Asymptotic Safety, Quantum Gravity and the Standard Model

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Based on work in collaboration with Daniel Litim

Asymptotic Safety Meets Particle Physics
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Content

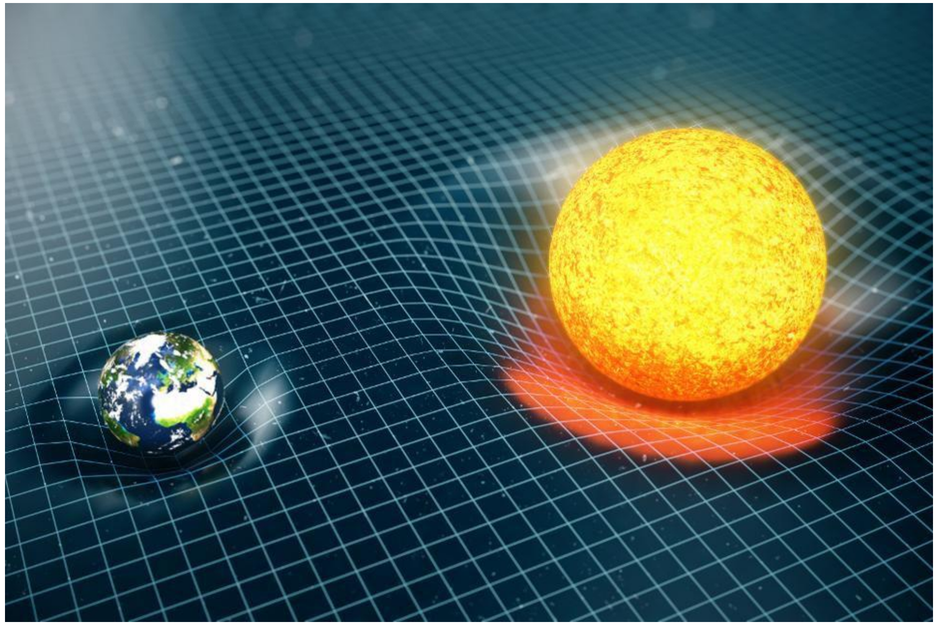
- Motivation
- Main framework
- Methodology
- Fixed points of Quantum Gravity with SM matter
- Conclusions & further work

Classical gravity

- General relativity provides the best description of gravity at classical scales

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

- Experimentally tested in the weak gravity regime:
    - Gravitational waves
    - Gravitational lensing
    - Redshift of light
- Breaks down at small scales (strong gravity regime)



Quantizing gravity the traditional way

- Dimensional regularization worked great for renormalizing QCD, however, it does not work for gravity
- At 1-loop:

  Pure gravity: power-counting renormalizable

('t Hooft, G., & Veltman, M.; 1974)
(Christensen, S. M., & Duff, M. J.; 1980)

$$\Delta L^{(1)} = \frac{1}{\epsilon} \int d^4x \sqrt{g} \left(\frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right)$$

- The 1-loop counter-term vanishes in **pure gravity**

Quantizing gravity the traditional way

- At 2-loop:
  - **Non-renormalizable**
- Counter-term is of the form:

(Goroff, M. H., Sagnotti, A., & Sagnotti, A.; 1985)

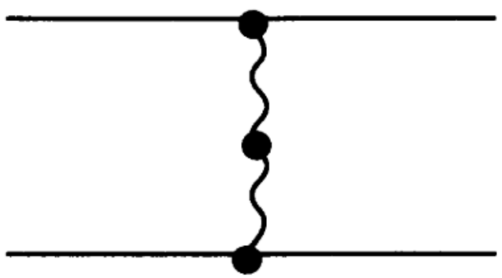
$$\Delta L^{(2)} = \frac{209}{2880\,(4\pi)^4}\frac{1}{\epsilon}\int d^4x\sqrt{g}R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\epsilon\zeta}R_{\epsilon\zeta}{}^{\alpha\beta}$$

- It gets worse, already at 1-loop:
  - **Gravity + matter: non-renormalizable**
- Not-straightforward to quantize gravity
- New directions were needed

(Deser, S., & van Nieuwenhuizen, P; 1974)
(Deser, S., Tsao, H.-S., & van Nieuwenhuizen, P.; 1974)
(Deser, S., & van Nieuwenhuizen, P.; 1974)

Some ideas

(Donoghue, J. F.; 1994)



- Do nothing: **Effective Field Theory**
  - Theory valid only up to a finite energy scale
  - Can derive 1-loop corrections to the Newtonian potential
  - Give up on UV-completeness
- Beyond Einstein gravity: **Higher derivative gravity**
  - Include terms with 4 derivatives: $(R^2, R_{\mu\nu}R^{\mu\nu})$
  - Renormalizable in dimensional regularization
  - Massive spin-2 ghost leads to loss of unitarity

(Stelle, K. S.; 1977).

$$S = \int d^4x\sqrt{g}\left(\underbrace{\frac{2\Lambda}{16\pi G_N} - \frac{1}{16\pi G_N}R}_{\text{Einstein gravity}} \underbrace{- \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}}_{\text{Higher derivative terms}}\right)$$

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A new idea: Asymptotic Safety

- Weinberg proposed that a **non-trivial UV Fixed Point** (FP) of the **Renormalization Group** (RG) can tame the divergences of gravity at high energies, making the theory **finite and non-perturbatively renormalizable** (Weinberg, S..; 1979)

- The RG flow vanishes at a FP:

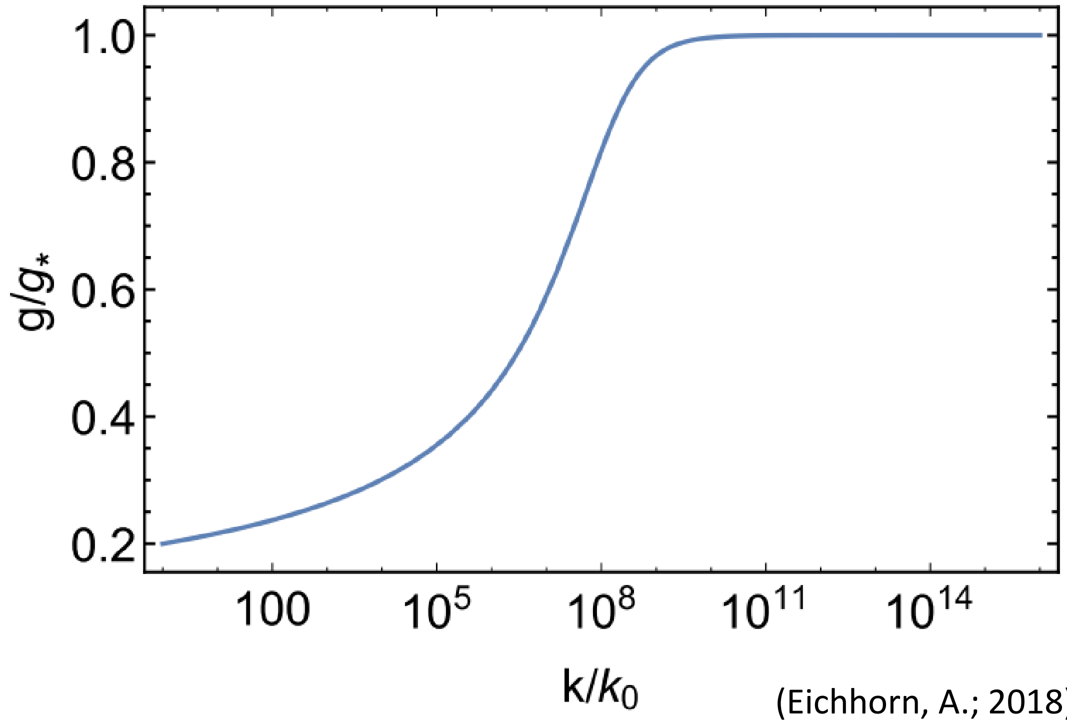
$$k\partial_k g(k)^* = 0$$

- Generalization of Asymptotic Freedom

- Interactive in the UV

- Theory remains **predictive** as long as only a finite number of RG trajectories flow into the FP in the UV

Asymptotic safety



(Eichhorn, A.; 2018)

Running Newton's coupling

- RG equation of a dimensionless running coupling:

$$k\partial_k g(k) = \beta_g = \underbrace{-d_g g}_{\text{Classical}} + \underbrace{f(g) g^2}_{\text{Quantum}}$$

- With $d_g = [\bar{g}]$ the canonical mass dimension $[G_N] = 2 - d$
- Gravity at one loop:

$$\beta_{g_N} = (d - 2 - b\, g_N) g_N$$

$$g_N^* = \frac{d-2}{b}$$

- Can the UV FP be found **perturbatively**?

Quantum gravity near 2d

- $d = 2 + \epsilon$ expansion

$$g^* = \frac{\epsilon}{b} \qquad b \propto \frac{2}{3}\{1, 19, 25\}$$

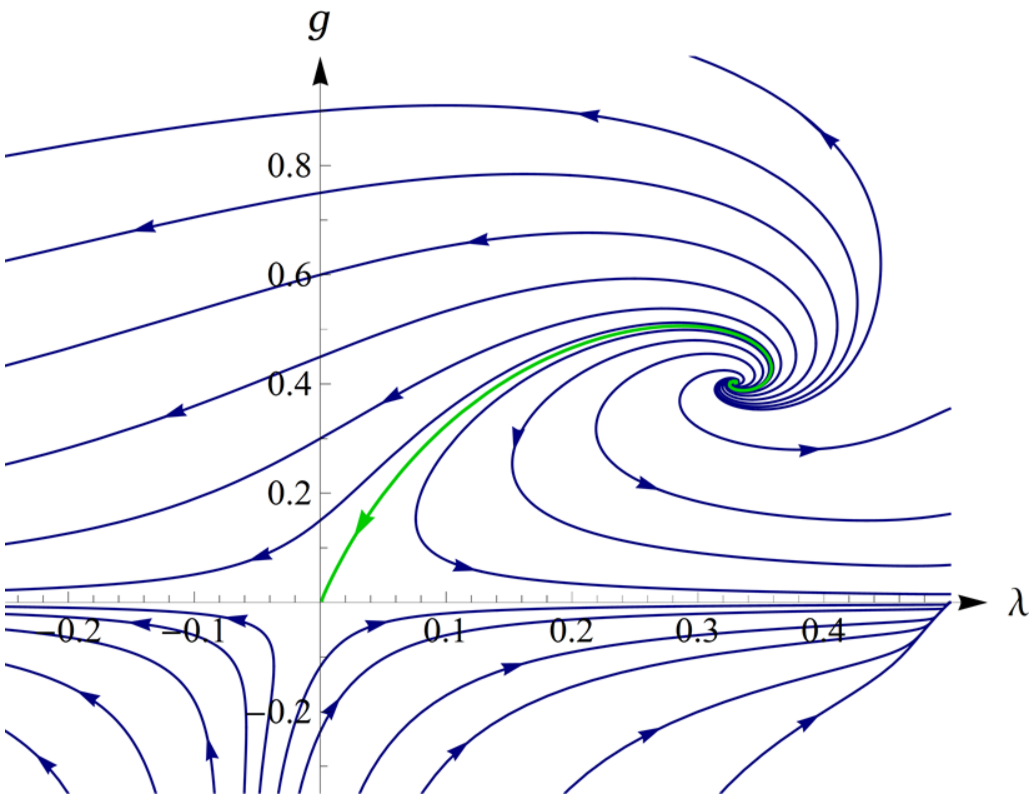
(Christensen, S. M., & Duff, M. J.; 1978)
(Gastmans, R., Kallosh, R., & Truffin, C.; 1978)
(Kawai, H., & Ninomiya, M.; 1990)
(Kawai, H., Kitazawa, Y., & Ninomiya, M.; 1993)

- Graviton contribution at one loop is **positive** (the exact number depends on technical choices, c.f. refs.)
- Key result: **Gravity is weakly coupled in the UV**
- If the results can be extrapolated to $\epsilon \to 2$, then the non-trivial FP may survive in four dimensions
- However, that's as far as perturbation theory gets us and progress was slow for a while

Asymptotically safe gravity

- The development of the Functional Renormalization Group (**FRG**) allowed the Asymptotic Safety programme to continue [1]
- There are strong indications for a non-perturbative **UV FP in pure gravity** [2]
- Extensions beyond Einstein-Hilbert are stable and reveal a finite number of free parameters (3)
- RG trajectories allow **GR** to be recovered in the IR

[1](Reuter, M.; 1998)
[1](Souma, W.; 1999)
[1](Reuter, M., & Saueressig, F.; 2002)



(Reuter M., Saueressig F.; 2002)

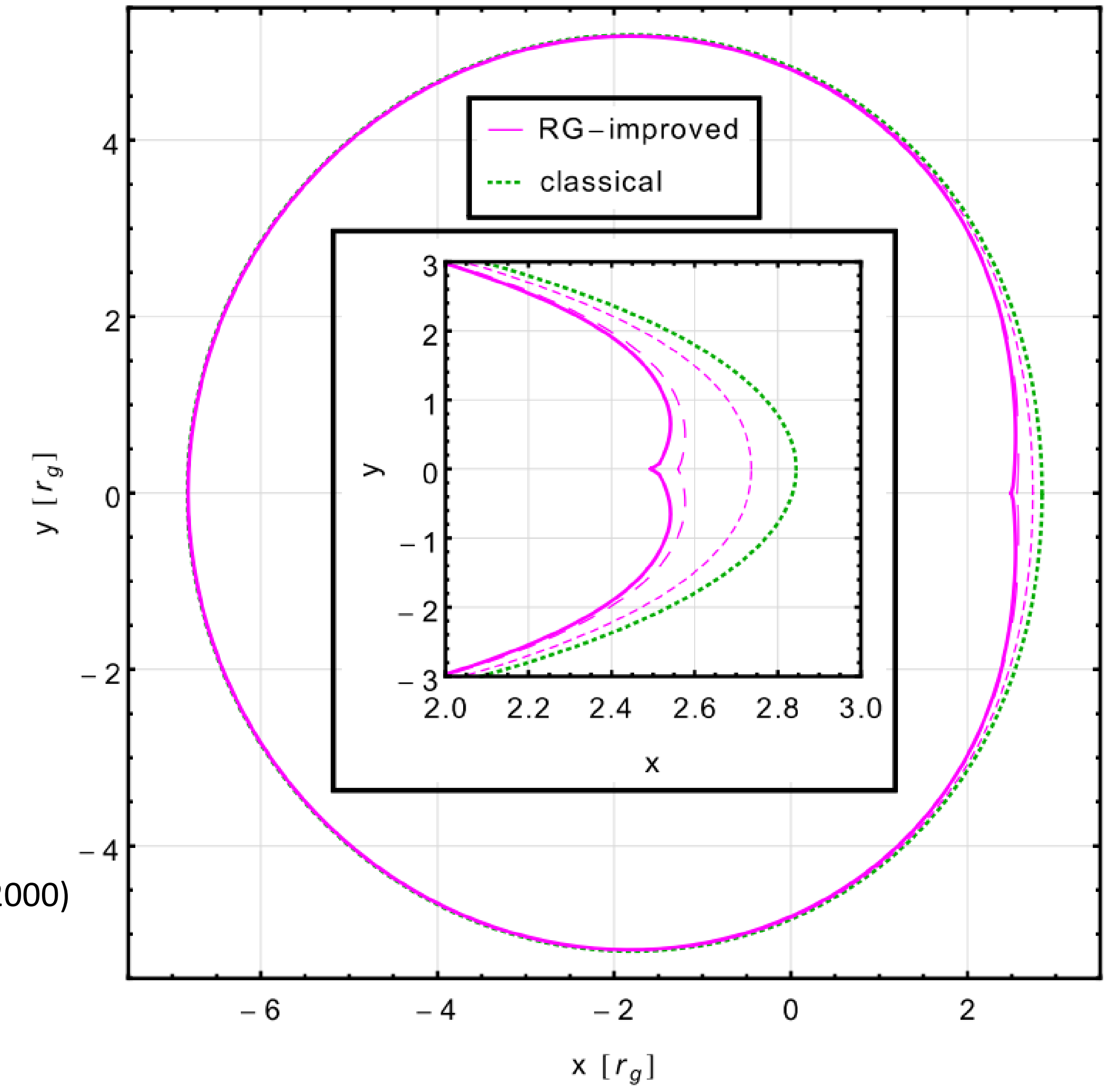
[2] For a complete review of the asymptotic safety paradigm in gravity see:
(Percacci, R.; 2017)
(Reuter, M., & Saueressig, Frank.; 2019)

For a more complete reading list see refs. at the end

Asymptotically safe gravity

- Properties of black-holes and cosmology have been studied in asymptotically safe gravity [1]
- Results from the discrete approaches (CDT, EDT, Lattice) show indications of a phase transition, i.e. a FP [2]

(Held, A., Gold, R., & Eichhorn, A.; 2019)



[1] For black-holes and cosmology see:
(Bonanno, A., & Reuter, M.; 2000)
(Bonanno, A., Contillo, A., & Percacci, R.; 2010)
(Falls, K., Litim, D. F., & Raghuraman, A.; 2010).
(Koch, B., & Saueressig, F.; 2013)
(Bonanno, A., & Saueressig, F.; 2017)

[2] For discrete approaches see:
(Ambjørn, J., Jurkiewicz, J., & Loll, R.; 2000)
(Hamber, H. W.; 2009)
(Laiho, J., & Coumbe, D.; 2011)

What about matter?

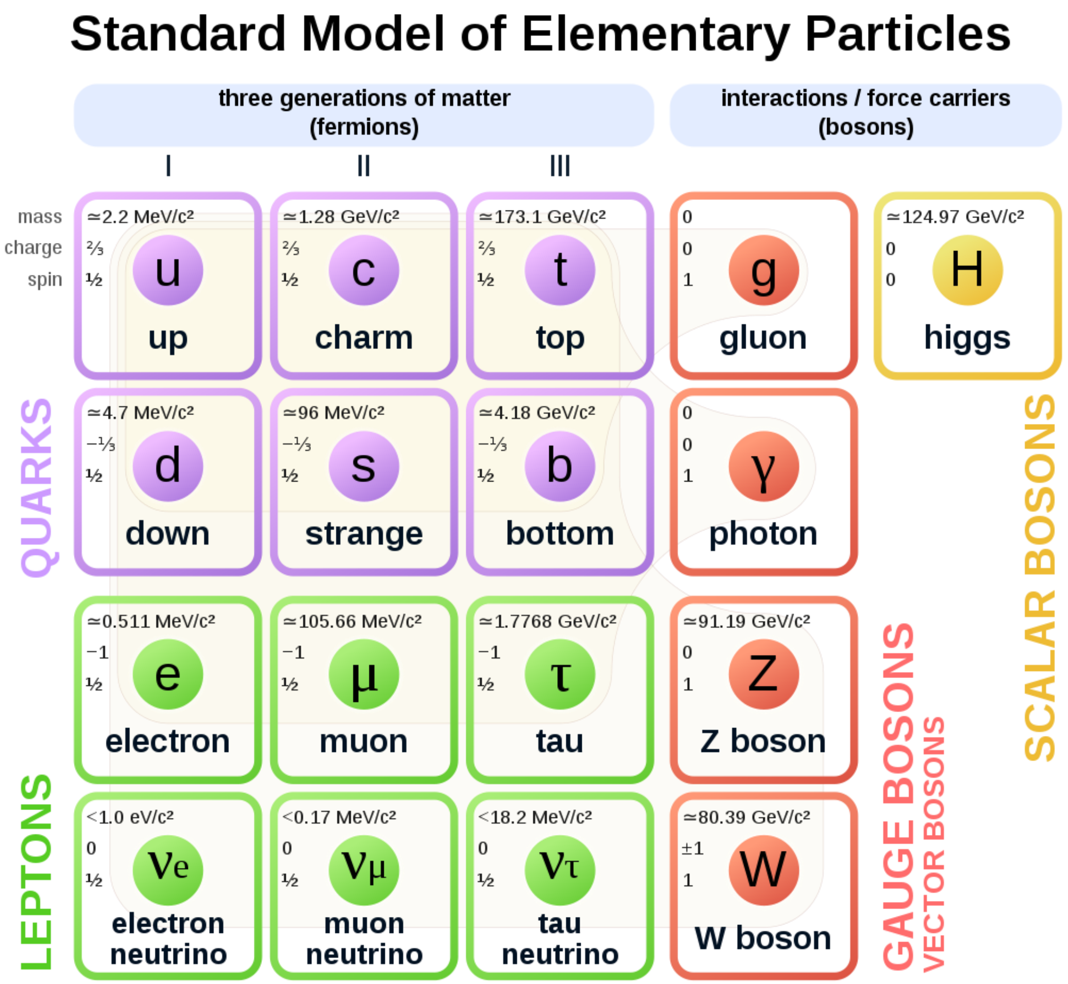
- However, our universe contains matter

$$N_S^{SM} = 4$$ **Real scalars**

$$N_M^{SM} = 12$$ **Gauge bosons**

$$N_D^{SM} = 45/2$$ **Dirac fermions**

- BSM fields ???
- Does the gravitational FP carry over straightforwardly when matter is included?



(Wikipedia CC v3.0, 2019)

Matter at 1-loop

- 1-loop counter-terms for gravity + matter were computed early on using perturbation theory
- In terms of the Weyl tensor and Gauss-Bonnet integrand:

$$\Delta L^{(1)} = \sqrt{g}\left(b_2^{(1)} C^2 + b_2^{(2)} E\right)$$

(Duff, M. J.; 1977)
(Dowker, J. S., & Critchley, R.; 1977)
(Tsao, H.-S.; 1977)

$$b_2^{(1)} \propto N_S + 6N_F + 12N_M$$

$$b_2^{(2)} \propto N_S + 11N_F + 62N_M$$

- All fields enter with **positive sign**
- No matter configuration can make the divergence vanish

Matter at 1-loop

- For the running of Newton's coupling:

$$b \propto (N_S + 2N_F - 4N_M)$$

(Fradkin, E. S., & Tseytlin, A. A.; 1982)

- Note that this time matter fields enter with different signs
- Scalars and fermions are screening
- Gauge bosons are anti-screening

Large N limit

- Large number **N** of non-interacting matter fields
- Matter quantum fluctuations dominate, gravitational quantum effects can be neglected: $b \propto N$

(Tomboulis, E.; 1977)
(Smolin, L.; 1982)

(Percacci R.; 2006)

$$g_N^* = -\frac{12\pi}{B\,N}$$

$$B = N_S + 2N_D - 4N_M$$

Scalars (N_S), Fermions (N_D), Gauge bosons (N_M)

- **N** copies of the SM: $B\big|_{SM} = 1$
- SM is on the tipping point where g_N^* changes sign

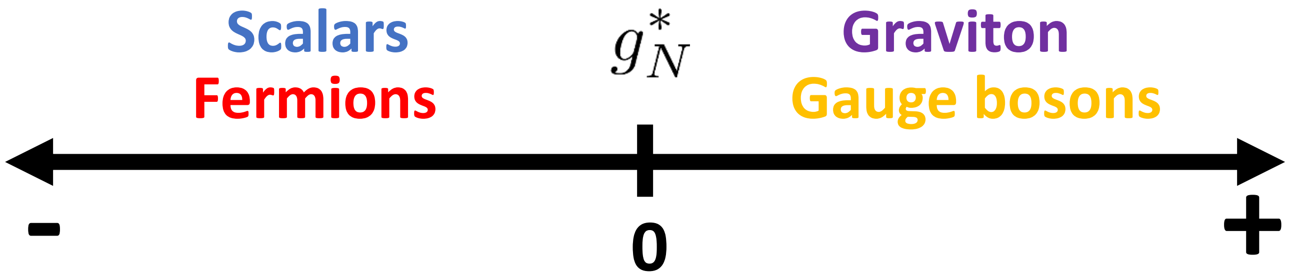
Weak gravity + matter

- In the perturbative gravity regime:

$$g_N^* = -\frac{12\pi}{N_S + 2N_D - 4N_M - 2N_G}$$

Graviton

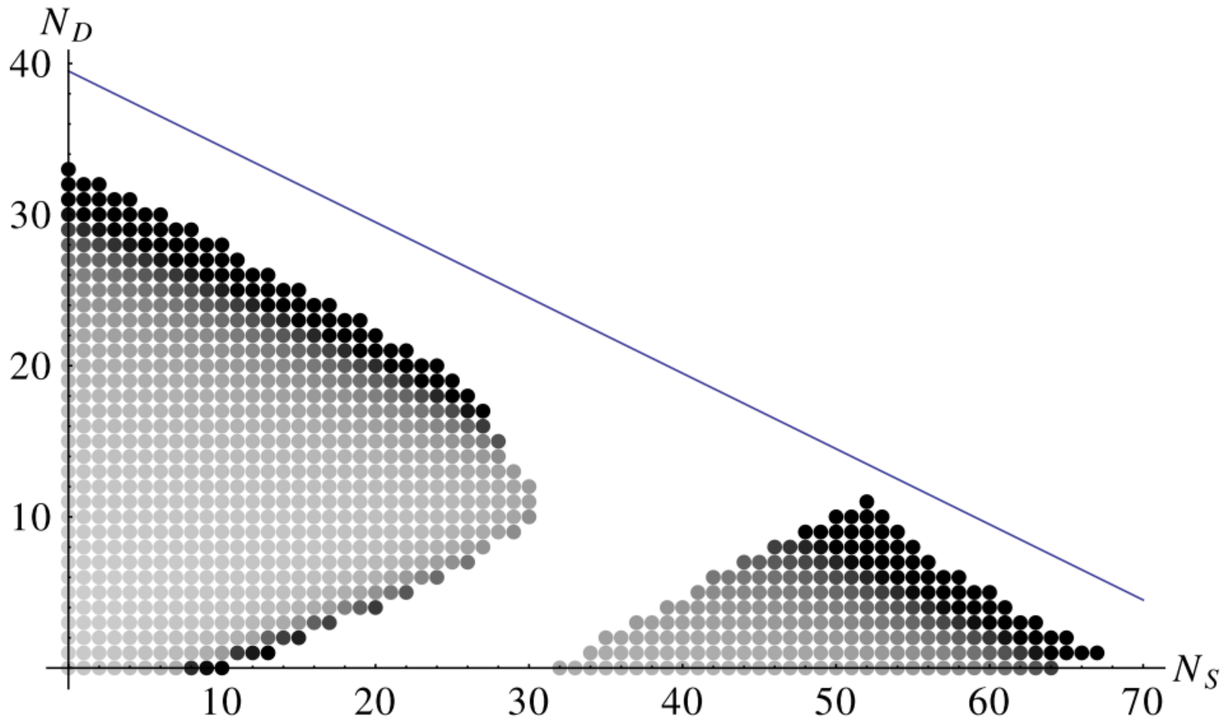
- **Graviton** and **gauge bosons** compete against **scalars** and **fermions**



- Fixed point is by no means guaranteed to exist beyond this approximation in the presence of matter

Asymptotically safe Gravity + Matter

- Bounds on the matter fields multiplicities have been derived in the Einstein-Hilbert truncation
- Higher derivative + matter has a FP at 1-loop
- UV FP in polynomial f(R) + SM matter up to R^{13} (depending on some technical choices)
- But, does the FP continues existing beyond these approximations?



(Donà, P., Eichhorn, A., & Percacci, R.; 2014)

(Percacci, R.; 2006)
(Codello, A., Percacci, R., & Rahmede, C.; 2009)
(Folkerts, S., Litim, D. F., & Pawlowski, J. M.; 2012)
(Donà, P., Eichhorn, A., & Percacci, R.; 2014)
(Christiansen, N., Litim, D. F., Pawlowski, J. M., & Reichert, M.; 2017)
(Alkofer, N., & Saueressig, F.; 2018)

Questions

- Is there a physical gravitational UV fixed point with SM matter in $d = 4$?
- What are its properties?
- Is it qualitatively different to the case without matter?
- Can GR be recovered in the IR limit?

Main framework

- Introduce **higher order** curvature invariants in the gravitational action

$$\Gamma_k = \int d^4x \sum_n \lambda_n \mathcal{O}_n[g_{\mu\nu}] + \Gamma_{matter}$$

- Gravitational interactions can **stabilize** the fixed point in the presence of **SM** matter

- Minimally coupled matter. Neglect all SM interactions and matter anomalous dimensions

- Main task: Find a UV FP of the RG equations of all couplings with the **SM** matter configuration

Methodology

Higher order curvature invariants

$$\Gamma_k = \int d^4x \sum_n \lambda_n \mathcal{O}_n$$

- Operators can be constructed with the quadratic curvature invariants

$$\mathcal{O}_0 \ni \{\mathbb{1}\}$$
$$\mathcal{O}_1 \ni \{R\}$$

Einstein-Hilbert

$$\mathcal{O}_2 \ni \{R^2,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\}$$
$$\mathcal{O}_3 \ni \{R^3,\ R\,R_{\mu\nu}R^{\mu\nu},\ R\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\}$$
$$\mathcal{O}_4 \ni \{R^4,\ R\,(R_{\mu\nu}R^{\mu\nu})^2,\ R\,(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^2\}$$

Higher derivative terms

Gravitational action

- In particular, I will restrict to **only** higher order terms of the form: $f(R, Riem^2)$

$$(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^n\,, \quad R\,(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^n$$

$$\Gamma_{f(R,Riem^2)} = \int d^4x\sqrt{g}\big(\lambda_0 + \lambda_1 R + \lambda_2 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \lambda_3 R\,(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) + \cdots\big)$$

- Action is evaluated in the 4-sphere

$$\mathcal{O}_n\Big|_{S^4} \propto R^n$$

- Flow of running couplings is computed within the **FRG**

$$\beta_n = \frac{\partial \lambda_n}{\partial \log k}$$

Functional methods

- The Functional Renormalization Group (**FRG**) is a framework useful for studying non-perturbative physics
- Analogue of Wilsonian renormalization by introducing the Effective Average Action (**EAA**) Γ_k

$$\Gamma \xleftarrow{k\to 0} \Gamma_k \xrightarrow{k\to\infty} S$$

- **EEA** follows an **exact** flow equation:

$$k\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]$$

(Wetterich, C.; 1993)
(Morris, T. R.; 1994)

- Can deal with infinitely many couplings at once

Bootstrap hypothesis

(Falls, K., Litim, D. F., Nikolakopoulos, K., & Rahmede, C.; (2013)

- Canonical mass dimension as an ordering principle

$$\underbrace{\lambda_0 + \lambda_1 R}_{\text{Relevant}} + \underbrace{\lambda_2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}}_{\text{Marginal}} + \underbrace{\lambda_3 R \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\right) + \cdots}_{\text{Irrelevant}}$$

- Systematic FP search up to high polynomial order ***N***

$$\begin{array}{llll} N=2 & \beta_0^* = 0, & \beta_1^* = 0 & \\ N=3 & \beta_0^* = 0, & \beta_1^* = 0, & \beta_2^* = 0 \\ N=4 & \beta_0^* = 0, & \beta_1^* = 0, & \beta_2^* = 0, \quad \beta_3^* = 0 \\ & \cdots & & \end{array}$$

- Look for convergence in the fixed point as ***N*** increases

Predictivity of asymptotic safety

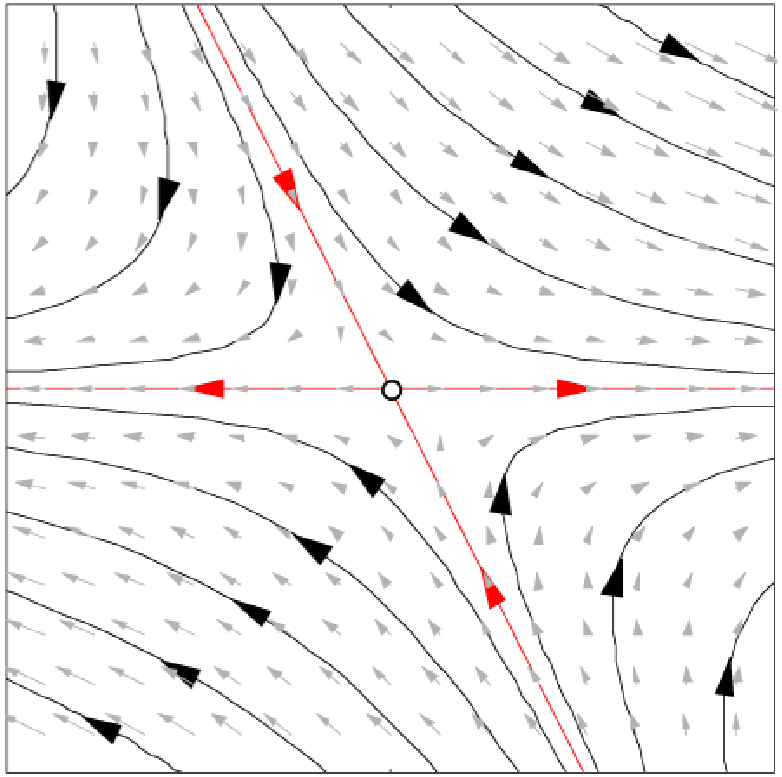
- Scaling exponents encode the power-law behaviour of an order parameter near a phase transition
- Close to a fixed point:

$$\beta_{\lambda_i} = \left.\frac{\partial \beta_{\lambda_i}}{\partial \lambda_j}\right|_{\lambda*} (\lambda_i - \lambda_i^*) + O\left(\lambda_i - \lambda_i^*\right)^2$$

- To linear order, the couplings behave as:

$$\lambda_i(k) = \lambda_i^* + \sum_j c_j v_i^{(j)} \left(\frac{k}{k_0}\right)^{\vartheta_j}$$

- Where ϑ_j are the eigenvalues of the stability matrix $M_{ij} = \left.\frac{\partial \beta_i}{\partial \lambda_j}\right|_{\lambda^*}$

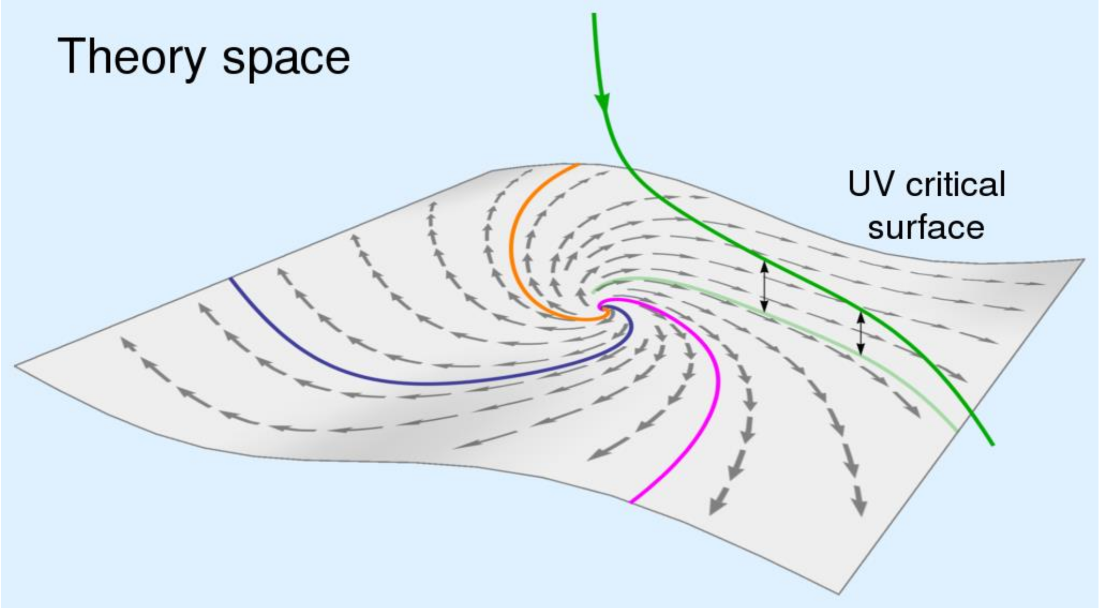

(Wikipedia, CC 2.5, 2005)

Predictivity of asymptotic safety

- To flow into the FP in the UV, set:

$$c_j = 0, \quad \mathrm{Re}(\vartheta_j) > 0$$
$$c_j = \text{free}, \quad \mathrm{Re}(\vartheta_j) < 0$$

- The theory is **<u>predictive</u>** as long as the UV critical surface is finite dimensional



(Nink, A. et al.; Scholarpedia 2013)

- Irrelevant couplings can be expressed in terms of relevant ones

Predictive power of Asymptotic Safety:
<u># of relevant directions ↔ # of free parameters (fixed by experiment)</u>

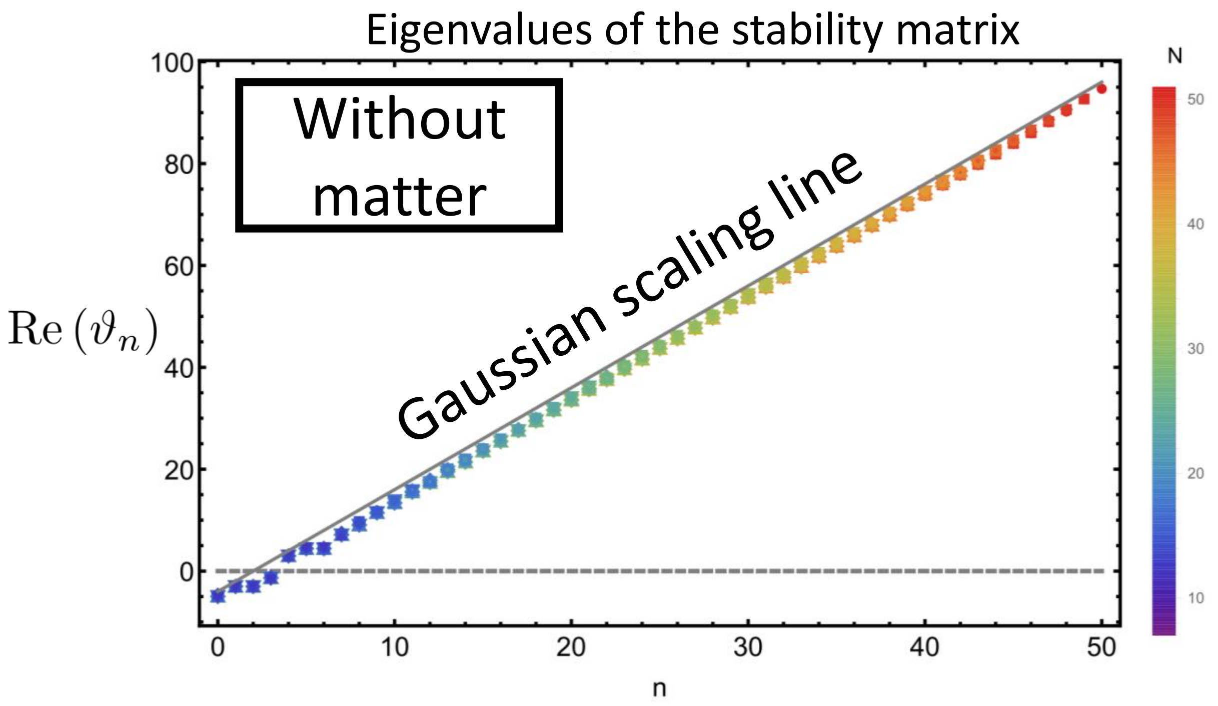
At a glance: $f(R, Riem^2)$ without matter

- For reference, consider the case without matter
- The UV critical surface is 4-dimensional

ϑ_1	$\vartheta_{2,3}$	ϑ_4	ϑ_5
-4.6919	$-3.0065 \pm 1.4580i$	-1.3299	2.9872

(Kluth Y. et al at Bad Honnef 2019)

- Complex eigenvalues
- Bootstrap hypothesis confirmed
- Near-Gaussian scaling



Fixed points with SM matter

Fixed points QG + SM $f(R, Riem^2)$

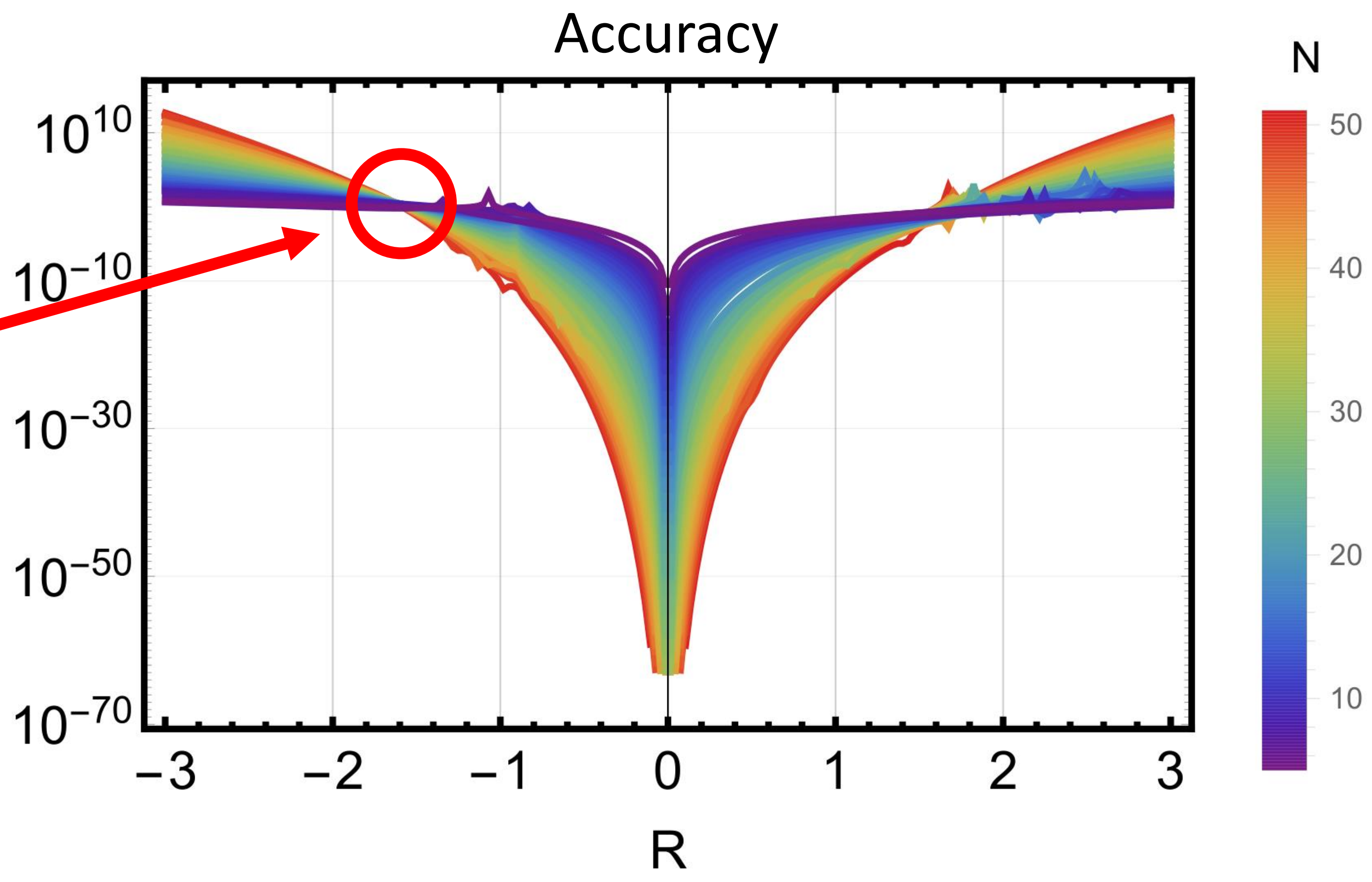
$$\Gamma_{f(R,Riem^2)} = \int d^4x\sqrt{g}\left(\lambda_0 + \lambda_1 R + \lambda_2 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \lambda_3 R\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) + \cdots\right)$$

- Numeric search up to ***N=51*** in the bootstrap with SM matter
- A **UV FP** with **SM** matter is found with high numeric accuracy
- FP mediated by higher order curvature invariants
- Coupling values and scaling dimensions quickly converge as ***N*** increases

Accuracy QG + SM

$f(R, Riem^2)$

- High numerical accuracy
- Polynomial solution bounded by radius of convergence r_C
- r_C inferred at the point where higher *N* yields lower accuracy.

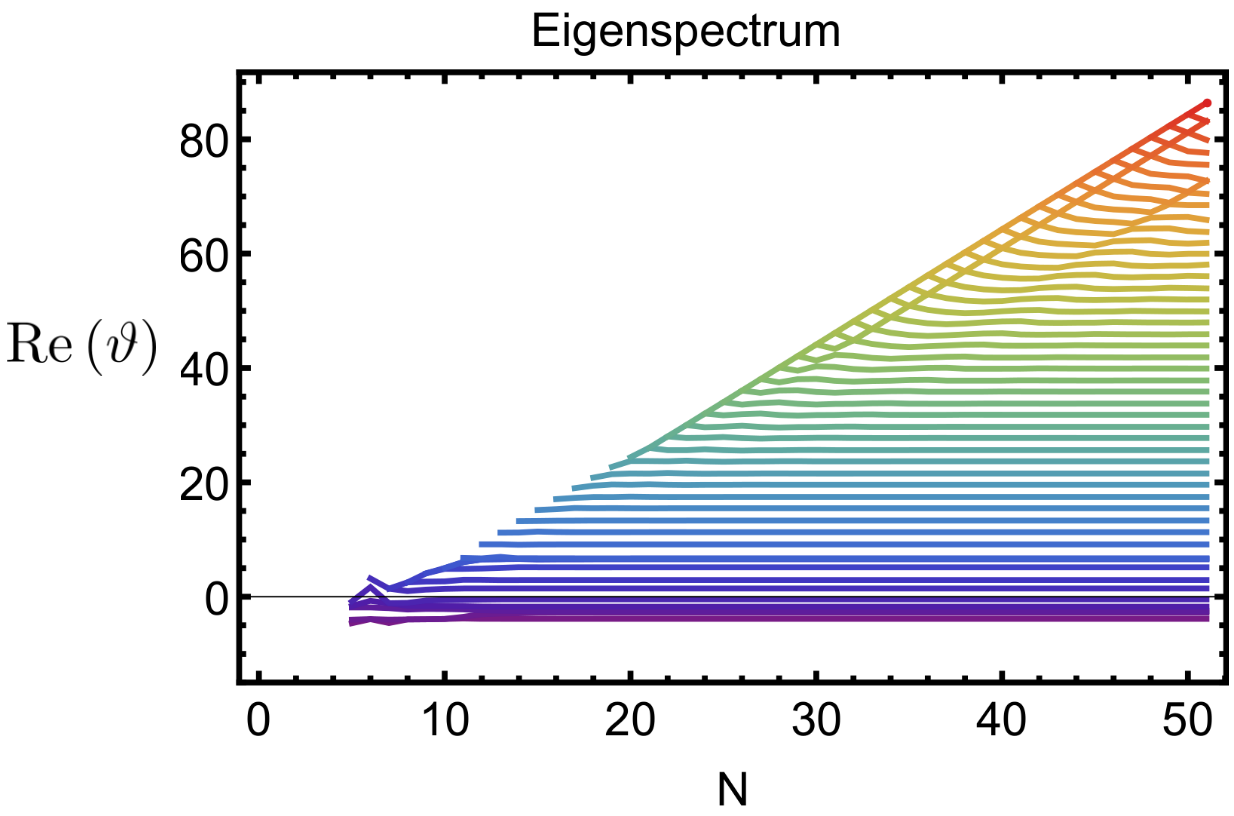


Scaling dimensions QG + SM

$f(R, Riem^2)$

ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6
-3.8688	-2.7985	-2.5150	-1.7398	-0.45897	1.4421

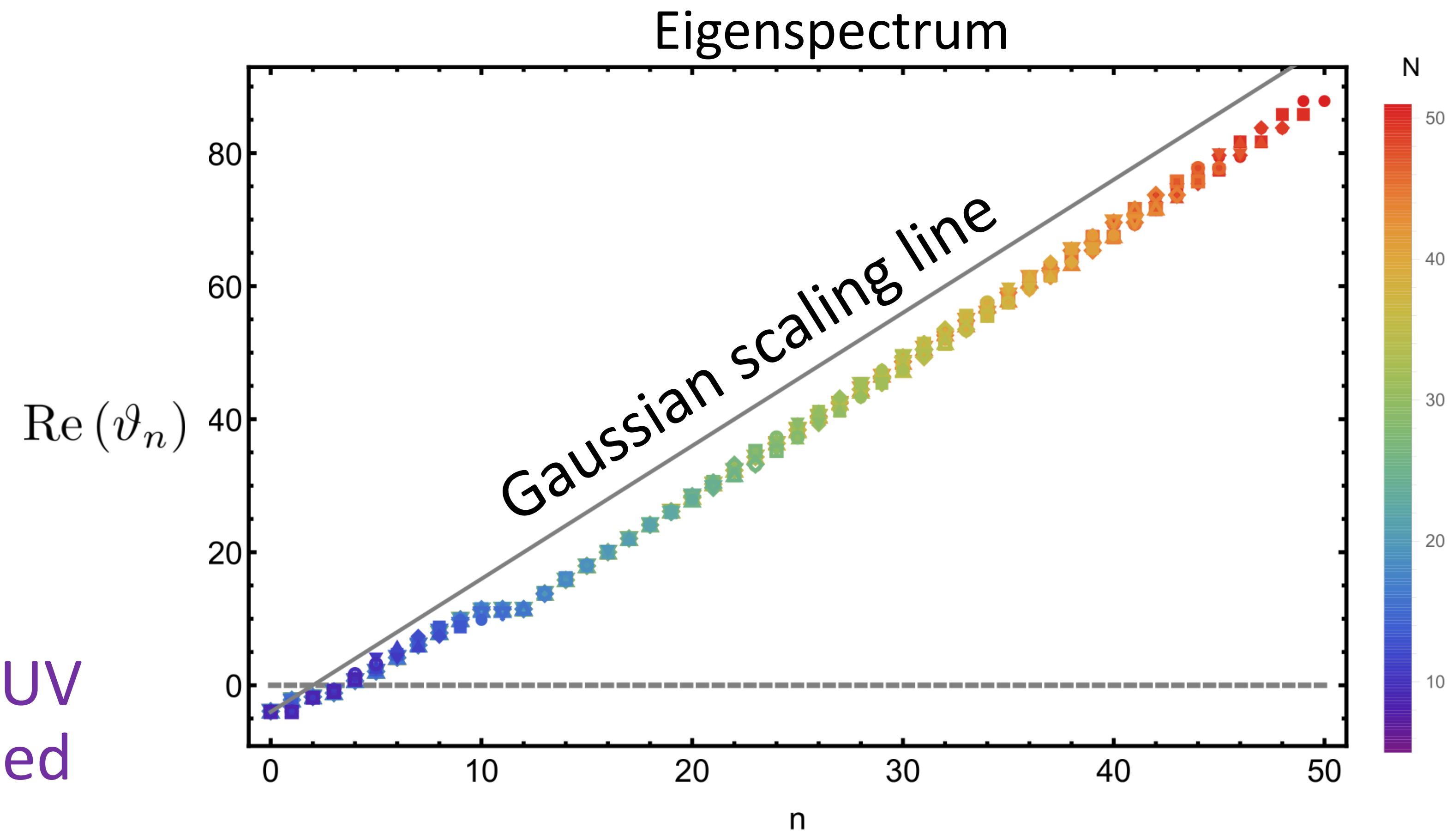
- 5-d UV critical surface
- Real relevant eigenvalues: SM lifts degeneracy
- Stability at high polynomial orders



Near-Gaussianity **QG** + **SM**

$f(R, Riem^2)$

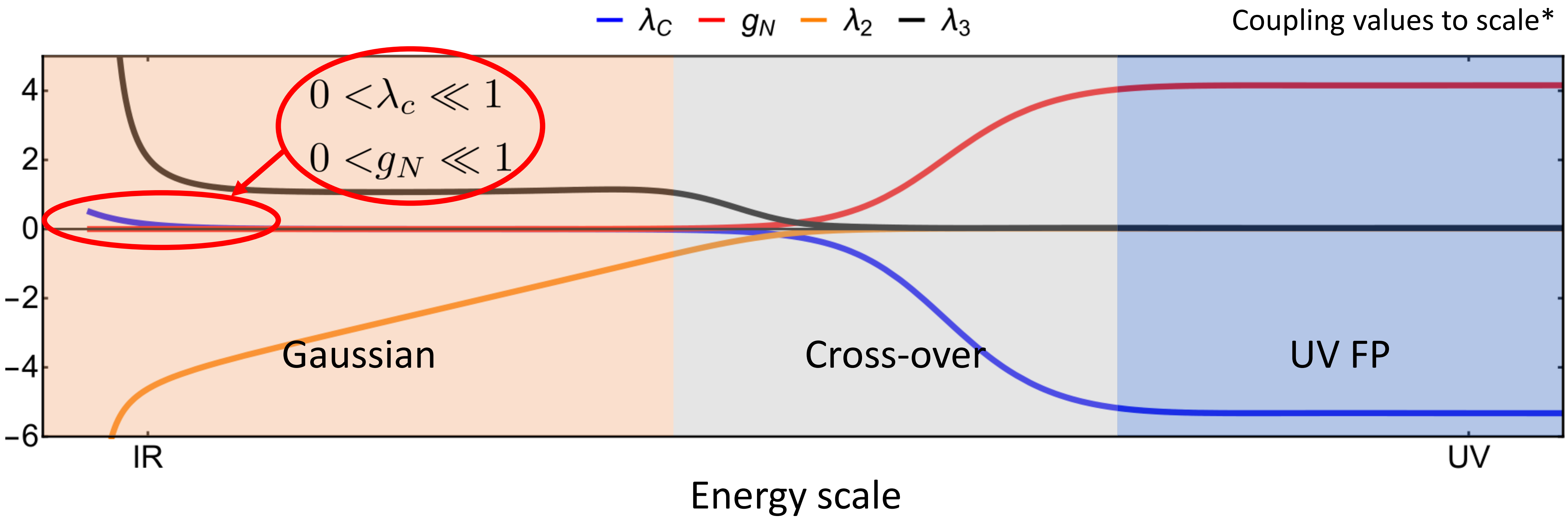
- Bootstrap hypothesis confirmed
- SM shifts scaling dimensions of all operators
- Dimensionality of the UV critical surface increased



RG flows **QG** + **SM**

$f(R, Riem^2)$

- Well defined UV-IR trajectory
- Can recover GR in the IR



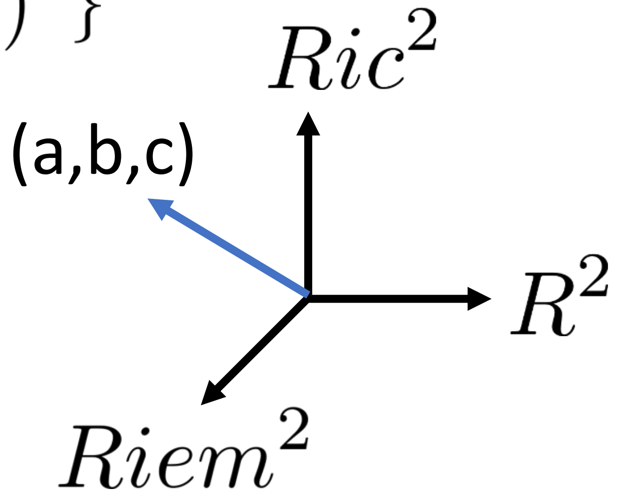
Other curvature invariants

Combinations of curvature invariants

- $f(R, Riem^2)$ is one example
- Other linear combinations of curvature invariants can be explored

$$\mathcal{O}_2 \ni \{aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\}$$
$$\mathcal{O}_3 \ni \{R\left(aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\}$$
$$\mathcal{O}_4 \ni \{R\left(aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)^2\}$$
$$\ldots$$

- With *a*, *b*, *c* free constant parameters that define the relative weight of each term



Linear combinations of curvature invariants

- Not any linear combination supports **SM UV FPs**
- Results **qualitatively similar** to $f(R, Riem^2)$
- **SM** matter **increases dimensionality** of **UV** critical surface
- **Degeneracy** can be **lifted** in scaling dimensions
- Well defined **UV-IR** trajectories exist

Conclusions & outlook

Conclusions

- **Matter** plays a crucial role in quantum gravity and can **destabilize** the FP
- **SM** is on the tipping point of a physical large ***N*** UV FP
- **UV FP** stable up to high polynomial orders mediated by the **interplay** of **QG** and **SM** matter
- **Different universality class** compared to **QG** without matter
- **SM matter lifts degeneracy** on scaling exponents and can increase the dimensionality of the UV critical surface
- Well defined **RG trajectories** connecting **UV-IR** regimes exist

Outlook

- Explore **further curvature invariants**
- Study can be extended **beyond SM** matter (MSSM, SU(5) GUT, SO(10) GUT etc.)
- SM **interactions** and matter **anomalous dimensions** can be incorporated
- Working in a general **Einstein space** to further distinguish different operators
- Further **stability tests** are required to assess how robust the solution is

Thank you

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