# Formalism of hydrodynamics with spin degrees of freedom

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## Abstract

Measurements made recently by the STAR collaboration show that the $\Lambda$ hyperons produced in relativistic heavy-ion collisions are subject to global spin polarization with respect to an axis coincident with the axis of rotation of the produced matter. Recently formulated formalism of relativistic hydrodynamics with spin, which is a generalization of the standard hydrodynamics, is a natural tool for describing the evolution of such systems. This approach is based on the conservation laws and the form of energy-momentum tensor and spin tensor postulated by de Groot, van Leeuwen, and van Weert (GLW). Using Bjorken symmetry we show how this formalism may be used to determine observables describing the polarization of particles measured in the experiment.

## Motivation

So far the studies of spin polarization of particles measured in heavy-ion collisions dealt mainly with the spin polarization of particles at freeze-out. In these approaches, the basic hydrodynamic quantity giving rise to spin polarization is so called thermal vorticity. These calculations lack however the dynamical evolution of the spin polarization which takes place in the system's evolution. In our approach we extend the standard hydrodynamical perfect-fluid framework to describe the dynamics of the spin degrees of freedom. Within this approach we aim at describing longitudinal polarization of particles as measured recently by the STAR experiment.

## Hydrodynamic equations

For perfect fluid hydrodynamics framework for spin $\frac{1}{2}$ particles we use the conservation laws for charge, energy, linear momentum and angular momentum with the GLW-based forms of the energy-momentum and spin tensor with the assumption that spin polarization is small ($|\omega_{\mu\nu}| < 1$)

$$\partial_\mu N^\mu = 0 \qquad \partial_\mu T^{\mu\nu}_{\rm GLW} = 0 \qquad \partial_\lambda J^{\lambda\mu\nu}_{\rm GLW} = 0$$

where

$$N^\alpha = n U^\alpha$$
$$T^{\alpha\beta}_{\rm GLW} = (\varepsilon + P) U^\alpha U^\beta - P g^{\alpha\beta}$$

The polarization tensor is decomposed w.r.t fluid flow vector $u^\beta$ in the following way

$$\omega_{\mu\nu} = \kappa_\mu u_\nu - \kappa_\nu u_\mu + \epsilon_{\mu\nu\beta\gamma} u^\beta \omega^\gamma$$

where $\kappa \cdot u = 0$ and $\omega \cdot u = 0$.
The energy-momentum tensor used in the GLW formalism is symmetric, thus the conservation of the angular momentum implies conservation of its spin part:

$$\partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0,$$
$$S^{\lambda,\mu\nu}_{\rm GLW} = \frac{\hbar w}{4\zeta} u^\lambda \omega^{\mu\nu} + \frac{2\hbar \sinh(\zeta)\cosh(\xi)}{m^2 \zeta} s^{\lambda,\mu\nu}_{\rm GLW}$$

where, $w = 4\sinh(\zeta)\cosh(\xi) n_0(T)$,
$s^{\lambda,\mu\nu}_{\rm GLW} = A u^\lambda u^\alpha u^{[\mu} w^{\nu]}{}_\alpha + B \Delta^{\lambda\alpha} u^{[\mu} w^{\nu]}{}_\alpha + B u^\lambda \Delta^{\alpha[\mu} w^{\nu]}{}_\alpha + B u^\alpha \Delta^{\lambda[\mu} w^{\nu]}{}_\alpha$,
$B = -\frac{1}{\beta}\left(\varepsilon_{(0)}(T) + P_{(0)}(T)\right)$,
$A = -3B + \frac{m^2}{T} P_{(0)}(T)$

## Spin polarization tensor and boost-invariant flow

For boost invariant and transversely homogeneous systems, we introduce the following basis:

$$u^\alpha = (\cosh\eta, 0, 0, \sinh\eta), \qquad X^\alpha = (0,1,0,0), \qquad Y^\alpha = (0,0,1,0), \qquad Z^\alpha = (\sinh\eta, 0, 0, \cosh\eta)$$

where, $\tau = \sqrt{t^2 - z^2}$ and $\eta = \frac{1}{2}\ln((t+z)/(t-z))$.
Using above basis, one can decompose the vectors $\kappa^\mu$ and $\omega^\mu$ as:
$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha$; $\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$,
where the coefficients $C_{\kappa X}$, $C_{\kappa Y}$, $C_{\kappa Z}$, $C_{\omega X}$, $C_{\omega Y}$, and $C_{\omega Z}$ are functions of the proper time $\tau$ only. Using the above forms of $\kappa^\alpha$ and $\omega^\alpha$ in conservation law of spin tensor and projecting the resulting tensor equation on $u_\mu X_\nu$, $u_\mu Y_\nu$, $u_\mu Z_\nu$, $Y_\mu Z_\nu$, $X_\mu Z_\nu$ and $X_\mu Y_\nu$, we obtain the set of the equations for the coefficients $C$. These coefficients turn out to evolve independently. The coefficients $C_{\kappa X}$ and $C_{\kappa Y}$ (and similarly $C_{\omega X}$ and $C_{\omega Y}$) obey the same differential equations due to the rotational invariance in the transverse plane.

## Information about spin polarization of particles at freeze-out

The average spin polarization per particle is given by

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

where,

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta\cdot p} \tilde{\omega}_{\mu\beta} p^\beta .$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta\cdot p} .$$

In the local rest frame of the particle, polarization vector $\langle \pi^\star_\mu \rangle$ can be obtained by using the canonical boost. Its longitudinal component is given as:

$$\langle \pi^\star_z \rangle = \frac{1}{8mK_1(\hat{m}_T)} \Bigg( \left( \frac{m\cosh(y_p) + m_T}{m_T\cosh(y_p) + m} \right) \Big( (K_0(\hat{m}_T) + K_2(\hat{m}_T))(C_{\kappa X} p_y - C_{\kappa Y} p_x) + 2C_{\omega Z} m_T K_1(\hat{m}_T) \Big) + \frac{m(K_0(\hat{m}_T) + K_2(\hat{m}_T))\sinh(y_p)(C_{\omega X} p_x + C_{\omega Y} p_y)}{m_T\cosh(y_p) + m} \Bigg)$$

## Results 1

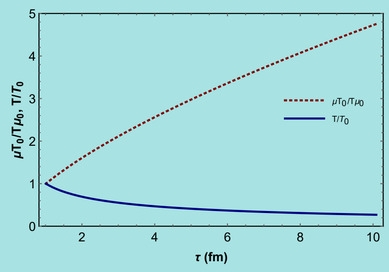


Fig 1: Proper-time dependence of temperature $T$ divided by its initial value $T_0$ (solid line) and the ratio of baryon chemical potential $\mu$ and $T$ rescaled by the initial ratio $\mu_0/T_0$ (dotted line).

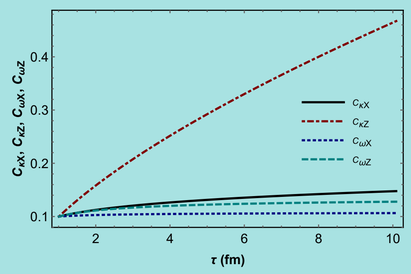


Fig 2: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$.

## Results 2

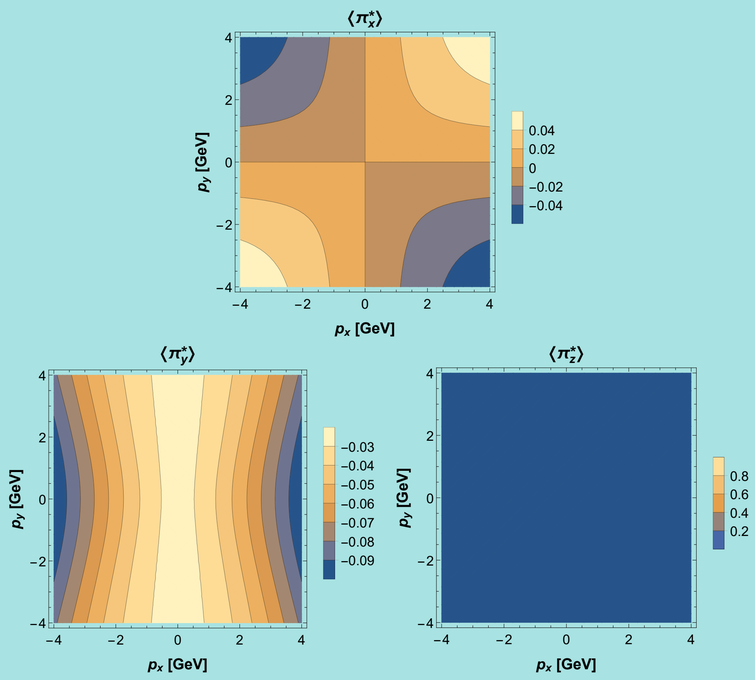


Fig 3: Components of mean polarization of $\Lambda$-hyperon in the rest frame of the particle at $y_p = 0$ obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\mathcal{C}_{\kappa,0} = (0,0,0)$, and $\mathcal{C}_{\omega,0} = (0, 0.1, 0)$.

## References

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