Hadron structure in Drell-Yan – Theory Overview

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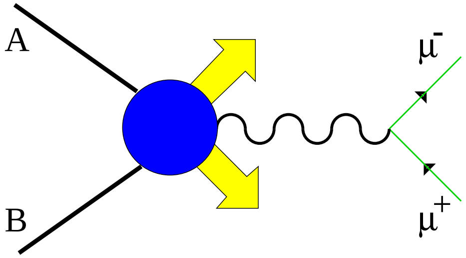
Outline

- Theoretical description of the DY cross section (σ, $d\sigma(Q_T)$ and $d\sigma(\boldsymbol{q}_T)$)
- Matching low and high transverse momentum descriptions
- Polarization dependence (mostly transverse spin)
- Partonic transverse momentum dependence
- Spin-orbit correlations
- Q dependence of asymmetries
- Process dependence

Drell-Yan process: $H_A + H_B \to \ell + \bar{\ell} + X$



In general, the virtual photon has a transverse momentum $\boldsymbol{q}_T$ w.r.t. P_A, P_B

Consider three cases (with each a different factorization):

- $\boldsymbol{q}_T$ integrated cross section

$$\frac{d\sigma}{dx_A dx_B} \sim \frac{d\sigma}{dQ^2 dy}$$

- $Q_T \equiv |\boldsymbol{q}_T|$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy dQ_T^2}$$

- $\boldsymbol{q}_T$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4 q d\Omega}$$

Collinear factorization

Leading twist factorization theorem in Drell-Yan:

$$\frac{d\sigma}{dQ^2dy} = \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;\mu) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;\mu)\, H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right)$$

$$x_A = e^y \sqrt{\frac{Q^2}{s}}, \quad x_B = e^{-y}\sqrt{\frac{Q^2}{s}}, \quad y = \frac{1}{2}\ln\frac{q\cdot P_A}{q\cdot P_B}$$

Q^2 is large, one deals with collinear factorization

A similar collinear factorization applies when Q_T is observed and large ($Q_T \sim Q$):

$$\frac{d\sigma}{dQ^2dy} \longrightarrow \frac{d\sigma}{dQ^2dy\,dQ_T^2}$$

$$H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right) \longrightarrow T_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, Q_T; \mu, \alpha_s(\mu)\right)$$

T_{ab} is singular as $Q_T \to 0$, one needs to resum large logarithms $(\log Q/Q_T)$

Collinear factorization plus resummation

$\Lambda^2 \ll Q_T^2 \ll Q^2$: Collins-Soper-Sterman (CSS) formalism

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(b, Q; x_A, x_B) + Y(Q_T, Q; x_A, x_B) \qquad b = |\boldsymbol{b}|$$

$$\begin{aligned} \tilde{W}(b, Q; x_A, x_B) &= \sum_j e_j^2 \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A; 1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B; 1/b) \\ &\quad \times e^{-S(b,Q)} C_{ja}\left(\frac{x_A}{\xi_A}; \alpha_s(1/b)\right) C_{\bar{j}b}\left(\frac{x_B}{\xi_B}; \alpha_s(1/b)\right) \end{aligned}$$

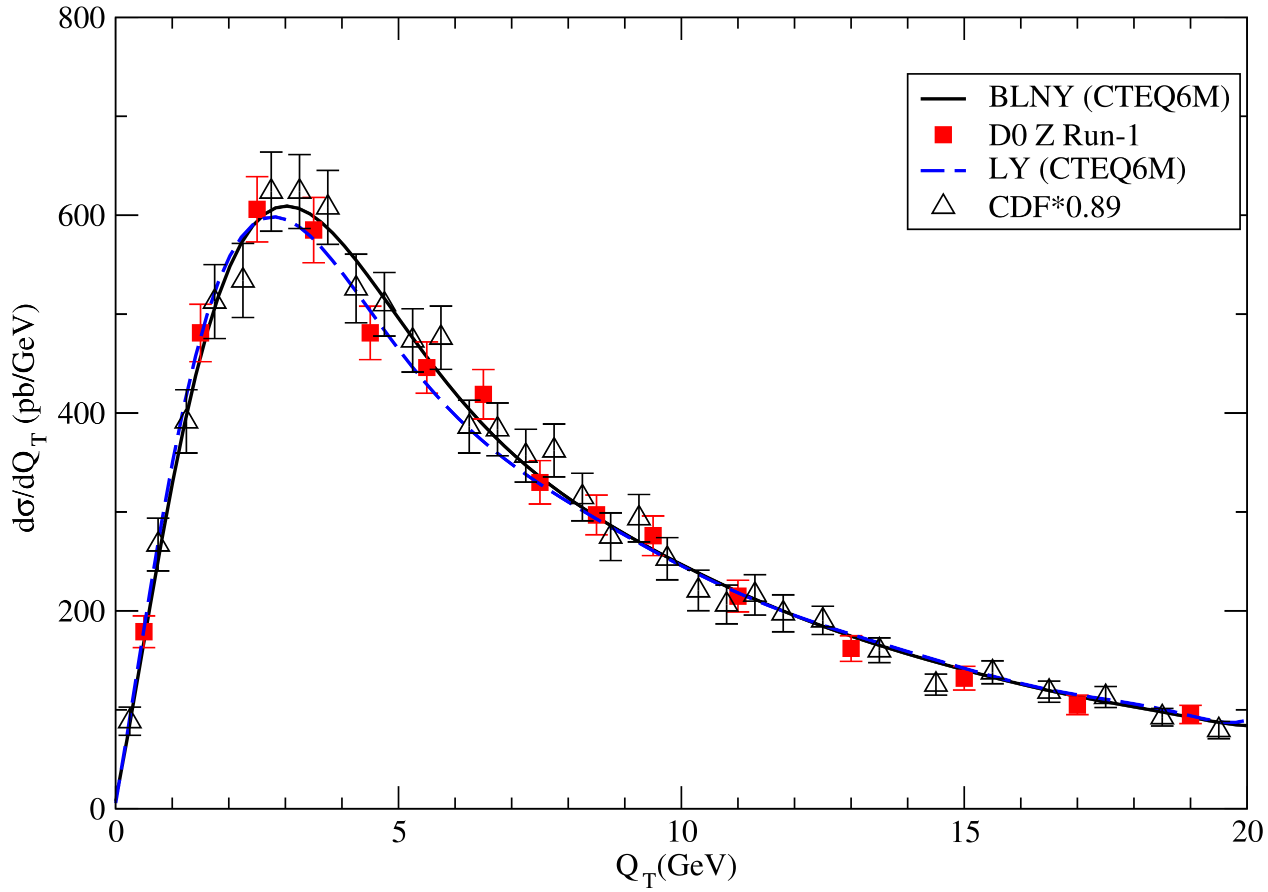
Collins, Soper & Sterman, NPB 250 (1985) 199

$Y(x_1, x_2, Q, Q_T)$ becomes important only when $Q_T \sim Q$

Introduced to match to fixed order pQCD calculations at large Q_T

$e^{-S(b,Q)}$ = Sudakov form factor, resums the large log's

Application of CSS formalism



At small Q_T one needs to include a nonperturbative Sudakov factor

$$e^{-S_{\text{pert}}(b,Q)-S_{NP}(b,Q)}$$

The Q independent part of S_{NP} can be viewed as the average intrinsic transverse momentum

Transverse momentum distribution of Z bosons at the Tevatron run-1 fitted using the CSS resummation formalism (includes low energy DY data in global fit)

Landry, Brock, Nadolsky, Yuan, PRD 67 (2003) 073016

Polarized scattering

Polarized DY will allow to probe the distributions of longitudinally and transversely polarized quarks inside polarized hadrons: g_1 and h_1

The helicity distributions: $g_1^q(x)$ and $g_1^{\bar{q}}(x)$

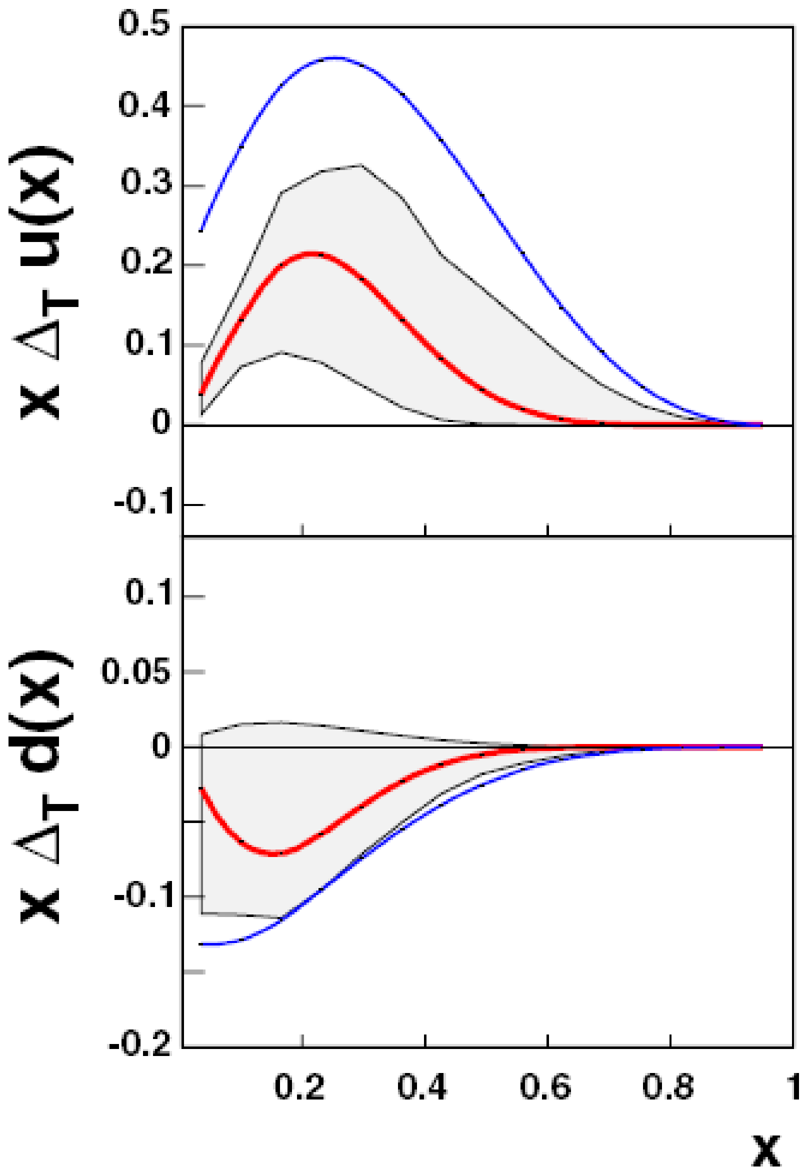
Already fairly well known

The transversity distributions: $h_1^q(x)$ and $h_1^{\bar{q}}(x)$

It is known that $h_1^q(x)$ is nonzero and a first extraction with rather large experimental and theoretical uncertainties has been obtained using SIDIS and e^+e^- data

Anselmino $et\ al.$, PRD 75 (2007) 054032

First transversity extraction



First extraction of transversity

Plot: $xh_1^{u,d}(x)$ at $Q^2 = 2.4$ GeV^2

Anselmino *et al.*, PRD 75 (2007) 054032

Best fit means $h_1(x) \approx f_1(x)/3$ and is about half its maximally allowed value

Transversity in DY

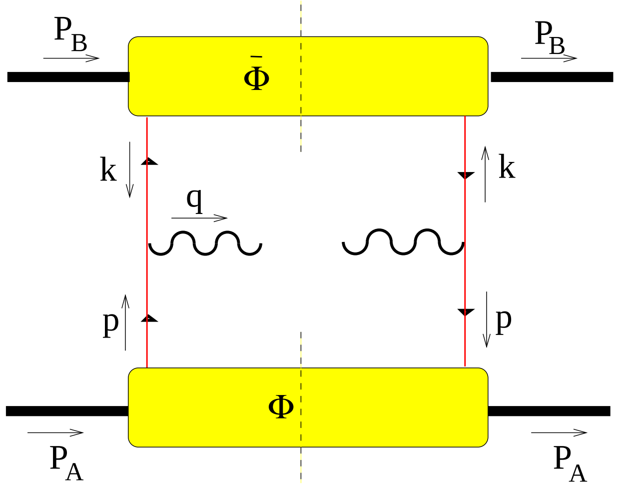
Transversity distribution first discussed more than 30 years ago

Ralston & Soper, NPB 152 (1979) 109

First suggestion was to measure it through the Drell-Yan process

The Drell-Yan Process

$H_A + H_B \to \ell + \bar{\ell} + X$



$$A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow \to \ell\,\ell'\,X) - \sigma(p^\uparrow p^\downarrow \to \ell\,\ell'\,X)}{\sigma(p^\uparrow p^\uparrow \to \ell\,\ell'\,X) + \sigma(p^\uparrow p^\downarrow \to \ell\,\ell'\,X)} = \frac{\sin^2\theta\cos 2\phi_S^\ell}{1+\cos^2\theta} \frac{\sum_q e_q^2\, h_1^q(x_1)\, \overline{h}_1^q(x_2)}{\sum_q e_q^2\, f_1^q\, \overline{f}_1^q}$$

Artru, Mekhfi, ZPC 45 ('90) 669; Jaffe, Ji, NPB 375 ('92) 527; Cortes, Pire, Ralston, ZPC 55 ('92) 409

However, polarized Drell-Yan is very demanding, still not done...

A_{TT} at RHIC

RHIC is at present the only place that can do double polarized hadron scattering

$$A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow \to \ell\,\bar{\ell}\,X) - \sigma(p^\uparrow p^\downarrow \to \ell\,\bar{\ell}\,X)}{\sigma(p^\uparrow p^\uparrow \to \ell\,\bar{\ell}\,X) + \sigma(p^\uparrow p^\downarrow \to \ell\,\bar{\ell}\,X)} \propto \sum_q e_q^2\, h_1^q(x_1)\, h_1^{\bar{q}}(x_2)$$

This involves two unrelated functions, for which likely holds:

$$h_1^{\bar{q}} \ll h_1^q$$

An upper bound can be obtained by using Soffer's inequality,

$$|h_1(x)| \leq \frac{1}{2}\left[f_1(x) + g_1(x)\right]$$

The upper bound on A_{TT} was shown to be small at RHIC (percent level)

Martin, Schäfer, Stratmann & Vogelsang, PRD 60 (1999) 117502

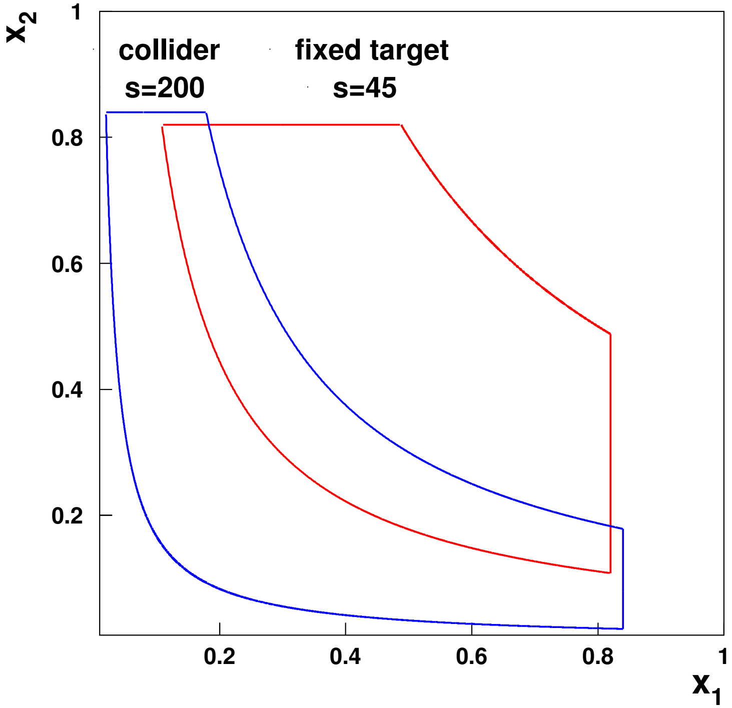
A_{TT} in $\bar{p}^\uparrow p^\uparrow$ Drell-Yan

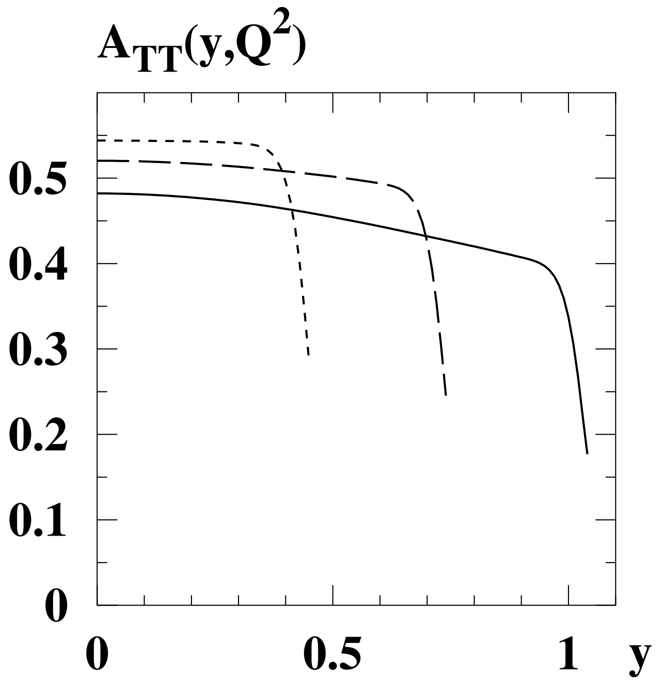
$\bar{p}^\uparrow p^\uparrow$ Drell-Yan is ideally suited for h_1 extraction, because $h_1^{\bar{q}/\bar{p}} = h_1^{q/p}$

$$A_{TT} = \frac{\sigma(\bar{p}^\uparrow p^\uparrow \to \ell\,\bar{\ell}\,X) - \sigma(\bar{p}^\uparrow p^\downarrow \to \ell\,\bar{\ell}\,X)}{\sigma(\bar{p}^\uparrow p^\uparrow \to \ell\,\bar{\ell}\,X) + \sigma(\bar{p}^\uparrow p^\downarrow \to \ell\,\bar{\ell}\,X)} \propto \sum_q e_q^2\, h_1^q(x_1)\, h_1^q(x_2)$$

Perhaps at GSI-FAIR

F. Rathmann, hep-ex/0412078





solid line: $Q^2 = 5$ GeV^2

dashed line: $Q^2 = 9$ GeV^2

dotted line: $Q^2 = 16$ GeV^2

$s = 45$ GeV^2 (fixed target mode)

Chiral quark soliton model for h_1

Efremov, Goeke & Schweitzer, EPJC 35 (2004) 207

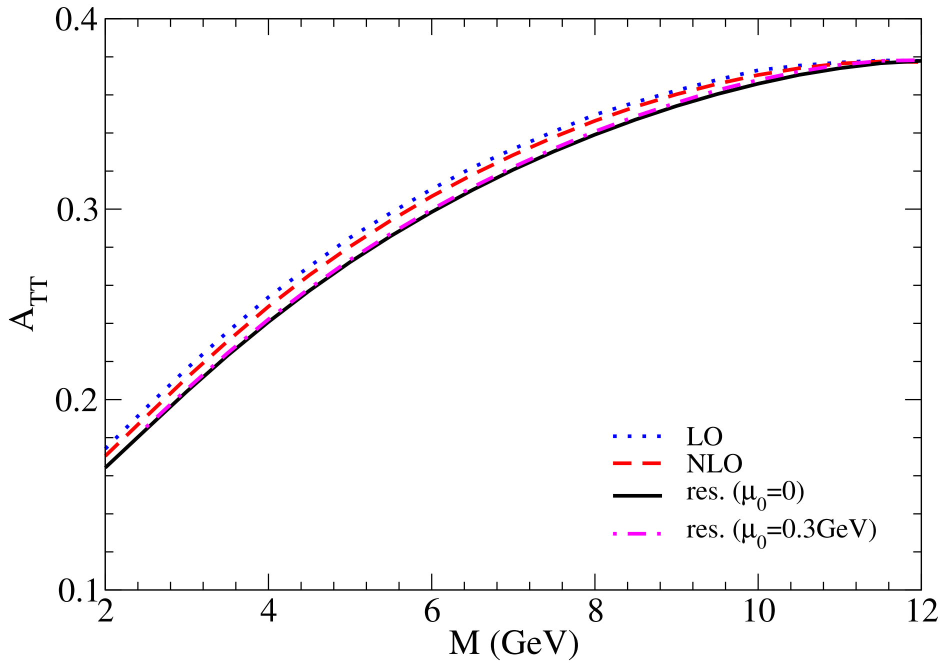
$M = Q$

$\sqrt{s} = 14.5$ GeV (collider mode)

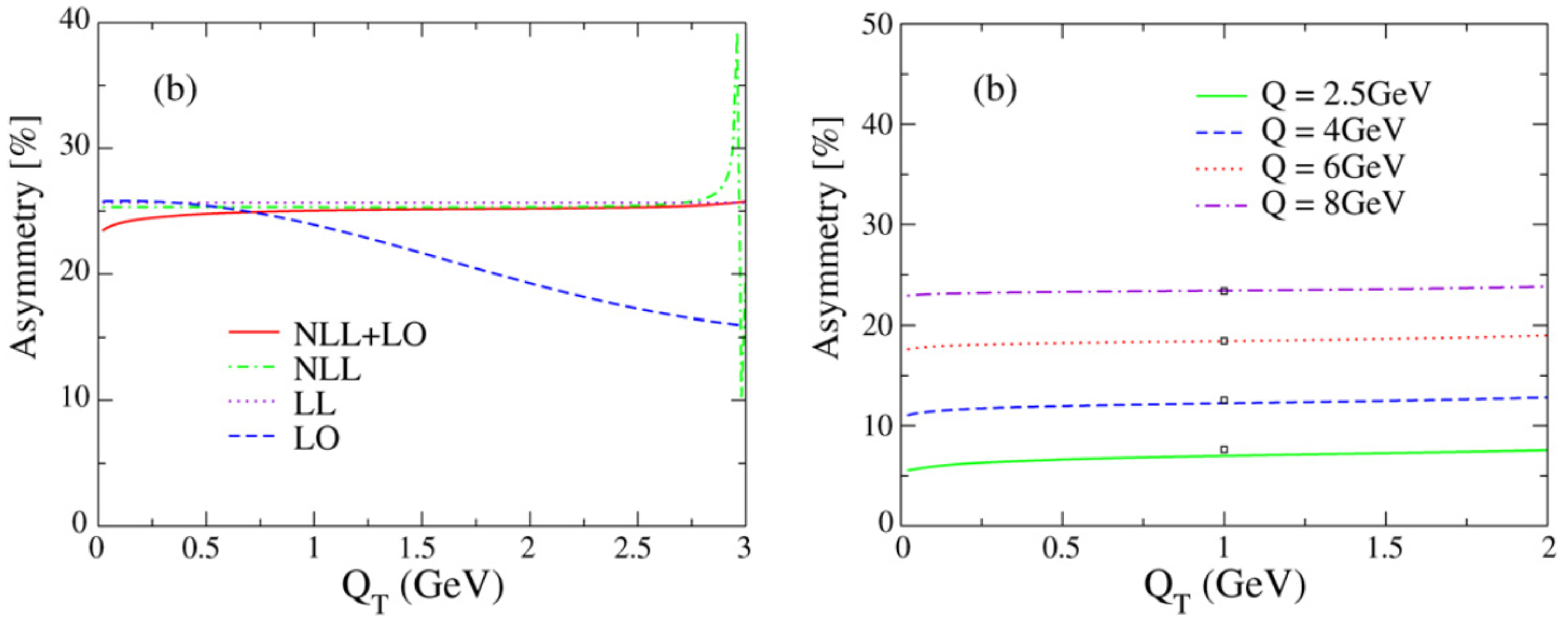
Upper bound for LO and NLO pQCD

Shimizu *et al.*, PRD 71 (2005) 114007

First extraction implies bound$/4$



Unintegrated $A_{TT}(Q_T)$ for $\bar{p}^{\uparrow}\, p^{\uparrow} \to \ell\, \bar{\ell}\, X$



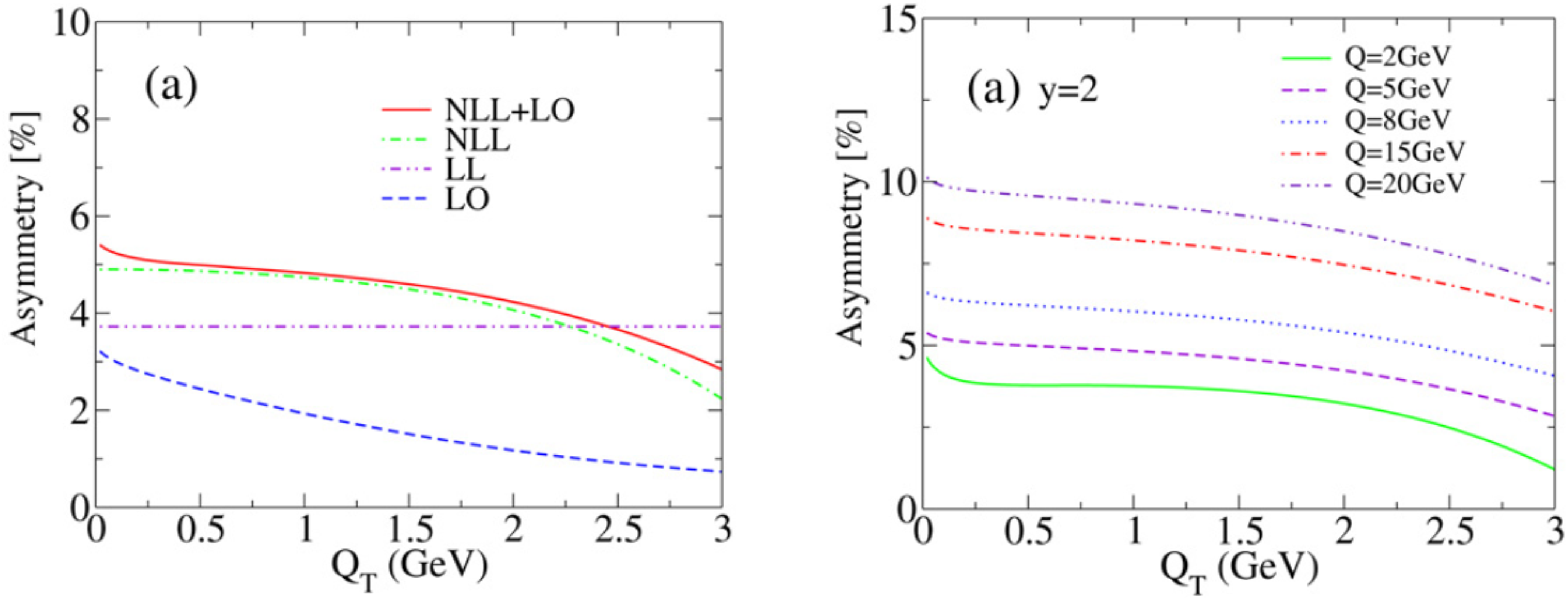
Upper bound for $\phi = \phi_S^{\ell} = 0$, $\sqrt{s} = 14.5$ GeV, $Q = 4$ GeV, $y = 0$

Kawamura, Kodaira & Tanaka, PLB 662 (2008) 139

Asymmetry is very flat and resummation beyond LL has small effect!

Applies specifically to $\bar{p}\, p$ in the valence region

$A_{TT}(Q_T)$ at RHIC

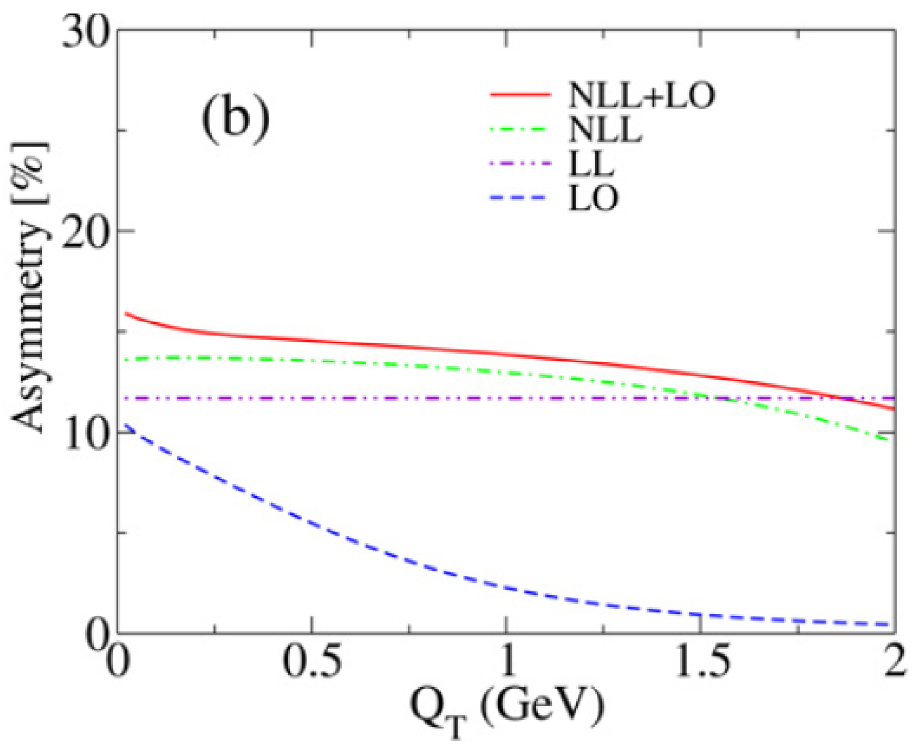


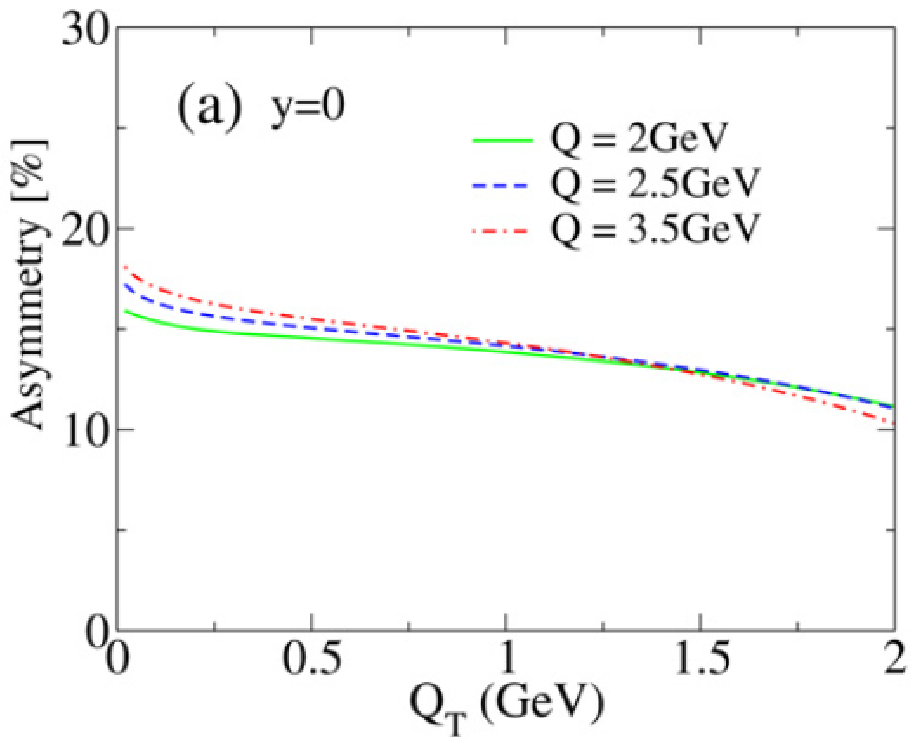
$\sqrt{s} = 200$ GeV, $Q = 5$ GeV, $y = 2$, $\phi = 0$

Kawamura, Kodaira & Tanaka, NPB 777 (2007) 203

Asymmetry not flat and resummation beyond LL matters

$A_{TT}(Q_T)$ at J-PARC





$\sqrt{s} = 10$ GeV, $Q = 2$ GeV, $y = 0$, $\phi = 0$

Kawamura, Kodaira & Tanaka, NPB 777 (2007) 203

Larger asymmetries at J-PARC than at RHIC

Conclusion: A_{TT} in $\bar{p}\,p$ in valence region robust under perturbative corrections

Single spin asymmetries

Asymmetry in $p\,p^{\uparrow} \rightarrow \ell\,\bar{\ell}\,X$ integrated over Q_T is not related to transversity

The Qiu-Sterman effect can contribute:

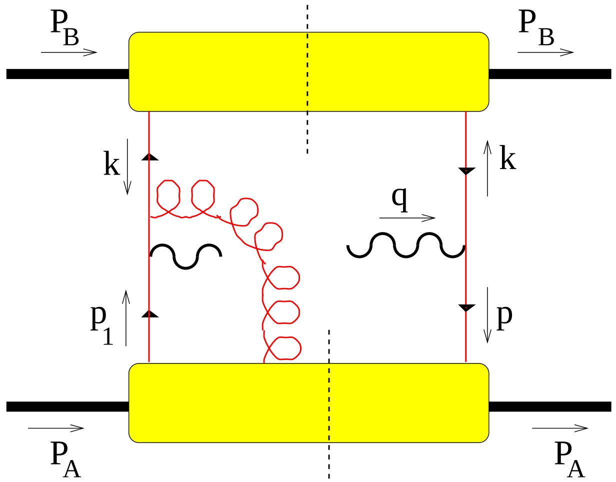
$$T(x, S_T) \overset{A^+=0}{\propto} \text{F.T.}\ \langle P|\ \overline{\psi}(0) \int d\eta^-\ F^{+\alpha}(\eta^-)\ \gamma^+\, \psi(\xi^-)\ |P\rangle$$

$$\overset{?}{\approx} \quad \text{constant} \times f_1(x)$$

Qiu & Sterman, PRL 67 (1991) 2264

The quark-gluon correlation function $T(x, S_T)$ is a collinear twist-3 function

The resulting SSA is $1/Q$ suppressed



Single spin asymmetries

Asymmetry in $p\,p^\uparrow \to \ell\,\bar{\ell}\,X$ integrated over Q_T

$$A_N = -\sin\phi_S^\ell\,\frac{g}{Q}\left[\frac{\sin 2\theta}{1+\cos^2\theta}\right]\frac{\sum_a e_a^2\int dx\,T^a(x,S_T)\,f_1^{\bar{a}}(Q^2/xs)}{\sum_a e_a^2\int dx\,f_1^a(x)\,f_1^{\bar{a}}(Q^2/xs)}$$

Hammon, Teryaev & Schäfer, PLB 390 (1997) 409

D.B., Mulders & Teryaev, PRD 57 (1998) 3057

D.B. & Qiu, PRD 65 (2002) 034008

Anikin & Teryaev, arXiv:1003.1482

Asymmetry expression equally applies to $\bar{p}\,p^\uparrow$ and $\pi\,p^\uparrow$ DY of course

Estimate of QS SSA in DY

Qiu-Sterman's Ansatz

$$T^a(x, S_T) \approx \kappa_a \, \lambda \, f_1^a(x), \qquad \kappa_u = 1 = -\kappa_d, \; \kappa_s = 0$$

From E704 $p\,p^\uparrow \to \pi\,X$ data which shows large SSA: $\lambda \sim 100$ MeV

$$|A_N| \sim 0.7 \, \frac{\lambda}{Q}$$

This leads to small SSA in DY; just below and above the J/ψ:

$$|A_N| \quad \sim \quad 3.5\% \qquad \text{at } Q = 2 \text{ GeV}$$

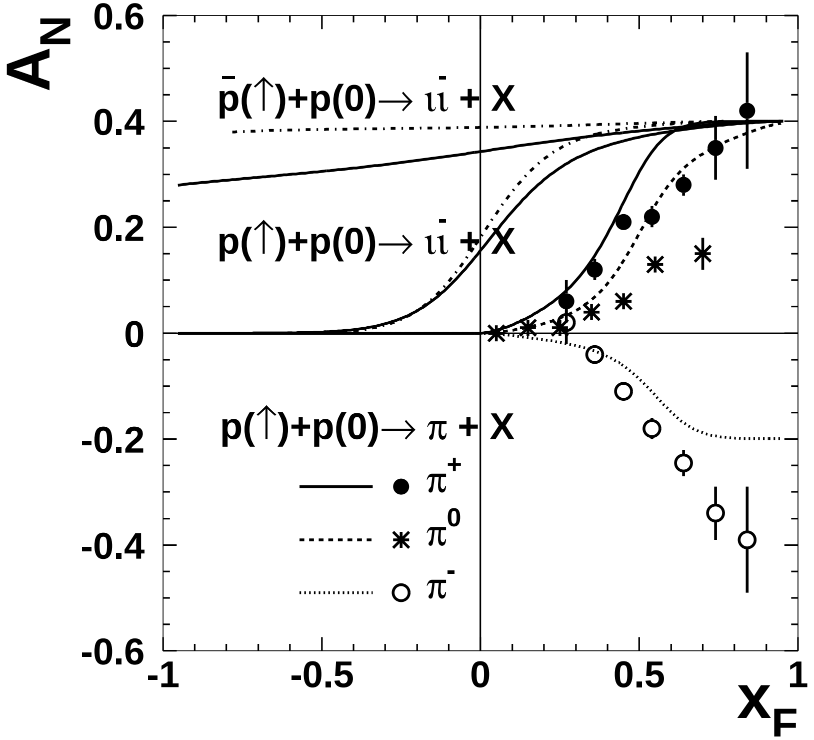
$$|A_N| \quad \sim \quad 1.75\% \qquad \text{at } Q = 4 \text{ GeV}$$

D.B. & Qiu, PRD 65 (2002) 034008

Approximately no x_F dependence

Other predictions for A_N in DY

This differs much from another SSA prediction:



A semi-classical model prediction

Boros, Liang, Meng, PRD 51 (1995) 4867

E704 π-production data

For DY, solid line is $Q = 4$ GeV

and dash-dotted is $Q = 9$ GeV

Both curves at $\sqrt{s} = 20$ GeV

Intermediate summary of hadron structure from DY

σ	$f_1(x)$
$d\sigma(Q_T)$	$f_1(x)$, S_{NP} or $\langle k_T^2 \rangle$
A_{LL}	$g_1(x)$
A_{TT}	$h_1(x)$
$A_{TT}(Q_T)$	$h_1(x)$, S_{NP}
A_N	$T(x, S_T)$

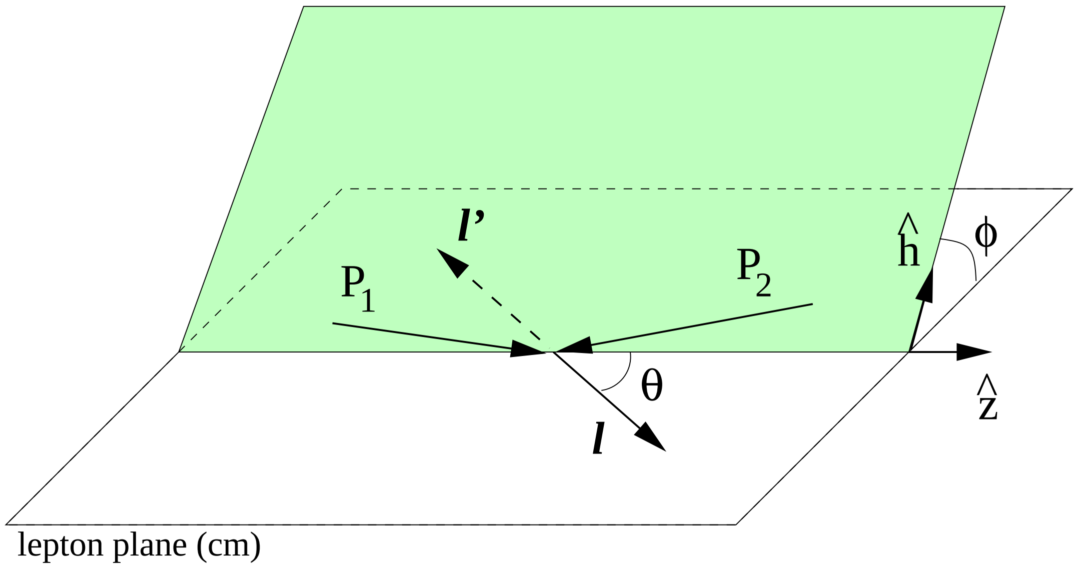
The Q_T-dependent SSA will be discussed after $d\sigma(\boldsymbol{q}_T)$

Angular dependences

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} \quad \longrightarrow \quad \frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4 q d\Omega}$$

$d\Omega = d\cos\theta d\phi^l$, where θ and ϕ^l are the angles of one of the leptons in the lepton-pair center of mass

$d^2\boldsymbol{q}_T = d\phi^h dQ_T^2/2$ and $\phi = \phi^h - \phi^l$



Angular asymmetries

For unpolarized scattering one has the general angular dependence

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\,\frac{1}{\lambda+3}\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Fixed order perturbative calculation at $\mathcal{O}(\alpha_s)$ as function of $\rho \equiv Q_T/Q$

Collins, PRL 42 (1979) 291

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\,\frac{1+\frac{3}{2}\rho^2}{1+\rho^2}\left[1 + \frac{1-\frac{1}{2}\rho^2}{1+\frac{3}{2}\rho^2}\cos^2\theta + \frac{\rho}{(1+\frac{3}{2}\rho^2)} f\left(\frac{\xi_A}{x_A}, \frac{\xi_B}{x_B}\right)\sin 2\theta\cos\phi + \frac{1}{2}\,\frac{\rho^2}{1+\frac{3}{2}\rho^2}\sin^2\theta\cos 2\phi\right]$$

This satisfies the Lam-Tung relation $1 - \lambda - 2\nu = 0$

Beyond fixed order perturbation theory

For small Q_T one finds from fixed order (LO) perturbation theory:

$$\lambda \to 1, \quad \mu \to 0, \quad \nu \to 0$$

Not a singular limit

But for small Q_T collinear and even CSS factorization is not the right starting point

D.B. & Vogelsang, PRD 74 (2006) 014004

Berger, Qiu & Rodriguez-Pedraza, PLB 656 (2007) 74 & PRD 76 (2007) 074006

Zhou, Yuan, Liang, PLB 678 (2009) 264

The CSS formalism applies to $d\sigma/dQ^2dydQ_T^2$, but it stems from a more general factorization theorem that applies to $d\sigma/dQ^2dyd^2\boldsymbol{q}_T d\Omega$

Collins & Soper, NPB 193 (1981) 381

Ji, Ma & Yuan, PRD 71 (2005) 034005 & PLB 597 (2004) 299

Collins-Soper or TMD factorization

Collins-Soper (CS) or TMD factorization in DY is schematically given by:

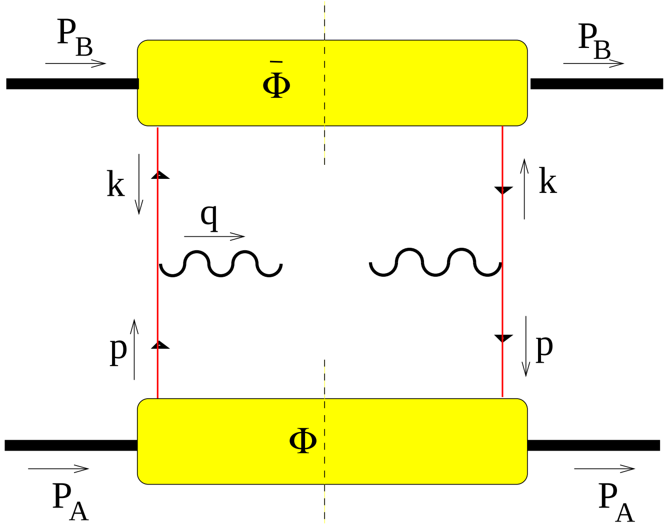
$$\Phi \otimes \bar{\Phi} \otimes H \otimes e^{-S} \otimes U$$

Collins & Soper '81; Ji, Ma & Yuan '04 & '05

U is called the soft factor

A correlator of Wilson lines

At tree level: $U = 1 \Rightarrow$



Another difference to CSS factorization is:

CS or TMD factorization includes partonic transverse momentum $\Phi(x, \boldsymbol{k}_T)$

Transverse Momentum of Quarks

TMD factorization: include partonic transverse momentum $\Phi(x) \to \Phi(x, \boldsymbol{k}_T)$

TMD = transverse momentum dependent parton distribution function

This is more than just an extension of $f_1^q(x) \to f_1^q(x, \boldsymbol{k}_T^2)$

$\boldsymbol{k}_T$-odd functions may arise, that vanish upon integration over all $\boldsymbol{k}_T$

And also new spin-dependent terms may arise

Ralston & Soper '79; Sivers '90; Kotzinian '95; Mulders & Tangerman '95; D.B. & Mulders '98

For unpolarized hadrons:

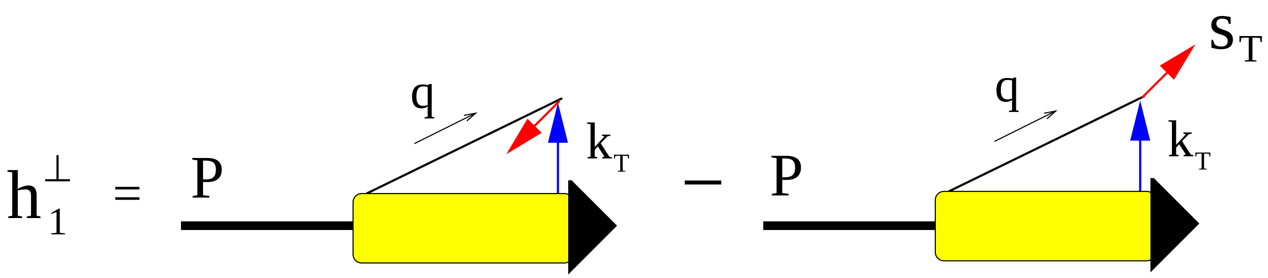
$$\Phi(x) = \frac{1}{2} f_1(x) \not{P},$$

but

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + h_1^\perp(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

Transverse quark polarization

Transversely polarized quarks inside an *unpolarized* hadron

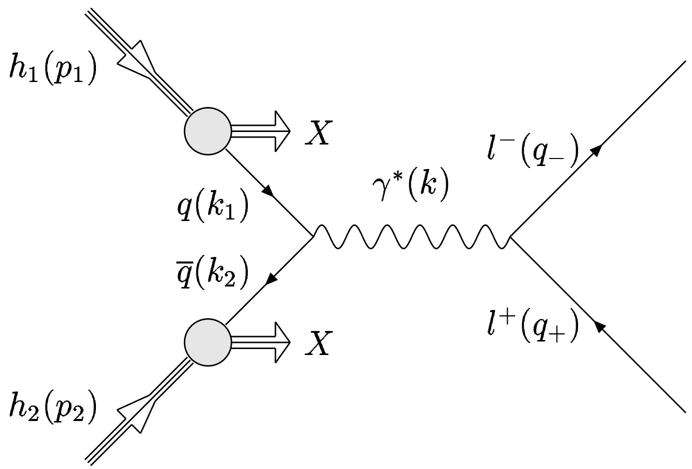


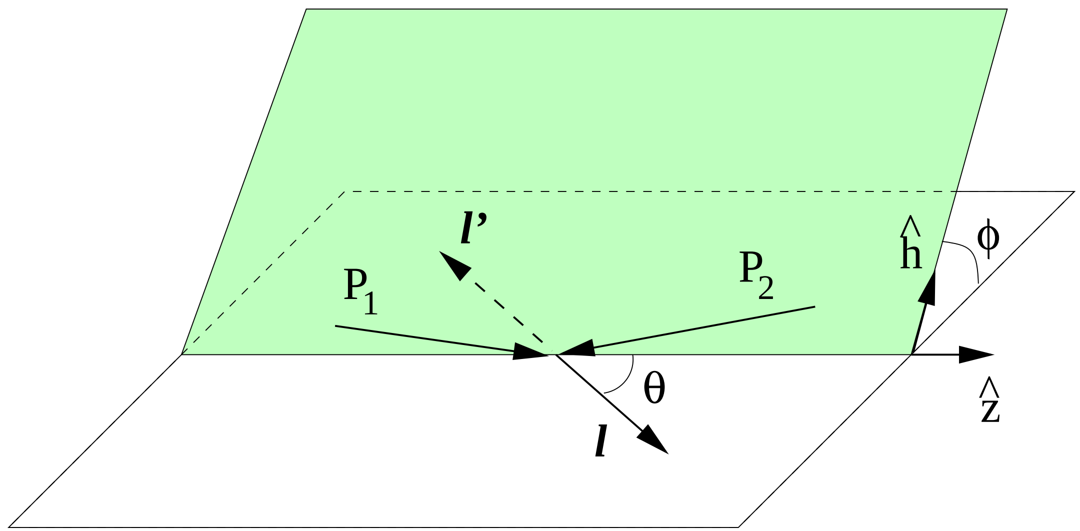
Allowed by the symmetries as long as $\boldsymbol{k}_T \neq 0$

It generates azimuthal asymmetries in unpolarized collisions, e.g. in DY

These have been measured in $\pi^- N$, $p\,p$, and $p\,d$ DY

Azimuthal asymmetries according to pQCD



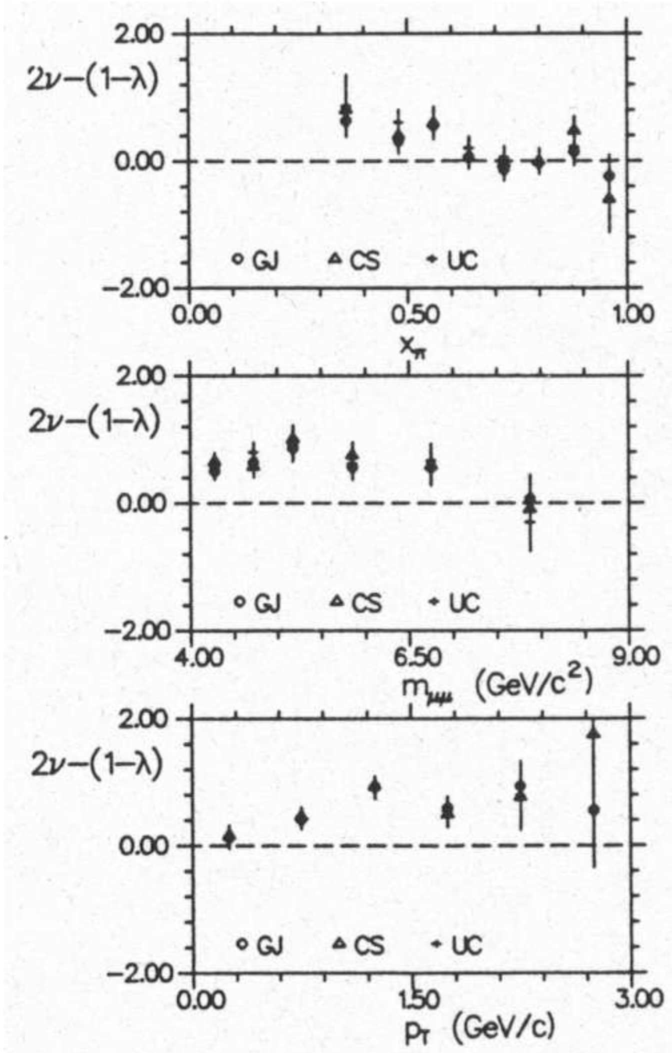


$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

Collinear factorization: Mirkes & Ohnemus '95

Parton Model	$\mathcal{O}(\alpha_s^0)$	$\lambda = 1,\ \mu = \nu = 0$	
LO pQCD	$\mathcal{O}(\alpha_s^1)$	$1 - \lambda - 2\nu = 0$	Lam-Tung relation
NLO	$\mathcal{O}(\alpha_s^2)$	$1 - \lambda - 2\nu \neq 0$	small and positive

Azimuthal asymmetries in Drell-Yan in experiment



Data: $1 - \lambda - 2\nu \neq 0$ large and negative!

NA10 Collab. ('86/'88) & E615 Collab. ('89)

Data for $\pi^- N \to \mu^+ \mu^- X$, with $N = D, W$

$\sqrt{s} \approx 20 \pm 3$ GeV

lepton pair invariant mass $Q \sim 4 - 12$ GeV

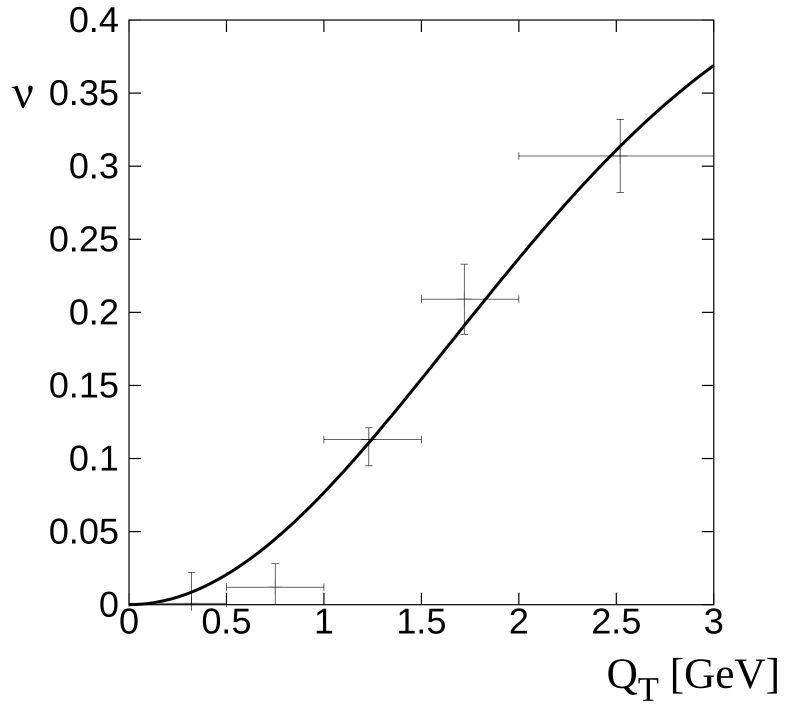
Nonzero $h_1^\perp$ offers an explanation of these anomalous Drell-Yan data

D.B., PRD 60 (1999) 014012

Explanation in terms of $h_1^\perp$

$$(1 - \lambda - 2\nu) \propto h_1^\perp(\pi)\, h_1^\perp(N)$$

Fit $h_1^\perp$ to data by assuming Gaussian TM dependence



Many model calculations of $h_1^\perp$ and its asymmetries have been performed

Goldstein & Gamberg '02, '07; D.B., Brodsky & Hwang '03

Lu & Ma '04, '05; Barone, Lu & Ma '07; Zhang, Lu, Ma & Schmidt '08

Courtoy, Scopetta & Vento '09; Lu & Schmidt '09

Allows to predict other observables, such as DY for $p\,p, \bar{p}\,p$, $p\,p^\uparrow$, $\pi\,p^\uparrow$, etc

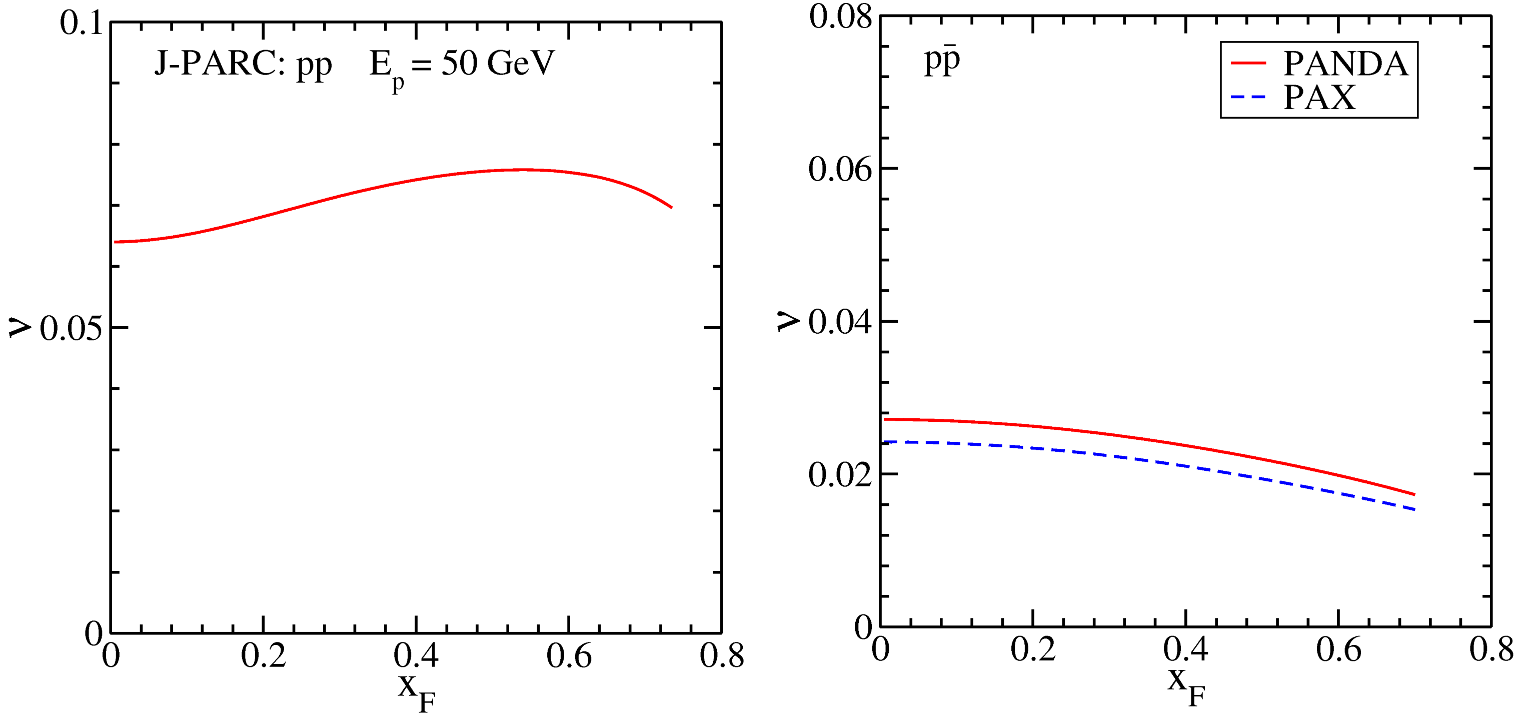
Hadron type dependence

Asymmetry for $p\,p$ and $p\,d$ expected to be smaller, as confirmed by recent Fermilab data

FNAL-E866/NuSea Collaboration, L.Y. Zhu *et al.* '07 & '09

Asymmetry for $\bar{p}\,p$ expected to be very similar to $\pi\,p$ (both have valence antiquarks)

Although this depends on the kinematics too of course:

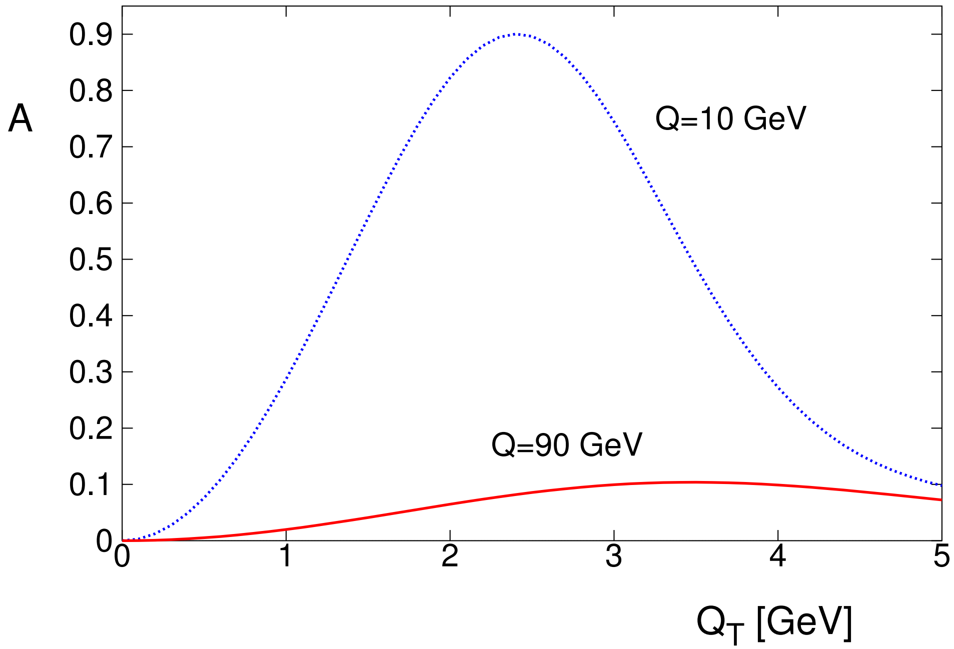


Lu & Schmidt '09

$\cos 2\phi$ asymmetry from $h_1^\perp$ beyond tree level

Assuming Gaussian k_T dependence for $h_1^\perp$, the $\cos(2\phi)$ asymmetry is proportional to

$$\mathcal{A}(Q, Q_T, Q_0) = M \frac{\int db\, b^3\, J_2(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}$$



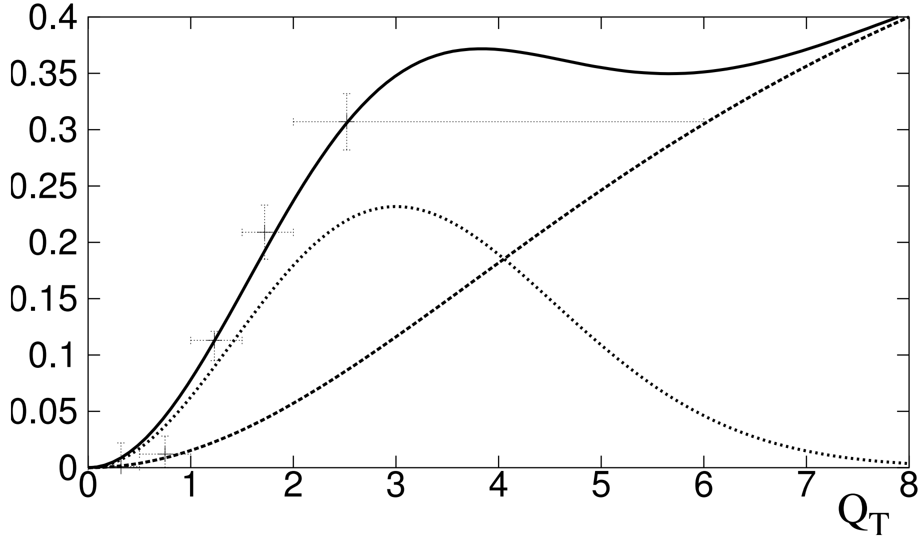
D.B., NPB 603 (2001) 195 & 806 (2009) 23

Considerable Sudakov suppression with increasing Q: $\sim 1/Q$ (effectively twist-3)

$\cos 2\phi$ asymmetry as function of Q_T

The high-p_T tail of $h_1^\perp$ is related to a chiral-odd QS effect (M^2/Q_T^2 suppressed)

The $\cos(2\phi)$ asymmetry ν at high Q_T is dominated by the perturbative contribution



These contributions can be added:

$$\nu = \nu_{h_1^\perp} + \nu_{\text{pert}} + \mathcal{O}\left(\frac{Q_T^2}{Q^2} \text{ or } \frac{M^2}{Q_T^2}\right)$$

Bacchetta, D.B., Diehl, Mulders,

JHEP 0808 (2008) 023

Q dependence at small Q_T approximately $1/Q$ and at high Q_T $1/Q^2$

The polarized Drell-Yan process

In the case of one transversely polarized hadron beam:

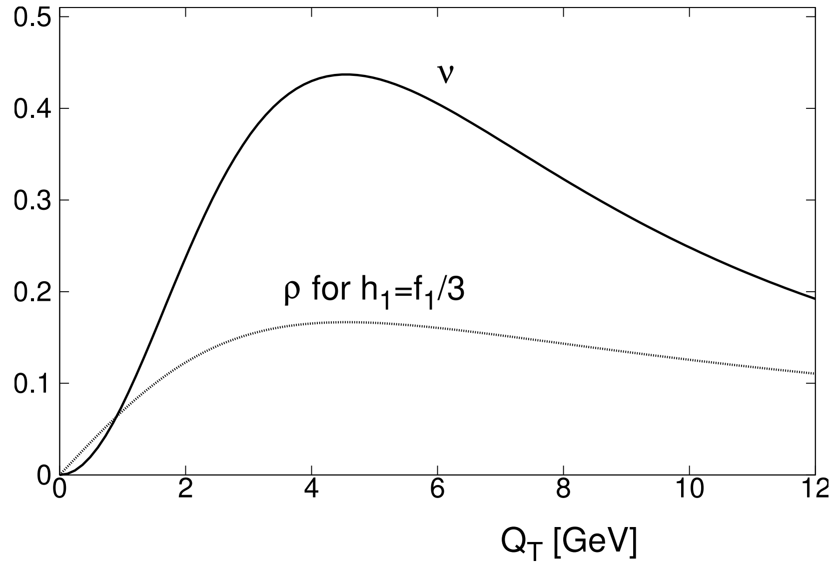
$$\frac{d\sigma}{d\Omega\, d\phi_S} \propto 1 + \cos^2\theta + \sin^2\theta \left[\frac{\nu}{2}\, \cos 2\phi - \rho\, |\boldsymbol{S}_T|\, \sin(\phi + \phi_S)\right] + \ldots$$

Assuming u-quark dominance and Gaussian k_T-dependence for $h_1^\perp$:

$$\nu \quad\propto\quad h_1^\perp\, h_1^\perp$$

$$\rho \quad\propto\quad h_1\, h_1^\perp$$

$$\rho = \frac{1}{2}\sqrt{\frac{\nu}{\nu_{\text{max}}}}\,\frac{h_1^u}{f_1^u}$$



First extraction of h_1 indicates $h_1 \approx f_1/3$, which leads to ρ of $\mathcal{O}(10 - 15\%)$

DY at Compass

Measurement of ν and ρ with only one polarized beam offers a probe of *transversity*

The COMPASS experiment plans to extract them using $\pi^\pm p^\uparrow$ Drell-Yan
Would provide valuable information on the flavor dependence of h_1 and $h_1^\perp$

Especially $\pi^+ p^\uparrow$ is of interest, since no data yet and it provides information on the d-quark ratio $h_1^{\perp d/p}/h_1^{d/p}$, without suppression by a charge-squared factor

Using the input on $h_1^\perp$ from for example unpolarized $p\,\bar{p}$ Drell-Yan would allow for an extraction of h_1 from $\pi^\pm p^\uparrow$ Drell-Yan at COMPASS

Azimuthal spin asymmetries

Besides the transversity asymmetry $\rho \propto h_1 h_1^\perp$, there are other asymmetries:

$$\frac{d\sigma}{d\Omega\, d\phi_S} \propto 1 + \cos^2\theta + \frac{\nu}{2}\, \cos 2\phi + A_{h_1^\perp}\, |\boldsymbol{S}_T|\, \sin(\phi + \phi_S) + A_{f_{1T}^\perp}\, |\boldsymbol{S}_T|\, \sin(\phi - \phi_S) + \ldots$$

Transversity asymmetry: $A_{h_1^\perp} \propto h_1 h_1^\perp$

Sivers asymmetry: $A_{f_{1T}^\perp} \propto f_{1T}^\perp f_1$

There is also a $\sin(3\phi - \phi_S)$ asymmetry which is $\propto h_{1T}^\perp h_1^\perp$ (pretzelosity)
A link between pretzelosity and orbital angular momentum of quarks found in models:

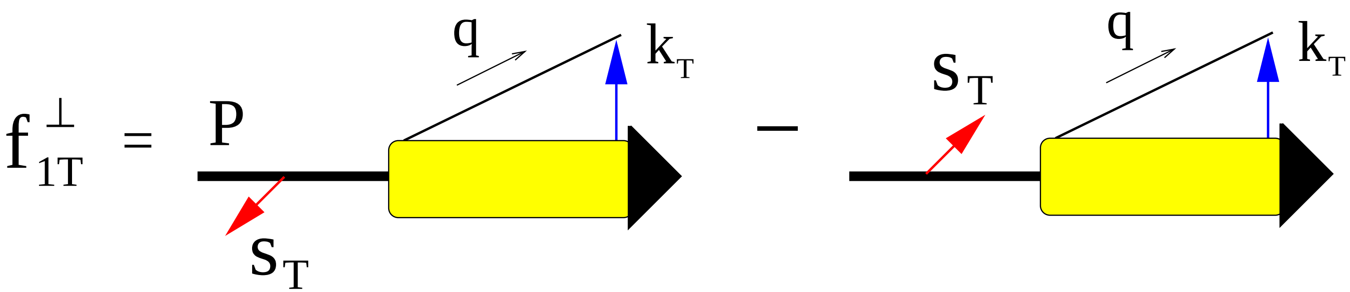
$$L_q^3 = -\int dx h_{1T}^{\perp(1)q}(x)$$

J. She, J. Zhu & B.Q. Ma, PRD 79 (2009) 054008
Avakian, Efremov, Schweitzer & Yuan, arXiv:1001.5467

Sivers effect

The Sivers effect is described by a $\boldsymbol{k}_T$ *and* $\boldsymbol{S}_T$ dependent distribution function

Sivers '90



Captures nonperturbative spin-orbit coupling effects inside a polarized proton

$$\Phi(x,\boldsymbol{k}_T) = \frac{1}{2}\, f_1(x,\boldsymbol{k}_T^2)\, \not{P} + i h_1^{\perp}(x,\boldsymbol{k}_T^2)\, \frac{\not{P}\,\not{k}_T}{M} + \frac{\boldsymbol{P}\cdot(\boldsymbol{k}_T \times \boldsymbol{S}_T)}{2M}\, f_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\, \not{P} + \ldots$$

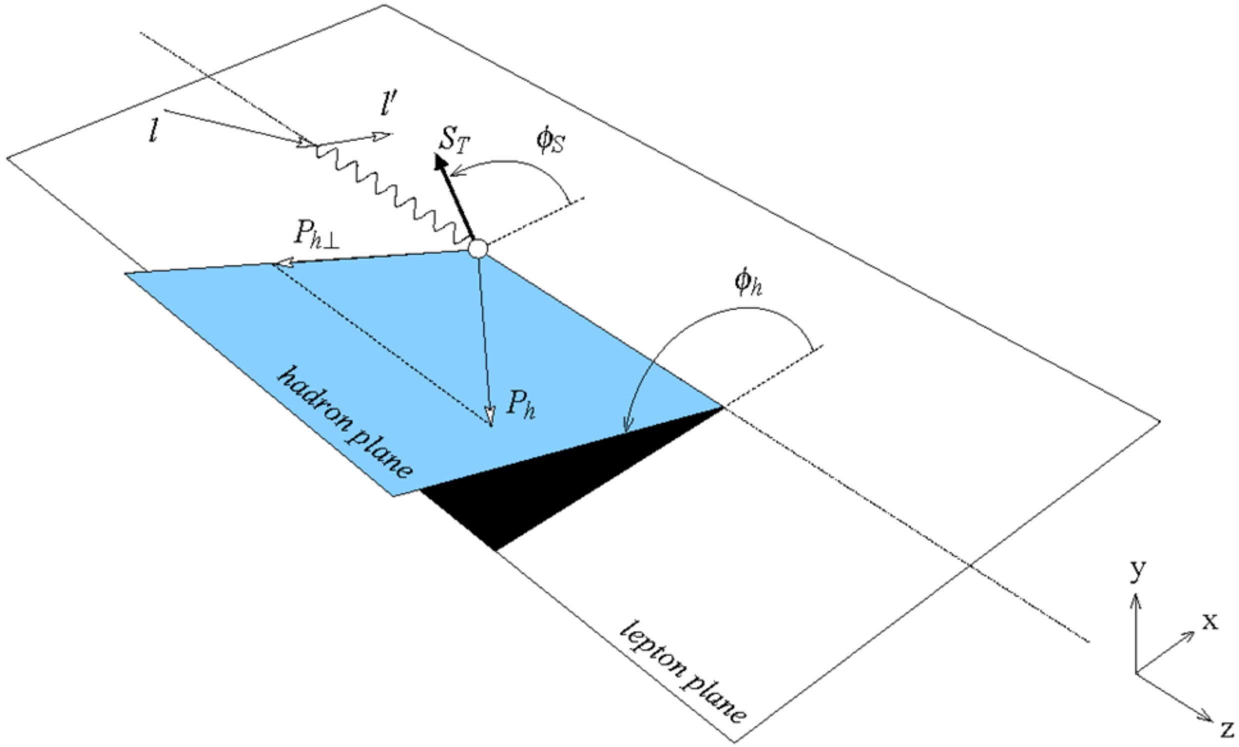
Sivers effect in semi-inclusive DIS

Sivers effect leads to an unsuppressed $\sin(\phi_h - \phi_S)$ asymmetry in $\ell\, p^{\uparrow} \to \ell'\, h\, X \propto f_{1T}^{\perp} D_1$

D.B. & Mulders '98

SIDIS

$\ell\, p \to \ell'\, h\, X$



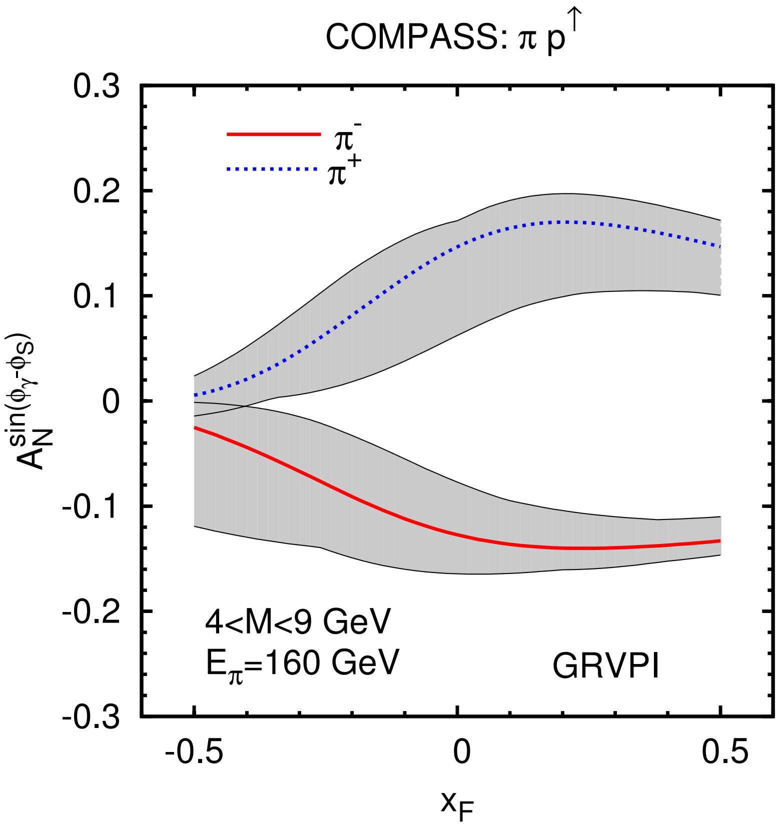
Such an asymmetry has been clearly observed by the HERMES Collaboration

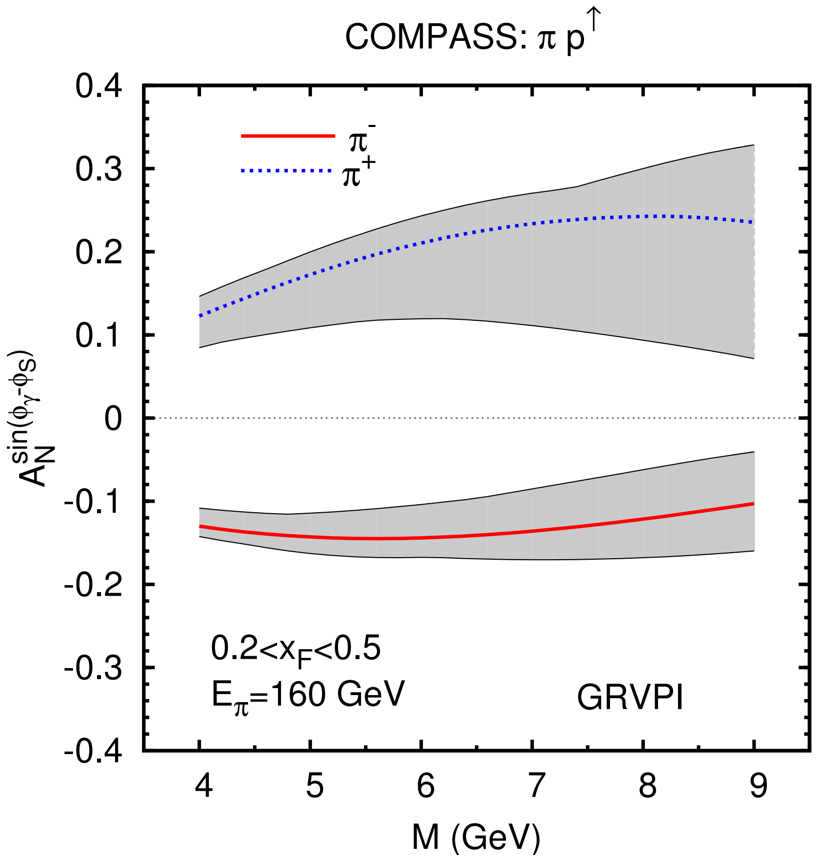
And recently also by the COMPASS Collaboration

Sivers effect in Drell-Yan

Sivers effect also leads to a $\sin(\phi - \phi_S)$ asymmetry in Drell-Yan $\propto f_{1T}^{\perp} \bar{f}_1$

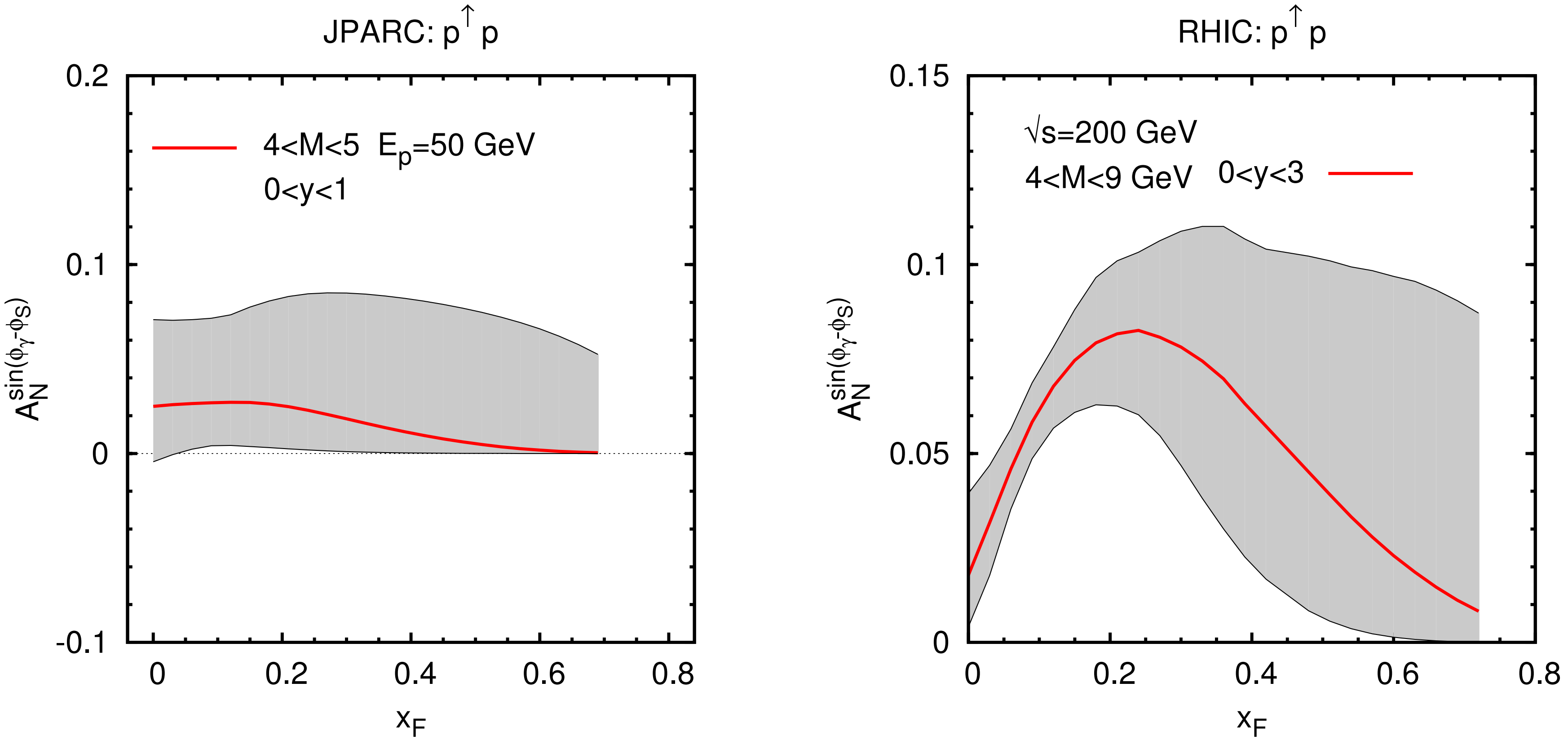
Some predictions based on fit to SIDIS data:





Anselmino *et al.* '09

Sivers effect in Drell-Yan



Anselmino *et al.* '09

$p^{\uparrow}p$ DY studies kinematically largely complementary to SIDIS data

These predictions take into account the *process dependence* of the Sivers function

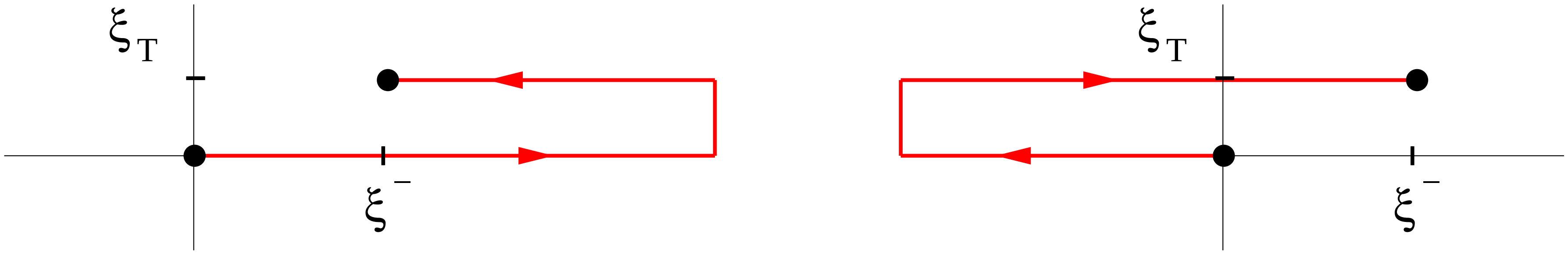
Link structure of TMDs

$\Phi(x, \boldsymbol{k}_T)$ is a matrix element of operators that are nonlocal *off* the lightcone

$$\Phi(x, \boldsymbol{k}_T) = \text{F.T.} \langle P| \, \overline{\psi}(0) \, \mathcal{L}[0,\xi] \, \psi(\xi) \, |P\rangle \big|_{\xi=(\xi^-,0^+,\boldsymbol{\xi}_T)}$$

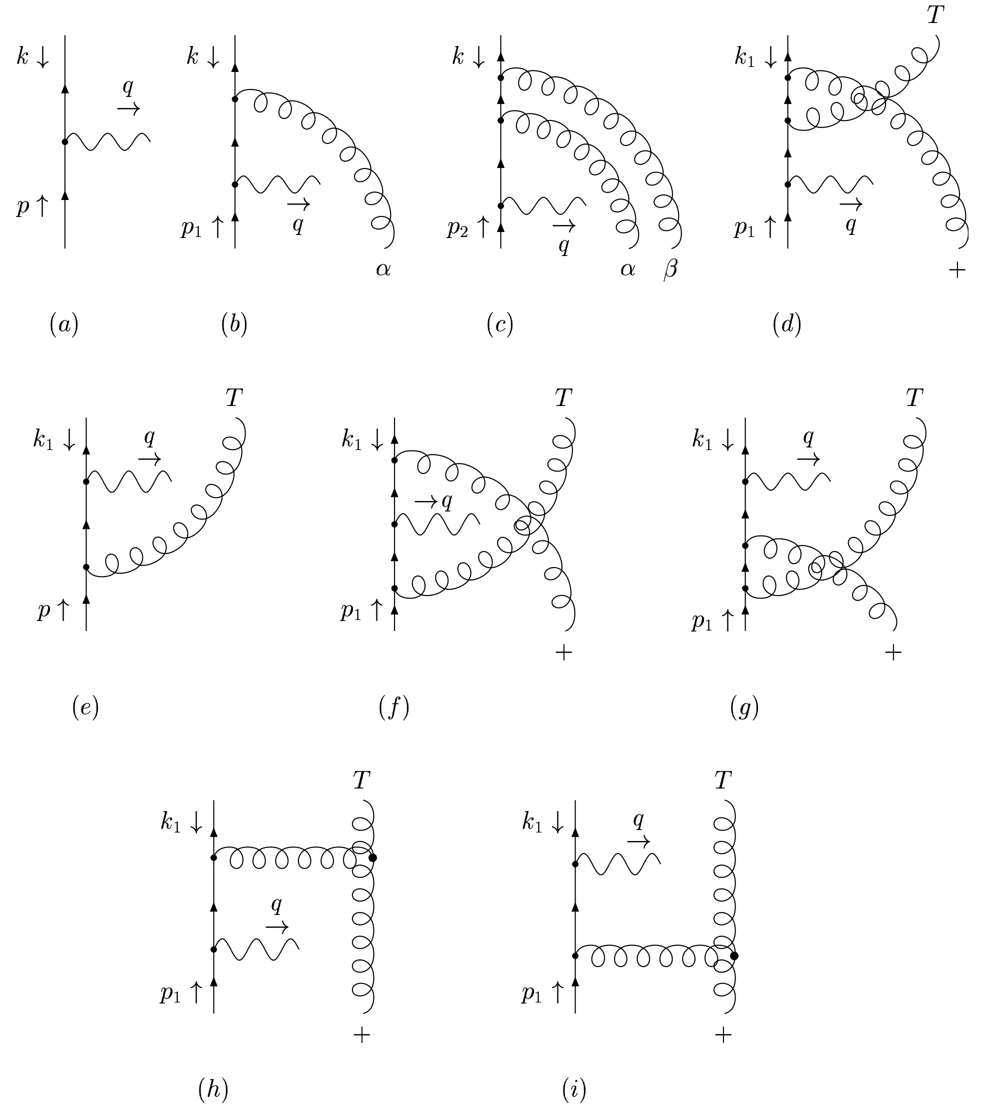
$$\mathcal{L}[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu \, A^\mu(s) \right)$$

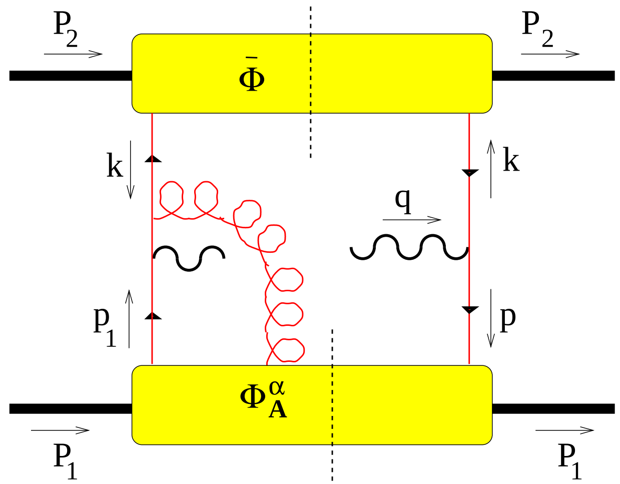
Proper gauge invariant definition of TMDs in SIDIS contains a future pointing Wilson line (FSI), whereas in Drell-Yan (DY) it is past pointing (ISI)



$$P\cdot(\boldsymbol{k}_T \times \boldsymbol{S}_T) \, f_{1T}^{\perp[\mathcal{C}]}(x, \boldsymbol{k}_T^2) \propto \text{F.T.} \langle P, S_T|\overline{\psi}(0) \, \mathcal{L}[0,\xi] \, \gamma^+ \, \psi(\xi)|P, S_T\rangle \big|_{\xi=(\xi^-,0^+,\boldsymbol{\xi}_T)}$$

Obtaining the link structure





path-ordered exponentials in off-lightcone non-local operators

D.B. & Mulders '00

Belitsky, Ji & Yuan '03

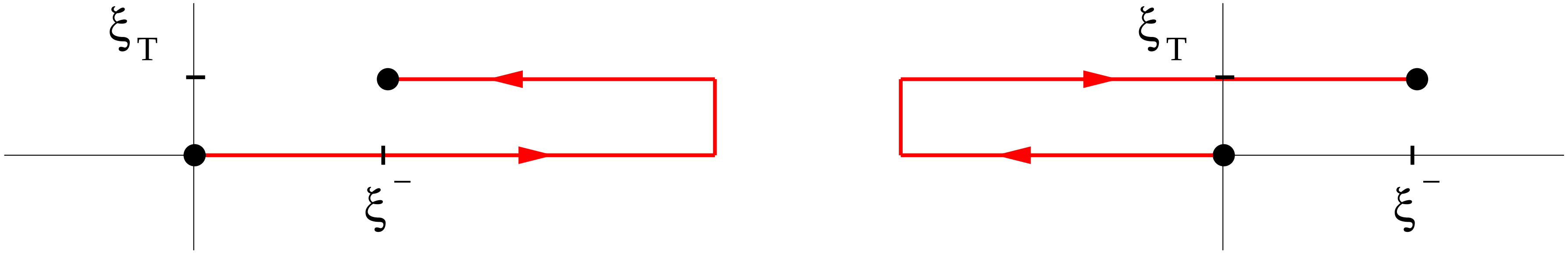
DY: ISI

SIDIS: FSI

Link structure of TMDs

Time reversal invariance relates $\Phi^{[+]}(x, p_T)$ of SIDIS to $\Phi^{[-]}(x, p_T)$ of Drell-Yan

Collins '02



Time reversal invariance does not yield a constraint on $\Phi^{[\pm]}$, but a relation

$$f_{1T}^{\perp[+]} = -f_{1T}^{\perp[-]}$$

Ignoring the link dependence yields $f_{1T}^{\perp} = 0$ because of time reversal invariance

$f_{1T}^{\perp[\pm]}$ could be called naive T-odd (since not exchanging ISI and FSI)

$\Phi(x, \boldsymbol{k}_T)$ contains parts that depend on H, universality is lost for those parts

But predictability is not lost!

Process dependence of TMDs

There is a *calculable* process dependence, which yields the relation (Collins '02):

$$(f_{1T}^{\perp})_{\rm SIDIS} \quad = \quad -(f_{1T}^{\perp})_{\rm DY} \qquad \text{to be tested}$$

The color flow of a process is crucial (usually not the case in high energy scattering!)
The more hadrons are observed, the more complicated the end result (ISI *and* FSI)
Bomhof, Mulders & Pijlman '04

This leads to trouble for processes like $p\,p \to$ jet jet X
TMD factorization fails for such processes
Not simply $\Phi \otimes \bar{\Phi} \otimes H \otimes \Delta \otimes \Delta$
Collins & Qiu '07; Collins '07; Rogers & Mulders '10

This does *not* cast doubt on the above sign relation

Large transverse momentum tails

What about the SSA at large Q_T where collinear factorization should apply?

$$f_1(x,\boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{1}{\boldsymbol{p}_T^2} \left(K \otimes f_1\right)(x)$$

$$f_{1T}^{\perp}(x,\boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{M^2}{\boldsymbol{p}_T^4} \left(K' \otimes f_{1T}^{\perp(1)}\right)(x)$$

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M^2} f_{1T}^{\perp}(x,\boldsymbol{k}_T^2) \propto T(x,S_T)$$

The first transverse moment of the Sivers function is the Qiu-Sterman function

D.B., Mulders & Pijlman, NPB 667 (2003) 201

The Qiu-Sterman effect determines the large p_T behavior of the Sivers effect

This yields precisely the high Q_T result! (adding the effects is double counting)

Ji, Qiu, Vogelsang, Yuan, PRL 97 (2006) 082002; PLB 638 (2006) 178

Evolution of the high-Q_T tail

What about the Q dependence of the SSA?

The high-Q_T tail of the asymmetry is given by $f_{1T}^{\perp(1)} \sim T_F$

Much recent progress on the evolution of T_F

It evolves just like f_1, i.e. logarithmically with Q^2

Kang, Qiu, PRD 79 (2009) 016003; Zhou, Yuan, Liang, PRD 79 (2009) 114022

Braun, Manashov, Pirnay, PRD 80 (2009) 114002

Ratcliffe, Teryaev, arXiv:0910.5348 & arXiv:0911.4306

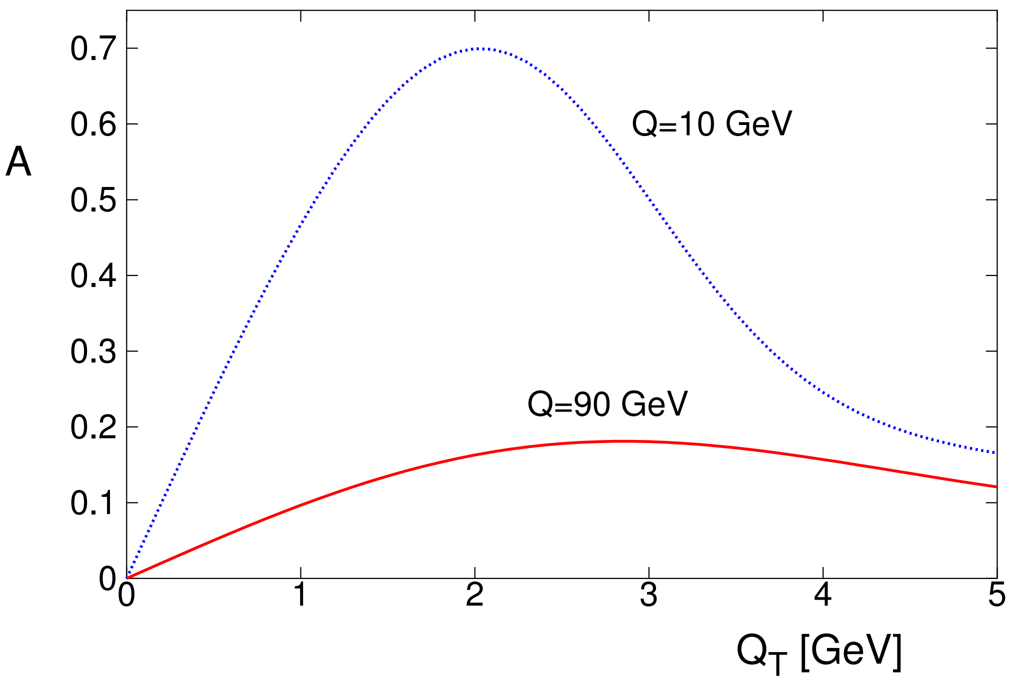
Vogelsang, Yuan, PRD 79 (2009) 094010

What about the Q dependence of the low Q_T asymmetry?

Sudakov suppression of Sivers asymmetry

$$\text{Sivers asymmetry} \propto \frac{f_{1T}^{\perp}(x)}{f_1(x)} \mathcal{A}(Q, Q_T, Q_0)$$

$$\mathcal{A}(Q, Q_T, Q_0) = M \frac{\int db\, b^2\, J_1(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}$$



The maximum of $\mathcal{A}$ decreases with Q^2 as $Q^{-0.6}$

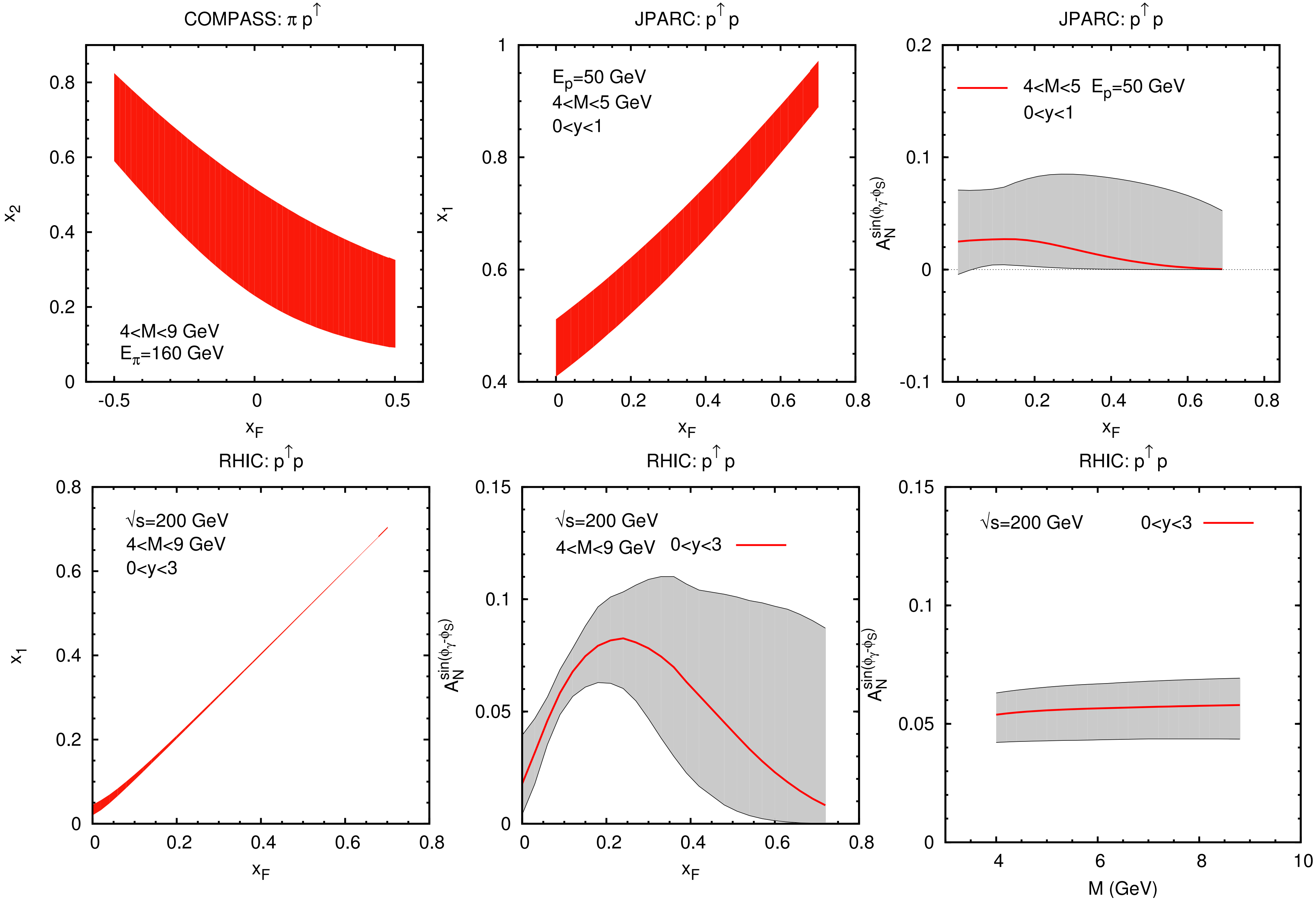
Summary of hadron structure from DY

DY	non-TMD	TMD
σ	$f_1,\ S_{NP}$ or $\langle k_T^2\rangle$	$h_1^\perp$
A_{UL}		$h_{1L}^\perp$
A_{LL}	g_1	
A_{UT}	$T(x, S_T)$	$h_1,\ f_{1T}^\perp,\ h_{1T}^\perp$
A_{TT}	h_1	$f_{1T}^\perp,\ g_{1T}$

Conclusions

- For the Drell-Yan cross section three types of factorization are relevant
- CS or TMD factorization applies when the direction of $\boldsymbol{q}_T$ of the photon matters
- Factorization and resummation determine the Q dependence
- Polarization adds many subtleties, especially spin-orbit correlations
- DY is the perfect process to test all these issues:
  - Verification of predicted Q_T and Q dependences
  - Hadron type and quark flavor dependences
  - Relation to other processes
- Hadron structure in DY is highly nontrivial, interesting and worth pursuing!

Back-up Slides


COMPASS: π p↑
4<M<9 GeV
Eπ=160 GeV
x2
xF
JPARC: p↑ p
Ep=50 GeV
4<M<5 GeV
0<y<1
x1
xF
JPARC: p↑ p
4<M<5 Ep=50 GeV
0<y<1
AN sin(φγ-φS)
xF
RHIC: p↑p
√s=200 GeV
4<M<9 GeV
0<y<3
x1
xF
RHIC: p↑p
√s=200 GeV
4<M<9 GeV 0<y<3
AN sin(φγ-φS)
xF
RHIC: p↑p
√s=200 GeV
0<y<3
AN sin(φγ-φS)
M (GeV)

Future DY data

Usually Drell-Yan data is taken in the safe region, cutting out the resonances (J/ψ and Υ)

They are however also vector particles

Anselmino, Barone, Drago & Nikolaev '04

Note that the NA10 data ('86) on the Υ is very similar to that above/below it