Recent developments in large-N β-functions

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I

Background

Why large N?

- For a theory with large flavour symmetry (like $O(N)$, $SU(N)$...), $1/N$ is a good expansion parameter
  - Reorganising perturbative expansion in terms of powers of $1/N$ can give non-perturbative information away from the Gaussian fixed point
- Example Gross–Neveu (GN) model in 3d

$$L_{\mathrm{GN}} = \bar{\psi} i \partial\!\!\!/ \psi + g^2 (\bar{\psi}\psi)^2$$

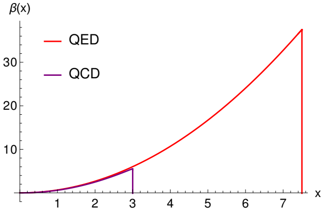
  - In 2d asymptotically free
  - In 3d not perturbatively renormalisable
  - But: In the large-N limit can be shown that it is non-perturbatively renormalisable and there is a UV fixed point Gawedzki & Kupiainen '85, de Calan, da Veiga, Magnen, Seneor '91
  - A prototype for quantum gravity?

What about 4d (without gravity)?

- Gauge-Yukawa theories in the Veneziano limit
  - $0 < \epsilon \equiv \frac{N_f}{N_c} - \frac{11}{2} \ll 0.1$ fixed for $N_c, N_f \to \infty$
  - Scalars needed

Litim & Sannino [1406.2337]

- The large-N beta functions have singularities
  $\Rightarrow$ speculations about a possible UV FP



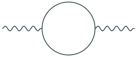
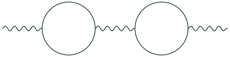
Mann et al., [1707.02942]
Pelaggi et al. [1708.00437]
Antipin & Sannino [1709.02354]
Molinaro, Sannino, Wang [1807.03669]
Cacciapaglia et al. [1812.04005]
Sannino, Smirnov, Wang [1902.05958]
Cai & Zhang [1905.04227]

In practise

- Define 't Hooft coupling $K = \frac{g^2 N}{4\pi^2}$ which is kept fixed at $N \to \infty$
- Any amplitude can then be expanded as

$$\mathcal{A}(K; p_i) = \mathcal{A}_0(K; p_i) + \frac{1}{N}\mathcal{A}_1(K; p_i) + \frac{1}{N^2}\mathcal{A}_2(K; p_i) + \dots$$

- For example, diagrams like

 and 

are both order $1/N^0$ ($g^2 N \sim K$ and $g^4 N^2 \sim K^2$)

- Infinitely many diagrams contribute at each order in $1/N$

In practise

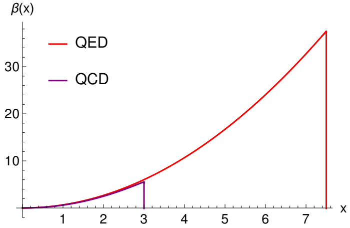
- Each fixed order in $1/N$ contains all-orders or non-perturbative information in the traditional perturbation-theory sense
- $1/N$ expansion of β-functions convenient: at fixed order in N, the diagrams grow polynomially only
  $\Rightarrow$ finite radius of convergence
- But: Need to resum an inifinite number of diagrams at each order or use some other methods

Direct resummation: History

- The $\mathcal{O}(1/N)$ coefficients of gauge β-functions known

Palanques-Mestre & Pascual (1984), Gracey [hep-ph/9602214]

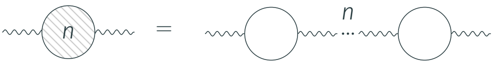
- The gauge β-function starts positive, but the $1/N$ coefficient has a negative singularity at $x_{\mathrm{QED}} = 15/2$ ($x_{\mathrm{QCD}} = 3$), $x \equiv \frac{\alpha}{\pi} N$



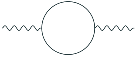
- Near the singularity $1/N$ coefficient exceeds $1/N^0$ one $\Rightarrow$ speculations about possible UV fixed point

Direct resummation: practise

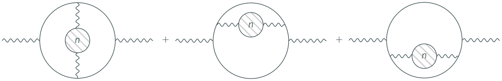
- Bubble chains have net effect: $\frac{1}{q^2} \to \frac{K^n \Pi_0^n}{(q^2)^{1+n\epsilon/2}}$



- Example: QED two-point function
  - $\Pi_0(p)$: one-loop



  - $\Pi_1(p)$: two-loop topologies, all orders



  - Corresponding $1/N$ expansion of the β-function

$$\beta_K = \frac{2}{3}K^2 + \frac{1}{N}F_1(K) + \dots$$

Computation

Task: compute $F_1(K)$

- The renormalisation factor can be written as

$$Z_A = 1 - \frac{2}{3}\frac{K}{\epsilon} + \sum_{n=0}^{\infty} \mathrm{div}\left\{ \frac{K^{n+2}}{N}\left(1 - \frac{2}{3}\frac{K}{\epsilon}\right)^{-n} \Pi_1^{(n)}(p^2, \epsilon) \right\} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

- Eventually, have to resum

$$\sum_{n=2}^{\infty} K^n \mathrm{div}\left\{ \sum_{j=0}^{\infty} \frac{\pi_j(p^2, \epsilon)}{\epsilon^{n-j-1}} \sum_{k=0}^{n-2} \binom{n-2}{k} (n-k)^{j-1}(-1)^k \right\}$$

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- Euler's finite difference theorem Palanques-Mestre & Pascual '84

$$\sum_{k=0}^{n-2} \binom{n-2}{k} (n-k)^{j-1}(-1)^k = \begin{cases} \frac{(-1)^n}{n(n-1)} & j = 0 \\ 0 & j \in (1, n-2) \\ a_{n,j} n! & j > n-2 \end{cases}$$

Computation

Task: compute $F_1(K)$

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- Eventually, have to resum

$$\sum_{n=2}^{\infty} K^n \text{div}\left\{ \sum_{j=0}^{\infty} \frac{\pi_j(p^2, \epsilon)}{\epsilon^{n-j-1}} \sum_{k=0}^{n-2} \binom{n-2}{k} (n-k)^{j-1}(-1)^k \right\}$$

- Finally

$$Z_A = 1 - \frac{2}{3}\frac{K}{\epsilon} + \frac{K^2}{N} \sum_{n=2}^{\infty} \left(-\frac{K}{3}\right)^{n-2} \text{div}\left\{ \frac{1}{\epsilon^{n-1}(n-1)n} \pi_0(\epsilon) \right\} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

  - Consistency check: $\pi_0(p^2, \epsilon) \equiv \pi_0(\epsilon)$ independent of p^2

The dust settles

- Only the $1/\epsilon$ part contributes to the β-function

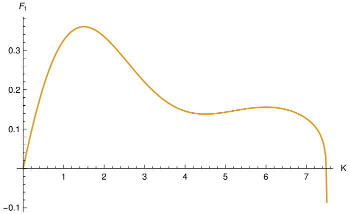
$$\sum_{n=1}^{\infty} \frac{K^n}{n\epsilon^n}\pi_0(\epsilon)\bigg|_{1/\epsilon} = \frac{1}{\epsilon}\sum_{n=0}^{\infty}\frac{K^{n+1}}{n+1}\pi_0^{(n)} = \frac{1}{\epsilon}\int_0^K \pi_0(\epsilon)d\epsilon$$

  - Coupling K and the dimension $d = 4 - \epsilon$ are exchanged as final outcome of the large-N resummation!

- Result:

$$F_1(K) = \int_0^K \mathrm{d}t\, \frac{(1-t)(1-t/3)(1+t/2)\Gamma(4-t)}{6\Gamma^2(2-t/2)\Gamma(3-t/2)\Gamma(1+t/2)}$$

  - First singularity at $K = 15/2$



Revival of interest

- Gauge contribution to the Yukawa β-function

  Kowalska & Sessolo [1712.06859]

- Semi-simple gauge groups — Antipin et. al [1803.09770]
- a-theorem at large N — Antipin et al. [1808.00482]
- Full gauge-Yukawa β-functions at large N

  TA & Blasi [1806.06954, 1808.03252]

  TA, Blasi, Dondi [1904.05751]

- Critical look at β-function singularities

  TA, Blasi, Dondi [1905.08709]

Critical point method

- So how about QCD?
  - Fermion bubble chains as in QED, but more basic topologies due to non-abelian vertices (double chains)
  - Direct resummation impossible, results from critical point method
- Exploits conformal properties of the theory in arbitrary dimension close to the Wilson–Fisher fixed point
- Developed by Vasiliev, Pismak & Honkonen in early 80’s
- Universality is used to connect theories in the same class (e.g. QCD and non-abelian Thirring Model)

Critical point method

- In arbitrary dimension $d = d_c - \epsilon$, the β-function for a one-coupling theory is

$$\beta(g) = -\epsilon g + bg^2 + \dots$$

- The critical coupling, g_c, at the WF fixed point satisfies

$$\beta(g_c) = 0 \quad \Leftrightarrow \quad g_c = \frac{\epsilon}{b} + \dots$$

- This signals a phase transition whose properties are encoded in the critical exponents, e.g.

$$\omega = \beta'(g_c), \quad \eta = \gamma_\phi(g_c)$$

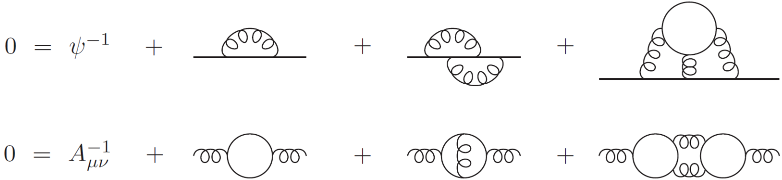
Critical point method: practise

The exponents ω, η are computed by:

- making a scaling ansatz for the propagators at the WF fixed point

$$\psi \sim A \frac{\not{p}}{(p^2)^{d/2-\alpha+1}}, \quad A_{\nu\sigma} \sim \frac{B}{(p^2)^{\mu-\beta}}$$

- solving the Schwinger-Dyson equation at large N, which yields algebraic equations for the critical exponents (d only variable)



- using the relations among the different exponents

Critical point method: some literature

- O(N) model: η up to $\mathcal{O}(1/N^3)$

  Vasiliev, Pismak, Honkonen ’81, ’82

- Gross–Neveu model, η up to $\mathcal{O}(1/N^3)$

  Gracey ’91, ’92, ’94, Vasiliev, Derkachov, Kivel, Stepanenko ’93, Valiliev & Stepanenko ’93

- Gross–Neveu–Yukawa model, ω up to $\mathcal{O}(1/N^2)$

  Gracey ’17, Manashov & Strohmaier ’18

- QED & QCD, ω up to $\mathcal{O}(1/N)$, η up to $\mathcal{O}(1/N^2)$

  Gracey ’93, ’96, Ciuchini, Derkachov, Gracey, Manashov ’00

- Wess–Zumino model, ω up to $\mathcal{O}(1/N^2)$

  Ferreira & Gracey ’98

II

Large N for Yukawa models

With Simone Blasi JHEP 1808 (2018), PRD 98 (2018)

and Simone Blasi & Nicola Dondi, EPJC 79 (2019)

Gross–Neveu–Yukawa model

- N massless fermion flavours, ψ, a massless real scalar, ϕ

$$\mathcal{L}_{\rm GNY} = \bar{\psi}\mathrm{i}\partial\!\!\!/\psi - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + g_1\phi\bar{\psi}\psi + g_2\phi^4.$$

- Same universality class that describes critical properties of the Mott transition in graphene
- Rescaled couplings: $y \equiv \frac{g_1^2}{8\pi^2}$, $K \equiv 2yN$, and $\lambda \equiv \frac{g_2}{8\pi^2}$
- β-functions at $\mathcal{O}(1/N)$

$$\begin{aligned}\beta_y =& (d-d_c)y + y^2(2N+3+F_1(yN))\\ \beta_\lambda =& (d-d_c)\lambda + y^2(-N+F_2(yN))\\ &+\lambda^2(36+F_3(yN)) + y\lambda(4N+F_4(yN)).\end{aligned}$$

  - Perturbatively known up to four loops   Zerf et al. [1709.05057]

Critical exponents for two-coupling case

- Two-coupling model $\Rightarrow$ two critical exponents, $\omega_\pm$
  - $\omega_\pm$ are the eigenvalues of the Jacobian $[\partial\beta_i/\partial g_i]$ at WFFP
  - $\frac{\partial\beta_y}{\partial\lambda} \equiv 0$ at $\mathcal{O}(1/N) \Rightarrow \omega_\pm$ directly correspond to $\frac{\partial\beta_\lambda}{\partial\lambda}$ and $\frac{\partial\beta_y}{\partial y}$
- Known up to $\mathcal{O}(1/N^2)$
  - Suggest shrinking radius of convergence $1/N \to 1/N^2$

Gracey [1707.05275], Manashov & Strohmaier [1711.02493]

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Gracey [1707.05275], Manashov & Strohmaier [1711.02493]

- Comparing with the β-function ansatz, we get

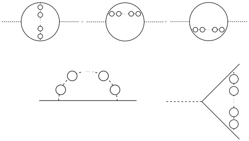
$$F_1(t) = \int_0^t \frac{\omega_-^{(1)}(2\epsilon)}{\epsilon^2} \mathrm{d}\epsilon, \quad \text{and}$$

$$30 - 2F_1(\epsilon/2) + F_3(\epsilon/2) + F_4(\epsilon/2) = 2\frac{\omega_+^{(1)}(\epsilon)}{\epsilon}$$

  - β_λ cannot be computed with the knowledge of $\omega_\pm$ only
  - in particular, F_2 is fully unconstrained

Direct resummations

- Direct resummation to get the missing information
- First the Yukawa coupling TA, Blasi [1806.06954]



  - $\ln Z_K \equiv \ln (Z_S^{-1} Z_F^{-2} Z_V^2)$
  - $Z_S = 1 - \text{div}\{Z_S \Pi_0(p^2, Z_K K, \epsilon)\}$
  - $Z_F = 1 - \text{div}\left\{\Sigma_0(p^2, Z_K K, \epsilon)\right\}$
  - $Z_V = 1 - \text{div}\left\{V_0(p^2, Z_K K, \epsilon)\right\}$

$\Rightarrow$

$$Z_S = 1 - \frac{K}{\epsilon} - \frac{1}{N_f}\sum_{n=2}^{\infty} K^n \left\{ \left(1 - \frac{K}{\epsilon}\right)^{1-n} \left(2\Pi_F^{(1)}\left[\Sigma^{(n-1)} - V^{(n-1)}\right] + \Pi^{(n)}\right)\right\}$$

+ a new summation rule

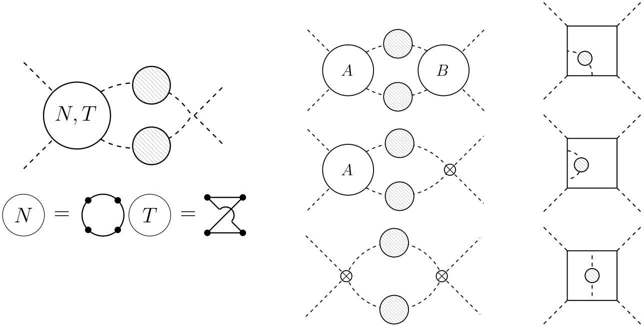
$$\sum_{i=0}^{n-2} \begin{pmatrix} n-2 \\ i \end{pmatrix} (-1)^i \frac{(n-i)^{j-1}}{(n-i-1)} = \begin{cases} \frac{(-1)^n}{n} & j = 0 \\ \frac{(-1)^n}{n-1} & j = 1, \dots, n-1 \end{cases}$$

- Straight-forward extension to gauge-Yukawa system

TA, Blasi [1808.03252]

Direct resummations

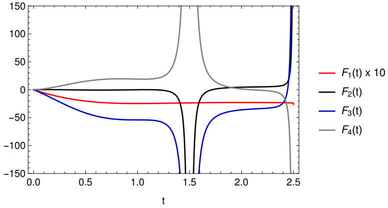
- The quartic a bit more complicated
    - First time resummation with three-loop basic topology!
    - Possible, because the double chains can be reduced to a single one



- We were able to compute the full systen of GNY β-functions at $\mathcal{O}(1/N)$
- The closer singularity at $\mathcal{O}(1/N^2)$ is actually already present at $\mathcal{O}(1/N)$ but is cancelled in the combinations of F_i entering $\omega_\pm$

$$F_1(t) = \int_0^t \frac{\omega_-^{(1)}(2\epsilon)}{\epsilon^2} \mathrm{d}\epsilon,$$

$$30 - 2F_1(\epsilon/2) + F_3(\epsilon/2) + F_4(\epsilon/2) = 2\frac{\omega_+^{(1)}(\epsilon)}{\epsilon}$$



III

Critical look at the β-function singularities

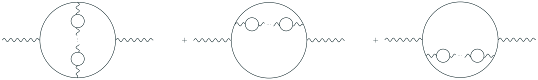
With Simone Blasi & Nicola Dondi, PRL123 (2019)

The large-N β-function

- Large-N ansatz

$$\beta(g) = (d - d_c)g + g^2 \left(bN + c + \sum_{n=1}^{\infty} \frac{F_n(gN)}{N^{n-1}} \right)$$

- Option 1: Compute F_n directly by resumming diagrams



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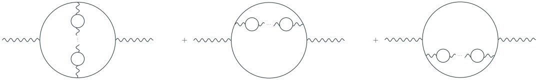


- Option 2: Get the slope of the β-function at WFFP
  - $1/N$ expansion of the critical exponent, ω, in arbitrary dimensions using CFT methods Vasiliev et al., Gracey...

$$\beta'(g_c) = \omega(d) \equiv \sum_{n=0}^{\infty} \frac{\omega^{(n)}(d)}{N^n}$$

- Computing β-function in terms of ω turns out convenient

Shadows on the fixed point

- For QED the fermion mass anomalous dimension, γ_m, diverges at the β-function singularity violating the unitarity bound Espriu et al. (1982), Antipin & Sannino [1709.02354]
- The same for the anomalous dimension of the glueball operator Ryttov & Tuominen (2019) [1903.09089]
- Similar arguments for 2d GN model would suggest an infinite number of IR fixed points
- Singularity structure of higher-order contributions? Example: O(N) model, where $\mathcal{O}(1/N^2)$ has a different sign nearer singularity wrt $\mathcal{O}(1/N)$ Gracey [hep-ph/9609409]
- Recent lattice studies suggest a Landau pole

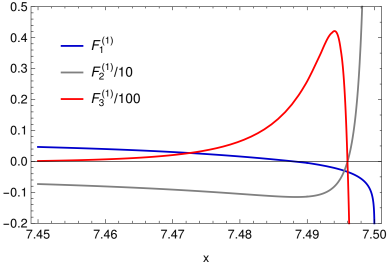
  Leino et al. [1908.04605]

Shadows on the fixed point

The $\mathcal{O}(1/N)$ critical exponent contributes to all F_n and generates a sequence of alternating-sign singularities

TA, Blasi, Dondi (2019), [1905.08709]

	ω_1	ω_2	ω_3	$\dots$
F_1	$F_1^{(1)}$			
F_2	$F_2^{(1)}$	$F_2^{(2)}$		
F_3	$F_3^{(1)}$	$F_3^{(2)}$	$F_3^{(3)}$	
$\vdots$				$\ddots$



$$F_1^{(1)}(x) = F_1(K) = \int_0^x \frac{\omega^{(1)}(d_c - bt)}{t^2} \mathrm{d}t,$$

$$F_2^{(1)}(x) = \int_0^x \frac{c + F_1(t)}{b}(2F_1'(t) + tF_1''(t))\mathrm{d}t,$$

$$F_3^{(1)}(x) = \int_0^x \frac{1}{2b^2}\left\{[2(c+F_1(t))^2 + 4bF_2^{(1)}(t)]F_1'(t) + [4t(c+F_1(t))^2 + 2btF_2^{(1)}(t)]F_1''(t) + t^2(c+F_1(t))^2F_1'''(t)\right\}\mathrm{d}t$$

Self-consistency equation

- Fixed-order ω produces a closed set of contributions to all higher-order β-function terms
- β-ansatz: $\beta(g) = (d - d_c)g + g^2 \left(bN + c + \mathcal{F}(x, N)\right), \quad \mathcal{F} \equiv \sum_{n=1}^{\infty} \frac{F_n}{N^{n-1}}$
- WFFP: relationship between coupling and dimension

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- WFFP: relationship between coupling and dimension
- $\beta'(g_c) = \omega(d) \Rightarrow$ a differential equation for $\mathcal{F}$

$$\partial_x \mathcal{F}(x, N) = \frac{1}{x^2}\omega(d) = \frac{1}{x^2}\omega\left(d_c - x\left(b + \frac{c + \mathcal{F}(x, N)}{N}\right)\right)$$

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- ω only known to fixed order: $\mathcal{O}(1/N)$ for QED/QCD
  $\Rightarrow$ truncate $\omega(d) = -(d - d_c) + \frac{1}{N}\omega^{(1)}(d)$

$$\partial_x \mathcal{F}^{(1)}(x, N) = \frac{1}{x^2}\omega^{(1)}\left(d_c - x\left(b + \frac{c + \mathcal{F}^{(1)}(x, N)}{N}\right)\right)$$

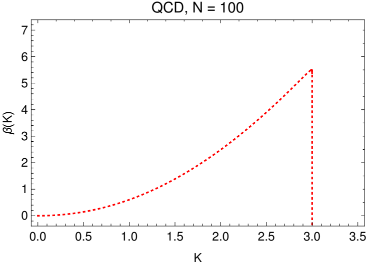
The large-N limit

$$\partial_x \mathcal{F}^{(1)}(x,N) = \frac{1}{x^2}\omega^{(1)}\left(d_c - x\left(b + \frac{c + \ \mathcal{F}^{(1)}(x,N)}{N} \quad \right)\right)$$

The large-N limit

$$\partial_x \mathcal{F}^{(1)}(x,N) = \frac{1}{x^2}\omega^{(1)}\left(d_c - x\left(b + \frac{c + \mathcal{F}^{(1)}(x,N)}{N} \to 0\right)\right)$$

Take first the limit, and then solve the DE:



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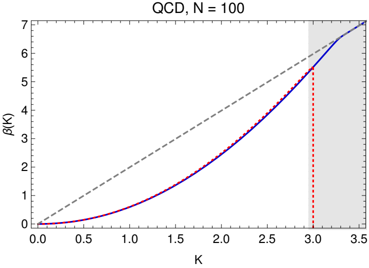
$$\partial_x \mathcal{F}^{(1)}(x,N) = \frac{1}{x^2}\omega^{(1)}\left(d_c - x\left(b + \frac{c + \mathcal{F}^{(1)}(x,N)}{N}\right)\right)$$

Solve the DE without taking the $N \to \infty$ limit first



- Includes the higher-order terms induced by $\omega^{(1)}$ that are not subleading!
- Away from the singularity (where expansion under control!) the two limits agree
- $\mathcal{F}^{(1)} = N\left(\frac{a}{x} - b\right) - c, \quad x \gtrsim x_s$

$$aN = -\omega^{(1)}(d_c - a)$$

Higher-order corrections

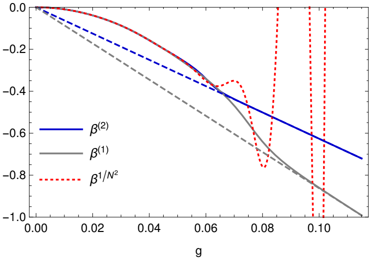
- When the $\mathcal{O}(1/N^2)$ term, $\omega^{(2)}$, is included, there are two possibilites:
    1. the closest singularity at $x = x_s^{(2)}$ is positive,
        - The β-function clearly grows faster than before close to $x_s^{(2)}$, so that no zero appears if not there with $\omega^{(1)}$
    2. the closest singularity at $x = x_s^{(2)}$ is negative.
        - Use the same procedure with ω truncated at $\mathcal{O}(1/N^2)$

$$\partial_x \mathcal{F}^{(2)}(x,N) == \frac{1}{x^2}\left[\omega^{(1)}\left(d_c - x\left(b + \frac{c + \mathcal{F}^{(2)}(x,N)}{N}\right)\right) + \frac{1}{N}\omega^{(2)}\left(d_c - x\left(b + \frac{c + \mathcal{F}^{(2)}(x,N)}{N}\right)\right)\right]$$

- Same reasoning applies to any fixed-order ω
    - For qualitative picture, the exact form of ω is not necessary

Gross–Neveu model in $d = 2$

- The GN β-function does not have singularities, but the same procedure applies for the wild oscillations
- Also $1/N^2$ coefficient of the critical exponent, λ, is known $\Rightarrow$ can compare the two truncations
  - The solid lines: Numerical solutions to the DE for $N = 100$
  - The dotted red line is the $\mathcal{O}(1/N^2)$ β-function.



Conclusions

- We computed the full set of gauge-Yukawa β-functions at $\mathcal{O}(1/N)$
  - Complementary information wrt critical exponents
  - First time resummation with three-loop basic topology
- A self-consistency equation takes into account the full available knowledge of the fixed-order critical exponents
  - We applied this method to QE(C)D and GN model
  - The singularity is removed and the wild oscillations tamed
  - In GN also the $\mathcal{O}(1/N^2)$ coefficient is known and taking that into account does not change the qualitative picture
- Near the singularity all the higher-order contributions are relevant and change the picture completely
  - Should not trust computations: expansion breaks down
  - No hint for a fixed point within the framework