. in real part $\ @ \ N_3^{3LO}$ ($\leftarrow$ dispersion integral)

- solution: non-relativistic Green-function in threshold range
  [real part renormalized, finite top width]  
  
  Melnikov, S., Yakovlev, ZPC 64 (1994) 401

- remainder?
$gg \rightarrow HH$ at NLO QCD | $\sqrt{s} = 14$ TeV | PDF4LHC15

$\frac{d\sigma}{dm_{HH}} [\text{fb}/\text{GeV}]$

$\mu_R = \mu_F = \frac{m_{HH}}{2}$

NLO scale uncertainty

$\text{Ratio to HTL}$

$m_{HH} [\text{GeV}]$

$HTL$
$HTL + full \text{ reals}$
$HTL + full \text{ virtuals}$
$\text{Full NLO}$