Single transversely polarized DY, observables, TMDs

Aram Kotzinian
Torino Uni&INFN
On leave in absence from YerPhI, Armenia

- Introduction
  - TMD DFs
  - SIDIS
- DY
  - General expression and LO parton model
- Intrinsic transverse parton momenta
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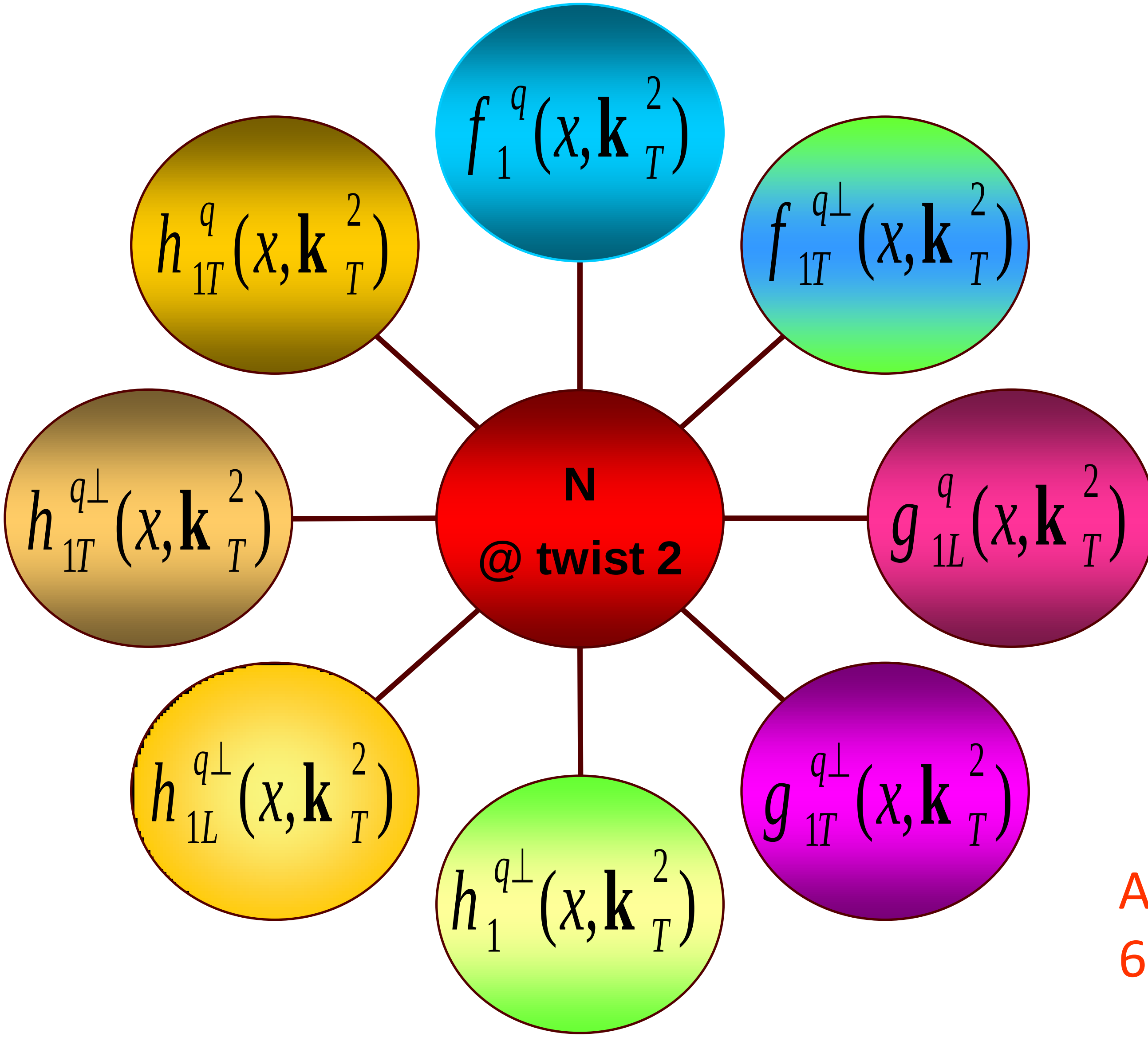
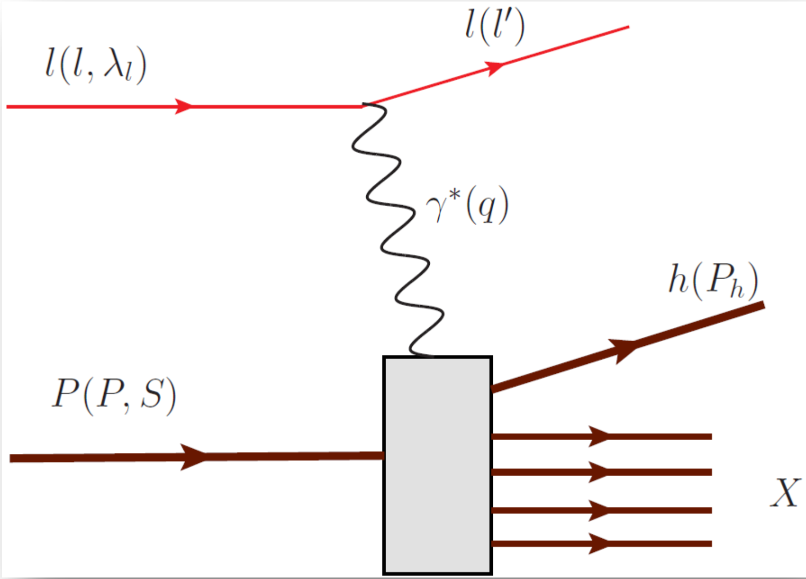
  - Application to Sivers effect in DY
- Discussion

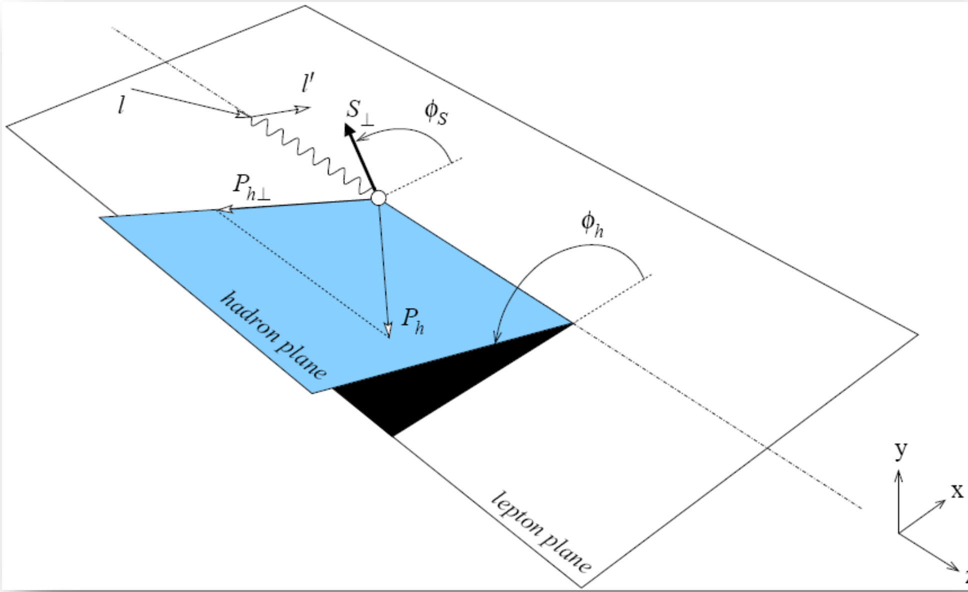
TMD quark DFs of Nucleon



At twist-two we have
6 T-even & 2 T-odd

SIDIS





Using current conservation + parity conservation + hermiticity one can show that 18 independent Structure Functions describe one particle SIDIS cross section in one photon exchange approximation.

Moreover, the dependences on azimuthal angle of produced hadron and of the target nucleon polarization are calculated explicitly and factorized

SIDIS, polarized lepton and target

$$\ell(l,\lambda_\ell)+N(P,S)\to\ell(l')+h(P_h)+X$$

$$\frac{d\sigma}{dx\,dy\,\varphi_{l'}\,dz\,d\phi_h\,dP_{h\perp}^2}=\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h}+S_L\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\Big]$$
$$+S_L\lambda_e\Big[\sqrt{1-\varepsilon^2}\;F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]+S_T\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)$$
$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]+S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\,F_{LT}^{\cos\phi_S}+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

18 structure functions $F_{AB}^{f(\phi_h,\phi_S)}$

$$\varepsilon=\left(1-y-\frac{1}{4}\gamma^2y^2\right)\Big/\left(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2\right),\;\gamma=\frac{2Mx}{Q}$$

SIDIS, unpolarized lepton

$$\ell(l) + N(P,S) \to \ell(l') + h(P_h) + X$$

$$\frac{d\sigma}{dxdy\varphi_{l'}dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{$$

$$F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}$$

$$+S_L\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\Big]$$

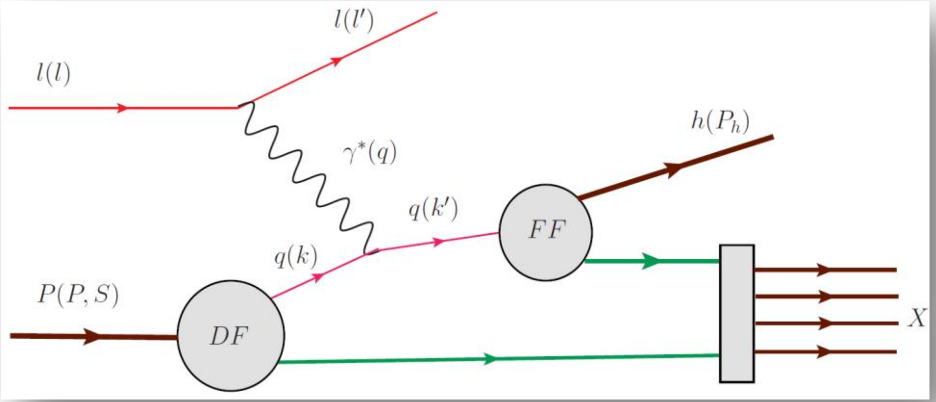
$$+S_T\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]\Big\}$$

12 structure functions $F_{AB}^{f(\phi_h,\phi_S)}$

SIDIS, unpolarized lepton, LO parton model



$$\frac{d\sigma}{dx\,dy\,\varphi_{l'}\,dz\,d\phi_h\,dP_{h\perp}^2}=\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}$$

$$+S_L\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}+S_T\Big[\sin(\phi_h-\phi_S)F_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]\Big\}$$

6 structure functions $F_{AB}^{f(\phi_h,\phi_S)}$

$$F_{UU,T}\propto f_1^q\otimes D_{1q}^h,\quad F_{UU}^{\cos(2\phi_h)}\propto h_1^{\perp q}\otimes H_{1q}^{\perp h},\quad F_{UL}^{\sin(2\phi_h)}\propto h_{1L}^{\perp q}\otimes H_{1q}^{\perp h}$$

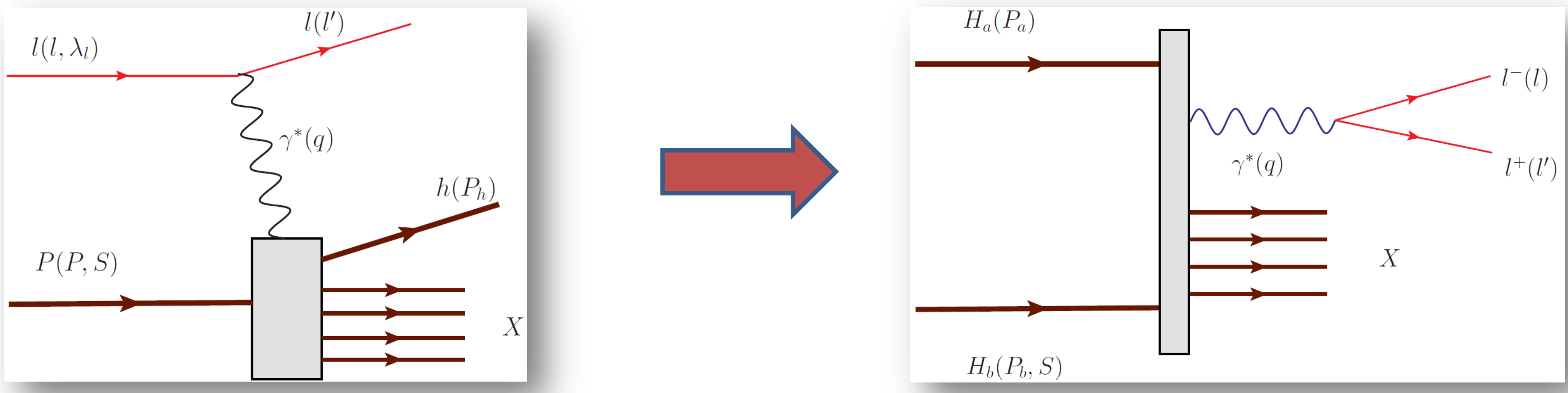
$$F_{UT}^{\sin(\phi_h-\phi_s)}\propto f_{1T}^{\perp q}\otimes D_{1q}^h,\quad F_{UT}^{\sin(\phi_h+\phi_s)}\propto h_1^q\otimes H_{1q}^{\perp h},\quad F_{UT}^{\sin(3\phi_h-\phi_s)}\propto h_{1T}^{\perp q}\otimes H_{1q}^{\perp h}$$

Crossing

$$\ell(l) + N(P,S) \rightarrow \ell(l') + h(P_h) + X$$

$$\Updownarrow$$

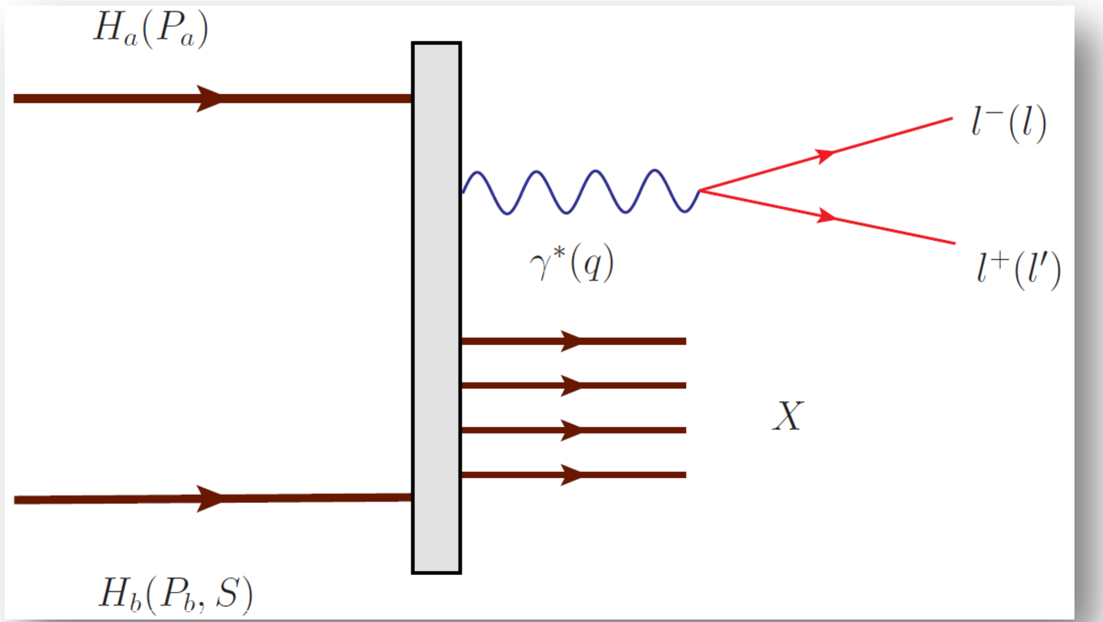
$$\bar{h}(P_h) + N(P,S) \rightarrow \ell(l') + \bar{\ell}(l) + X$$



DY processes

$$H_a(P_a) + H_b(P_b, S) \to \gamma^*(q) + X \to l^-(l) + l^+(l') + X$$

$$\pi^-(P_a) + p(P_b, S) \to \mu^+ + \mu^- + X$$

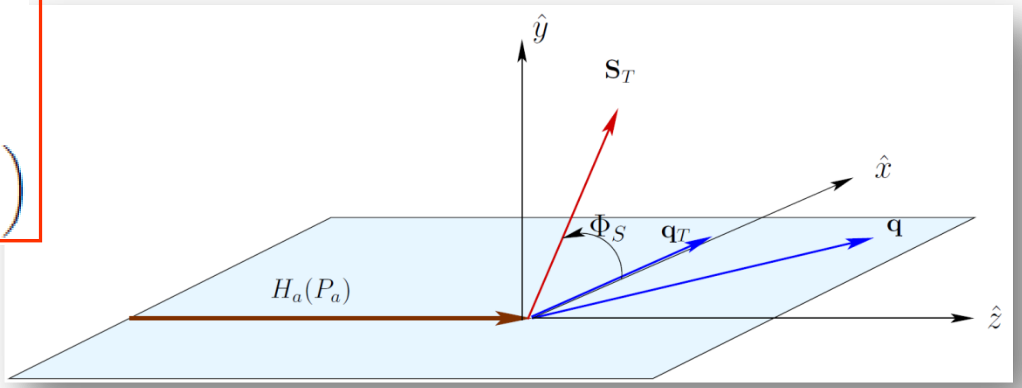


$$q = l + l',\ Q^2 = q^2,\ x_a = \frac{q^2}{2P_a \cdot q},\ x_b = \frac{q^2}{2P_b \cdot q},\ x_F = x_a - x_b$$

Angular variables

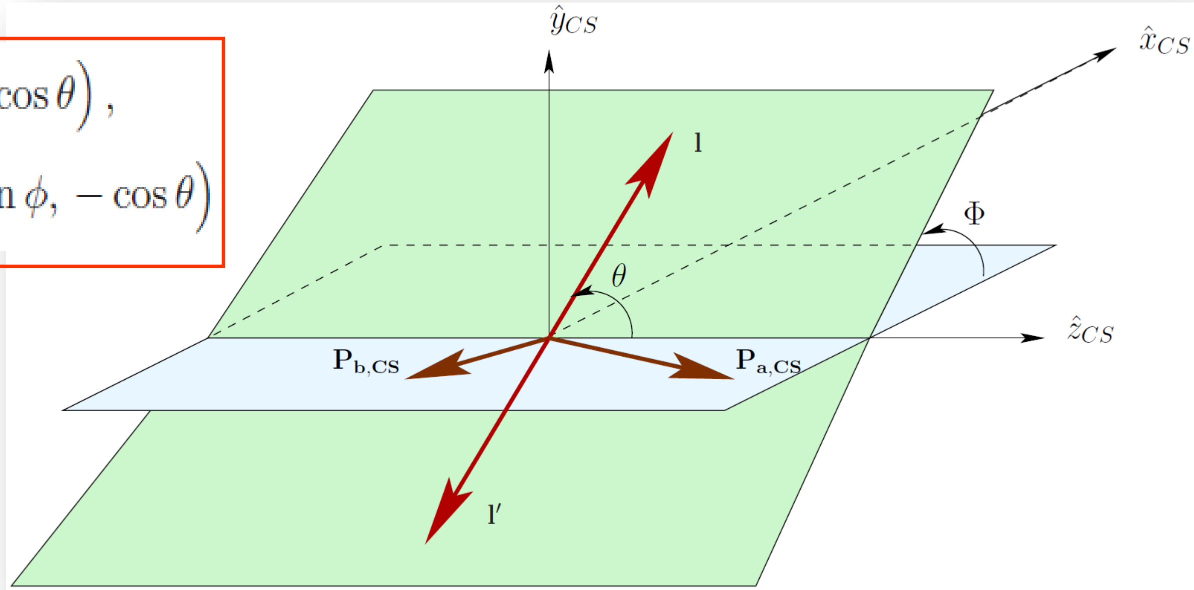
$$
\begin{aligned}
P^{\mu}_{a,TF} &= (E,\, 0,\, 0,\, P^{3}_{a,TF})\,, \\
P^{\mu}_{b,TF} &= (M_b,\, 0,\, 0,\, 0)\,, \\
q^{\mu}_{TF} &= (q_{0,TF},\, q_T,\, 0,\, q_{L,TF})\,, \\
S^{\mu}_{TF} &= \Big(0,\, |\vec{S}_T|\, \cos\phi_S,\, |\vec{S}_T|\, \sin\phi_S,\, S_L\Big)
\end{aligned}
$$

Target rest frame (TF)



$$
\begin{aligned}
l^{\mu}_{CS} &= \frac{q}{2}\Big(1,\, \sin\theta\, \cos\phi,\, \sin\theta\, \sin\phi,\, \cos\theta\Big)\,, \\
l'^{\mu}_{CS} &= \frac{q}{2}\Big(1,\, -\sin\theta\, \cos\phi,\, -\sin\theta\, \sin\phi,\, -\cos\theta\Big)
\end{aligned}
$$

Collins-Soper frame (CS)



General expression for DY cross section (only target is polarized)

S. Arnold, A. Metz and M. Schlegel, PRD 79, 034005 (2009)

48 structure functions if both spin-1/2 hadrons are polarized

$$\pi^-(P_a) + p(P_b, S) \to \mu^+ + \mu^- + X$$

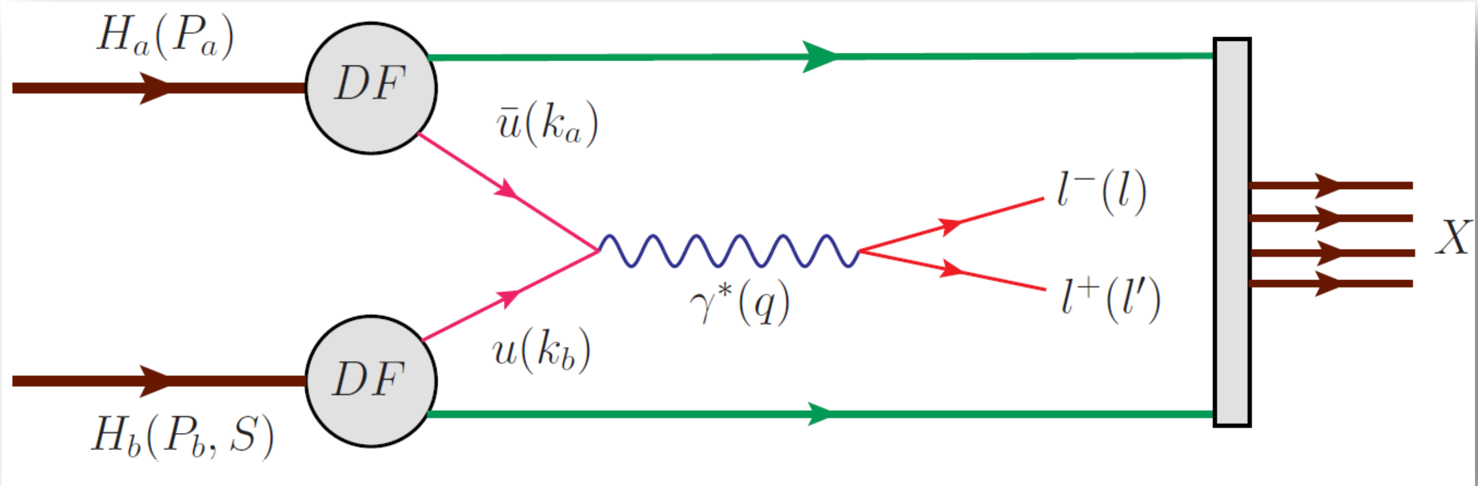
$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{\Phi q^2}\Big\{\Big((1+\cos^2\theta)F_U^1 + (1-\cos^2\theta)F_U^2 + \sin 2\theta F_U^{\cos\phi}\cos\phi + \sin^2\theta F_U^{\cos 2\phi}\cos 2\phi\Big)$$
$$+S_L\Big(\sin 2\theta F_L^{\sin\phi}\sin\phi + \sin^2\theta F_L^{\sin 2\phi}\sin 2\phi\Big)$$
$$+S_T\Big[\Big(F_T^{\sin\phi_S} + \cos^2\theta \tilde{F}_T^{\sin\phi_S}\Big)\sin\phi_S + \sin 2\theta\Big(F_T^{\sin(\phi+\phi_S)}\sin(\phi+\phi_S) + F_T^{\sin(\phi-\phi_S)}\sin(\phi-\phi_S)\Big)$$
$$+\sin^2\theta\Big(F_T^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S) + F_T^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Big\}$$

$\Phi = 4\sqrt{(P_a \cdot P_b)^2 - M_a^2 M_b^2}$ - flux, $F_A^{f(\phi,\phi_S)}(x_a, x_b, q_T, Q^2)$

12 structure functions $F_A^{f(\phi,\phi_S)}$

as expected from crossing symmetry

DY-LO, only transversely polarized target



$$\frac{d\sigma}{d^4q d\Omega} \overset{LO}{=} \frac{\alpha_{em}^2}{Fq^2}\hat{\sigma}_U \Big\{1 + D_{[\sin^2\theta]} A_U^{\cos 2\phi}\cos 2\phi$$

$$+ S_T\Big[A_T^{\sin\phi_S}\sin\phi_S + D_{[\sin^2\theta]}\Big(A_T^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S) + A_T^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Big\}$$

$$\hat{\sigma}_U \overset{LO}{=} F_U^1(1+\cos^2\theta),\ D_{[f(\theta)]} \overset{LO}{=} \frac{f(\theta)}{1+\cos^2\theta}$$

Only 5 structure functions $F_A^{f(\phi,\phi_S)}$

DY-LO asymmetries, transversely polarized target

$$A_U^{\cos 2\phi} \overset{LO}{=} \mathcal{C}\left[\left(2\left(\mathbf{h}\cdot\mathbf{k}_{aT}\right)\left(\mathbf{h}\cdot\mathbf{k}_{bT}\right)-\mathbf{k}_{aT}\cdot\mathbf{k}_{bT}\right) h_1^{\perp} \bar{h}_1^{\perp} \right] \Big/ M_a M_b F_U^1$$

$$A_T^{\sin \phi_S} \overset{LO}{=} \tilde{A}_T^{\sin \phi_S} \overset{LO}{=} \mathcal{C}\left[\mathbf{h}\cdot\mathbf{k}_{bT}\ f_1 \bar{f}_{1T}^{\perp} \right] \Big/ M_b F_U^1$$

$$A_T^{\sin(2\phi+\phi_S)} \overset{LO}{=} -\mathcal{C}\left[2(\mathbf{h}\cdot\mathbf{k}_{bT})[2(\mathbf{h}\cdot\mathbf{k}_{aT})(\mathbf{h}\cdot\mathbf{k}_{bT})-\mathbf{k}_{aT}\cdot\mathbf{k}_{bT}]-\mathbf{k}_{bT}^2(\mathbf{h}\cdot\mathbf{k}_{aT})\ \ h_1^{\perp} \bar{h}_{1T}^{\perp} \right] \Big/ 4 M_a M_b^2 F_U^1$$

$$A_T^{\sin(2\phi-\phi_S)} \overset{LO}{=} -\mathcal{C}\left[\mathbf{h}\cdot\mathbf{k}_{aT} h_1^{\perp} \bar{h}_1 \right] \Big/ 2 M_a F_U^1$$

$$F_U^1 \overset{LO}{=} \mathcal{C}\left[f_a \bar{f}_a \right] \qquad\qquad \mathbf{h} \equiv \frac{\mathbf{q}_T}{q_T}$$

$$\mathcal{C}\left[w(\mathbf{k}_{aT},\mathbf{k}_{bT}) f_1 \bar{f}_2 \right] \equiv \frac{1}{N_c}\sum_q e_q^2 \int d^2\mathbf{k}_{aT} d^2\mathbf{k}_{bT}\, \delta^{(2)}(\mathbf{q}_T-\mathbf{k}_{aT}-\mathbf{k}_{bT}) w(\mathbf{k}_{aT},\mathbf{k}_{bT})$$
$$\times \left[f_1^q(x_a,\mathbf{k}_{aT}^2) f_2^{\bar{q}}(x_b,\mathbf{k}_{bT}^2) + f_1^{\bar{q}}(x_a,\mathbf{k}_{aT}^2) f_2^q(x_b,\mathbf{k}_{bT}^2) \right]$$

Intrinsic transverse momentum in SIDIS

P. Schweitzer, T. Teckentrup and A. Metz, arXiv:1003.2190v1 [hep-ph,]
Comprehensive analysis of existing unpolarized SIDIS and DY data

SIDIS, factorized Gauss model:

$$f_1^a(x, p_T) = f_1^a(x) \frac{\exp(-p_T^2/\langle p_T^2\rangle)}{\pi\langle p_T^2\rangle},$$
$$D_1^a(z, K_T) = D_1^a(z) \frac{\exp(-K_T^2/\langle K_T^2\rangle)}{\pi\langle K_T^2\rangle}$$

$$\frac{\mathrm{d}^4\sigma_{UU}(x,y,z,P_{h\perp})}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{h\perp}^2} = \frac{4\pi^2\alpha^2\,s}{Q^4}\left(1-y+\frac{1}{2}y^2\right)\sum_a e_a^2\,x\,f_1^a(x)D_1^a(z)\,\mathcal{G}(P_{h\perp})$$

$$\mathcal{G}(P_{h\perp}) = \frac{1}{\pi\kappa_T^2(z)}\exp\left(-\frac{P_{h\perp}^2}{\kappa_T^2(z)}\right), \quad \kappa_T^2(z) = z^2\langle p_T^2\rangle + \langle K_T^2\rangle$$

Old extractions: $\langle p_T^2\rangle = \begin{cases} 0.25\,\mathrm{GeV}^2 \\ 0.33\,\mathrm{GeV}^2 \end{cases}$ $\langle K_T^2\rangle = \begin{cases} 0.20\,\mathrm{GeV}^2 \\ 0.16\,\mathrm{GeV}^2 \end{cases}$ Torino: EMC data
Bochum: HERMES

In many analyses, for example, extraction of transversity, BM function from SIDIS and pp and pd DY processes this values are applied

New data from JLab

$$R(P_{h\perp}) \equiv \frac{d^4\sigma_{UU}(x,y,z,P_{h\perp})/dx\,dy\,dz\,dP_{h\perp}^2}{d^4\sigma_{UU}(x,y,z,0)/dx\,dy\,dz\,dP_{h\perp}^2} = \exp\left(-\frac{P_{h\perp}^2}{\kappa_T^2(z)}\right)$$

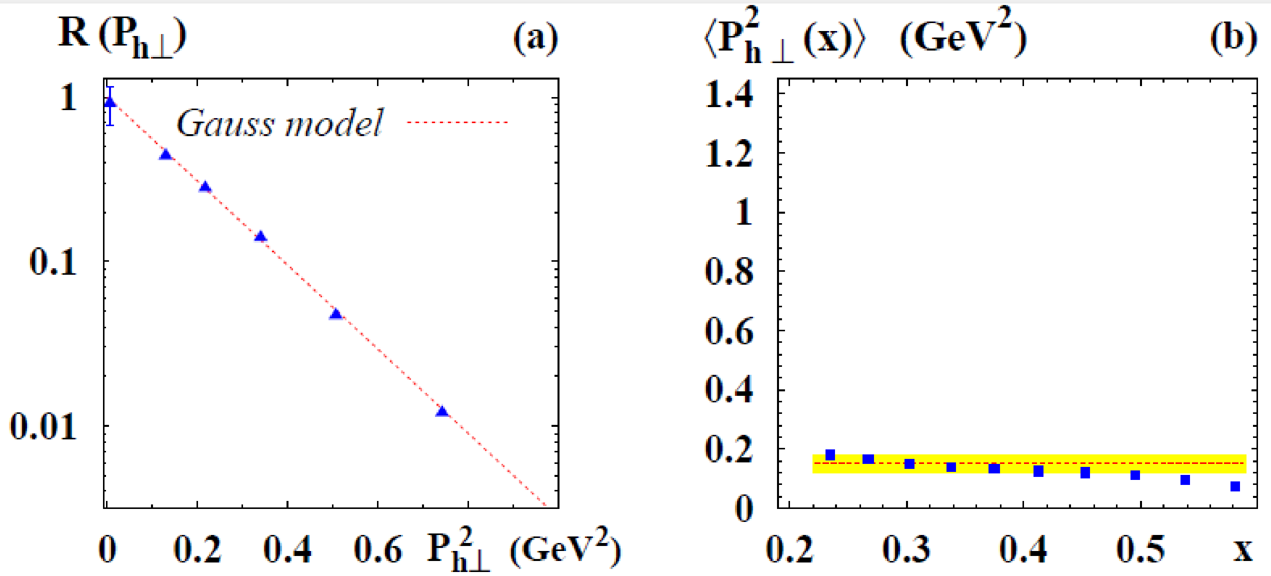


FIG. 2: (a) The ratio $R(P_{h\perp})$ as defined in Eq. (9) as function of the hadron transverse momentum square $P_{h\perp}^2$. The data are for π^+ from CLAS [33]. The dotted line is an effective description in the Gauss model, see text. (b) The average transverse momentum square $\langle P_{h\perp}^2\rangle$ of π^+ produced at $z = 0.34$ and $Q^2 = 2.37\,\mathrm{GeV}^2$ in SIDIS at CLAS [33] as function of x. The dotted line is an effective description in the Gauss model assuming the Gauss width of $f_1^a(x, p_T)$ to be x-independent. This describes data within 20% in the region $0.2 < x < 0.5$ as the shaded region shows.

Test of Gauss model at HERMES

$$\langle P_{h\perp}(z)\rangle^2 = \frac{\pi}{4}\,\langle P_{h\perp}^2(z)\rangle$$

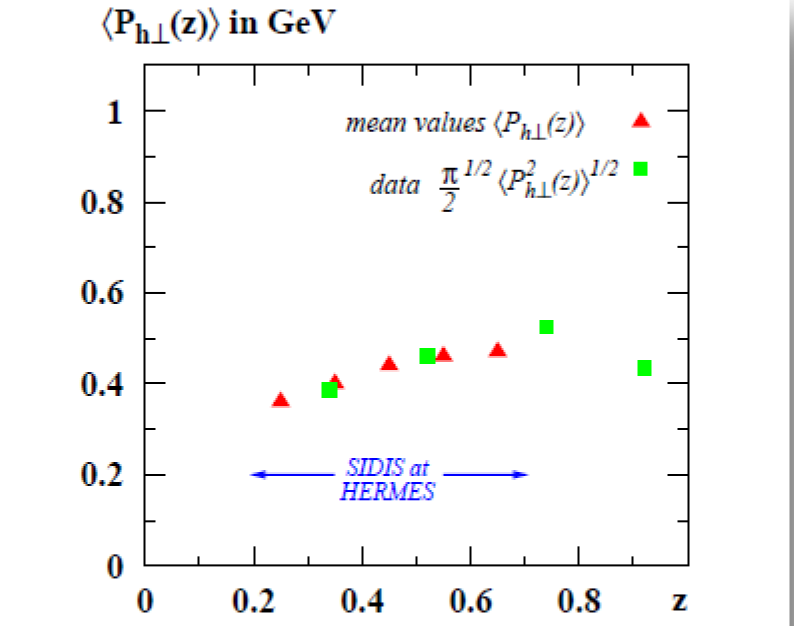


FIG. 4: Transverse momenta of hadrons in SIDIS off deuterium at HERMES vs. z. We compare $\langle P_{h\perp}(z)\rangle$ (triangles) from [39], with $\frac{1}{2}\sqrt{\pi}\,\langle P_{h\perp}^2(z)\rangle^{1/2}$ (squares) from [35]. In the indicated SIDIS range of HERMES these quantities are predicted to coincide in the Gauss model, see Eq. (12).

Parameters from HERMES and cross check with CLAS

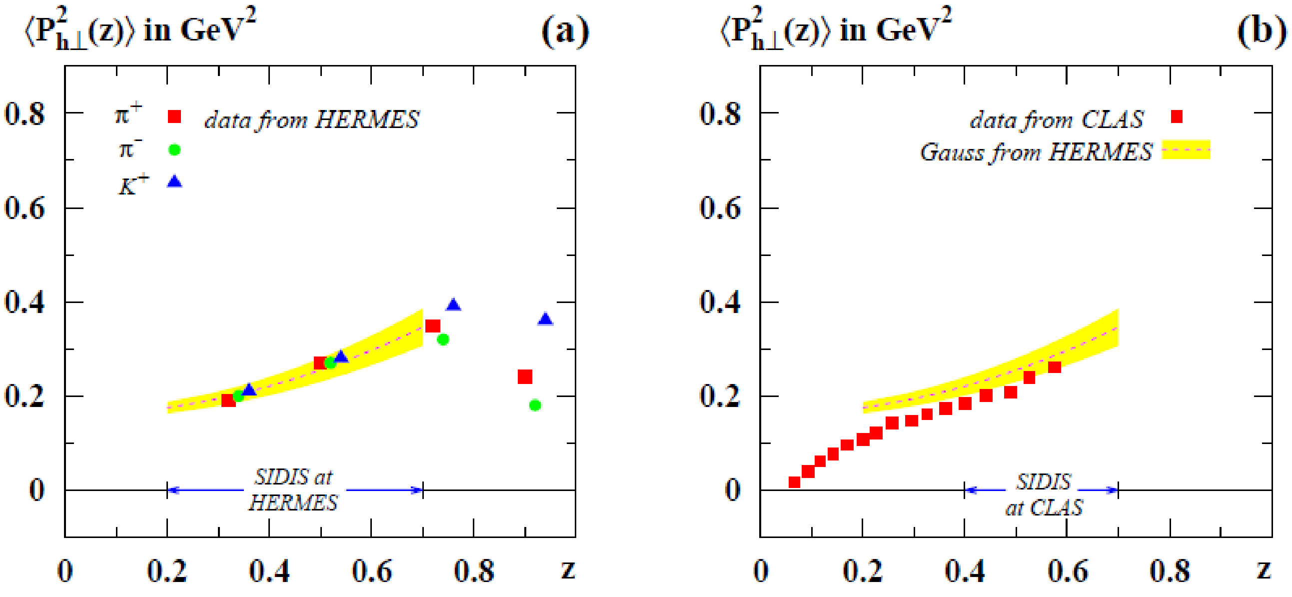


FIG. 5: (a) $\langle P^2_{h\perp}(z)\rangle$ of hadrons in SIDIS off deuterium at HERMES vs. z from [35]. The dotted line and the shaded region are the best fit and its 1–σ region from Eq. (13), see text. (b) $\langle P^2_{h\perp}(z)\rangle$ of π^+ in SIDIS off proton at CLAS vs. z [33]. Dotted line (shaded region) are the best fit (its 1–σ region) to HERMES data from Fig. 5a, see text.

$$\langle p_T^2\rangle = (0.38 \pm 0.06)\,\mathrm{GeV}^2$$
$$\langle K_T^2\rangle = (0.16 \pm 0.01)\,\mathrm{GeV}^2$$

Cahn effect at EMC

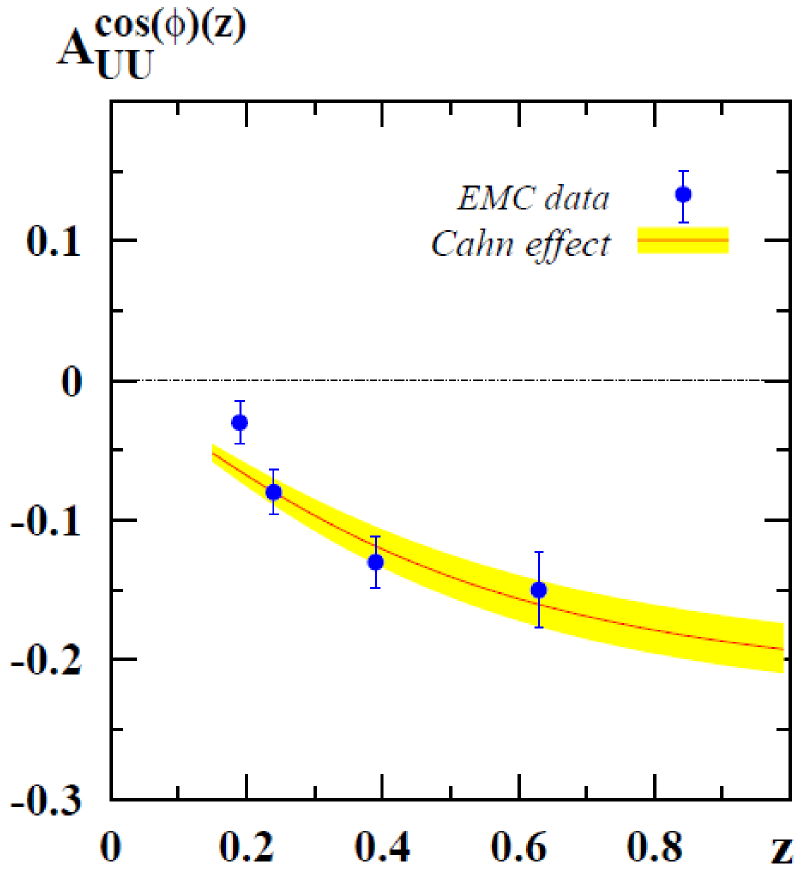


FIG. 6: Azimuthal asymmetry $A_{UU}^{\cos\phi}$ in charged hadron production vs. z. The data are from the EMC experiment [31]. The theoretical curve is the "Cahn-effect-only" approximation for this observable, which is justified under certain assumptions (see text), using the Gauss model with parameters fixed from HERMES, Eq. (13).

Intrinsic transverse momentum in DY: testing Gauss

$$\frac{d^3\sigma_{UU}}{dx_1\,dx_2\,dq_T} = \frac{4\pi\alpha^2}{9Q^2} \sum_{a=u,d,\bar{u},\dots} e_a^2 f_1^a(x_1) f_1^{\bar{a}}(x_2)\, 2q_T\, \frac{\exp(-q_T^2/\kappa_{DY}^2)}{\kappa_{DY}^2}\,, \quad \kappa_{DY}^2 \equiv \langle p_{1T}^2\rangle + \langle p_{2T}^2\rangle$$

$$\langle q_T^2\rangle \stackrel{\text{Gauss}}{=} \frac{4}{\pi}\langle q_T\rangle^2$$

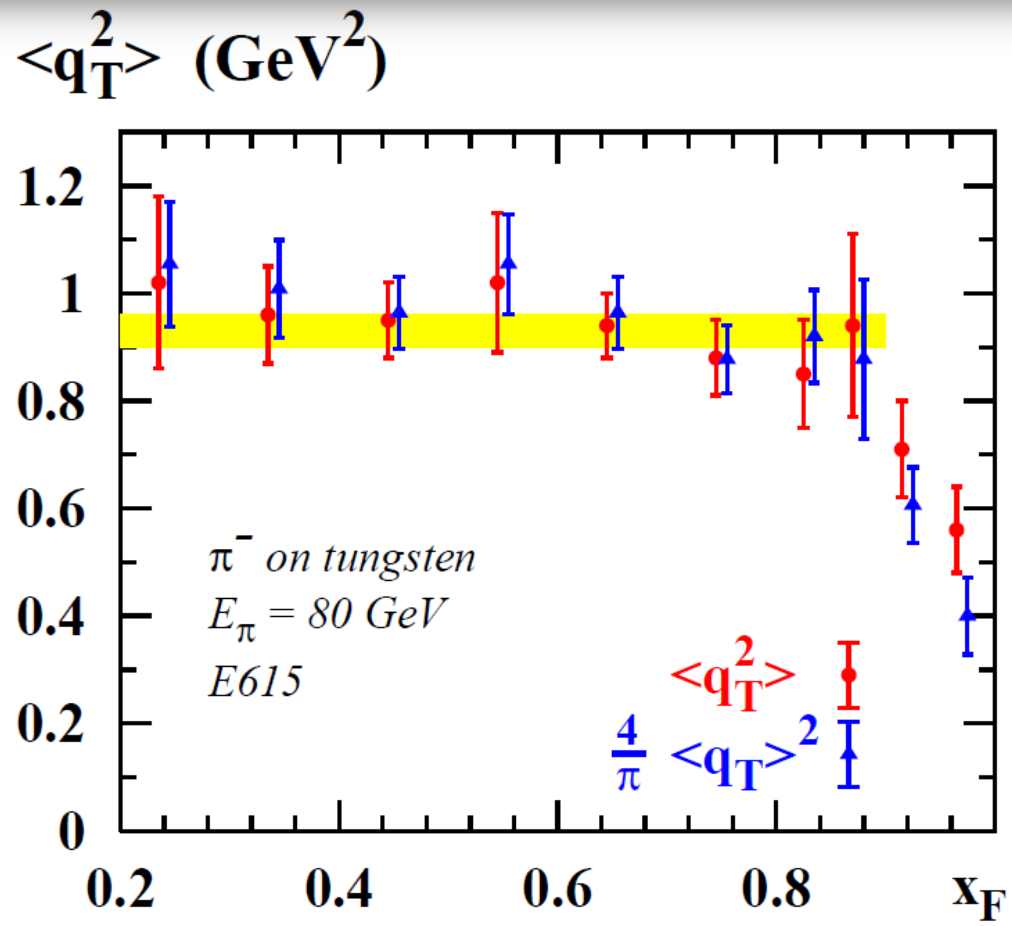


FIG. 7: The mean dimuon transverse momentum square $\langle q_T^2\rangle$ vs. x_F as measured in the Fermilab E615 experiment [52]. The data points for $\langle q_T^2\rangle$ are marked by circles, the data points for $\frac{4}{\pi}\langle q_T\rangle^2$ are marked by triangles. Both quantities are predicted to be equal in the Gauss model, see Eq. (28), which is the case within the statistical accuracy of the data.

Intrinsic transverse momentum in DY: pion vs nucleon

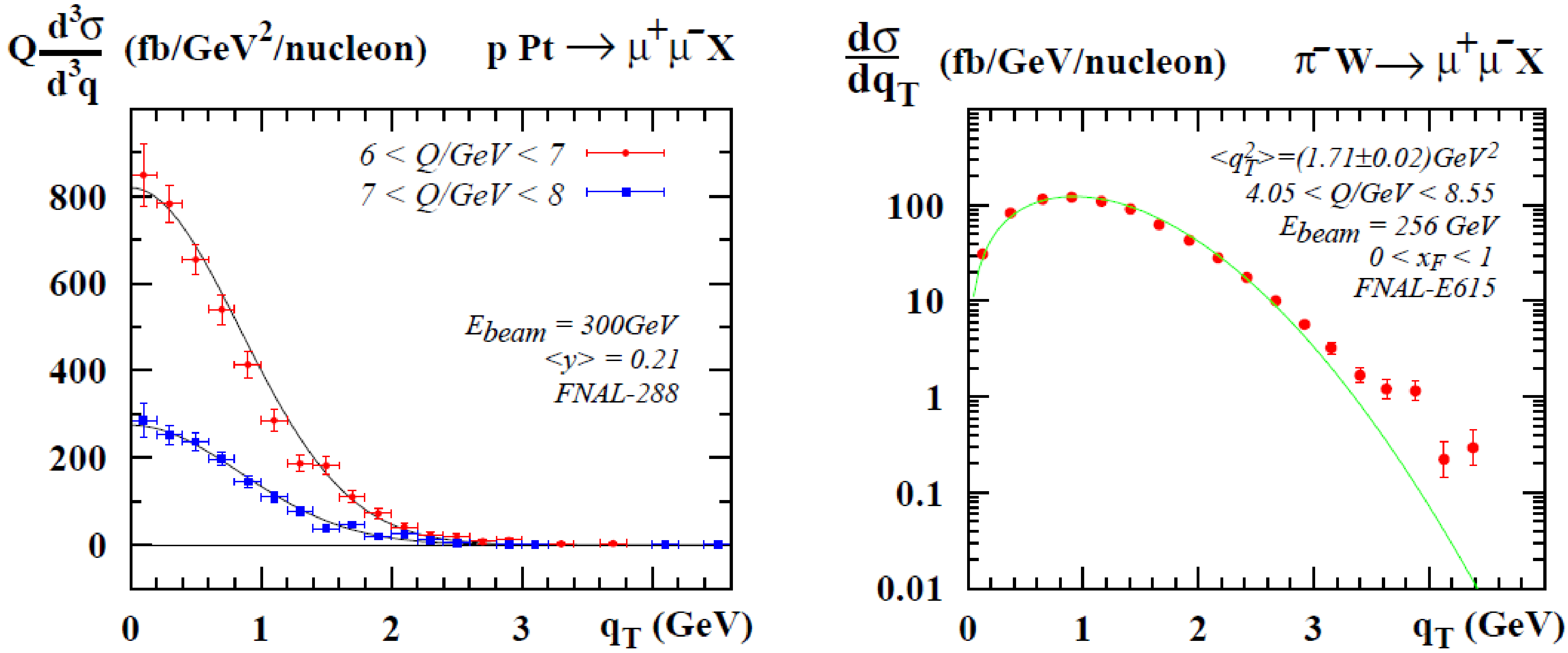


FIG. 8: Left: The invariant differential cross section $Q\frac{d^3\sigma}{d^3q}$ for $p\ Pt \to \mu^+\mu^- X$ at $\langle y\rangle = 0.21$ for two different Q-bins from FNAL-288 [50]. The Gauss model, Eq. (29), provides a good description of the data for $\langle q_T^2\rangle = 1.4\,\text{GeV}^2$.
Right: The differential cross section $\frac{d\sigma}{dq_T}$ for $\pi^- W \to \mu^+\mu^- X$ from FNAL-E615 [55]. Here the Gauss model, Eq. (29), provides a good description of the data up to $q_T \lesssim 3\,\text{GeV}$ with $\langle q_T^2\rangle = 1.7\,\text{GeV}^2$.

$$\kappa_{DY}^2 \overset{\text{here}}{=} 2\,\langle p_{NT}^2\rangle = 1.4\,\text{GeV}^2$$

$$\kappa_{DY}^2 \overset{\text{here}}{=} \langle p_{\pi T}^2\rangle + \langle p_{NT}^2\rangle = 1.7\,\text{GeV}^2$$

$$\langle p_{NT}^2\rangle = 0.7\,\text{GeV}^2\,, \quad \langle p_{\pi T}^2\rangle = 1.0\,\text{GeV}^2 \quad \text{at} \quad \sqrt{s} \sim 23\,\text{GeV}$$

Energy dependence of intrinsic transverse momenta

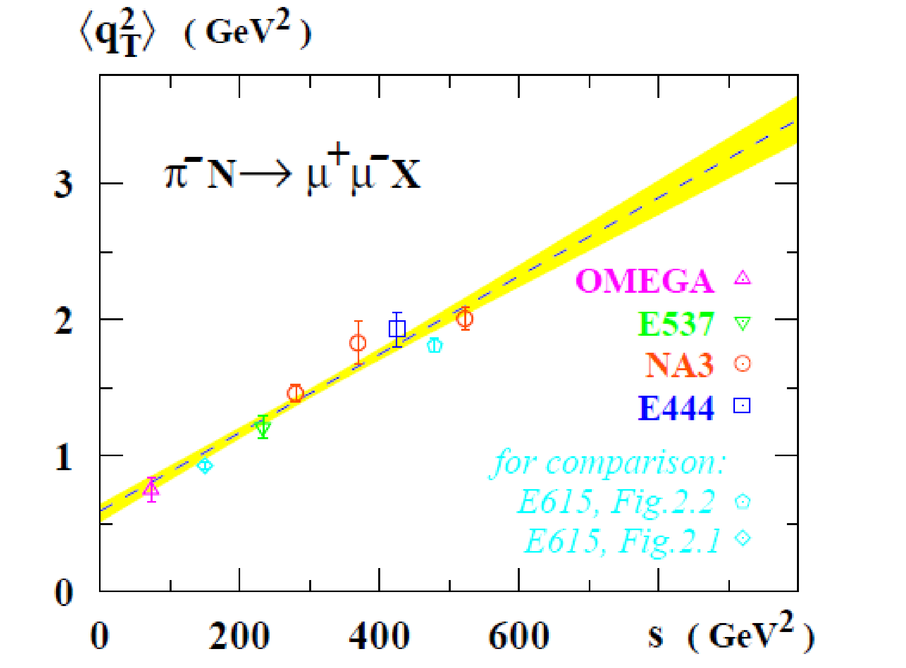


FIG. 10: Mean dimuon transverse momentum square $\langle q_T^2 \rangle$ as function of the center of mass energy square, s, in $\pi^- N$ induced Drell-Yan. Following [125].

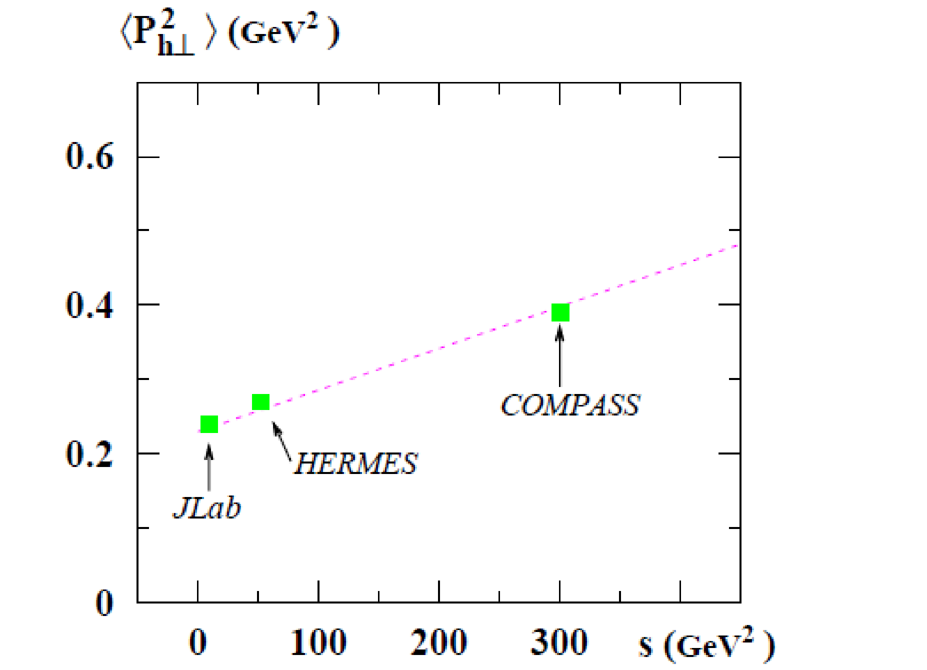


FIG. 11: Mean square transverse momenta $\langle P_{h\perp}^2(z) \rangle$ in SIDIS around $z \sim 0.5$ as function of s from Jefferson Lab [33, 34], HERMES [35], COMPASS [40].

$$\langle p_T^2(s) \rangle_h \approx \langle p_T^2(0) \rangle + C_h \, s$$

$$\langle p_T^2(0) \rangle = 0.3 \, \mathrm{GeV}^2$$

$$C_h = 10^{-3} \times \begin{cases} 2.1 & \text{for } h = \pi \\ 0.7 & \text{for } h = p \end{cases}$$

STM conclusions

- "Gauss model works very well in SIDIS and DY"
  - No evidence for flavor- or x- or z-dependence of Gauss widths
  - Gauss width increase with energy in DY
  - This property holds in SIDIS too
    - "More precise data from SIDIS are needed…"

"The Gauss Anzatz remains to be tested, in particular, when polarization phenomena are included, and future data may demand to refine it, which will improve our understanding of intrinsic transverse parton momenta."

Exercise for Sivers effect in DY@COMPASS

$$\frac{d\sigma}{d^4q\,d\Omega} \overset{LO}{=} \frac{\alpha_{em}^2}{F q^2}\hat{\sigma}_U \Big\{1 + D_{[\sin^2\theta]} A_U^{\cos 2\phi} \cos 2\phi$$

$$+ S_T \Big[A_T^{\sin\phi_S} \sin\phi_S + D_{[\sin^2\theta]} \Big(A_T^{\sin(2\phi+\phi_S)} \sin(2\phi+\phi_S) + A_T^{\sin(2\phi-\phi_S)} \sin(2\phi-\phi_S) \Big) \Big] \Big\}$$

$$A_T^{\sin\phi_S} \overset{LO}{=} \tilde{A}_T^{\sin\phi_S} \overset{LO}{=} \mathrm{C}\left[\mathbf{h}\cdot\mathbf{k}_{bT}\; f_1\, \bar{f}_{1T}^{\perp} \right] \Big/ M_b F_U^1$$

$$F_U^1 \overset{LO}{=} \mathrm{C}\left[f_a\, \bar{f}_a \right]$$

$$\mathrm{C}\left[w(\mathbf{k}_{aT}, \mathbf{k}_{bT})\, f_1 \bar{f}_2 \right] \equiv \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{aT}\, d^2\mathbf{k}_{bT}\, \delta^{(2)}(\mathbf{q}_T - \mathbf{k}_{aT} - \mathbf{k}_{bT})\, w(\mathbf{k}_{aT}, \mathbf{k}_{bT})$$

$$\times \left[f_1^q(x_a, \mathbf{k}_{aT}^2)\, f_2^{\bar{q}}(x_b, \mathbf{k}_{bT}^2) + f_1^{\bar{q}}(x_a, \mathbf{k}_{aT}^2)\, f_2^q(x_b, \mathbf{k}_{bT}^2) \right]$$

Unpolarized PDFs: GRV-P1 pion PDFs (Zeit.Phys.C53(1992)651) and GRV98 LO for proton

Parameterization of TMDs

$$f_1^q(x,\mathbf{k}_\mathbf{T}^2) = f_1^q(x)\frac{\exp\left(-\mathbf{k}_\mathbf{T}^2/\mathbf{k}_{UT}^2\right)}{\pi\mathbf{k}_{UT}^2}$$

$$f_{1T}^{q\perp}(x,\mathbf{k}_\mathbf{T}^2) = f_{1T}^{q\perp}(x)\frac{\exp\left(-\mathbf{k}_\mathbf{T}^2/\mathbf{k}_{ST}^2\right)}{\pi\mathbf{k}_{ST}^2}$$

Torino parameterization for Sivers function (with changed sign for DY case)

$$f_{1T}^{q\perp}(x,\mathbf{k}_\mathbf{T}^2) = \frac{M}{|\mathbf{k}_\mathbf{T}|}\mathcal{N}_q(x)h(\mathbf{k}_\mathbf{T})f_1^q(x,\mathbf{k}_\mathbf{T}^2)$$

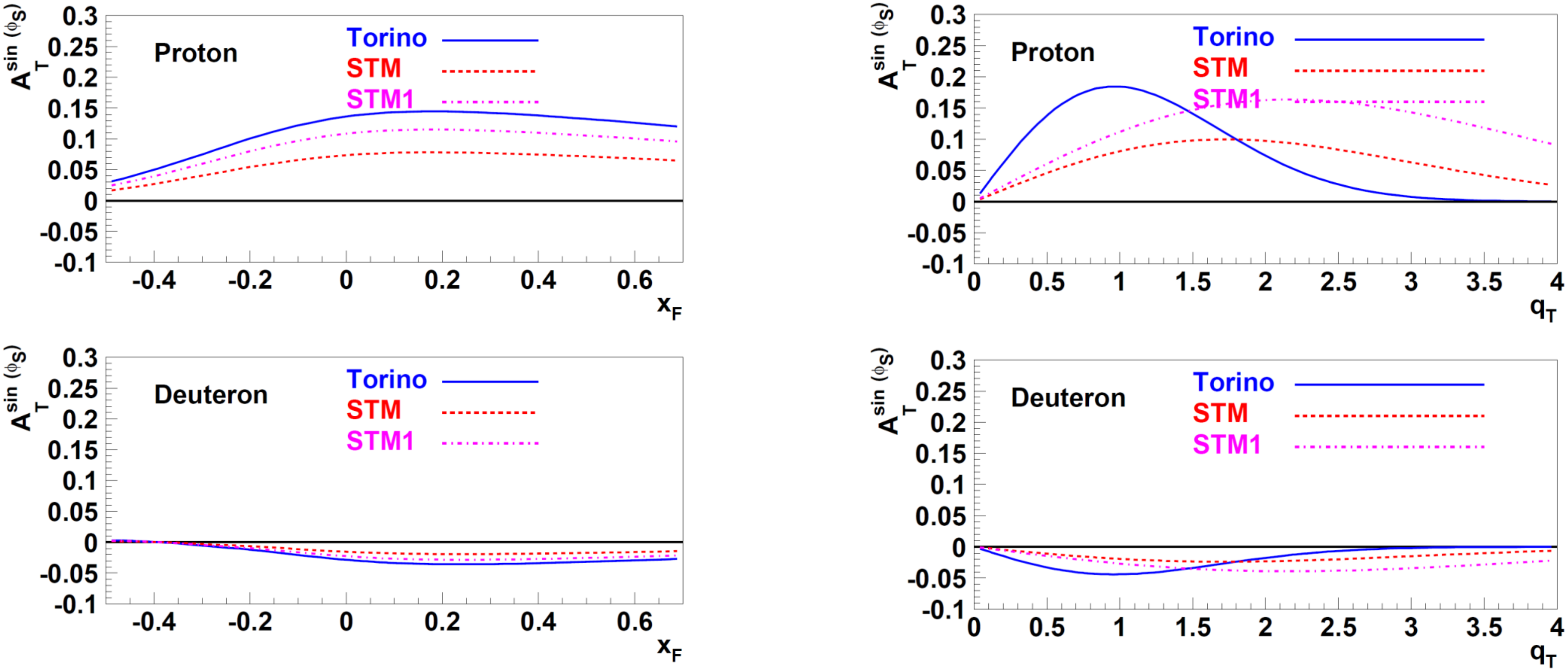
$$\mathcal{N}_q(x) = N_q\left(\frac{\alpha_q+\beta_q}{\alpha_q^{\alpha_q}+\beta_q^{\beta_q}}\right)^{\alpha_q+\beta_q} x^{\alpha_q}\left(1-x\right)^{\beta_q},\; h(\mathbf{k}_\mathbf{T}) = \sqrt{2e}\frac{|\mathbf{k}_\mathbf{T}|}{M_1}\exp\left(-\mathbf{k}_\mathbf{T}^2/M_1^2\right)$$

$$f_{1T}^{q\perp}(x,\mathbf{k}_\mathbf{T}^2) = \sqrt{2e}\frac{MM_1}{M_1^2+\mathbf{k}_{UTb}^2}N_q\left(\frac{\alpha_q+\beta_q}{\alpha_q^{\alpha_q}+\beta_q^{\beta_q}}\right)^{\alpha_q+\beta_q} x^{\alpha_q}\left(1-x\right)^{\beta_q} f_1^q(x)\frac{\exp\left(-\mathbf{k}_\mathbf{T}^2/\mathbf{k}_{ST}^2\right)}{\pi\mathbf{k}_{ST}^2}$$

$$\mathbf{k}_{ST}^2 = \frac{M_1^2\mathbf{k}_{UTb}^2}{M_1^2+\mathbf{k}_{UTb}^2}$$

Extracted parameters (Melis talk) $\mathbf{k}_{UTb}^2,\ M_1,\ N_q,\ \alpha_q,\ \beta_q$

Torino-STM



Torino	STM	STM1
$\mathbf{k}_{UTb}^2 = 0.25\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{UTb}^2 = 0.55\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{UTb}^2 = 0.55\ (\mathrm{GeV/c})^2$
$\mathbf{k}_{UTa}^2 = 0.25\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{UTa}^2 = 1.05\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{UTa}^2 = 1.05\ (\mathrm{GeV/c})^2$
$M_1^2 = 0.34\ (\mathrm{GeV/c})^2$	$M_1^2 = 0.34\ (\mathrm{GeV/c})^2$	$M_1^2 = 0.748\ (\mathrm{GeV/c})^2$
$\mathbf{k}_{ST}^2 = 0.144\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{ST}^2 = 0.21\ (\mathrm{GeV/c})^2$	$\mathbf{k}_{ST}^2 = 0.317\ (\mathrm{GeV/c})^2$

Discussion

- What is the origin of energy dependence of Gauss width?
- SIDIS at COMPASS can provide at higher than HERMES and Jlab energy
  - new acceptance corrected data on unpolarized TMD DFs
  - new data on Sivers asymmetry
    - to check the energy dependence of the Gauss width of unpolarized DFs and FFs
    - Energy dependence of parameters of Sivers DF
- First DY measurements with transversely polarized target at COMPASS will allow to study the universality of spin dependent TMD DFs
    - Size and shape of Sivers DY asymmetry dependence on x_F and $\boldsymbol{q}_T$ will are sensitive to Gaussian width of Sivers function at high energy and Q^2 and allow to understand if
    - this width is energy dependent?
      - What is universal: the Sivers PDF or the ratio of Sivers PDF to unpolarized one?