

Formalism of hydrodynamics with spin degrees of freedom

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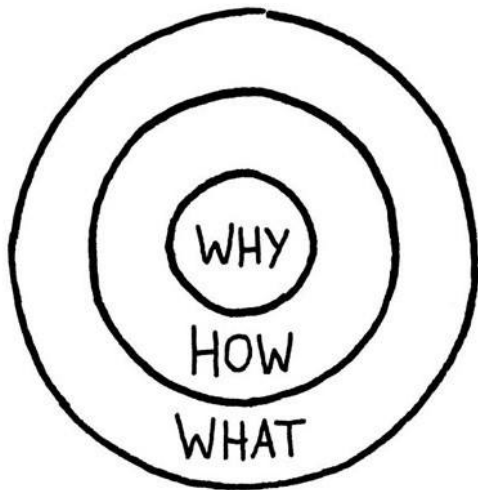
Primary References:

[Phys. Rev. C 99, 044910 \(2019\)](#)

[Prog. Part. Nucl. Phys. 108 \(2019\) 103709](#)

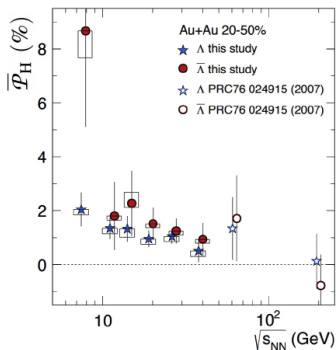
25-30 May, 2020

8th Edition of the Large Hadron Collider Physics Conference
LHCP2020



Motivation:

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach \rightarrow

$$P_{\Lambda} \approx \frac{1}{2} \frac{\omega}{T} + \frac{\mu_{\Lambda}^B}{T} \quad P_{\bar{\Lambda}} \approx \frac{1}{2} \frac{\omega}{T} - \frac{\mu_{\Lambda}^B}{T}$$

Becattini, F., Karpenko, I., Lisa, M., Upszal, I., Voloshin, S., PRC 95, 054902 (2017)

... the hottest, least viscous – and now, most vortical – fluid produced in the laboratory ...

$$\omega = (P_{\Lambda} + P_{\bar{\Lambda}}) k_B T / \hbar \sim 0.6 - 2.7 \times 10^{22} \text{ s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Motivation:

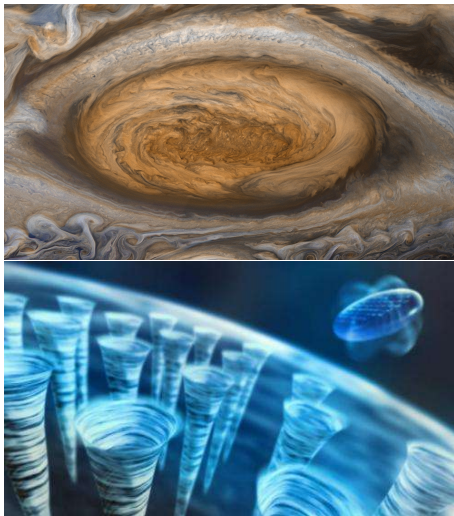


Figure: Jupiter great red spot ($10^{-4}s^{-1}$) and Nanodroplets of superfluid helium (10^7s^{-1})

Motivation:

- Non-central relativistic heavy ion collisions creates global rotation of matter. This may induce spin polarization reminding us of Einstein and De-Haas effect and Barnett effect.

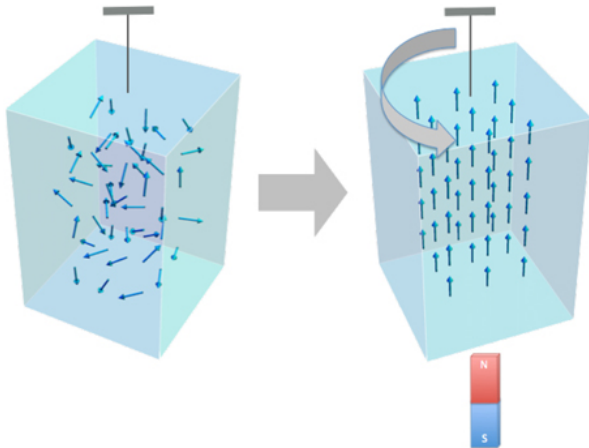


Figure: Einstein-De Haas Effect

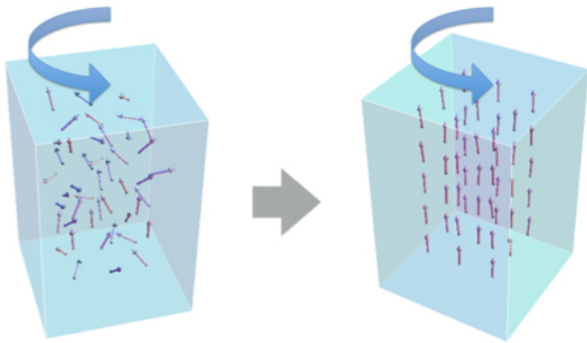


Figure: Barnett Effect

Motivation:

- Non-central relativistic heavy ion collisions creates global rotation of matter. This may induce spin polarization reminding us of Barnett effect and Einstein and de-Haas effect.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

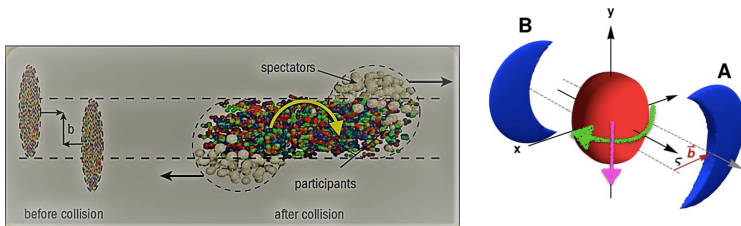


Figure: Schematic view of non-central heavy-ion collisions.

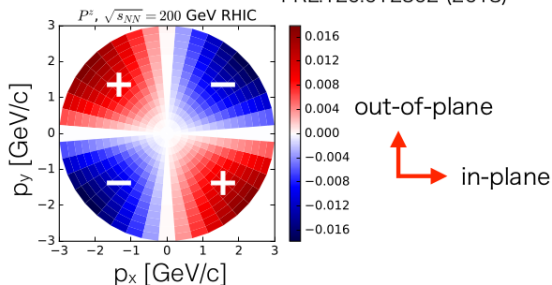
Source: CERN Courier

Other works:

- Other theoretical models used for the heavy-ions data interpretation dealt mainly with the spin polarization of particles at freeze-out, where the basic hydrodynamic quantity giving rise to spin polarization is the 'thermal vorticity' expressed as $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$.

F. Becattini *et al.*(Annals Phys. 338 (2013)), F. Becattini, L. Csernai, D. J. Wang (PRC 88, 034905), F. Becattini *et al.*(PRC 95, 054902), Iu. Karpenko, F. Becattini (EPJC (2017) 77: 213), F. Becattini, Iu. Karpenko(PRL 120, 012302 (2018))

Hydro calculation of P_z
F. Becattini and I. Karpenko,
PRL.120.012302 (2018)



Our hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin

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- Solving the standard perfect-fluid hydrodynamic equations without spin.
- Determination of the spin evolution in the hydrodynamic background.
- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Calculation of the spin polarization of particles in their rest frame. The spin polarization obtained is a function of the three-momenta of particles and can be directly compared with the experiment.

Our hydrodynamic framework:

- In this work, we use relativistic hydrodynamic equations for polarized spin $1/2$ particles to **determine the space-time evolution of the spin polarization** in the system using forms of the energy-momentum and spin tensors proposed by **de Groot, van Leeuwen, and van Weert (GLW)**.

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- The calculations are done in a **boost-invariant and transversely homogeneous setup**. We show how the formalism of hydrodynamics with spin can be used to determine physical observables related to the spin polarization required for the modelling of the experimental data.

[Wojciech Florkowski et al. \(Phys. Rev. C 99, 044910\)](#), [Wojciech Florkowski et al. \(Phys. Rev. C 97, 041901\)](#), [Wojciech Florkowski et al. \(Phys. Rev. D 97, 116017\)](#).

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- Our hydrodynamic formulation does not allow for arbitrary large values of the spin polarization tensor, hence we have **restricted ourselves to the leading order terms in the $\omega_{\mu\nu}$** .

Spin polarization tensor:

The spin polarization tensor $\omega_{\mu\nu}$ is anti-symmetric and can be defined by the four-vectors κ^μ and U^μ ,

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta,$$

Note that, any part of the 4-vectors κ_μ and ω_μ which is parallel to U_μ does not contribute, therefore κ_μ and ω_μ satisfy the following orthogonality conditions:

$$\kappa \cdot U = 0, \quad \omega \cdot U = 0$$

We can express κ_μ and ω_μ in terms of $\omega_{\mu\nu}$, namely

$$\kappa_\mu = \omega_{\mu\alpha} U^\alpha, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} \omega^{\alpha\beta} U^\gamma$$

Conservation of charge:

$$\partial_\alpha N^\alpha(x) = 0,$$

$$\text{where, } N^\alpha = nU^\alpha, \quad n = 4 \sinh(\xi) n_{(0)}(T).$$

The quantity $n_{(0)}(T)$ defines the number density of spinless and neutral massive Boltzmann particles,

$$n_{(0)}(T) = \langle \mathbf{p} \cdot \mathbf{U} \rangle_0 = \frac{1}{2\pi^2} T^3 \hat{m}^2 K_2(\hat{m})$$

where, $\langle \dots \rangle_0 \equiv \int dP (\dots) e^{-\beta \cdot P}$ denotes the thermal average, $\hat{m} \equiv m/T$ denotes the ratio of the particle mass (m) and the temperature (T), and $K_2(\hat{m})$ denotes the modified Bessel function.

The factor, $4 \sinh(\xi) = 2(e^\xi - e^{-\xi})$ accounts for spin degeneracy and presence of both particles and antiparticles in the system and the variable ξ denotes the ratio of the baryon chemical potential μ and the temperature T , $\xi = \mu/T$.

Conservation of energy and linear momentum:

$$\partial_\alpha T_{GLW}^{\alpha\beta}(x) = 0$$

where the energy-momentum tensor $T_{GLW}^{\alpha\beta}$ has the perfect-fluid form:

$$T_{GLW}^{\alpha\beta}(x) = (\varepsilon + P)U^\alpha U^\beta - P g^{\alpha\beta}$$

with energy density $\varepsilon = 4 \cosh(\xi)\varepsilon_{(0)}(T)$ and pressure

$$P = 4 \cosh(\xi)P_{(0)}(T)$$

The auxiliary quantities are:

$$\varepsilon_{(0)}(T) = \langle (p \cdot U)^2 \rangle_0 \text{ and } P_{(0)}(T) = -(1/3)\langle p \cdot p - (p \cdot U)^2 \rangle_0$$

are the energy density and pressure of the spin-less ideal gas respectively.

In case of **ideal relativistic gas** of classical massive particles,

$$\varepsilon_{(0)}(T) = \frac{1}{2\pi^2} T^4 \hat{m}^2 \left[3K_2(\hat{m}) + \hat{m}K_1(\hat{m}) \right], \quad P_{(0)}(T) = Tn_{(0)}(T)$$

Above conservation laws provide closed system of five equations for five unknown functions: ξ , T , and three independent components of U^μ .

Conservation of total angular momentum:

$$\partial_\mu J^{\mu,\alpha\beta}(x) = 0, \quad J^{\mu,\alpha\beta}(x) = -J^{\mu,\beta\alpha}(x)$$

Total angular momentum consists of orbital and spin parts:

$$J^{\mu,\alpha\beta}(x) = L^{\mu,\alpha\beta}(x) + S^{\mu,\alpha\beta}(x),$$

$$L^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x)$$

Since the energy-momentum tensor is symmetric, the conservation of the angular momentum implies the conservation of its spin part.

$$\partial_\lambda J^{\lambda,\mu\nu}(x) = 0, \quad \partial_\mu T^{\mu\nu}(x) = 0 \quad \implies \quad \partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$$

Hence, the spin tensor $S^{\mu,\alpha\beta}(x)$ is separately conserved in GLW formulation.

Conservation of spin angular momentum:

$$\partial_\alpha S_{GLW}^{\alpha, \beta\gamma}(x) = 0$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S_{GLW}^{\alpha, \beta\gamma} = \cosh(\xi) \left(n_{(0)}(T) U^\alpha \omega^{\beta\gamma} + S_{\Delta GLW}^{\alpha, \beta\gamma} \right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S_{\Delta GLW}^{\alpha, \beta\gamma}$ is:

$$S_{\Delta GLW}^{\alpha, \beta\gamma} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]_\delta} + \mathcal{B}_{(0)} \left(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]_\delta} + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]_\delta} + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]_\delta} \right),$$

with,

$$\mathcal{B}_{(0)} = -\frac{2}{\hat{m}^2} s_{(0)}(T)$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$$

Basis for boost invariant and transversely homogeneous systems:

For our calculations, it is useful to introduce a local basis consisting of following 4-vectors,

$$\begin{aligned}U^\alpha &= \frac{1}{\tau} (t, 0, 0, z) = (\cosh(\eta), 0, 0, \sinh(\eta)), \\X^\alpha &= (0, 1, 0, 0), \\Y^\alpha &= (0, 0, 1, 0), \\Z^\alpha &= \frac{1}{\tau} (z, 0, 0, t) = (\sinh(\eta), 0, 0, \cosh(\eta)).\end{aligned}$$

where, $\tau = \sqrt{t^2 - z^2}$ is the **longitudinal proper time** and $\eta = \ln((t+z)/(t-z))/2$ is the **space-time rapidity**.

The basis vectors satisfy the following normalization and orthogonal conditions:

$$\begin{aligned}U \cdot U &= 1 \\X \cdot X &= Y \cdot Y = Z \cdot Z = -1, \\X \cdot U &= Y \cdot U = Z \cdot U = 0, \\X \cdot Y &= Y \cdot Z = Z \cdot X = 0.\end{aligned}$$

Boost-invariant form for the spin polarization tensor:

We use the following decomposition of the vectors κ^μ and ω^μ ,

$$\kappa^\alpha = C_{\kappa U} U^\alpha + C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha,$$

$$\omega^\alpha = C_{\omega U} U^\alpha + C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.$$

Here the scalar coefficients are functions of the proper time (τ) only due to boost invariance. Since $\kappa \cdot U = 0$, $\omega \cdot U = 0$, therefore

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha,$$

$$\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.$$

$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$ can be written as,

$$\begin{aligned} \omega_{\mu\nu} = & C_{\kappa Z} (Z_\mu U_\nu - Z_\nu U_\mu) + C_{\kappa X} (X_\mu U_\nu - X_\nu U_\mu) + C_{\kappa Y} (Y_\mu U_\nu - Y_\nu U_\mu) \\ & + \epsilon_{\mu\nu\alpha\beta} U^\alpha (C_{\omega Z} Z^\beta + C_{\omega X} X^\beta + C_{\omega Y} Y^\beta) \end{aligned}$$

In the plane $z = 0$ we find:

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & C_{\kappa X} & C_{\kappa Y} & C_{\kappa Z} \\ -C_{\kappa X} & 0 & -C_{\omega Z} & C_{\omega Y} \\ -C_{\kappa Y} & C_{\omega Z} & 0 & -C_{\omega X} \\ -C_{\kappa Z} & -C_{\omega Y} & C_{\omega X} & 0 \end{bmatrix}$$

Boost-Invariant form of fluid dynamics with spin:

- **Conservation law of charge** can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

Therefore, for Bjorken type of flow we can write,

$$\dot{n} + \frac{n}{\tau} = 0$$

- **Conservation law of energy-momentum** can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

Hence for the Bjorken flow,

$$\dot{\varepsilon} + \frac{(\varepsilon + P)}{\tau} = 0$$

Boost-Invariant form of fluid dynamics with spin:

Using the equations,

$$S_{\Delta GLW}^{\alpha, \beta \gamma} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma] \delta} \\ + \mathcal{B}_{(0)} \left(U^{[\beta} \Delta^{\alpha \delta} \omega^{\gamma] \delta} + U^\alpha \Delta^{\delta [\beta} \omega^{\gamma] \delta} + U^\delta \Delta^{\alpha [\beta} \omega^{\gamma] \delta} \right),$$

and

$$S_{GLW}^{\alpha, \beta \gamma} = \cosh(\xi) \left(n_{(0)}(T) U^\alpha \omega^{\beta \gamma} + S_{\Delta GLW}^{\alpha, \beta \gamma} \right)$$

in

$$\partial_\alpha S_{GLW}^{\alpha, \beta \gamma}(x) = 0$$

Contracting the final equation with $U_\beta X_\gamma$, $U_\beta Y_\gamma$, $U_\beta Z_\gamma$, $Y_\beta Z_\gamma$, $X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2} \mathcal{A}_2 - \mathcal{A}_3,$$

$$\mathcal{P}(\tau) = \mathcal{A}_1,$$

$$\mathcal{Q}_1(\tau) = - \left[\dot{\mathcal{L}} + \frac{1}{\tau} \left(\mathcal{L} + \frac{1}{2} \mathcal{A}_3 \right) \right],$$

$$\mathcal{Q}_2(\tau) = - \left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau} \right),$$

$$\mathcal{R}_1(\tau) = - \left[\dot{\mathcal{P}} + \frac{1}{\tau} \left(\mathcal{P} - \frac{1}{2} \mathcal{A}_3 \right) \right],$$

$$\mathcal{R}_2(\tau) = - \left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau} \right).$$

$$\mathcal{A}_1 = \cosh(\xi) \left(n_{(0)} - \mathcal{B}_{(0)} \right),$$

$$\mathcal{A}_2 = \cosh(\xi) \left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)} \right),$$

$$\mathcal{A}_3 = \cosh(\xi) \mathcal{B}_{(0)}$$

Background evolution:

Initial baryon chemical potential $\mu_0 = 800$ MeV

Initial temperature $T_0 = 155$ MeV

Particle (Lambda hyperon) mass $m = 1116$ MeV

Initial and final proper time is $\tau_0 = 1$ fm and $\tau_f = 10$ fm, respectively.

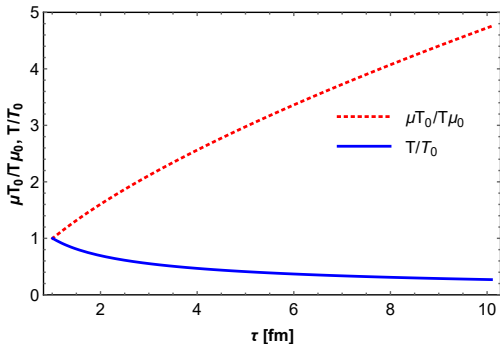


Figure: Proper-time dependence of T divided by its initial value T_0 (solid line) and the ratio of baryon chemical potential μ and temperature T re-scaled by the initial ratio μ_0/T_0 (dotted line) for a boost-invariant one-dimensional expansion.

Spin polarization evolution:

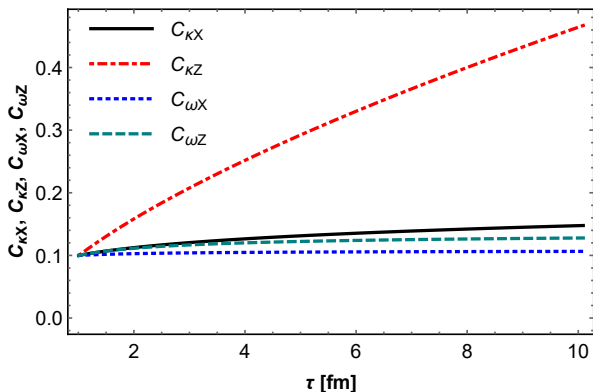


Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{\kappa Y}$ and $C_{\omega Y}$ satisfy the same differential equations as the coefficients $C_{\kappa X}$ and $C_{\omega X}$.

Spin polarization of particles at the freeze-out:

Average spin polarization per particle $\langle \pi_\mu(\mathbf{p}) \rangle$ is given as:

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(\mathbf{p})}{d^3p}}{E_p \frac{d\mathcal{N}(\mathbf{p})}{d^3p}}$$

where, the total value of the Pauli-Lubański vector for particles with momentum \mathbf{p} is:

$$E_p \frac{d\Pi_\mu(\mathbf{p})}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\mu\beta} p^\beta$$

momentum density of all particles is given by:

$$E_p \frac{d\mathcal{N}(\mathbf{p})}{d^3p} = \frac{4 \cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p}$$

and freeze-out hypersurface is defined as:

$$\Delta\Sigma_\lambda = U_\lambda dx dy \tau d\eta$$

Assuming that freeze-out takes place at a constant value of τ and parameterizing the particle four-momentum p^λ in terms of the transverse mass m_T and rapidity y_p , we get:

$$\Delta\Sigma_\lambda p^\lambda = m_T \cosh(y_p - \eta) dx dy \tau d\eta$$

Boost to the local rest frame (LRF) of the particle:

Polarization vector $\langle \pi_{\mu}^* \rangle$ in the local rest frame of the particle can be obtained by using the canonical boost. Using the parametrizations $E_p = m_T \cosh(y_p)$ and $p_z = m_T \sinh(y_p)$ and applying the appropriate Lorentz transformation we get,

$$\langle \pi_{\mu}^* \rangle = -\frac{1}{8m} \begin{bmatrix} 0 \\ \left(\frac{\sinh(y_p) p_x}{m_T \cosh(y_p) + m} \right) [\chi (C_{K_X} p_y - C_{K_Y} p_x) + 2C_{\omega_Z} m_T] + \frac{\chi p_x \cosh(y_p) (C_{\omega_X} p_x + C_{\omega_Y} p_y)}{m_T \cosh(y_p) + m} + 2C_{K_Z} p_y - \chi C_{\omega_X} m_T \\ \left(\frac{\sinh(y_p) p_y}{m_T \cosh(y_p) + m} \right) [\chi (C_{K_X} p_y - C_{K_Y} p_x) + 2C_{\omega_Z} m_T] + \frac{\chi p_y \cosh(y_p) (C_{\omega_X} p_x + C_{\omega_Y} p_y)}{m_T \cosh(y_p) + m} - 2C_{K_Z} p_x - \chi C_{\omega_Y} m_T \\ - \left(\frac{m \cosh(y_p) + m_T}{m_T \cosh(y_p) + m} \right) [\chi (C_{K_X} p_y - C_{K_Y} p_x) + 2C_{\omega_Z} m_T] - \frac{\chi m \sinh(y_p) (C_{\omega_X} p_x + C_{\omega_Y} p_y)}{m_T \cosh(y_p) + m} \end{bmatrix}$$

where,

$$\chi(\hat{m}_T) = (K_0(\hat{m}_T) + K_2(\hat{m}_T)) / K_1(\hat{m}_T),$$

$$\hat{m}_T = m_T / T$$

Momentum dependence of polarization:

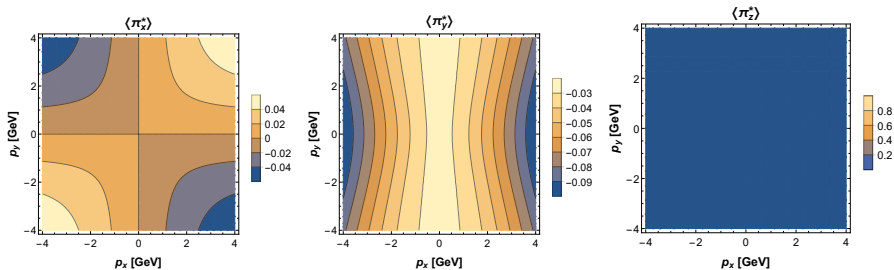
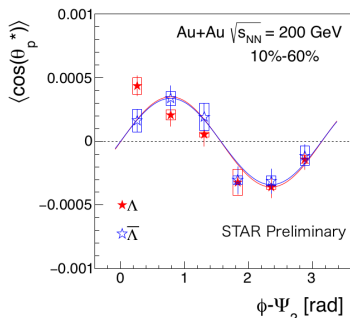


Figure: Components of the PRF mean polarization three-vector of Λ 's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\mathbf{C}_{\kappa,0} = (0, 0, 0)$, and $\mathbf{C}_{\omega,0} = (0, 0.1, 0)$ for $y_p = 0$.

Summary:

- We have discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- For boost invariant and transversely homogeneous set-up we show how our hydrodynamic framework with spin can be used to determine the spin polarization observables measured in heavy ion collisions.
- Since we worked with 0+1 dimensional expansion, our results cannot be compared with the experimental data. So we have to extend our hydrodynamic approach for 1+3 dimensions and interpret the experimental data correctly.



Grazie per l'attenzione!

All **truths** are easy to understand
once they are discovered;
the point is to **discover them.**

– Galileo Galilei

AZ QUOTES

Back-Up Slides

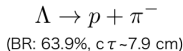
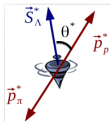
Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} (1 + \alpha_H \mathbf{P}_H \cdot \mathbf{p}_p^*)$$

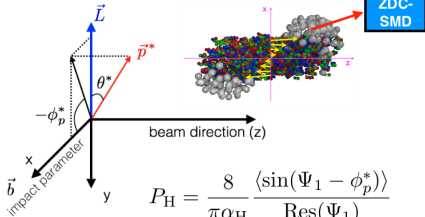
P_H : Λ polarization
 p_p^* : proton momentum in the Λ rest frame
 α_H : Λ decay parameter
 ($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)
 - S. Voloshin and TN, PRC94.021901(R)(2016)



$$P_H = \frac{8}{\pi\alpha_H} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{\text{Res}(\Psi_1)}$$

Ψ_1 : azimuthal angle of b

ϕ_p^* : ϕ of daughter proton in Λ rest frame
 STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

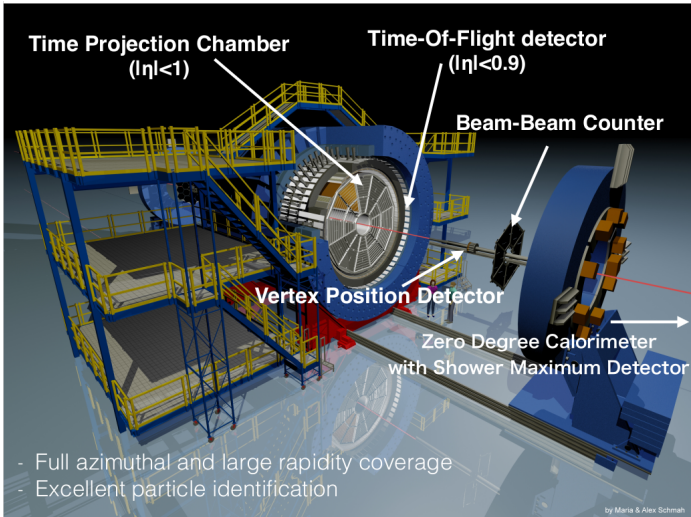


Figure: Schematic view of STAR Detector