Critical fermions in three dimensions

asymptotic safety, weak solutions & the origin of mass

Asymptotic safety meets particle physics 2019

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Fermionic systems

Systems with many fermionic d.o.f. are broadly applicable…

in condensed matter:
-low energy EFTs for graphene, Dirac materials

and high energy:
-toy models for strongly coupled QCD, composite Higgs

Simplified models allow insight into the workings of…
-nonperturbative asymptotic safety
-chiral symmetry breaking
-dynamical mass generation

Gross-Neveu

- classical action in d euclidean dimensions $a \in \{1,\ldots,N_\mathrm{f}\}$

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 \right\}$$

- discrete "chiral" symmetry: $\psi \to \gamma^5 \psi,\ \bar{\psi} \to -\bar{\psi}\gamma^5$

- four-fermion coupling: $[G] = 2 - d$

- *asymptotically…* ***free*** in $d = 2$ [*Gross, Neveu, PRD '74*]

  safe in $d = 3$

[*Gawędzki, Kupiainen, Nucl.Phys.B '85*]
[*Rosenstein, Warr, Park, PRL '88*]
[*de Calan et al., PRL '91*]

Gross-Neveu

- classical action in d euclidean dimensions $\qquad a \in \{1,\ldots,N_\mathrm{f}\}$

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a\right)^2 + \frac{1}{3!} H \left(\bar{\psi}_a \psi_a\right)^3 \right\}$$

- discrete “chiral” symmetry: $\psi \to \gamma^5 \psi, \;\; \bar{\psi} \to -\bar{\psi}\gamma^5 \qquad [H = 0]$

- four-fermion coupling: $[G] = 2 - d$

- *asymptotically…* **free** in $d = 2$ *[Gross, Neveu, PRD ’74]*

  safe in $d = 3$

[Gawędzki, Kupiainen, Nucl.Phys.B ’85]
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[de Calan et al., PRL ’91]

Phase structure

$$d = 3,\ N_{\mathrm{f}} \to \infty$$

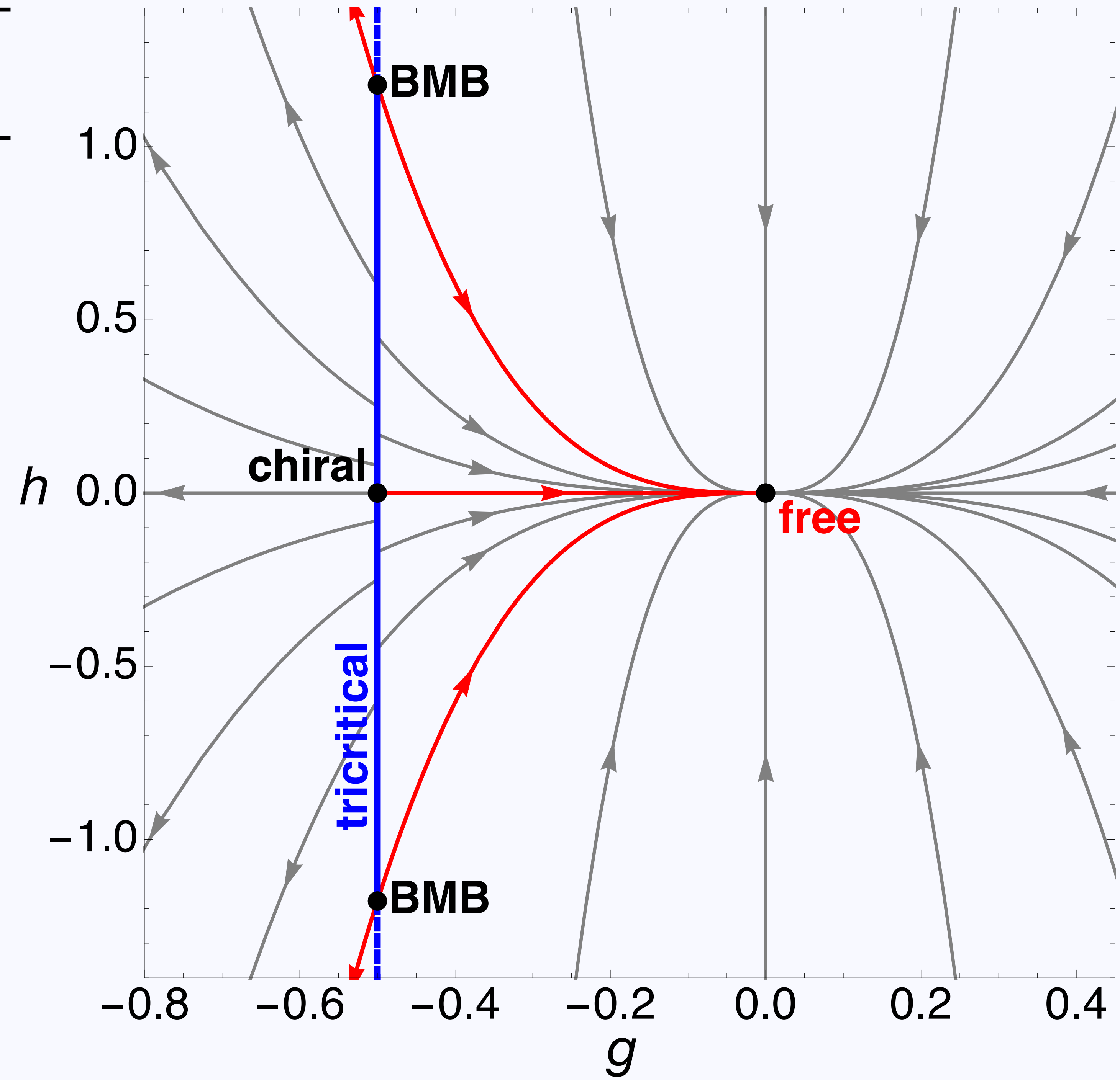
free in IR:

- classical scaling
- m relevant
- IR sink for $m = 0$

tricritical line in UV:

- nontrivial scaling
- m & g relevant
- h exactly marginal
- chiral symmetry at $h = 0$

[g ~ 4-fermion, h ~ 6-fermion]



chiral FP well studied:
[Rosenstein, Warr, Park, PRL ’89]
[de Calan et al., PRL ’91]
[Braun, Gies, Scherer, 1011.1456]
[Jakovác, Patkós, 1306.2660]
[Jakovác, Patkós, Pósfay, 1406.3195]
[Ihrig, Mihaila, Scherer, 1806.04977]

IR limit

Possibilities…

massless	massive
-weak coupling	-strong coupling
-free theory	-interacting
-chiral sym.	-broken sym.

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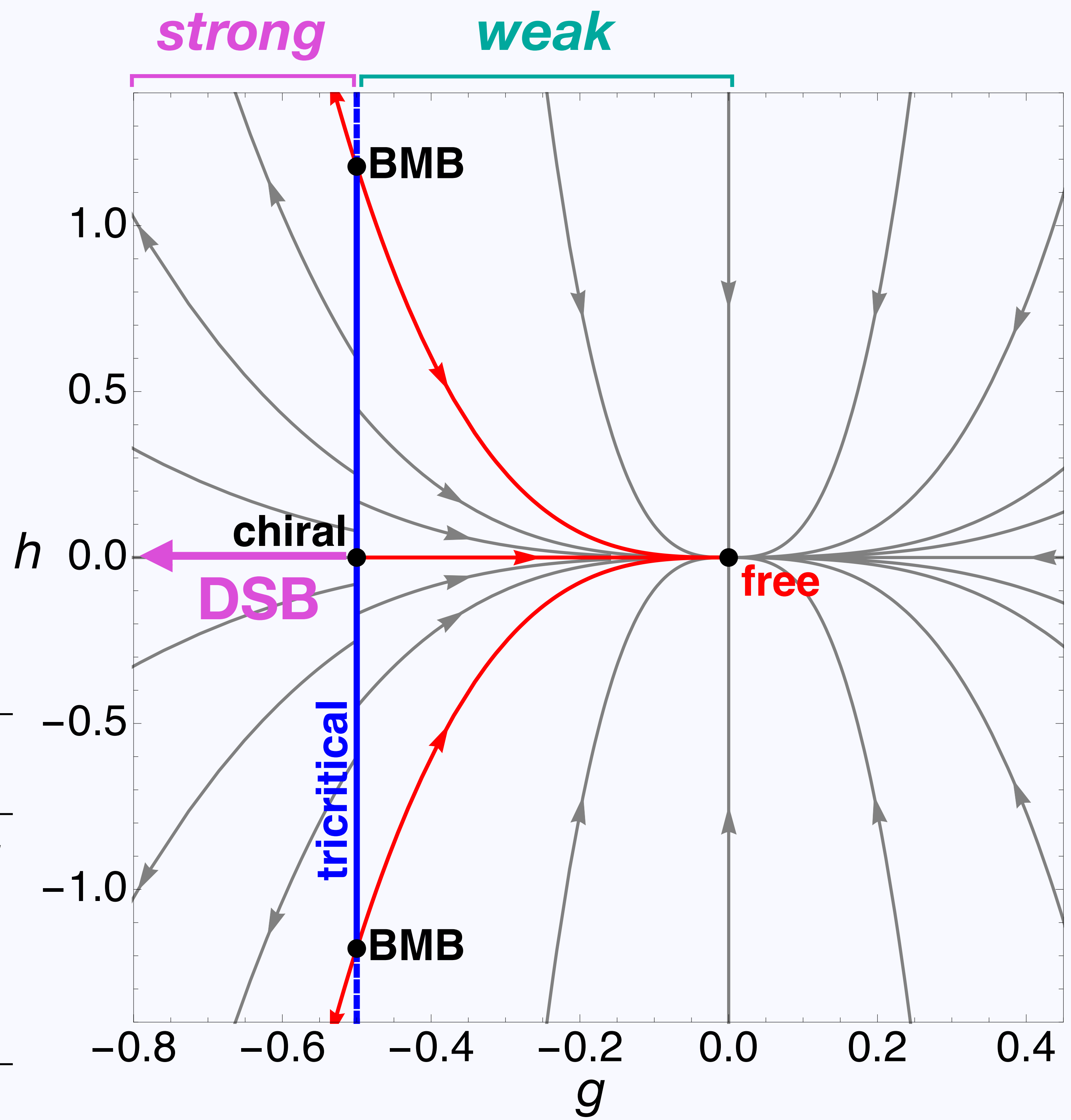
dynamical sym. breaking

visible with auxiliary scalar field: "partial bosonisation"

vev generates fermion mass



Q: how can mass generation be seen with fermions only?

Effective action

Flowing effective action Γ_k interpolates UV to IR

$$\underset{S_\text{class}}{k=\Lambda} \longrightarrow \underset{\Gamma_\text{1PI}}{k\to 0}$$

Exact functional flow equation:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{str}\left\{ [\Gamma_k^{(2)} + R_k]^{-1} \cdot \partial_t R_k \right\}$$

regulated for small and large momenta

Ansatz:

$$\Gamma_k = \int \mathrm{d}^d x \, \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a \psi_a) \right\}$$

closed under RG when $N_\text{f} \to \infty$

Effective potential

Parameterise interactions with effective potential

$$V_k(\tilde{z}) \equiv V(\tilde{z}, t) \quad \tilde{z} = (\bar{\psi}_a \psi_a)$$

think of fields averaged on patches

Exact flow equation has leading term:

$$\partial_t V = -\frac{(\Lambda e^t)^{d+2}}{(\Lambda e^t)^2 + (\partial_{\tilde{z}} V)^2} + \mathcal{O}(1/N_\mathrm{f})$$

with $t = \ln k/\Lambda$

Exact solution possible in infinite N limit

Series solution

Expand potential in powers of field (dimensionless)

$$v(z,t) = \sum_{n=1}^{\infty} \frac{\lambda_n(t)}{n!} z^n$$

identify $\lambda_n \sim$ *2n-fermion coupling*

beta-functions reproduce known results

4-fermion coupling blows up in IR for initial values

$$\lambda_{2,0} < \lambda_{2,*} = -\frac{1}{2}$$

critical scale $k_{\text{crit}} = \left(1 + \frac{1}{2\lambda_{2,0}}\right)\Lambda$

Claim: divergence is signal of mass generation & DSB

Effective potential - DSB

Evolve effective potential from initial condition

$$V_\Lambda(\tilde{z}) = \frac{1}{2}\frac{g_0}{\Lambda}\tilde{z}^2$$

strong coupling in UV: $g_0 < g_*$

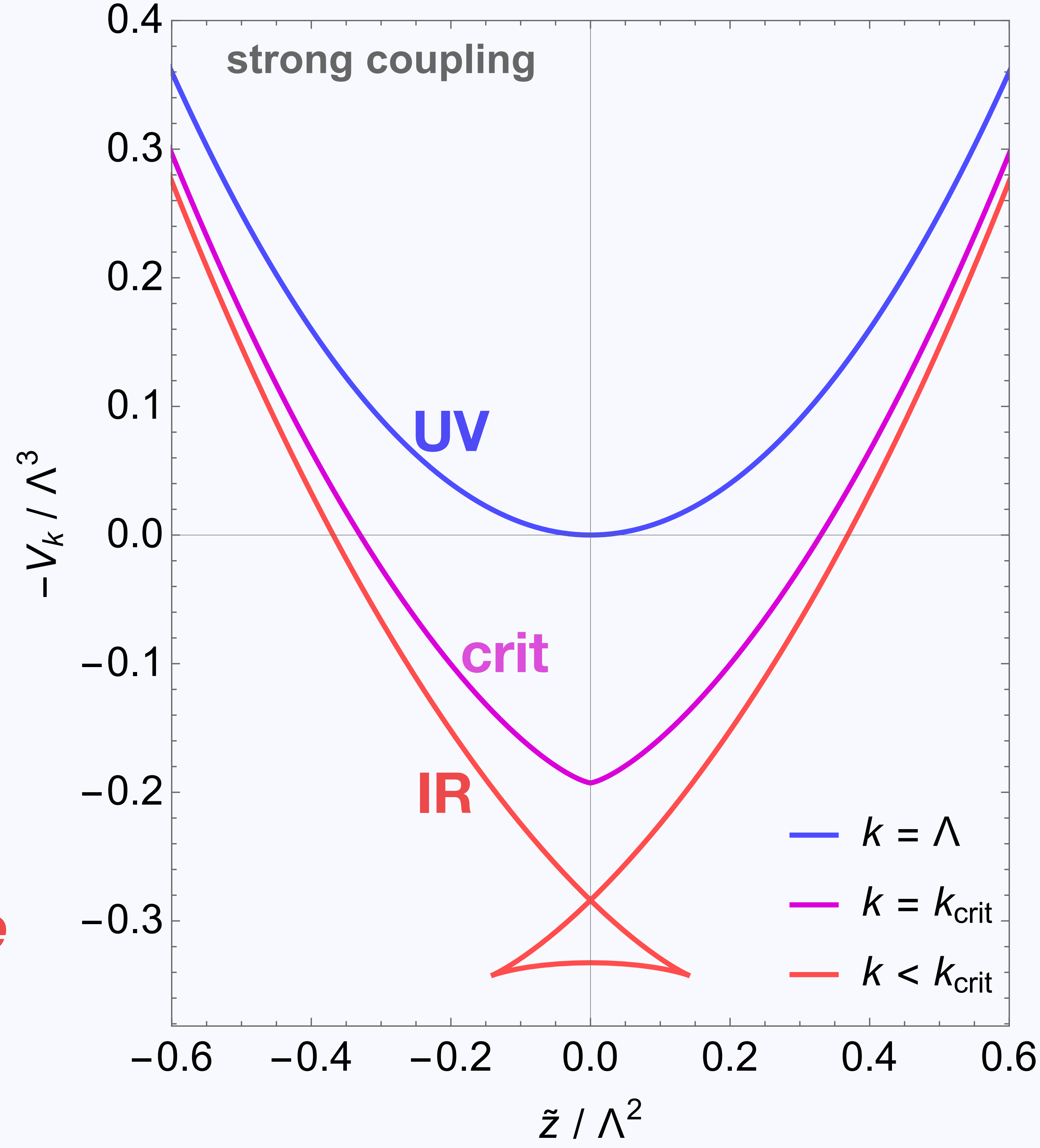
-gradient at zero field defines mass

cusp develops at critical scale

-gradient nonzero when $\tilde{z} \to 0^\pm$

multivalued below critical scale

-physical branch?



Characteristics

At infinite N, flow for V' has conservation law form

$$\partial_t u + \partial_x f(u,t) = 0$$

over domain $\Omega = \{(x,t)\,|\,x \in \mathbb{R}, t \in \mathbb{R}^-\}$

nonlinear flux $f(u,t) = \dfrac{(\Lambda e^t)^5}{(\Lambda e^t)^2 + u^2}$

identifying $u(x,t) \sim \partial_{\tilde{z}} V(\tilde{z},t)$

Solution constant on ***characteristic curves*** $\bar{x} = \bar{x}(t)$

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = \partial_u f(\bar{u},t)$$

$$\bar{u} = u|_{\bar{x}} = u(x_0, 0)$$

$$\bar{x}(0) = x_0$$

Linear case

Consider linear conservation law (transport equation)

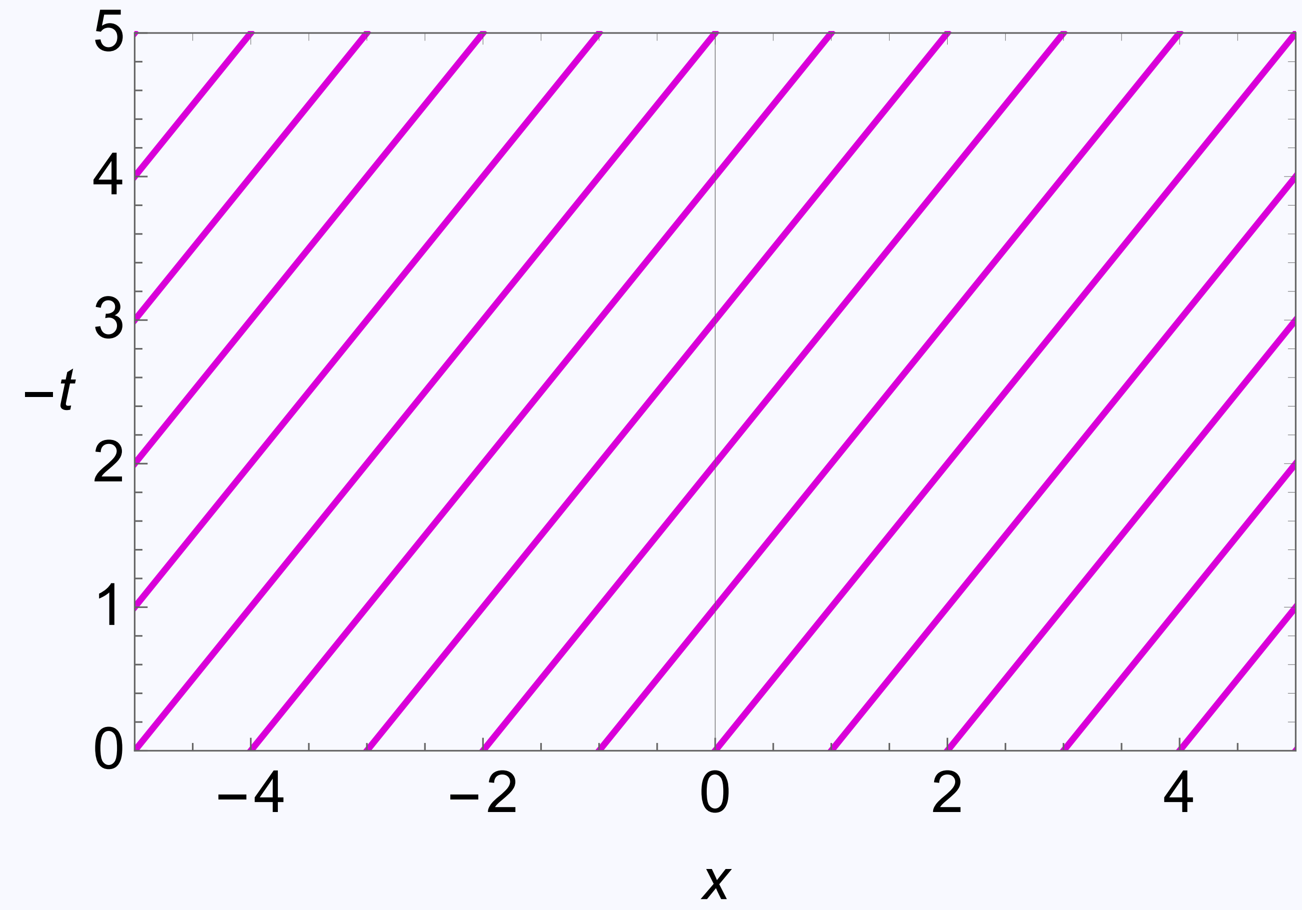
$$\partial_t u + c \cdot \partial_x u = 0 \qquad u(x,0) = g(x)$$

solved by $u(x,t) = g(x - ct)$

describes transport of initial waveform g *with constant speed* c

Characteristics:
lines of slope c *in* (x,t)*-plane*

$$\bar{x}(t) = ct + x_0$$

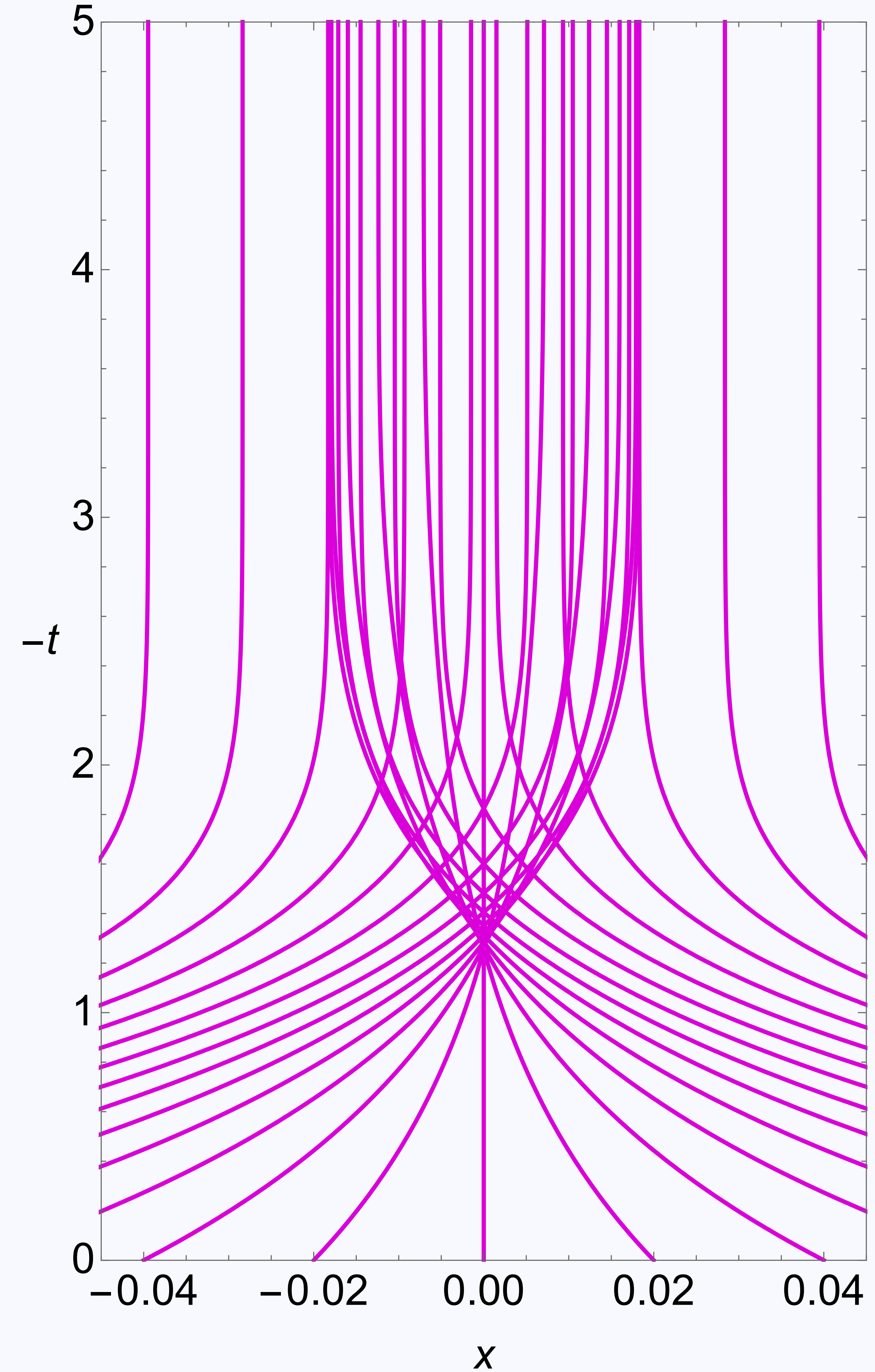


Nonlinear case

$$\partial_t u + c(u,t) \cdot \partial_x u = 0$$
$$u(x,0) = g(x)$$

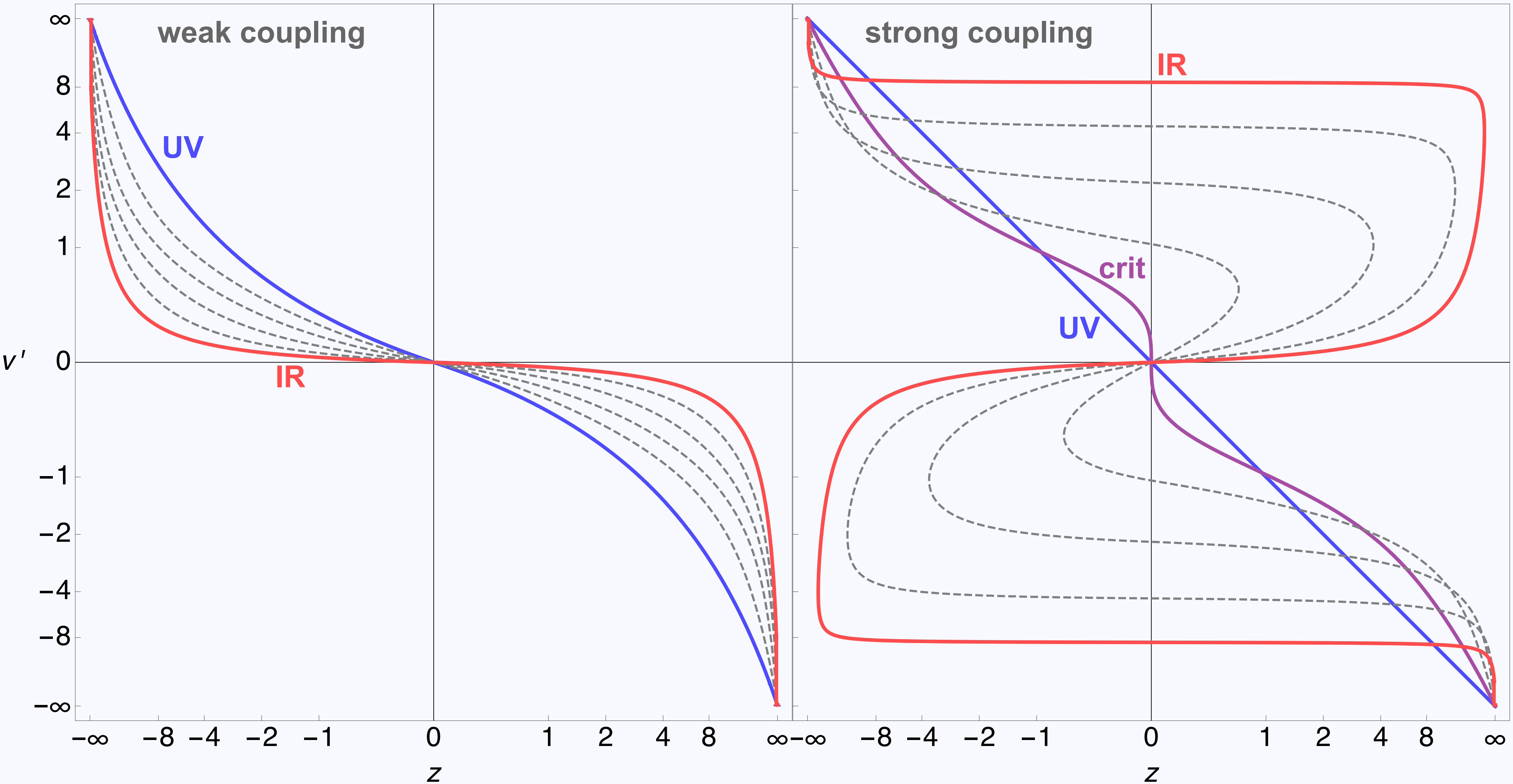
Nonlinear transport:
wave speed depends on u, t
$$c(u,t) = \partial_u f(u,t)$$

Characteristics intersect!



Mass function

IR evolution of mass "function" $V_k'(\tilde{z})$ $\left[z = \tilde{z}/k^2 \quad v'(z,t) = V_k'(\tilde{z})/k^3\right]$



Weak solutions

Applications to:
4d fermions [*Aoki, Kumamoto, Sato, '14*]
3d scalars [*Grossi Wink, '19*]

Take test function $\phi = \phi(x, t)$
-smooth: at least $C^1(\Omega)$
-compact support on Ω

Integrate PDE over Ω *to obtain* ***weak form***

$$\star \iint_\Omega \left\{ u \cdot \partial_t \phi + f \cdot \partial_x \phi \right\} \, \mathrm{d}x \, \mathrm{d}t - \int_{-\infty}^{\infty} (u \cdot \phi)|_{t=0} \, \mathrm{d}x = 0$$

functions u *satisfying* $\star \ \forall \phi \in C^1(\Omega)$ *are called* ***weak solutions***

Properties:

Weak solutions may be non-differentiable at some points

All "strong" solutions are automatically weak solutions

In our case, can think of piecewise "strong" solutions

Not unique…

Rankine-Hugoniot condition

Further condition required to ensure uniqueness

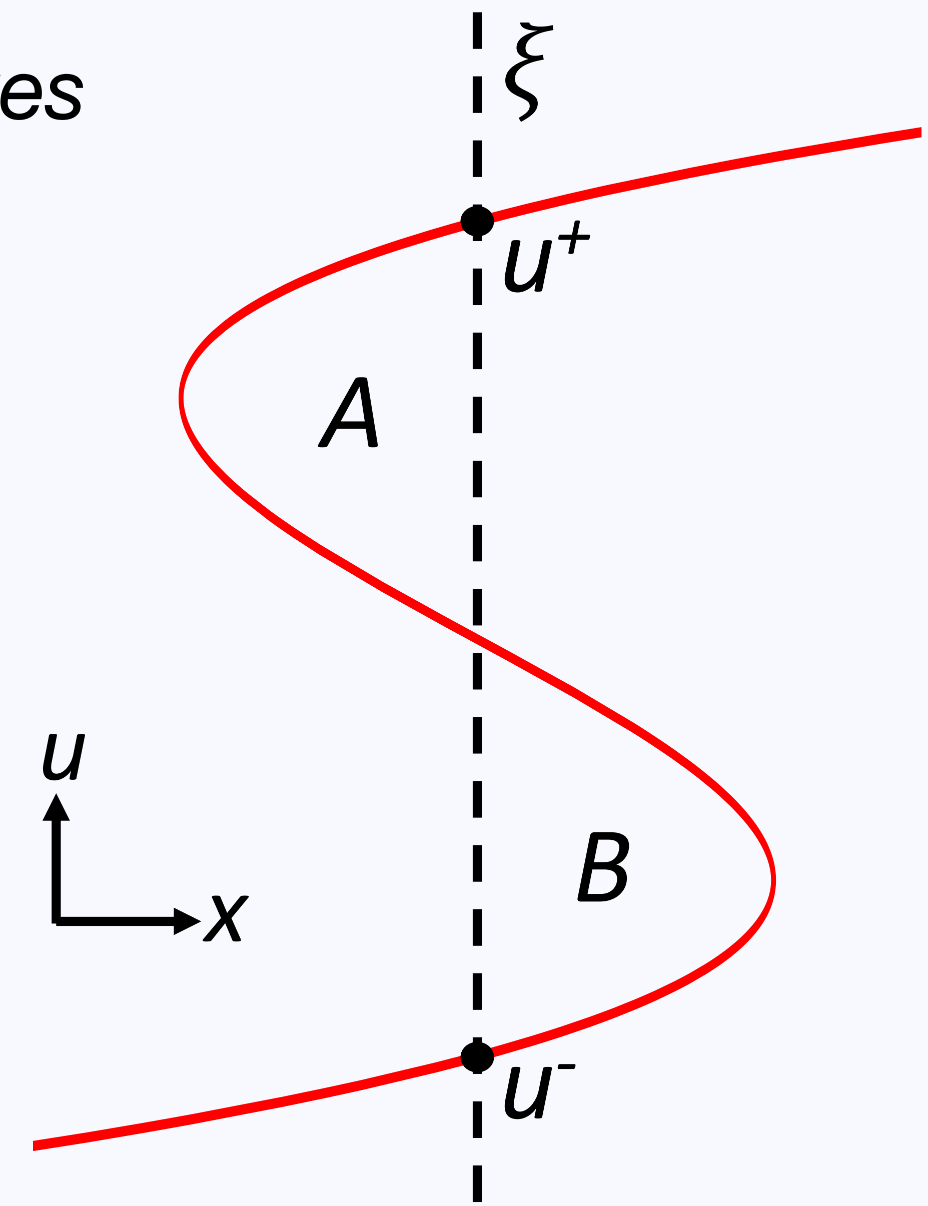
Can construct weak solution by cutting multivalued solution

Discontinuity at position $x = \xi(t)$ *propagates according to RH condition*

$$\xi'(t) = \frac{f(u^+) - f(u^-)}{u^+ - u^-}$$

[*Rankine, 1870*]
[*Hugoniot, 1887, 1889*]

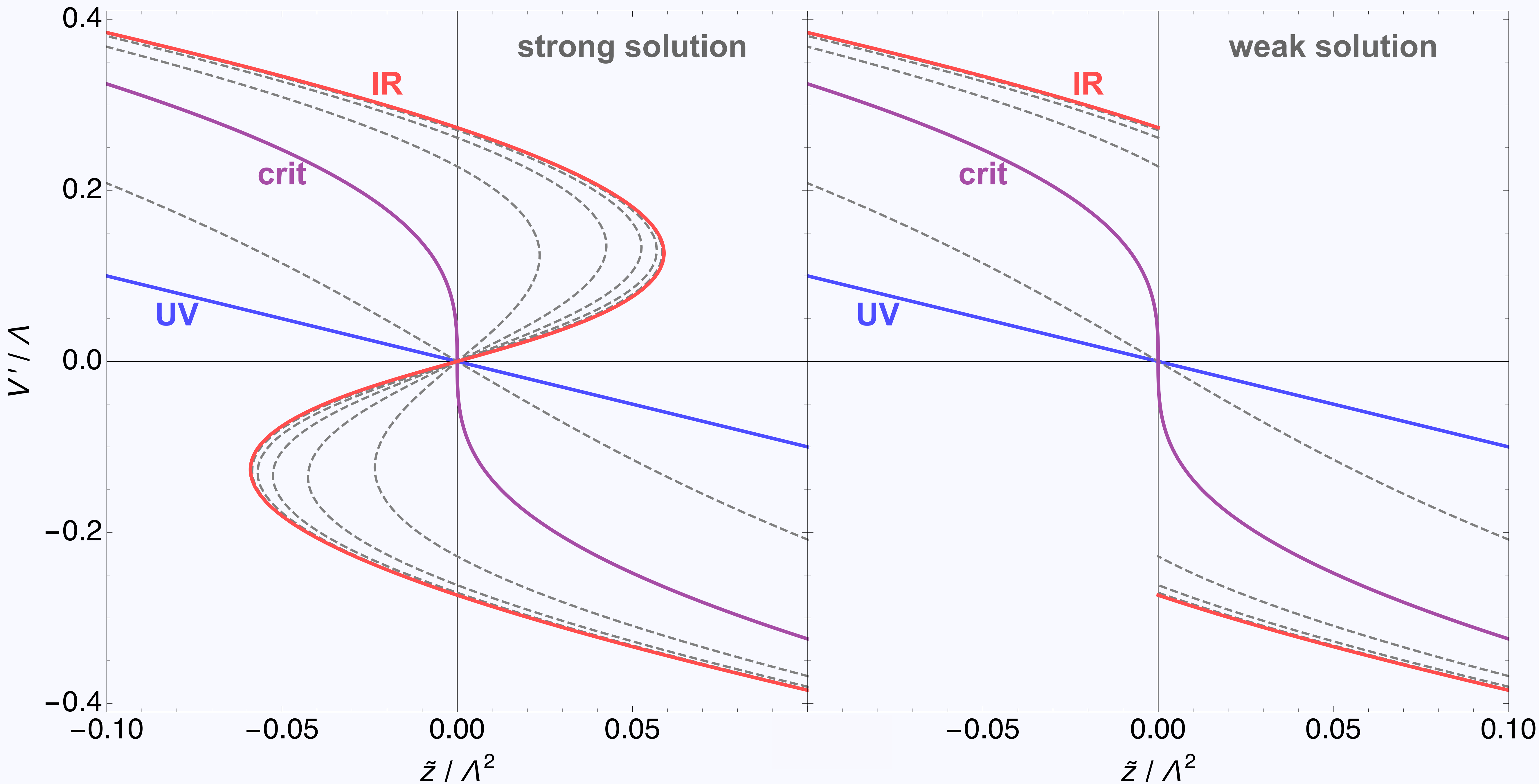
Geometric interpretation:
area A = area B



Putting it all together

For initial conditions with chiral symmetry, $\xi = 0 \ \forall t$

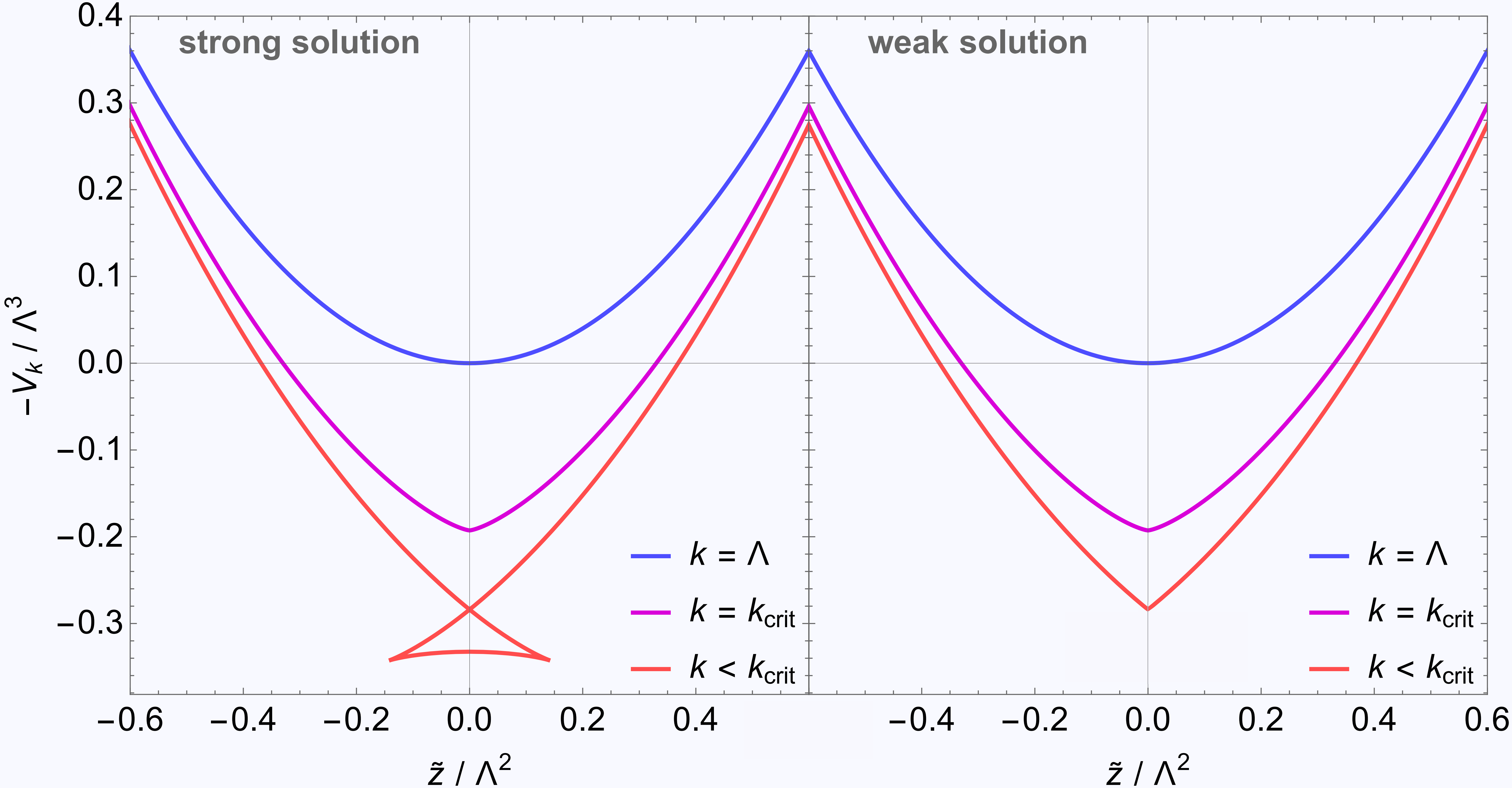
Below critical scale, V' *discontinuous* $\Rightarrow$ *mass generation!*



Effective potential

Potential becomes single-valued at all scales

Cusp survives in IR limit



Chiral phase transition

Physical fermion mass $M_\psi = \lim_{t \to -\infty} V_k'(0)$

2nd order quantum PT as function of initial coupling

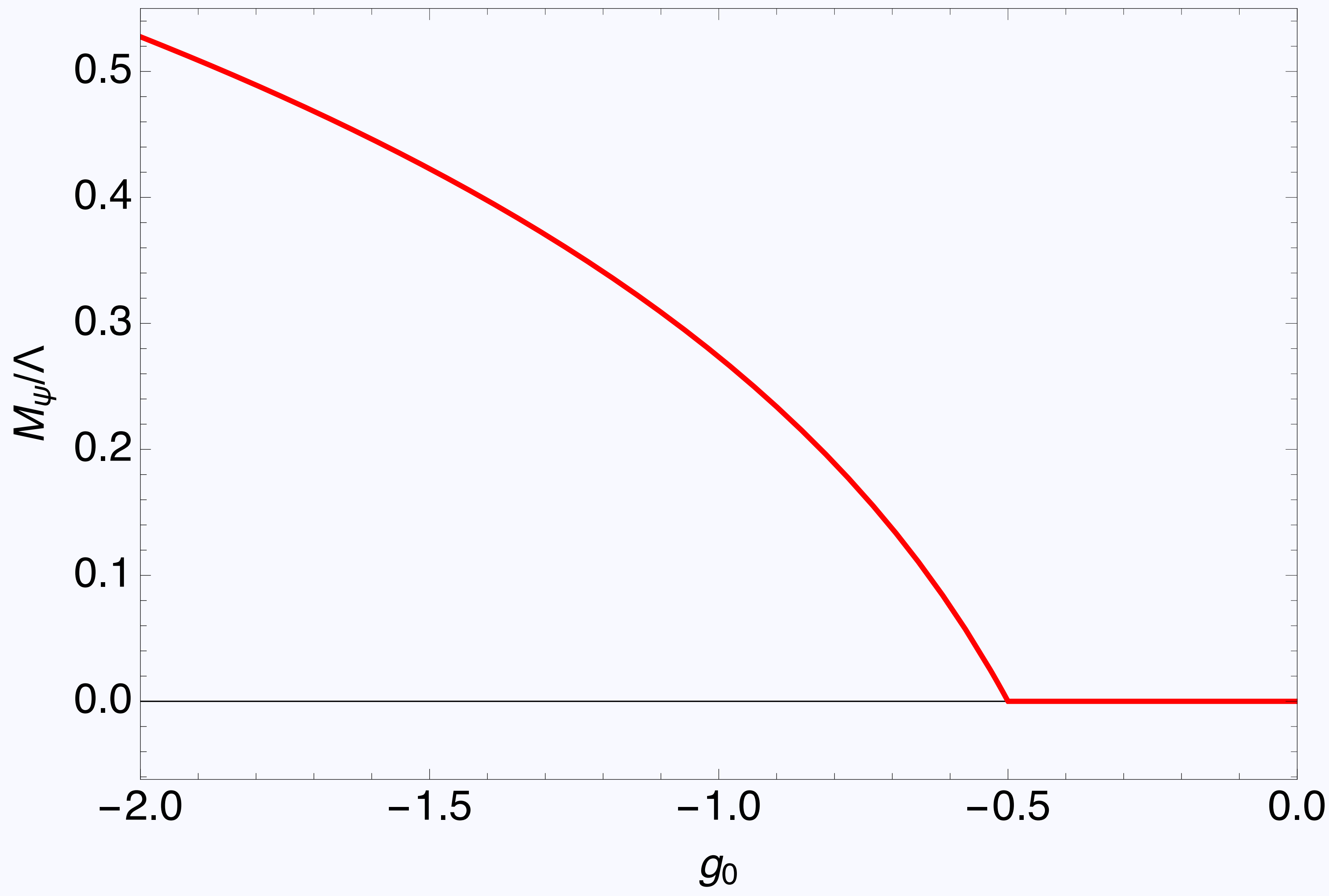
Near critical point

$M_\psi \sim |g_0 - g_*|^\nu$

critical exponent

$\nu = 1$

at infinite-N



Wider phase plane

With 6-fermion coupling…

1st order PTs

-for h > "BMB"

BMB phenomenon

-spontaneous breaking of scale invariance

GN:

[*CCH, Litim, in preparation*]

O(N):

[*Bander, Bardeen, Moshe, PRL '83*]

[*David, Kessler, Neuberger, PRL '85*]

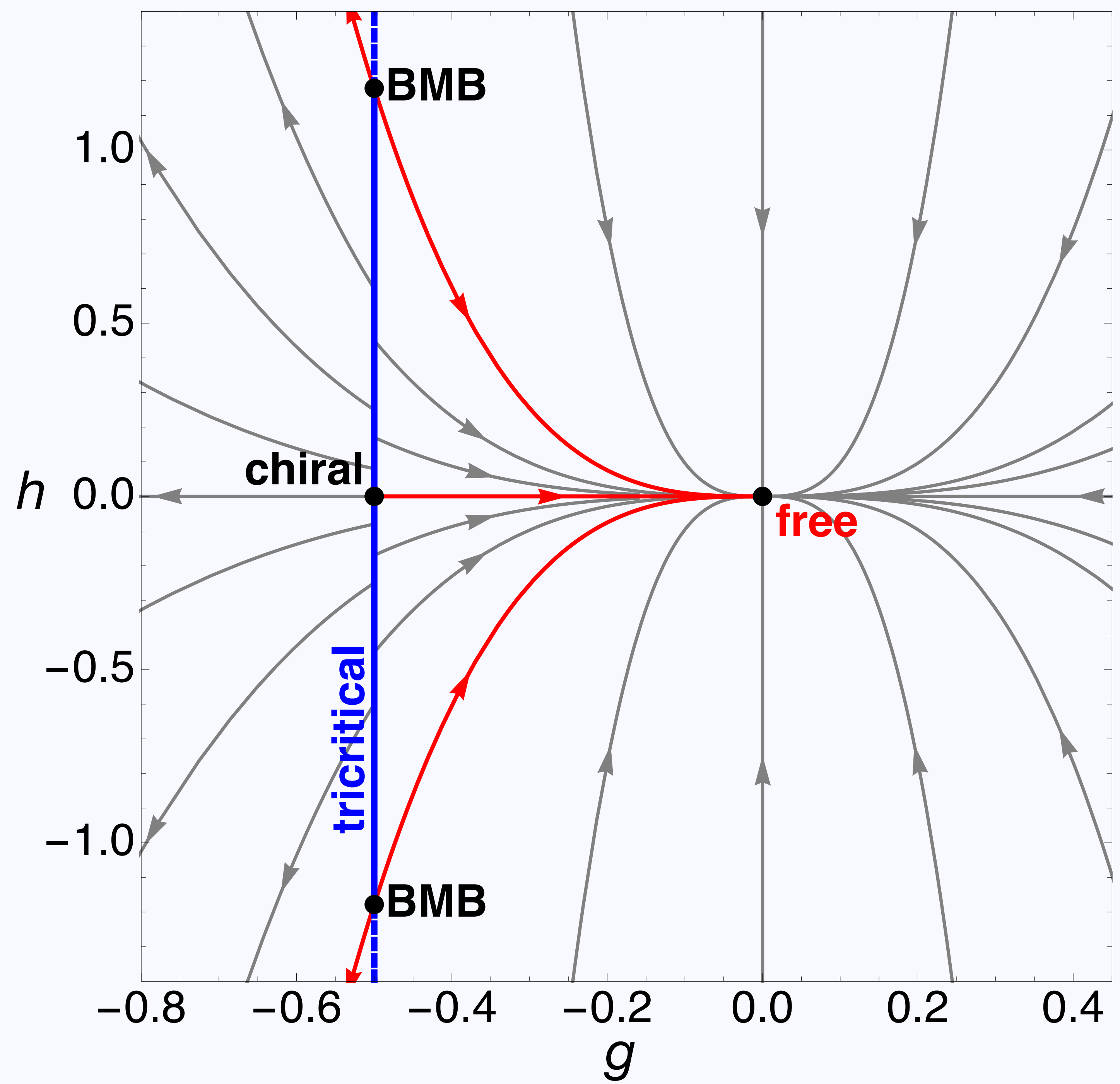
boson-fermion duality

-consistent with CS-matter

see, e.g.:

[*Maldacena, Zhiboedov, 1204.3882*]

[*Aharony, Jain, Minwalla, 1808.03317*]



So…

GN can give analytical insight into nonperturbative phenomena
-asymptotic safety
-dynamical symmetry breaking & mass generation

Novel approaches required in pure fermionic formulation
-weak solutions

1st order phase transitions possible in wider phase plane
-controlled by 6-fermion coupling

Challenges at finite-N
-believe same mechanism to be operative