QCD corrections for the Drell-Yan process

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Drell-Yan is probably theoretically best explored process in hadronic scattering:

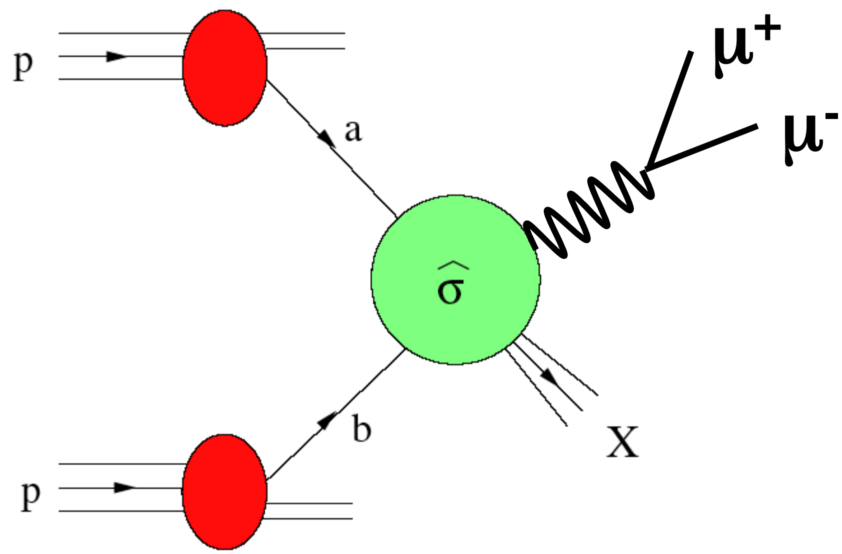
- in pp, pN: probe of anti-quark distributions
- in πN: probe of pion structure
- important spin phenomena
- interface of QCD and QED/el.weak interactions
- LO is color-singlet annihilation $q\bar{q} \longrightarrow \gamma^*$
  - higher-orders under control
  - higher-order computations "easier"
- techniques relevant for $gg \longrightarrow H$

Outline:

- Introduction: Factorized hadronic scattering
- Fixed-order calculations
- Resummation
- Threshold resummation:
  some phenomenology for COMPASS
- Conclusions

Introduction:

Factorized hadronic scattering



hard scale Q

$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a(x_a, \mu) \, f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

universal pdfs

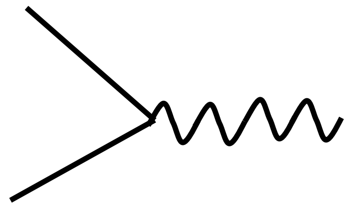
partonic hard scatt.
perturbative QCD

$\mu \sim Q$ factorization scale

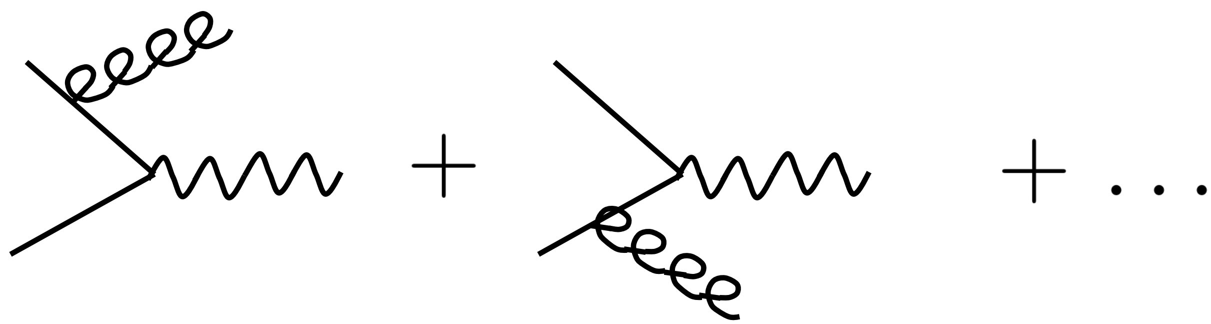
$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{\mathrm{LO}} + \alpha_s \, d\hat{\sigma}_{ab}^{\mathrm{NLO}} + \ldots$$

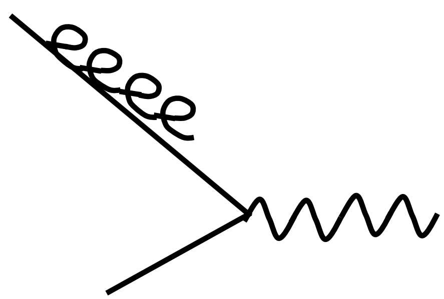
up to power corrections $1/Q^2$

LO:



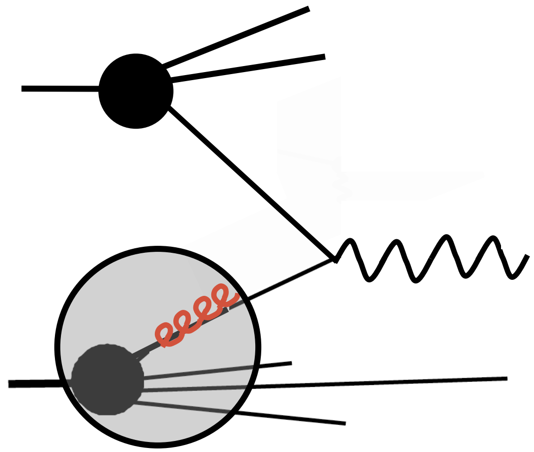
NLO:





Collinear singularity

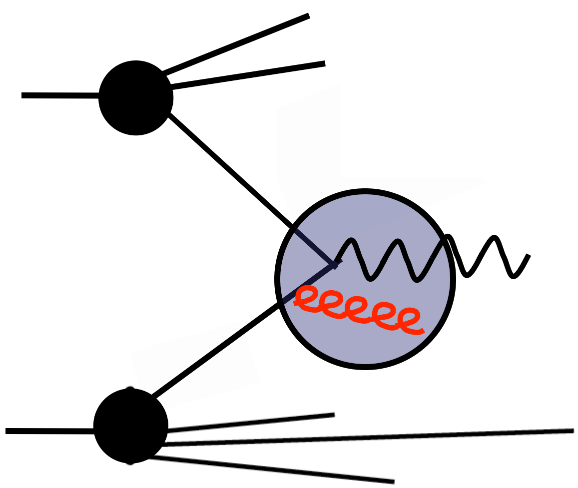
- introduce “factorization” scale μ



“Part of quark distribution”

$$k_\perp < \mu$$

$$\alpha_s \int_m^\mu \frac{dk_\perp}{k_\perp} = \alpha_s \log\left(\frac{\mu}{m}\right)$$



“Part of hard scattering”

$$k_\perp > \mu$$

$$\alpha_s \int_\mu^Q \frac{dk_\perp}{k_\perp} = \alpha_s \log\left(\frac{Q}{\mu}\right)$$

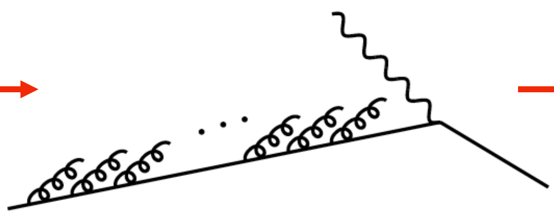
- factorization allows to systematically do this to all orders in α_s :

$$d\sigma = q\left(\frac{\mu}{m}, \alpha_s(\mu)\right) \otimes d\hat{\sigma}\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \otimes \bar{q}\left(\frac{\mu}{m}, \alpha_s(\mu)\right)$$

- $d\sigma$ is physical :

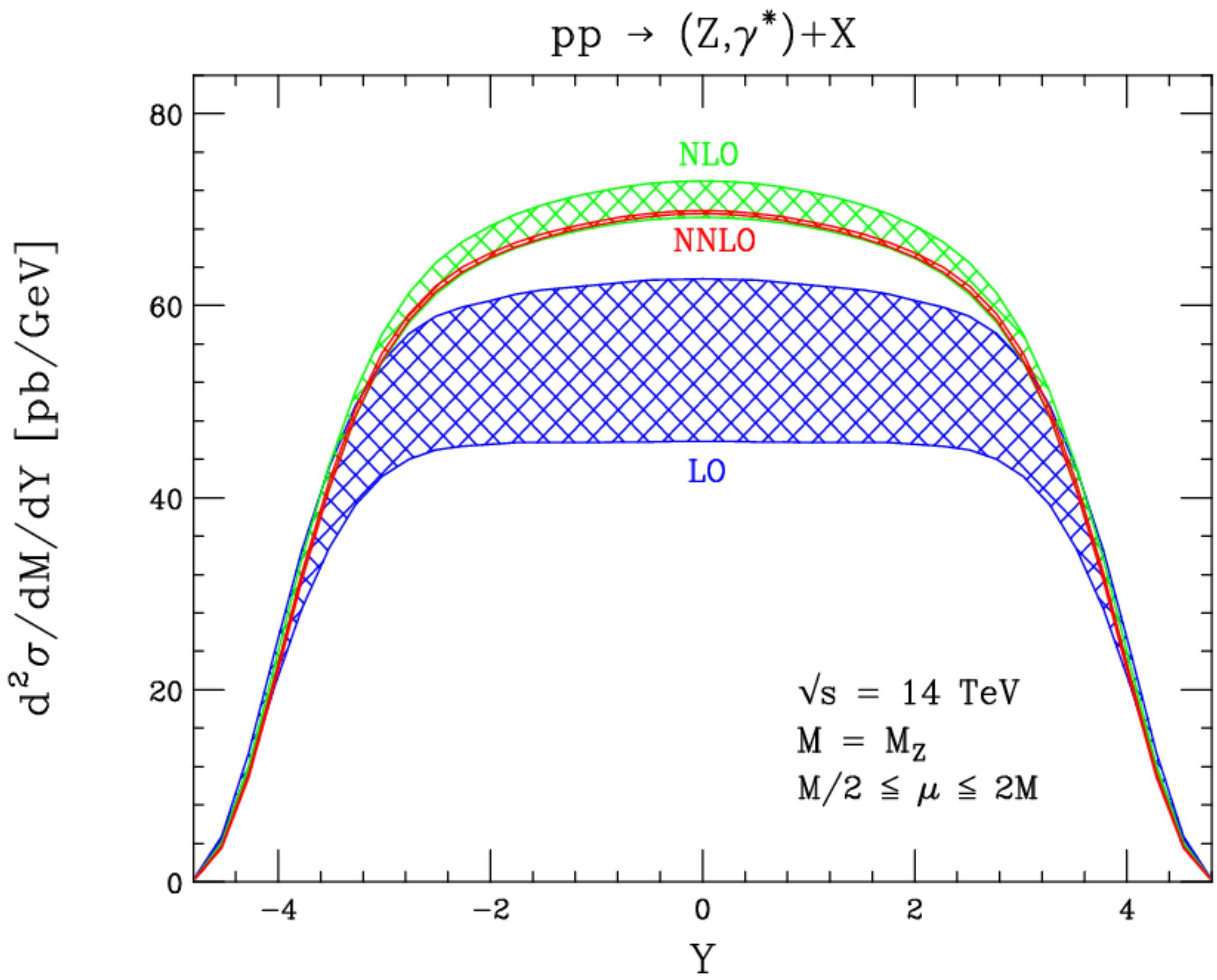
$$\mu \frac{d\sigma}{d\mu} = 0$$

- gives evolution equations for parton distribution functions

$q(\mu_0)$ →  → $q(\mu)$

resums terms $\alpha_s^k \log^k(\mu/\mu_0)$

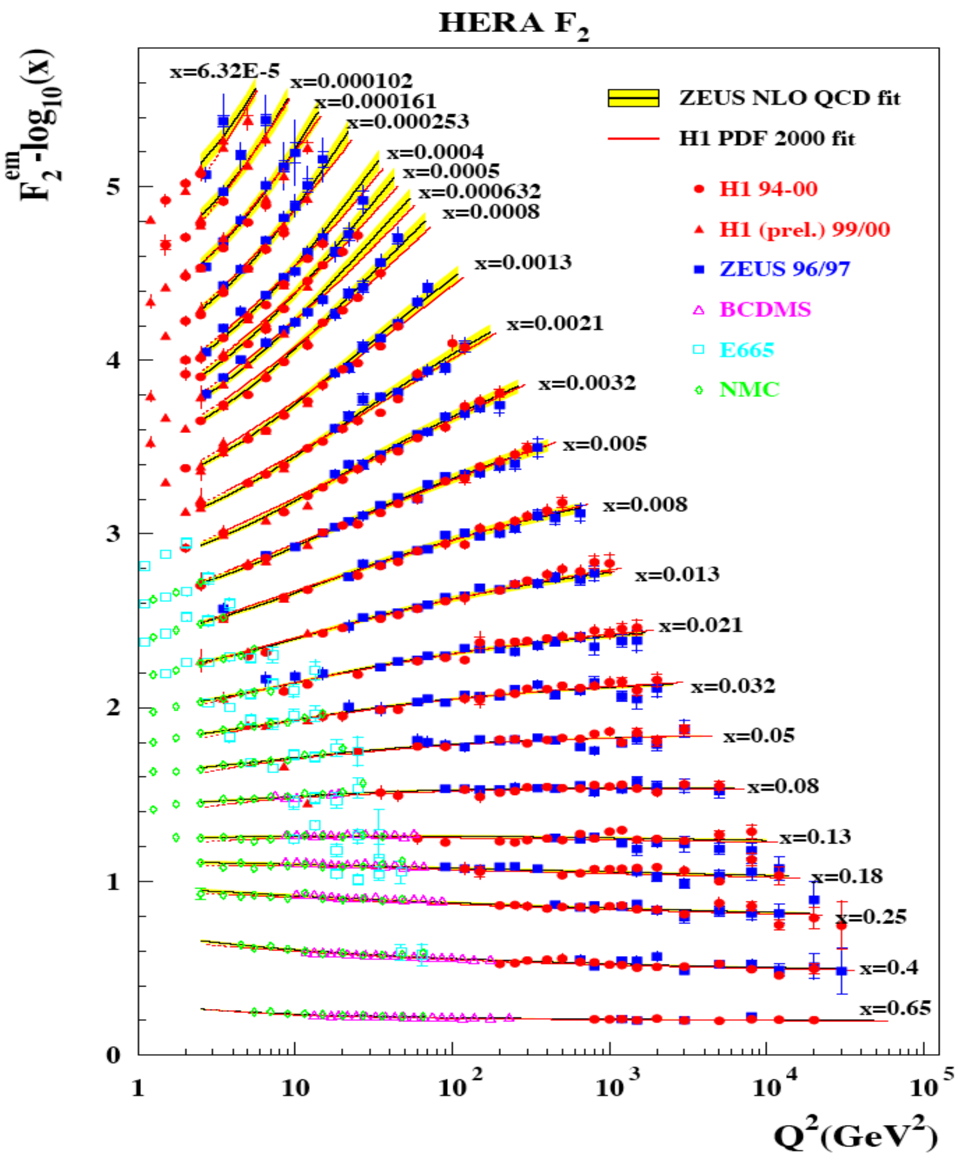
- choose μ ~ Q. Dependence on scale μ decreases order by order

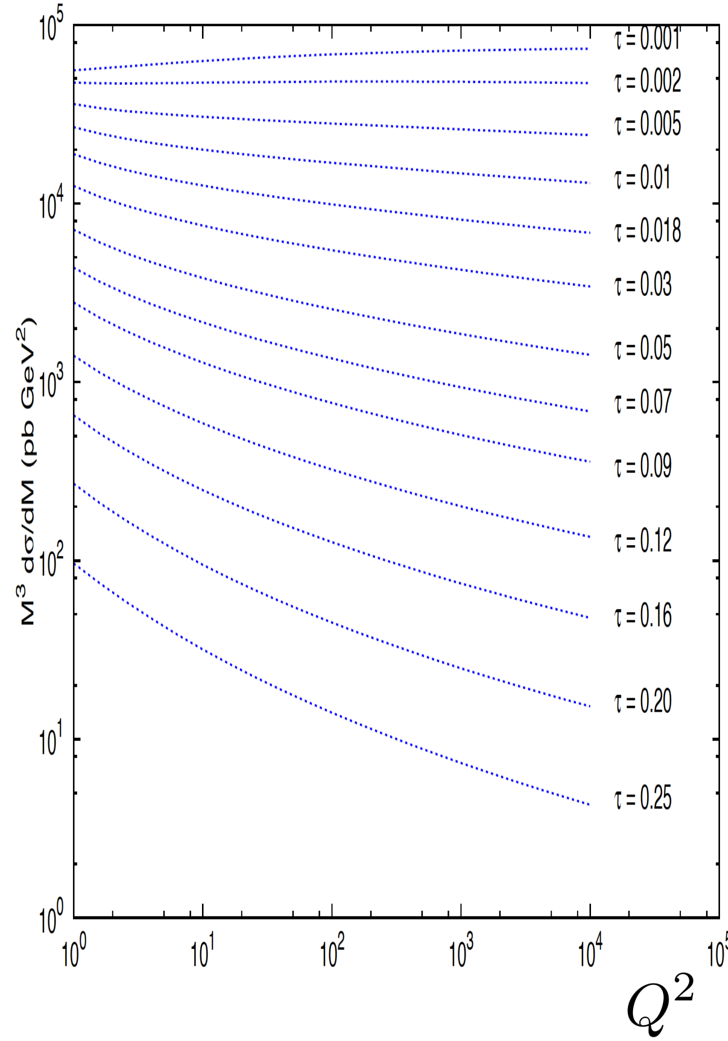


Anastasiou et al.

Skalenverletzungen in DY at NLO

$$Q^3 \frac{d\sigma}{dQ}$$





(M.Aicher)

Fixed-order calculations

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{\rm LO} + \alpha_s\, d\hat{\sigma}_{ab}^{\rm NLO} + \alpha_s^2\, d\hat{\sigma}_{ab}^{\rm NNLO} + \ldots$$

	Unpol.	Long. pol.	Trans. pol.
NLO	Kubar et al. Altarelli, Ellis, Martinelli Harada et al.	Ratcliffe Weber Gehrmann Kamal de Florian, WV	Weber, WV WV Contogouris et al. Barone et al.
NNLO	Hamberg, van Neerven, Matsuura Harlander, Kilgore Anastasiou, Dixon, Melnikov, Petriello Catani et al.	Smith, v.Neerven, Ravindran	

NLO:

$$\frac{d\hat{\sigma}}{dQ^2} \sim C_F \left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z \right.$$

$$\left. + \left(\frac{2}{3}\pi^2 - 8\right)\delta(1-z) + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2} \right]$$

$$z = \frac{Q^2}{\hat{s}}$$

- DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} \left(\frac{x}{z}, \mu^2\right)$$

$$\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \mathcal{P}_{ij}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}_{ij}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \mathcal{P}_{ij}^{\mathrm{NNLO}} + \ldots$$

$\mathcal{P}_{ij}^{\mathrm{LO}}$ ↑
Ahmed,Ross
Altarelli,Parisi,…

$\mathcal{P}_{ij}^{\mathrm{NLO}}$ ↑
Curci, Furmanski, Petronzio
Antoniadis,Kounnas, Lacaze
Mertig, van Neerven
WV
Kumano et al.
Koike et al.
WV

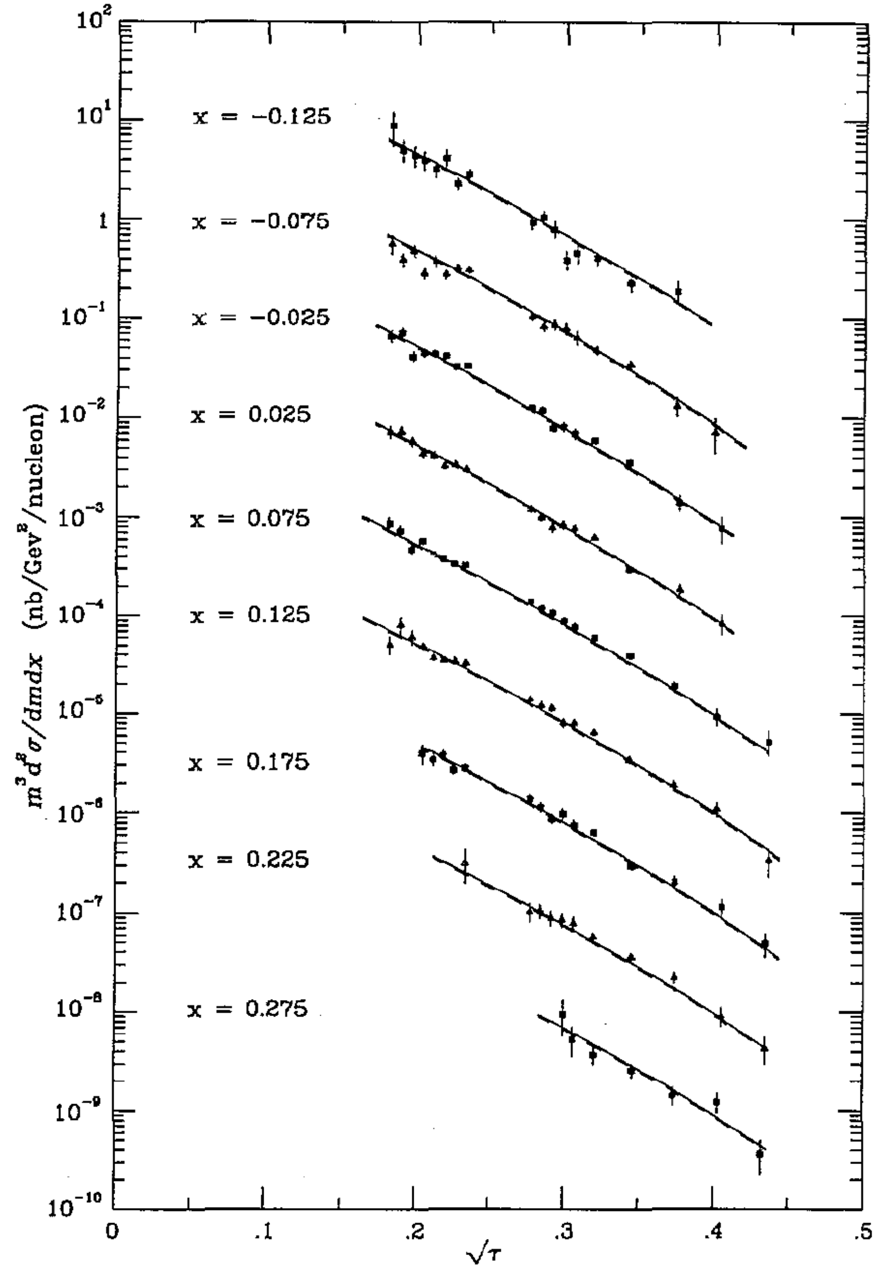
$\mathcal{P}_{ij}^{\mathrm{NNLO}}$ ↑
Moch,Vermaseren, Vogt, Rogal

NLO quite successful overall for inclusive DY:

E605 (800 GeV pC)

NLO scaled by 1.071

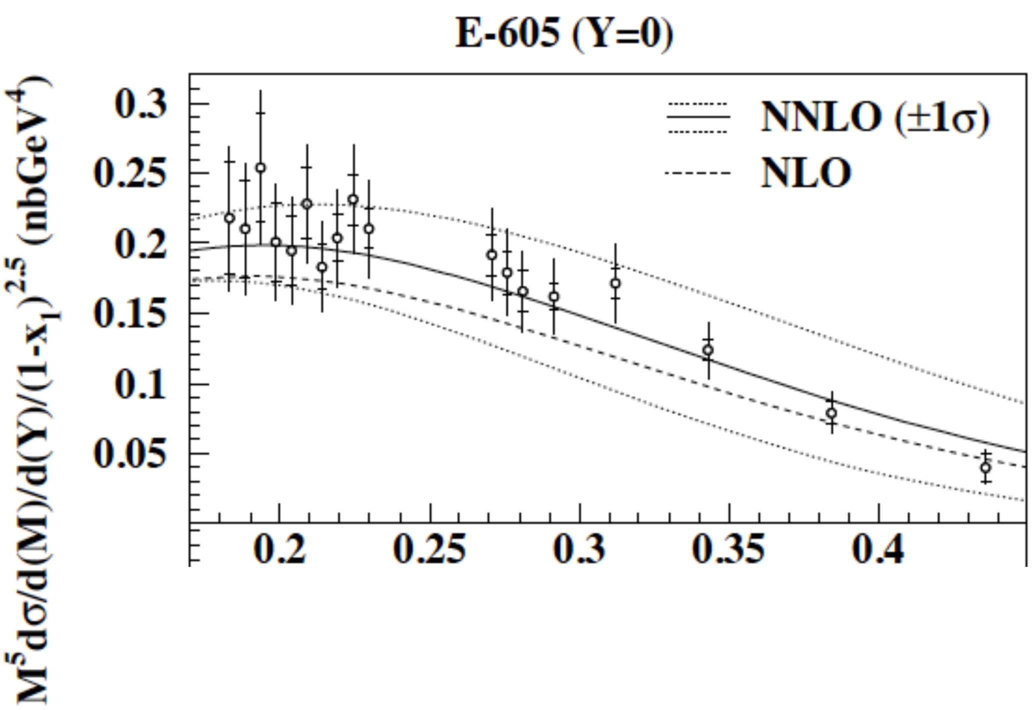
$$x = \frac{2p_L}{\sqrt{S}}$$



Stirling/Whalley

Alekhin, Melnikov, Petriello:

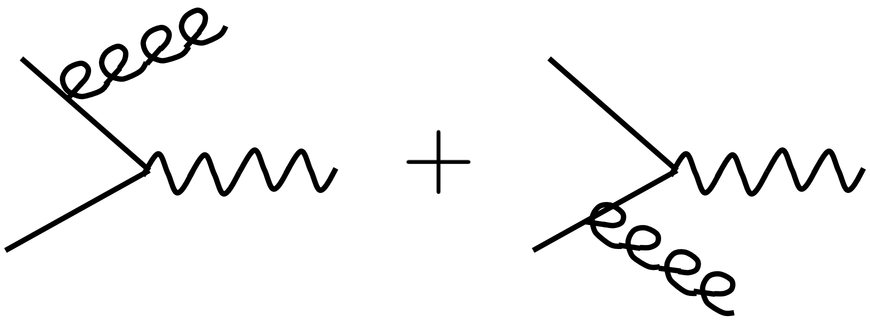
NNLO important for improving precision of sea quark distributions in pdf fits:



Note: also a lot of interest in DY process with measured photon transverse momentum q_T

Here LO already has one gluon:

LO:

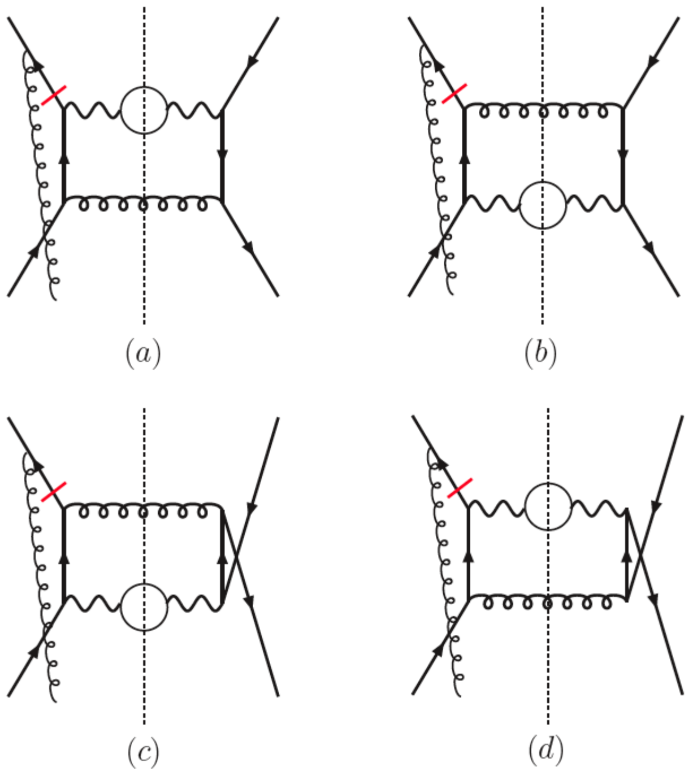


NLO: Gonzalves, Pawlowski, Wai; Mirkes, Ohnemus
Berger, Coriano, Gordon
de Florian et al.

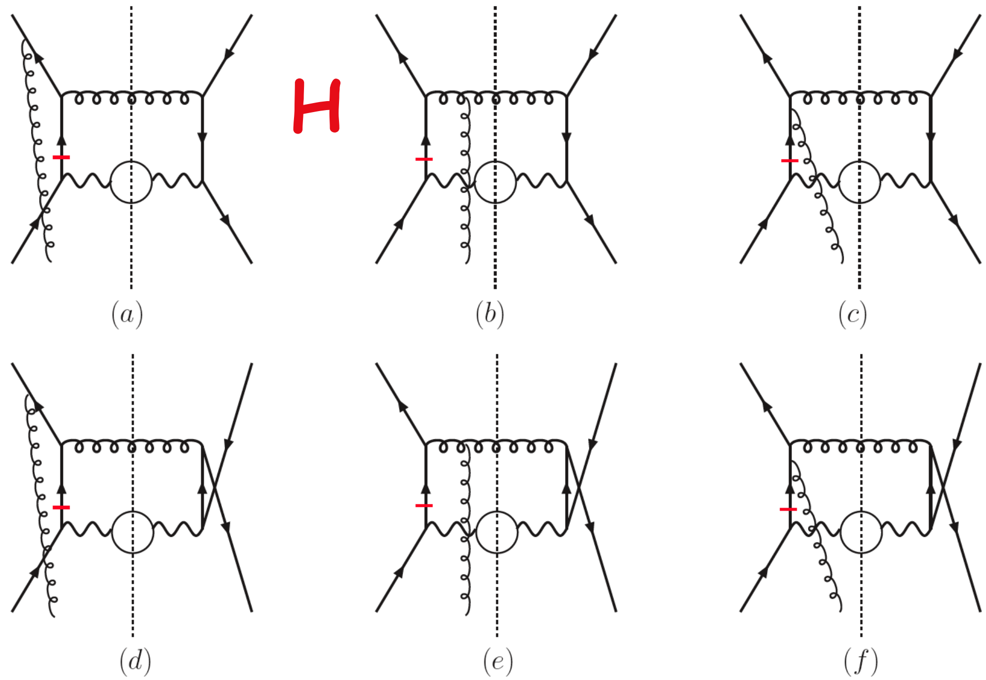
also: NLO correction to DY single-spin asymmetry Yuan, WV

Real emission:

S



H



Plus virtual

Final finite result …

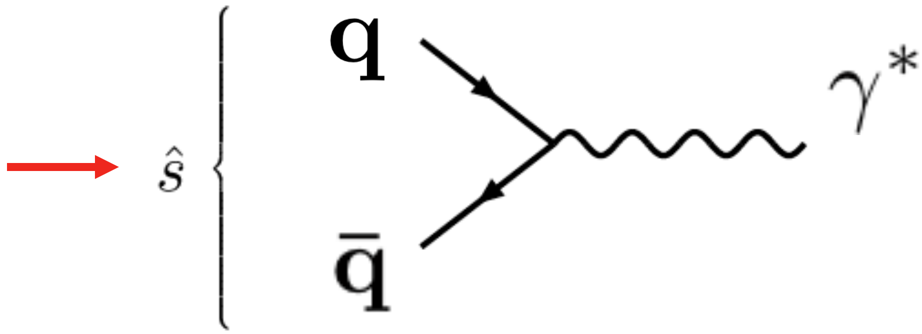
WV,Yuan

$$
\begin{aligned}
\frac{d\langle q_\perp \Delta\sigma(S_\perp)\rangle}{dQ^2} = & \ \sigma_0 \int \frac{dx}{x}\frac{dx'}{x'} T_F(x,x;\mu)\bar{q}(x';\mu) \\
& + \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x}\frac{dx'}{x'} \left\{ \bar{q}(x') \left[\ln\frac{Q^2}{\mu^2} \left(C_F \mathcal{P}_{qq} + \mathcal{P}_{qg\to qg} \otimes T_F(x,xz) \right) \right. \right. \\
& + \frac{1}{2N_c}\left(x\frac{\partial}{\partial x}T_F(x,x)\right)(1+z^2)\ln\frac{(1-z)^2}{z} + \left(2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{\ln z}{1-z} \right) \\
& \times \left(\left(C_F(1+z^2) + \frac{2z^3-3z^2-1}{2N_c}\right) T_F(x,x) + \left(\frac{1}{2N_c} + C_F\right)(1+z) T_F(x,xz) \right) \\
& \left. + T_F(x,x) \left(\left(C_F + \frac{z}{2N_c}\right)(1-z) + C_F\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) \right) \right] \\
& + \int \frac{dv}{1-z} \left(\frac{1}{v_+} + \frac{1}{(1-v)_+} \right) T_F\left(x, x\frac{z}{1-v(1-z)}\right)\bar{q}(x') \\
& \left. \times \left[(1-v(1-z))^3 + z \right] \left(\frac{1}{2N_c} + C_F \frac{1}{1-v(1-z)} \right) \right\}
\end{aligned}
$$

+ gluon terms + $\tilde{T}_F$ terms

Resummation

- **LO partonic cross section :**

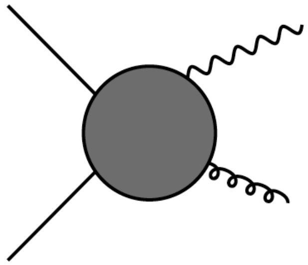


mass Q

$$\hat{s} = Q^2 \quad , \text{ or } \quad z \equiv \frac{Q^2}{\hat{s}} = 1$$

$$\frac{d\hat{\sigma}^{(0)}_{q\bar{q}}}{dQ^2} \propto \delta(1-z)$$

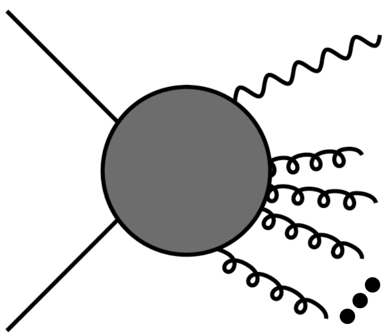
- **NLO correction:**



$$\text{as} \quad \frac{Q^2}{\hat{s}} \equiv z \to 1 \qquad\qquad \frac{d\hat{\sigma}^{(1)}_{q\bar{q}}}{dQ^2} \propto \alpha_s \frac{\ln(1-z)}{1-z} + \ldots$$

(same for unpolarized, long. & transv. polarized)

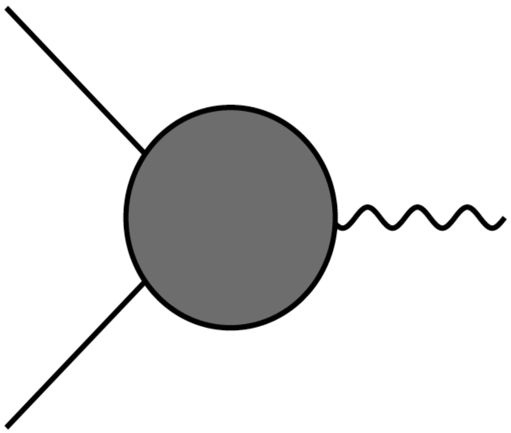
- **higher orders :**



$$\frac{d\hat{\sigma}^{(k)}_{q\bar{q}}}{dQ^2} \propto \alpha_s^k \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$

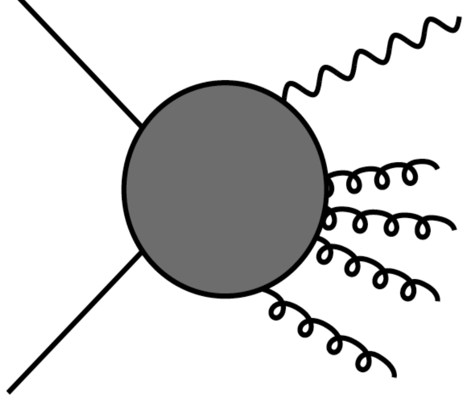
"threshold logarithms"

What's the origin of the logarithms ?



virtual corrections

$z = 1$



real emission

$z \neq 1$

For $z \to 1$ real radiation is inhibited,
only soft emission is allowed: affects IR cancellations

- of course, one doesn't really measure partonic energy

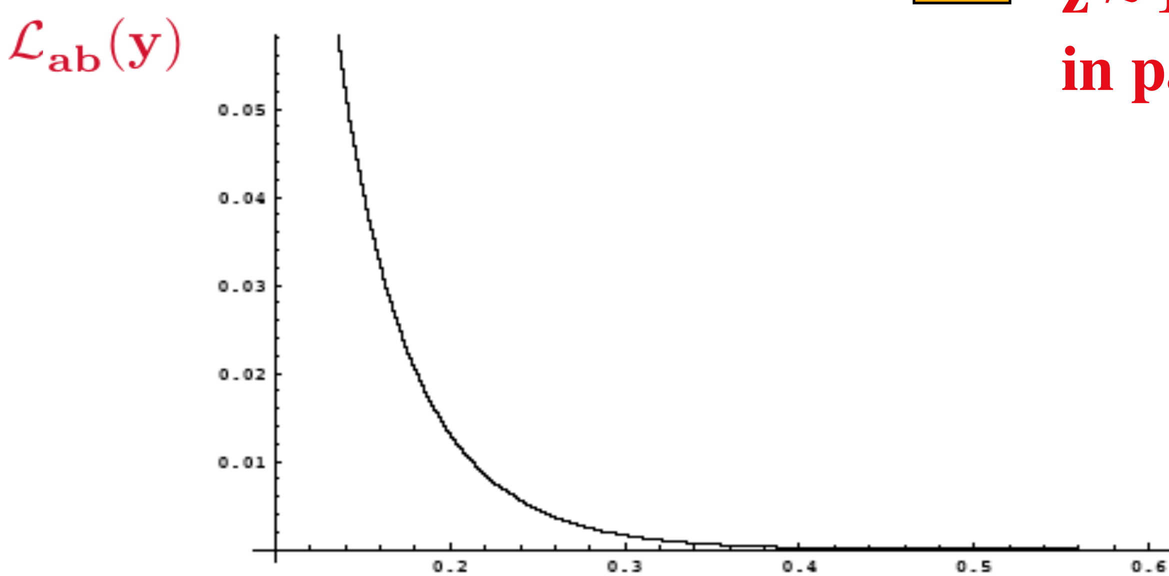
- nevertheless :

$$\tau = Q^2/S$$

$$\sigma^{DY} \propto \sum_{a,b} \int_{\tau}^{1} \frac{dx_a}{x_a} f_a(x_a) \int_{\tau/x_a}^{1} \frac{dx_b}{x_b} f_b(x_b)\, \hat{\sigma}_{ab}(z = \tau/x_a x_b)$$

$$= \sum_{a,b} \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \hat{\sigma}_{ab}(z)$$

z ~ 1 emphasized,
in particular as τ→1



- large logs will spoil perturbative series, unless taken into account to all orders

= (Threshold) Resummation !

- particularly relevant for fixed-target regime

- work began in the '80s with Drell-Yan process

Sterman; Catani, Trentadue

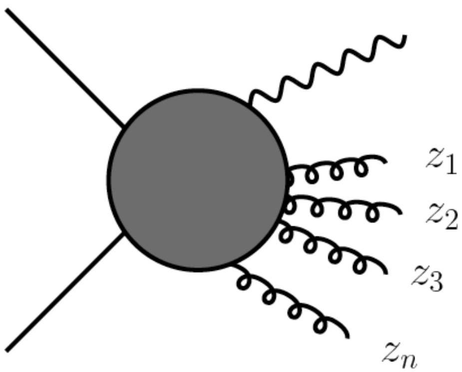
various new techniques: Forte, Ridolfi; Becher, Neubert
van Neerven, Smith, Ravindran
Laenen, Magnea

Resummation relies on two things:

- simplification of QCD matrix elements in soft/collinear limits

- factorization of phase space when integral transform is taken:

sc. variable of the process



$$\delta\left(1 - z - \sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, e^{N(1-z-\sum_{i=1}^{n} z_i)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$$\left(\frac{\ln^{2k-1}(1-z)}{1-z}\right)_+ \leftrightarrow \ln^{2k} N$$

- typically leads to exponentiation of large logs

- **General structure ?**

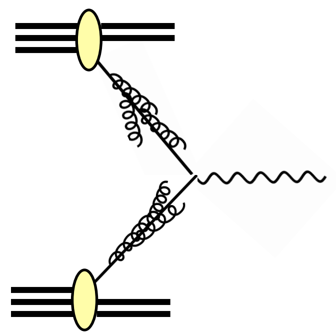
$$\mathbf{L \equiv \ln(N)}$$

Fixed order →

Resummation ↓

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	$+\dots$
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L \quad +\dots$
	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3 \quad +\dots$
	$\alpha_s^4 L^8$	$\alpha_s^4 L^7$	$\alpha_s^4 L^6$	$\alpha_s^4 L^5 \quad +\dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3} \quad +\dots$
	LL	**NLL**	**NNLL**	

Drell-Yan:



$\overline{\text{MS}}$ scheme:

$$\hat{\sigma}_{q\bar{q}} \propto \exp\left[2\int_0^1 dy\, \frac{y^N - 1}{1-y} \int_{\mu_F^2}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) + \ldots\right]$$

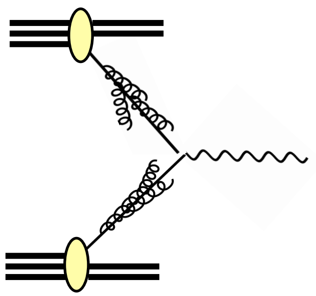
$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \ldots$$

$$A_q^{(1)} = C_F \qquad A_q^{(2)} = \frac{1}{2} C_F \left[C_A\left(\frac{67}{18} - \zeta(2)\right) - \frac{10}{9} T_R N_f\right]$$

Leading logs: $\hat{\sigma}_{q\bar{q}} \propto \exp\left[+\frac{2\,C_F}{\pi}\alpha_s \ln^2(N)\right] > 1$

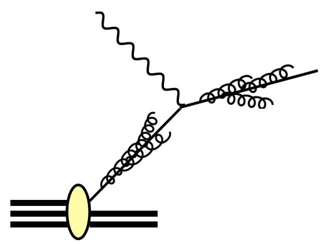
- role of parton distribution functions and factorization scheme:

DY:



$$\exp\left[+ \frac{2\,C_F}{\pi}\,\alpha_s \ln^2(N) \right]$$

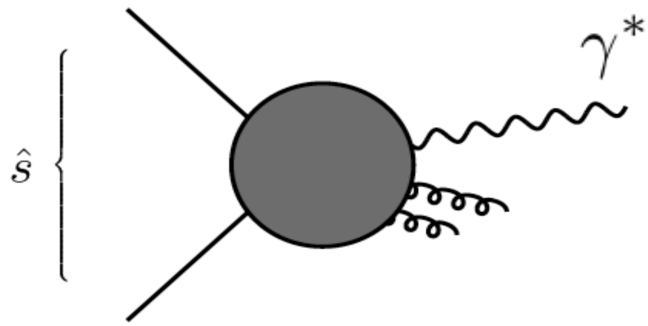
DIS:



$$\exp\left[\frac{1}{2}\,\frac{C_F\,\alpha_s}{\pi}\,\ln^2(N) \right]$$

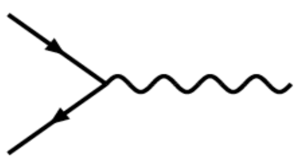
- would in principle need "resummed" parton distributions, but effects in DIS smaller

Large logarithms also appear in transverse-momentum distribution:



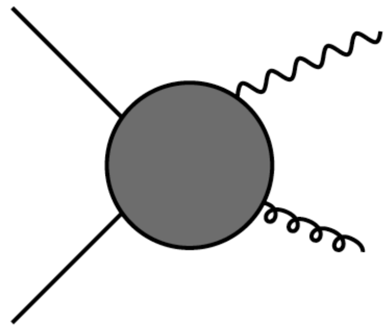
mass Q, transv. momentum q_T

- LO partonic cross section :



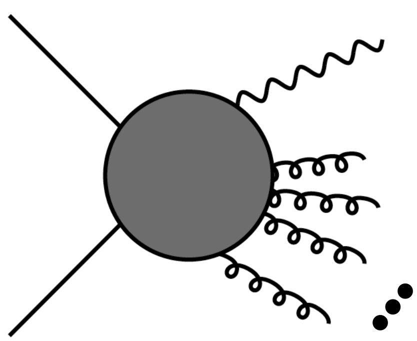
$$\frac{d^2\hat{\sigma}^{(0)}_{q\bar{q}}}{dQ^2 d^2 q_T} \sim \delta(\vec{q}_T)$$

- first-order correction :



$$\mathbf{as} \quad \mathbf{q_T} \;\rightarrow\; \mathbf{0} \qquad \frac{\mathbf{d^2 \hat{\sigma}^{(1)}_{q\bar{q}}}}{\mathbf{dQ^2 d^2 q_T}} \sim \alpha_s \frac{\ln(\mathbf{q_T}/\mathbf{Q})}{\mathbf{q_T^2}} + \ldots$$

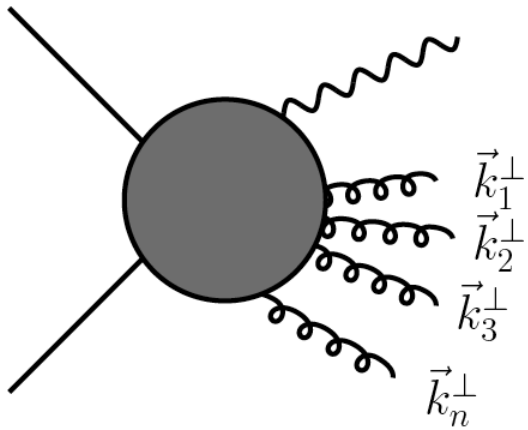
- higher orders :



$$\frac{\mathbf{d^2 \hat{\sigma}^{(k)}_{q\bar{q}}}}{\mathbf{dQ^2 d^2 q_T}} \sim \alpha_s^{\mathbf{k}} \frac{\ln^{\mathbf{2k-1}}(\mathbf{q_T}/\mathbf{Q})}{\mathbf{q_T^2}} + \ldots$$

"recoil logarithms"

- can be resummed with similar techniques:

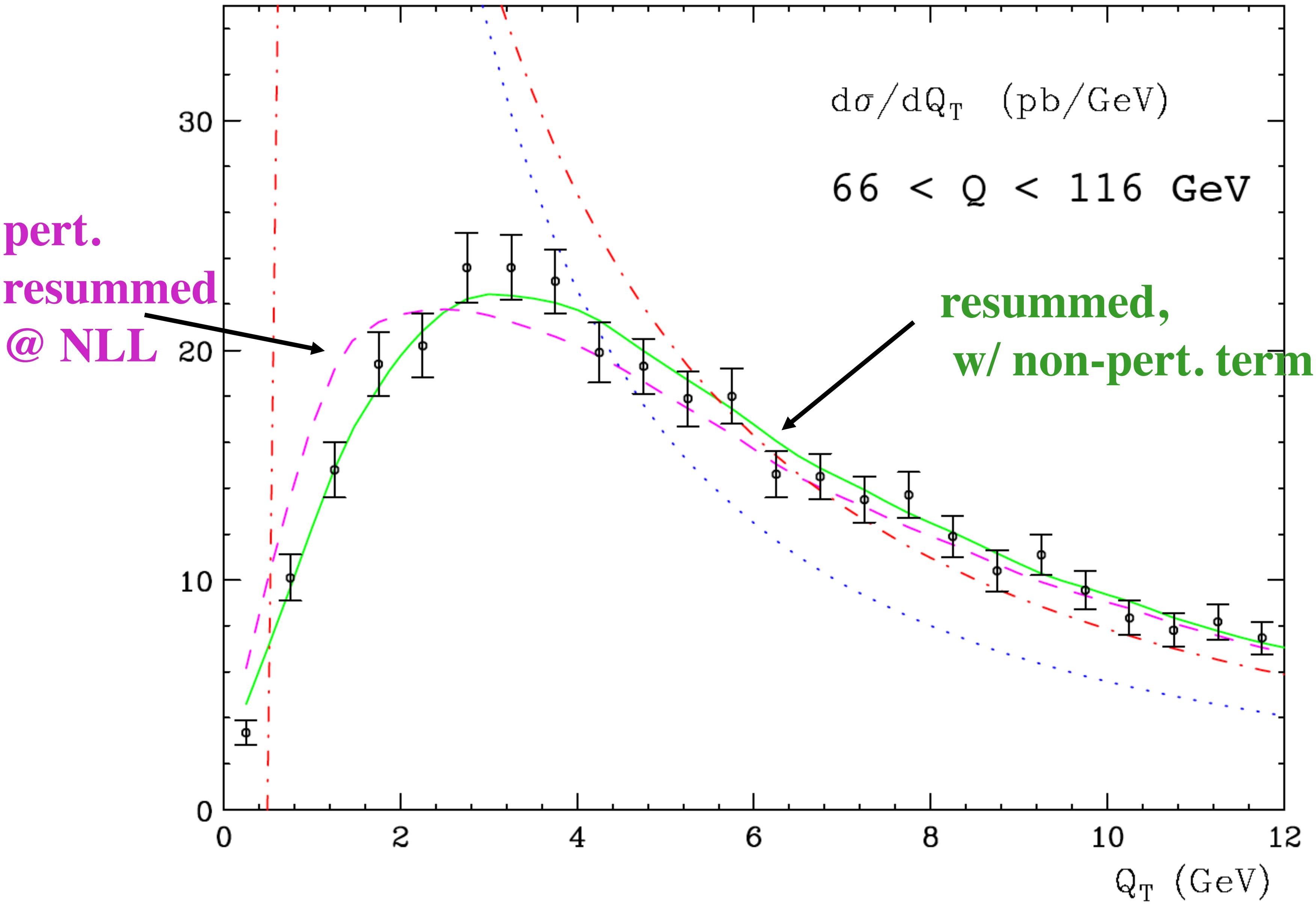


$$\delta^2\left(\vec{\mathbf{q}}_{\mathbf{T}}+\sum_{\mathbf{i}}\vec{\mathbf{k}}_{\mathbf{T}}^{\mathbf{i}}\right) = \frac{\mathbf{1}}{(\mathbf{2}\pi)^{\mathbf{2}}}\int \mathbf{d^2b}\ \mathbf{e}^{-\mathbf{i}\vec{\mathbf{b}}\cdot\left(\vec{\mathbf{q}}_{\mathbf{T}}+\sum_{\mathbf{i}}\vec{\mathbf{k}}_{\mathbf{T}}^{\mathbf{i}}\right)}$$

$$\frac{\ln^{\mathbf{2k-1}}(\mathbf{q_T/Q})}{\mathbf{q_T}} \leftrightarrow \ln^{\mathbf{2k}}(\mathbf{bQ})$$

Collins, Soper, Sterman; Altarelli,Ellis,Greco,Martinelli;
Davies, Stirling;…
Weber; Kawamura,Kodaira,Tanaka;…

Kulesza, Sterman, WV



Why non-perturbative piece ?

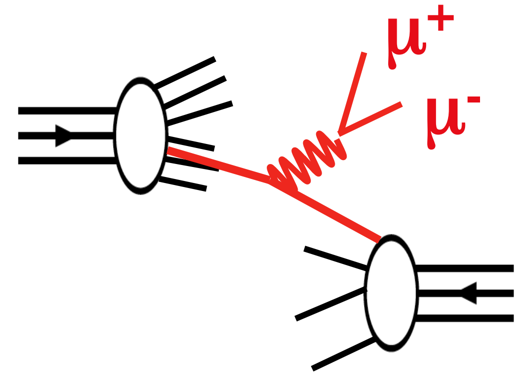
$$\exp\left[\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left(J_0(bk_\perp)-1\right)\left\{\frac{2C_F}{\pi}\alpha_s(k_\perp^2)\ln\left(\frac{Q^2}{k_\perp^2}\right)+\dots\right\}\right]$$

Contribution from very low $k_\perp$

$$\exp\left[-b^2\,\frac{C_F}{\pi}\underbrace{\int dk_\perp^2\,\alpha_s(k_\perp^2)\ln\left(\frac{Q}{k_\perp}\right)}\right]$$

$$g_1 + g_2\ln(Q^2/Q_0^2)$$

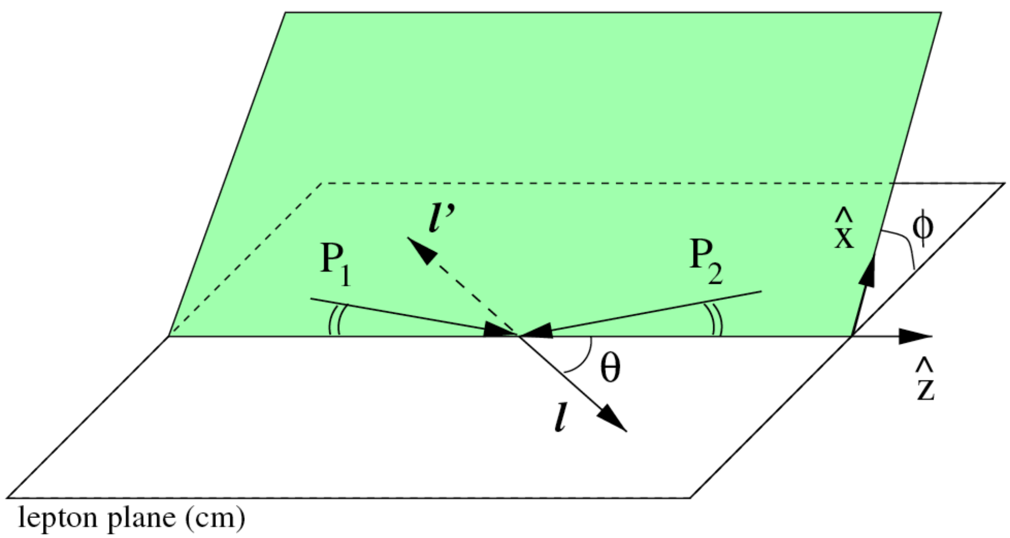
Relevant for TMD studies !



$$\frac{d\sigma}{d\Omega d^4q} = \frac{\alpha^2}{2(2\pi)^4 Q^2 s^2}\{W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta)$$

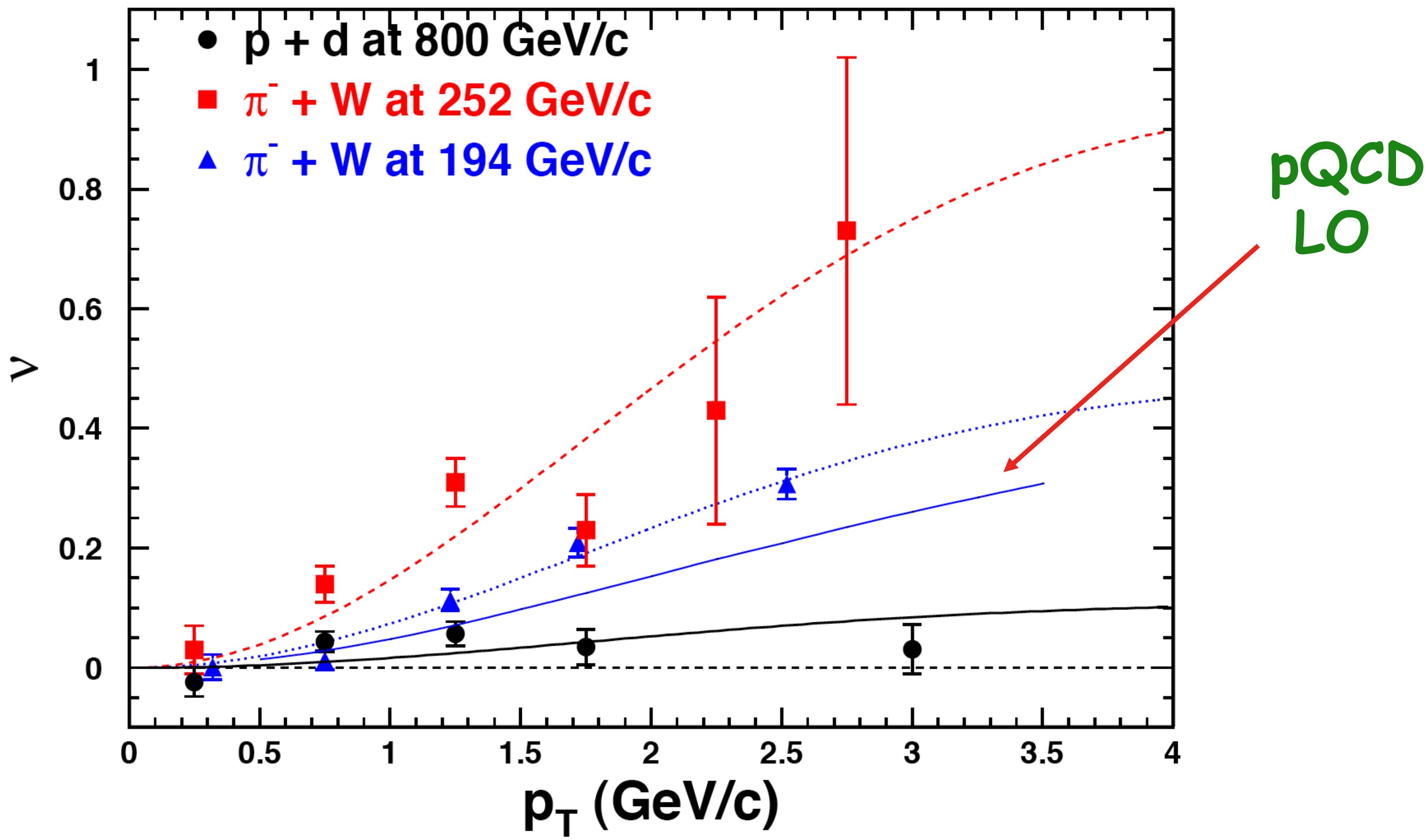
$$+ W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta} \sin^2\theta \cos 2\phi\}$$

Lam, Tung; Collins



$$1 - \lambda - 2\nu = 0$$

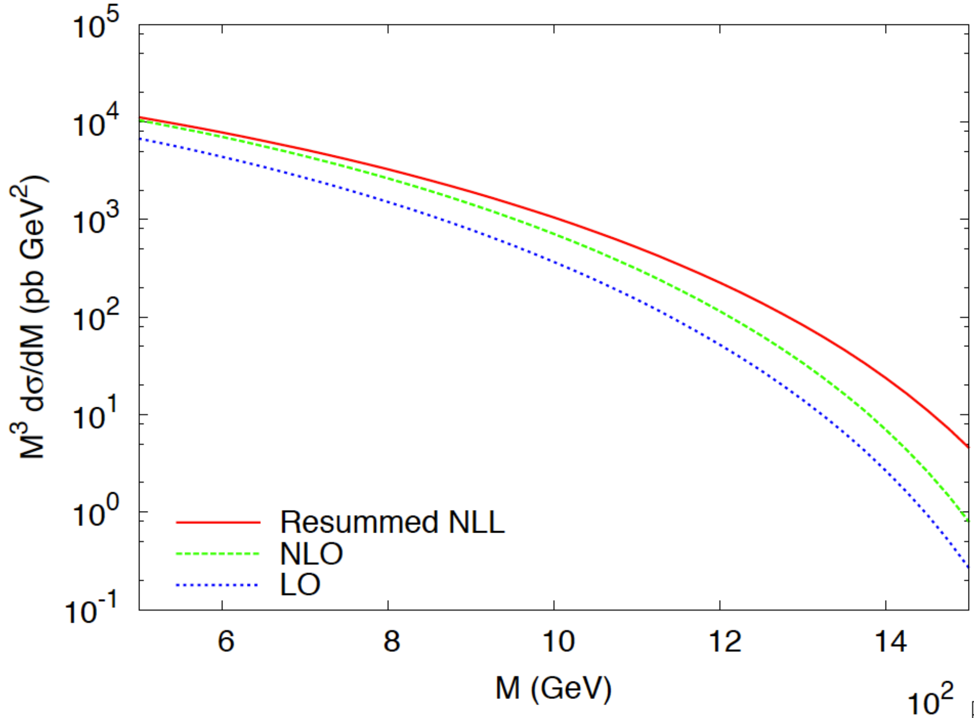
Lam-Tung relation



- large Sudakov logs $\alpha_s^k \ln^{2k-1}(p_T^2/Q^2)$ in numerator *and* denominator of ν (same LL resummation)

Boer, WV; Berger, Qiu, Rodriguez-Pedraza

Threshold resummation: some phenomenology for COMPASS

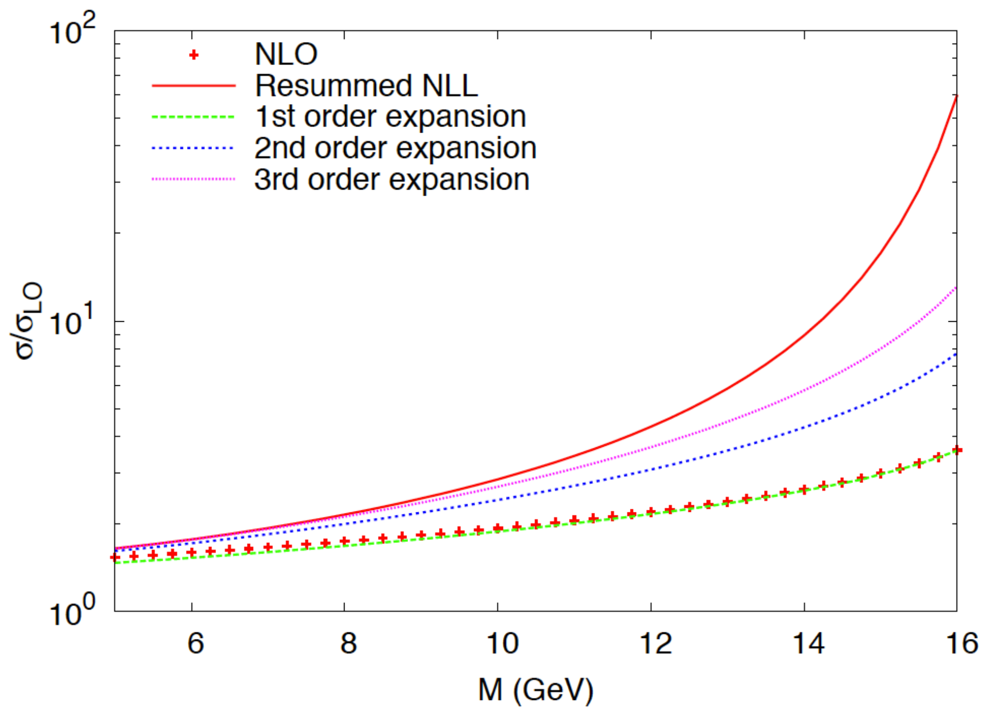


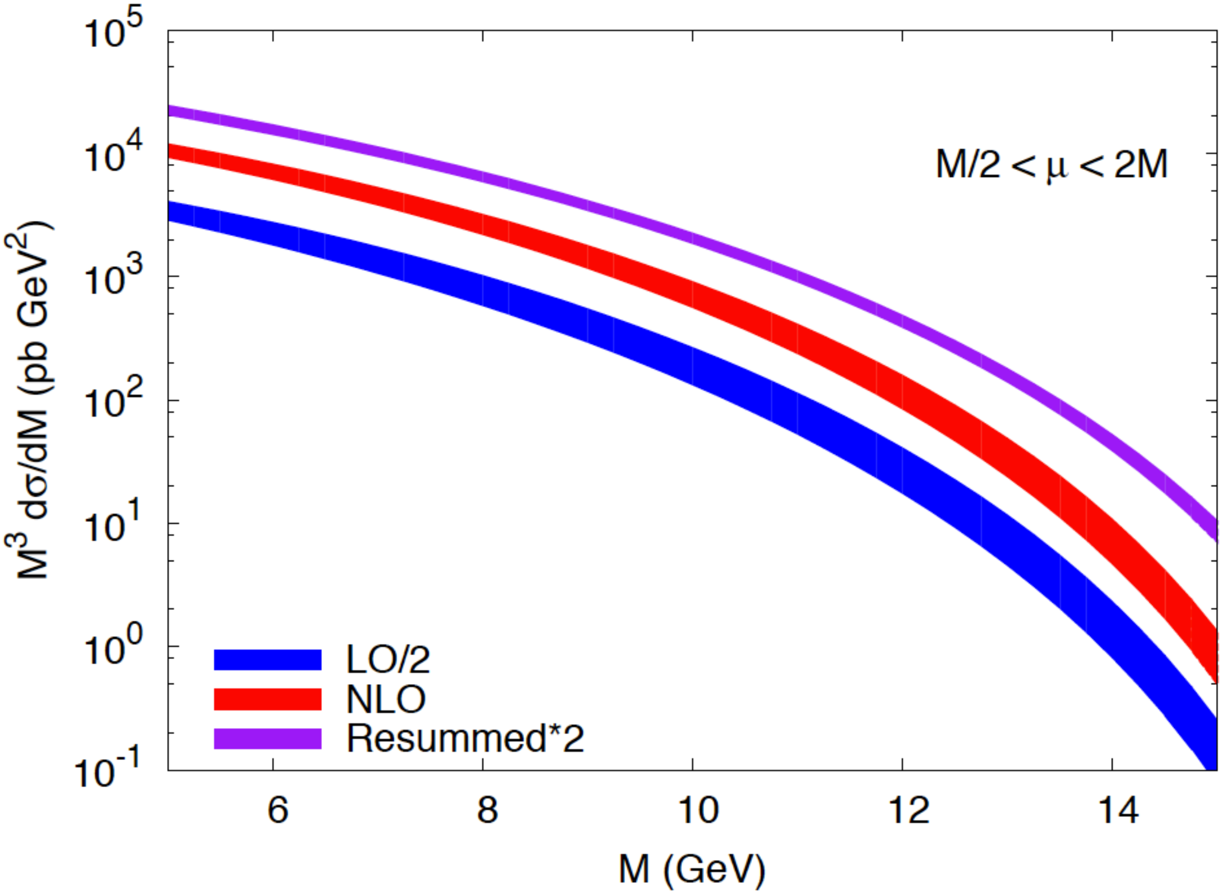
Aicher,Schäfer,WV

Pion parton distributions:
From new fit to NA10 data

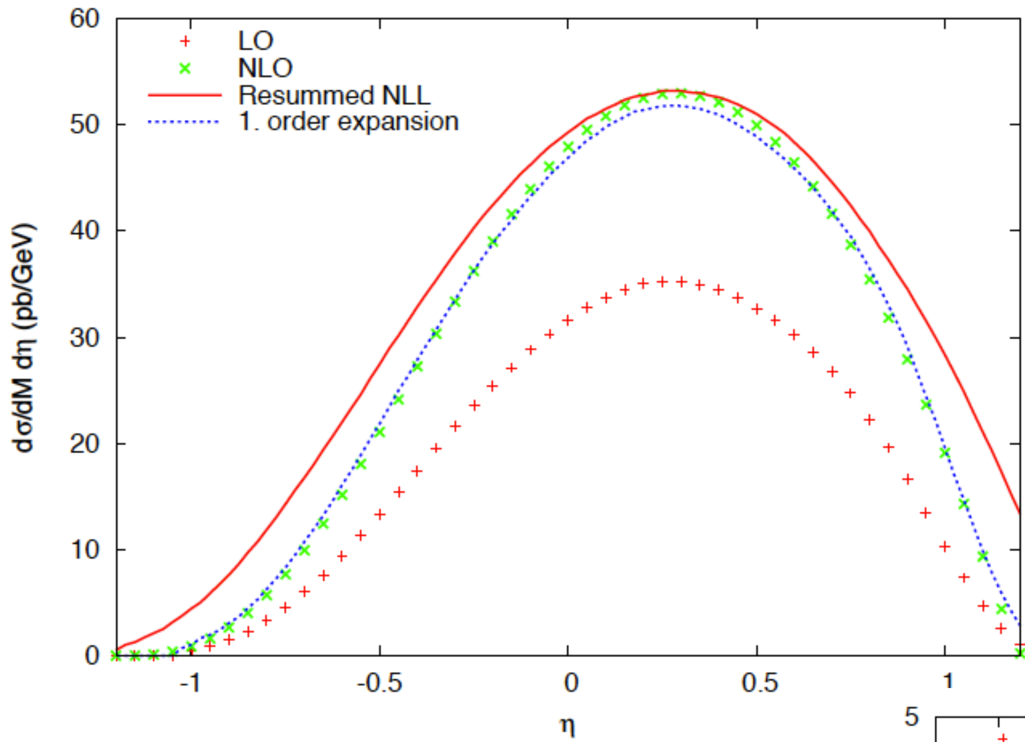
$$\pi^- p$$

$$s = 300\,\mathrm{GeV}^2$$





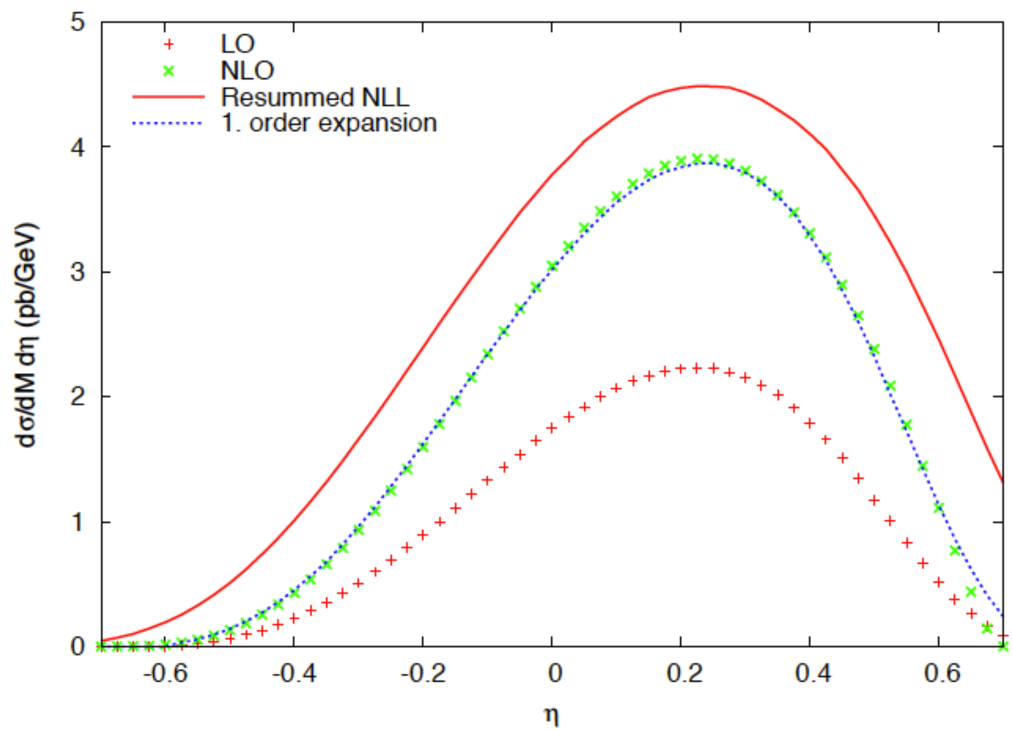
M/2 < μ < 2M
LO/2
NLO
Resummed*2
M (GeV)
M^3 dσ/dM (pb GeV^2)



$\sqrt{\tau} = 0.3$

Not uniform in η

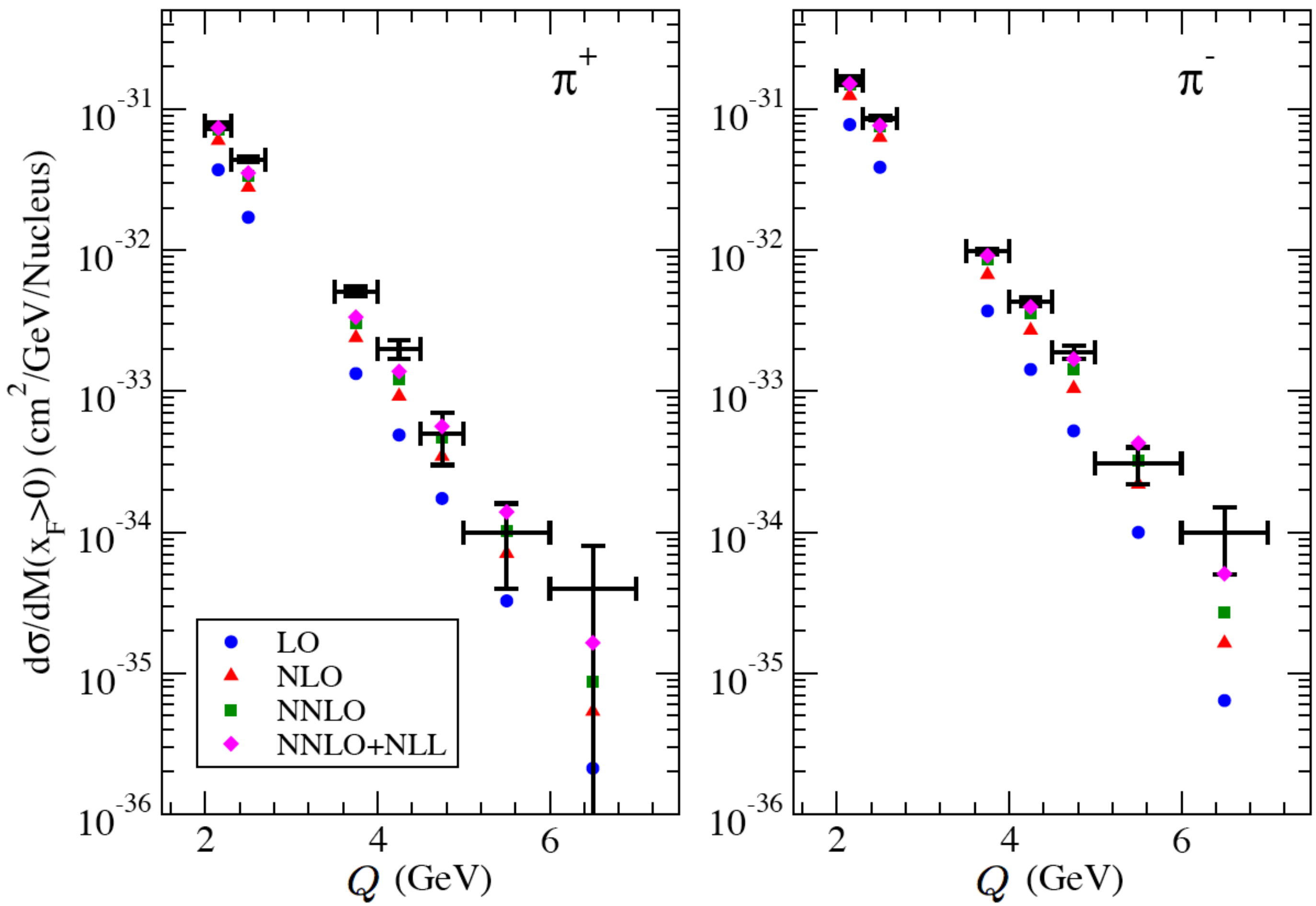
$\sqrt{\tau} = 0.5$



Any evidence for large effects in Drell-Yan ?

$$\pi^{\pm}\mathbf{N} \to \mu^{+}\mu^{-}\mathbf{X} \qquad \mathbf{E}_{\pi} = \mathbf{39.5\ GeV}$$

CERN WA39



Shimizu,Sterman,Yokoya,WV

Conclusions

- understanding of higher-order QCD corrections in Drell-Yan cross section very advanced: NLO, NNLO, resummations to NLL, NNLL

- QCD corrections important, in particular in fixed-target regime

- some open ends: resummation for angular dependence <-> TMDs

- many opportunities for fixed-target DY experiments:
  - pion structure
  - nuclear pdfs?
  - test of higher orders in pert. Theory
  - TMDs