General form of the DY-cross section

Marc Schlegel

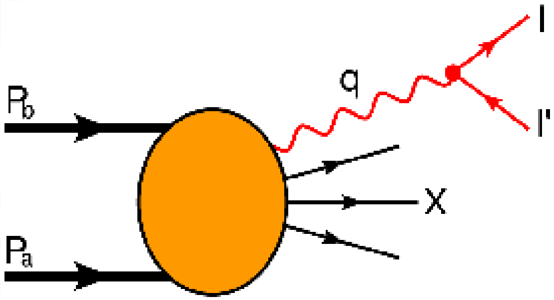
Institute for Theoretical Physics
University of Tuebingen

in collaboration with Werner Vogelsang

"Studying the hadron structure in DY-reactions", COMPASS

The Drell-Yan process

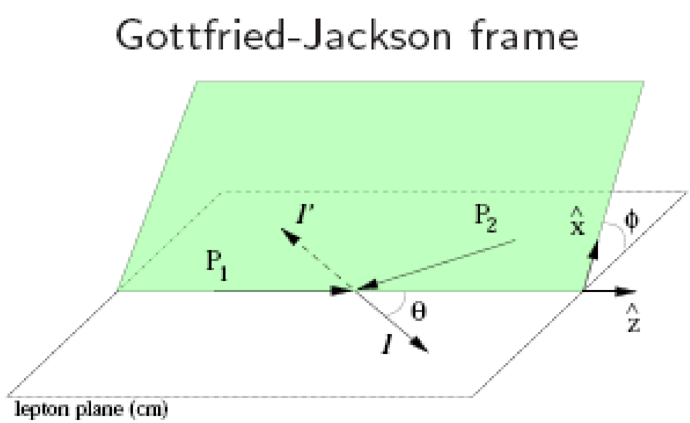
Kinematics (lepton pair produced by one decaying gauge boson):

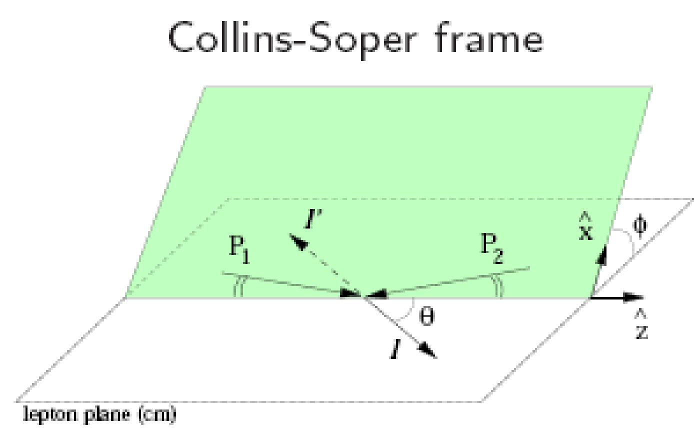


$$\frac{d\sigma}{d^4l\,d^4l'} = \frac{d\sigma}{d^4q\,d^4l} \propto \frac{\delta^+(l^2)\delta^+((q-l)^2)}{4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Disentangling the δ-functions → Dilepton rest frame

Gottfried-Jackson frame and Collins-Soper frame:





Lepton angles:

$$d^4l \to d\Omega = d\phi\, d\cos\theta$$

Diff. CS including angular dependences:

$$\frac{d^6\sigma}{d^4q\,d\Omega} = 2\frac{d^6\sigma}{dy\,dQ^2\,d^2\vec{q}_T d\Omega}$$

Angular structure functions

Separation of the leptonic part (generated by one photon):

$$\frac{d\sigma}{d^4q\,d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

with: $L_{\mu\nu} = 4\Big(l_\mu l'_\nu + l_\nu l'_\mu - \frac{Q^2}{2} g_{\mu\nu}\Big)$ → **Limited number of structure function**

Hadronic Tensor:

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \int \frac{d^4x}{(2\pi)^4}\, \mathrm{e}^{iq\cdot x} \langle a,b | J^\mu(0) J^\nu(x) | a,b\rangle$$

Parameterization constraint by current conservation, hermiticity and parity

Decomposition into 4 + 8 +8 + 28 = 48 structure functions $F(x_a, x_b, q_T^2, Q^2)$

[Arnold, Metz, M.S., PRD 79, 034005] **e.g. unpolarized Drell-Yan**

$$\frac{d\sigma_{UU}}{dx_a\, dx_b\, d^2q_T\, d\Omega} = \frac{\alpha^2 s}{2Q^2 F}\Big((1+\cos^2\Theta) F_{UU}^1 + (1-\cos^2\Theta) F_{UU}^2 + \sin 2\Theta \cos\phi F_{UU}^{\cos\phi} + \sin^2\Theta \cos 2\phi F_{UU}^{\cos 2\phi}\Big)$$

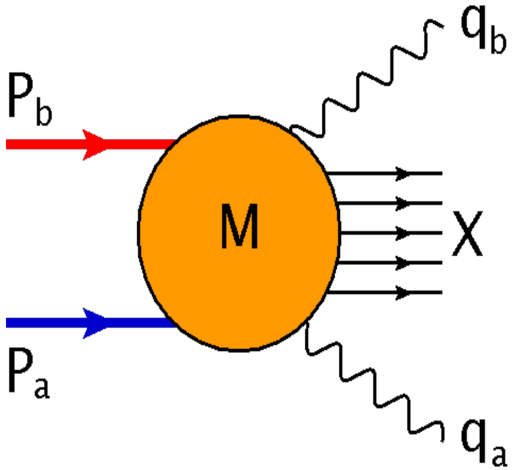
Classification of structure functions helpful for data analysis
→ *Parton model*: 24 leading twist structure functions

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2}{Fq^2}\{&((1+\cos^2\theta)F^1_{UU} + (1-\cos^2\theta)F^2_{UU} + \sin 2\theta\cos\phi F^{\cos\phi}_{UU} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{UU})\\
&+ S_{aL}(\sin 2\theta\sin\phi F^{\sin\phi}_{LU} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{LU}) + S_{bL}(\sin 2\theta\sin\phi F^{\sin\phi}_{UL} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{UL})\\
&+ |\vec{S}_{aT}|[\sin\phi_a((1+\cos^2\theta)F^1_{TU} + (1-\cos^2\theta)F^2_{TU} + \sin 2\theta\cos\phi F^{\cos\phi}_{TU} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{TU})\\
&+ \cos\phi_a(\sin 2\theta\sin\phi F^{\sin\phi}_{TU} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{TU})] + |\vec{S}_{bT}|[\sin\phi_b((1+\cos^2\theta)F^1_{UT} + (1-\cos^2\theta)F^2_{UT}\\
&+ \sin 2\theta\cos\phi F^{\cos\phi}_{UT} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{UT}) + \cos\phi_b(\sin 2\theta\sin\phi F^{\sin\phi}_{UT} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{UT})]\\
&+ S_{aL}S_{bL}((1+\cos^2\theta)F^1_{LL} + (1-\cos^2\theta)F^2_{LL} + \sin 2\theta\cos\phi F^{\cos\phi}_{LL} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{LL})\\
&+ S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1+\cos^2\theta)F^1_{LT} + (1-\cos^2\theta)F^2_{LT} + \sin 2\theta\cos\phi F^{\cos\phi}_{LT} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{LT})\\
&+ \sin\phi_b(\sin 2\theta\sin\phi F^{\sin\phi}_{LT} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{LT})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)F^1_{TL} + (1-\cos^2\theta)F^2_{TL}\\
&+ \sin 2\theta\cos\phi F^{\cos\phi}_{TL} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{TL}) + \sin\phi_a(\sin 2\theta\sin\phi F^{\sin\phi}_{TL} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{TL})]\\
&+ |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a+\phi_b)((1+\cos^2\theta)F^1_{TT} + (1-\cos^2\theta)F^2_{TT} + \sin 2\theta\cos\phi F^{\cos\phi}_{TT} + \sin^2\theta\cos 2\phi F^{\cos 2\phi}_{TT})\\
&+ \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}^1_{TT} + (1-\cos^2\theta)\bar{F}^2_{TT} + \sin 2\theta\cos\phi \bar{F}^{\cos\phi}_{TT} + \sin^2\theta\cos 2\phi \bar{F}^{\cos 2\phi}_{TT})\\
&+ \sin(\phi_a+\phi_b)(\sin 2\theta\sin\phi F^{\sin\phi}_{TT} + \sin^2\theta\sin 2\phi F^{\sin 2\phi}_{TT})\\
&+ \sin(\phi_a-\phi_b)(\sin 2\theta\sin\phi \bar{F}^{\sin\phi}_{TT} + \sin^2\theta\sin 2\phi \bar{F}^{\sin 2\phi}_{TT})]\}.
\end{aligned}
$$

: Leading twist in TMD – parton model $F = F(y, Q^2, q_T^2)$

- p – p scattering: relations between structure functions, e.g. $A_{UT} = -A_{TU}$
- p – _ scattering: no double polarization
- Integration over CS-angles: 8 structure functions survive

Diphoton production



Two highly energetic real photons produced with

$$q \equiv q_a + q_b$$

$$\frac{d\sigma}{d^4q_a\, d^4q_b} = \frac{d\sigma}{d^4q\, d^4q_a} \propto \frac{\delta^+(q_a^2)\delta^+((q-q_a)^2)}{2\times 4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Convenient choice: Diphoton rest frame → **Collins-Soper frame** $\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a}$

Unfortunately: No separation into ***hadronic – photonic*** parts possible!
→ *all* angular modulations are allowed, in principle.

$$\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm}(y, Q^2, q_T^2)\, Y_{lm}(\Omega_a)$$

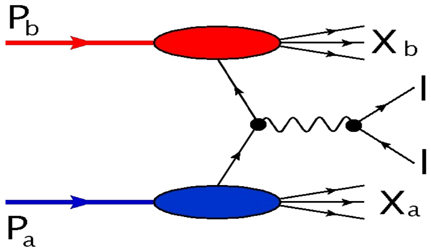
$$C_{00} = \frac{d^4\sigma}{dy\, dQ^2\, d^2q_T}, \ldots$$

However, we can calculate the cross section in the parton model.

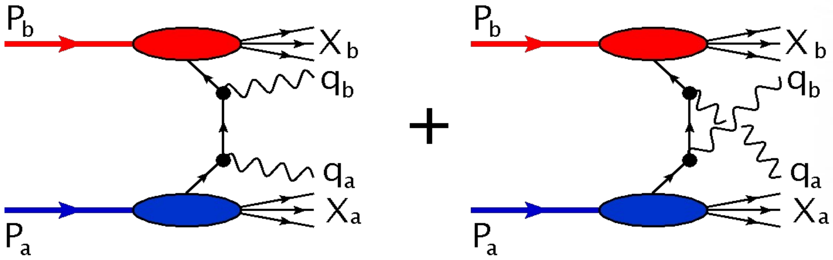
TMD tree-level formalism

Parton model tree-level at $O(\alpha_s^0)$:

Drell-Yan dilepton production:



Diphoton production:



Only relevant at very small q_T: $\Lambda_{QCD} \sim q_T \ll Q$

$$\left(\frac{d\sigma}{d^4q\, d\Omega}\right) \propto \int d^2k_{aT} \int d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T)\, \mathrm{Tr}\left[\Phi(x_a, \vec{k}_{aT})\, H(x_a, x_b, q_a, q_b)\, \bar{\Phi}(x_b, \vec{k}_{bT})\, H^\dagger\right] + \mathcal{O}(\tfrac{M}{Q})$$

k_T - correlator:

$$\Phi_{ij}(x, \vec{k}_T) = \int \frac{dz^- d^2z_T}{(2\pi)^2}\, e^{ik\cdot z} \langle P, S|\, \bar{q}_j(0)\, \mathcal{W}^{?/DY}[0\,;\,z]\, q_i(z)\, |P, S\rangle\Big|_{z^+=0}$$

→ can be parameterized in terms of TMDs according to quark / nucleon spin

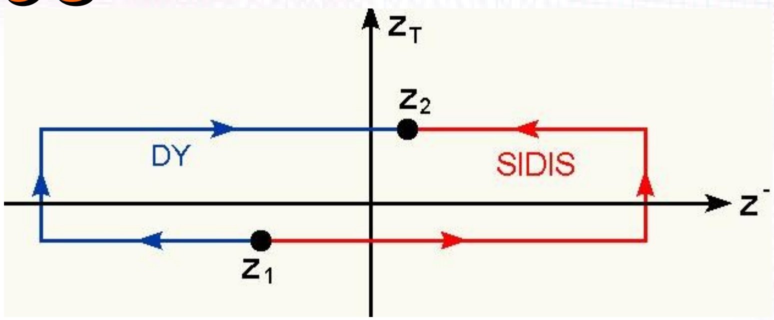
Main result of the TMD tree-level formalism:

$$\left(\frac{d^6\sigma^{hh\to\gamma\gamma X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q) = \frac{2}{\sin^2\theta}\left(\frac{d\sigma^{hh\to l^+l^-X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q\,|\, e_q \to e_q^2)$$

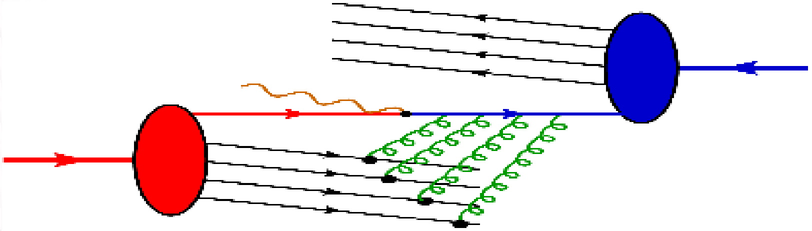
Wilson lines

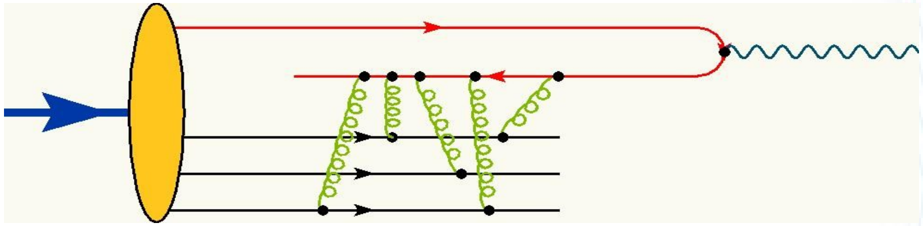
Wilson line process-dependent in DY/SIDIS:

$$\mathcal{W}[z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$

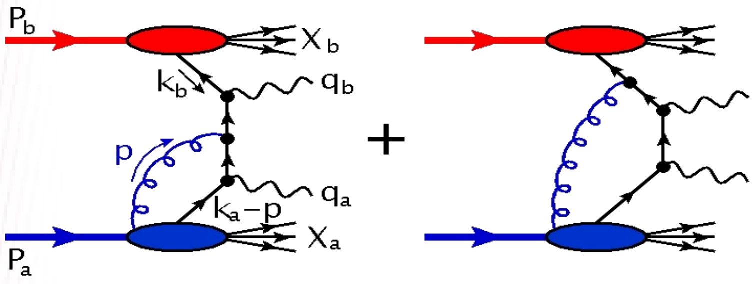


Physics: Initial / Final state interactions





Wilson line in diphoton production:



+crossed

Check for $A^+,\ A_T^i(z^- = -\infty)$

Diagrams topologically different to DY, *but* cancellations between diagrams

$$\mathcal{W}^{\gamma\gamma}[0;z]\Big|_{z^+=0} = 1 - ig\int_0^{-\infty} d\lambda\, A^+(\lambda n) - ig\int_{0_T}^{z_T} d\vec{y}_T\cdot\vec{A}_T(-\infty,0,\vec{y}_T) - ig\int_{-\infty}^{0} d\lambda\, A^+(\lambda n + z_T) + \mathcal{O}(g^2) = \mathcal{W}^{DY}[0;z]\Big|_{z^+=0}$$

Example: Sivers effect

k_T – correlator for unpolarized quarks:

$$\frac{1}{2}\mathrm{Tr}[\Phi(x,\vec{k}_T)\,\gamma^+] = f_1(x,\vec{k}_T^2) - \frac{\epsilon_T^{ij}\,k_T^i\,S_T^j}{M}\,f_{1T}^\perp(x,\vec{k}_T^2)$$

Sivers function → time-reversal odd → sign switch:

$$f_{1T}^\perp\Big|_{DIS} = -f_{1T}^\perp\Big|_{DY}$$

Can be determined from SIDIS data of a transverse target SSA (HERMES, COMPASS):

$$A_{UT}^{Siv} \sim \frac{f_{1T}^\perp \otimes D_1}{f_1 \otimes D_1}$$

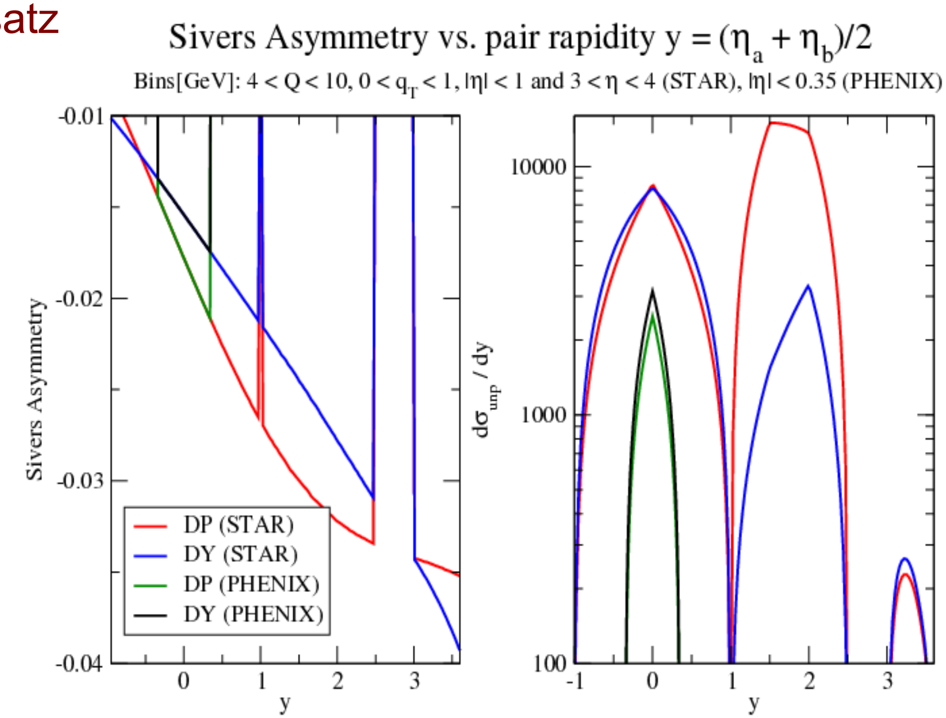
k_T – deconvolution through Gaussian ansatz

$$f(x,\vec{k}_T^2) \;=\; f(x)\exp\left[-\vec{k}_T^2/\langle k_T^2\rangle\right]$$

Fit of the Sivers function to data:
[Anselmino et al., EPJA 39, 89],
[Schweitzer, M.S., 0805.2137 and in prep.]

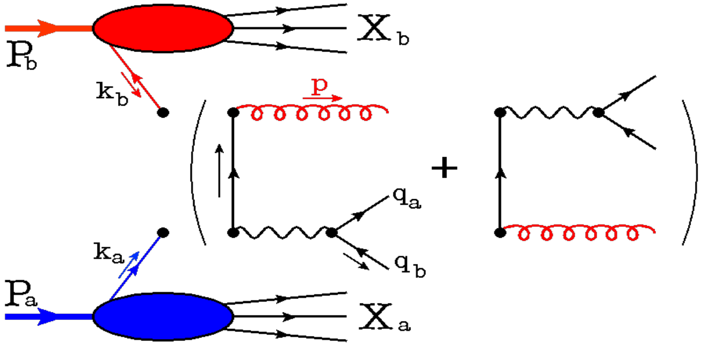
Sivers effect in Diphoton/DY process:

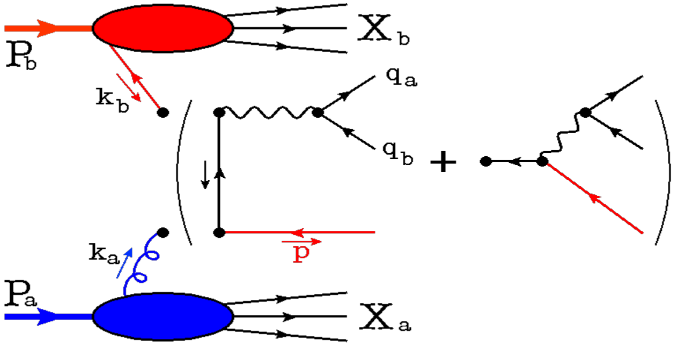
$$A_{TU}^{Siv,DP/DY} \sim \frac{2/\sin^2\theta\; f_{1T}^\perp \otimes \bar{f}_1}{2/\sin^2\theta\; f_1 \otimes \bar{f}_1}$$



High - q_T behaviour

At large $q_T \sim Q \rightarrow$ transverse momentum generated by gluon radiation





Unpolarized angular dependencies in Drell-Yan:

$$\frac{d\sigma_{UU}}{dx_a\, dx_b\, d^2q_T\, d\Omega} = \frac{\alpha^2 s}{2Q^2 F}\Big((1+\cos^2\Theta)F_{UU}^1 + (1-\cos^2\Theta)F_{UU}^2 + \sin 2\Theta\cos\phi F_{UU}^{\cos\phi} + \sin^2\Theta\cos 2\phi F_{UU}^{\cos 2\phi}\Big)$$

Collinear parton model applicable [Boer, Vogelsang, PRD74, 014004]

$$\frac{F^i}{x_a x_b} \sim \int\frac{d\xi_a}{\xi_a}\int\frac{d\xi_b}{\xi_b}\,\delta\Big((\xi_a - x_a)(\xi_b - x_b) - \frac{q_T^2}{s})\Big)\sum_{q,\bar{q}} e_q^2\Big(f_1^q(\xi_a)f_1^{\bar{q}}(\xi_b)\hat{F}^i_{q\bar{q}} + f_1^q(\xi_a)f_1^g(\xi_b)\hat{F}^i_{qg} + f_1^g(\xi_a)f_1^{\bar{q}}(\xi_b)\hat{F}^i_{g\bar{q}}\Big)$$

Partonic structure functions automatically obey angular decomposition

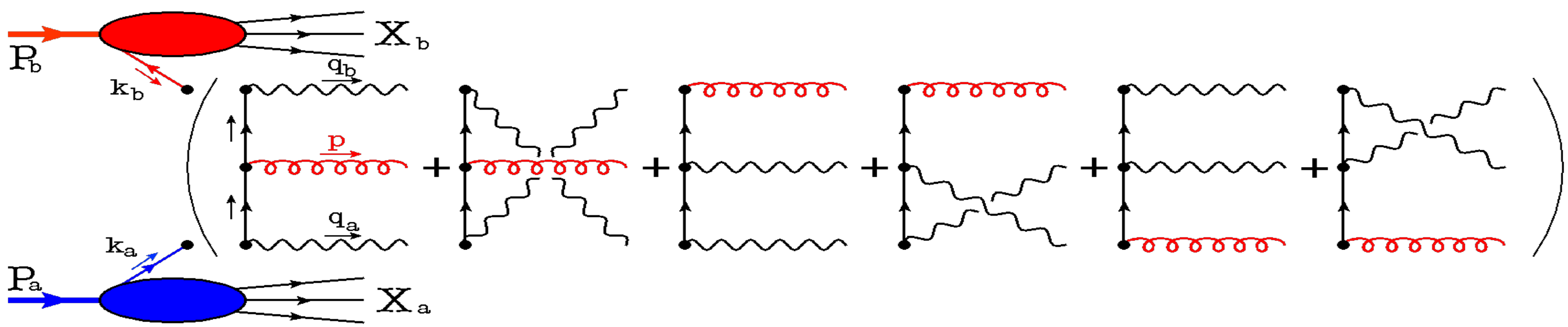
Lam-Tung relation in pQCD $\quad F_{UU}^2 = 2F_{UU}^{\cos 2\phi} + \mathcal{O}(\alpha_s^2)$

Similar to Callan-Gross relation in DIS

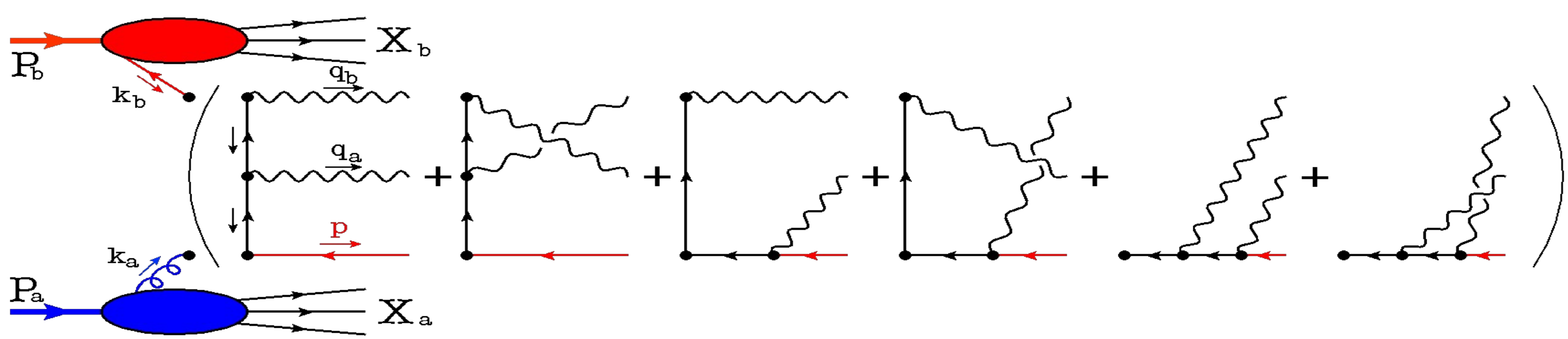
High - q_T of the photon pair

Same strategy as for DY:

quark – antiquark scattering:



quark – gluon scattering:



However: No model-independent angular decomposition!

Diphoton angles enter the partonic cross section in numerator and denominator
→ All angular depedencies are allowed.

Analytical results

What happens if q_T becomes smaller? → Expansion in $1/q_T$...

Drell-Yan:

$$d\sigma_{UU} \sim \left((1+\cos^2\Theta)F^1_{UU} + (1-\cos^2\Theta)F^2_{UU} + \sin 2\Theta\cos\phi F^{\cos\phi}_{UU} + \sin^2\Theta\cos 2\phi F^{\cos 2\phi}_{UU}\right)$$

Order $1/q_T^2$:

$$F^1_{UU} \propto \frac{\alpha_s}{q_T^2}\left[-C_F(2\ln\frac{q_T^2}{Q^2}+3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq}\otimes\bar{q} + P_{qg}\otimes g)(x_2) + \{x_1 \leftrightarrow x_2\}\right]$$

Order $1/q_T$:

$$F^{\cos\phi}_{UU} \propto \frac{\alpha_s}{q_T}\left[q(x_1)(\tilde{P}_{qq}\otimes\bar{q} + \tilde{P}_{qg}\otimes g)(x_2) - \{x_1 \leftrightarrow x_2\}\right]$$

Order $1/q_T^0$:

$$F^2_{UU} = 2F^{\cos 2\phi}_{UU} = -C_F(2\ln\frac{q_T^2}{Q^2}+3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq}\otimes\bar{q} + P'_{qg}\otimes\bar{q})(x_2) + \{x_1 \leftrightarrow x_2\}$$

DiPhoton production:

leading order follows "TMD - rule"

$$\sigma^{DP} = \frac{2}{\sin^2\theta}\sigma^{DY}(e_q \to e_q^2) + \mathcal{O}(1/q_T)$$

Order $1/q_T$:

$$\sigma^{DP} \sim (\sin 2\theta\cos\phi)\frac{\alpha_s}{q_T}\left[\frac{4}{\sin^4\theta}q(x_1)(\tilde{P}_{qq}\otimes\bar{q} + \tilde{P}_{qg}\otimes g)(x_2) + \frac{2}{\sin^2\theta}q(x_1)(\tilde{P}_{qq}\otimes\bar{q} + \tilde{P}^a_{qg}\otimes g)(x_2) - \{x_1 \leftrightarrow x_2\}\right]$$

Order $1/q_T^0$:

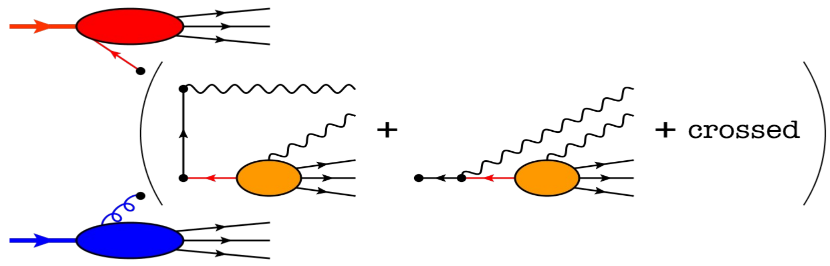
quark – antiquark contr. to diphoton production

$$\sigma^{DP}_{q\bar{q}} \propto \frac{(4-\cos^2\theta\sin^2\theta)\cos 2\phi + (3\sin^2\theta + 2\cos^2\theta)}{\sin^4\theta}F^{2,DY}_{UU,q\bar{q}}$$

quark – gluon scattering → collinearly divergent → need photon fragmentation function

Isolation of direct photons

Hide collinear divergence in photon fragmentation function:



- Potentially endangers TMD-factorization
- Photon FF unknown

Circumvent the problem → Isolation [Frixione PLB 429,369; Frixione, Vogelsang NPB 568, 60]

Define "cone" in rapidity – azimuthal angle space:

$$\mathcal{C}_\gamma(R_0) \equiv \left\{(\eta,\phi)\,|\,\sqrt{(\eta-\eta_\gamma)^2+(\phi-\phi_\gamma)^2} \le R_0\right\}$$

1. "Traditional" Criterium: allow certain percentage of hadronic energy inside the cone

$$E_T(R_0) \le \epsilon q_{T\gamma}$$

- Boost-invariant criterium.
- Infra-red safe.
- Allows certain contribution from fragmentation photons.

2. "Improved" Criterium: dynamically generated cone $R < R_0$

$$E_T(R) \le \epsilon_\gamma\, q_{T\gamma}\, f(R)$$

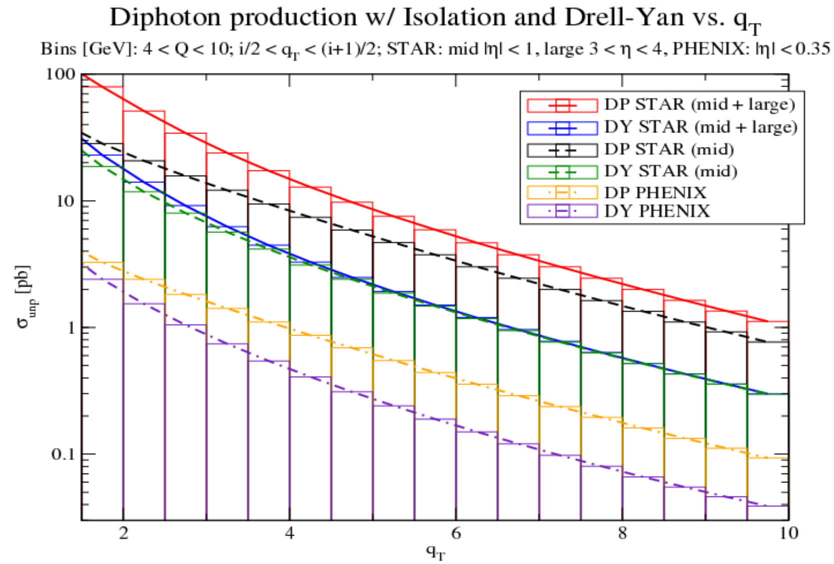
$$\lim_{R\to 0} f(R) = 0$$

- Boost-invariant criterium.
- Infra-red safe.
- Cuts out *all* fragmentation photons.
- Experimentally harder→ needs high resolution in η and ϕ.

Numerical results

Predictions for the qT-tail for STAR and PHENIX:

$$\sigma_{\text{unp}} = \int_{\text{cuts}} dy\, dQ^2\, dq_T\, d\varphi_q\, d\Omega\, \frac{d\sigma}{dy\, dQ^2\, dq_T\, d\varphi_q\, d\Omega}$$

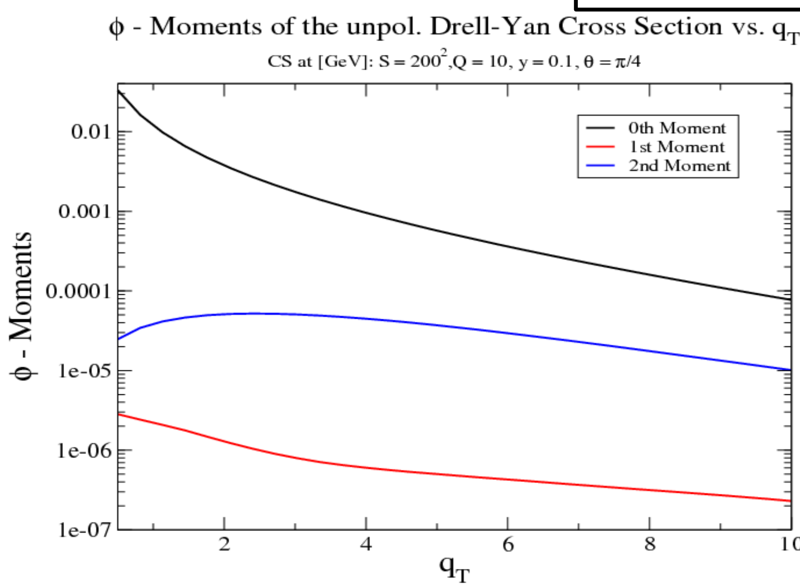


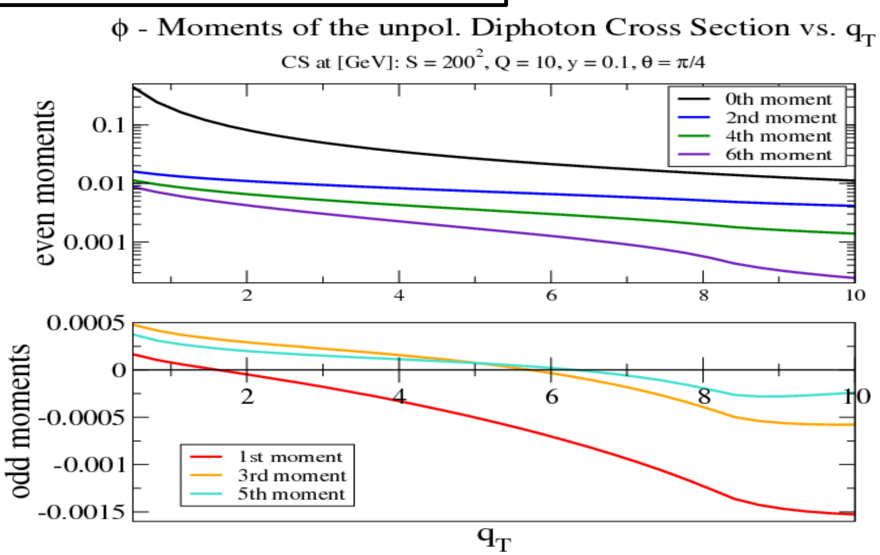
Also at larger qT:

→ Diphoton production rate larger than Drell-Yan

Define phi moments:

$$\langle \cos(n\phi) \rangle = \int_0^{2\pi} d\phi \cos(n\phi) \frac{d\sigma}{dy\, dQ^2\, d^2q_T\, d\Omega}$$





Summary:

- Drell-Yan cross section can be decomposed model-independently into angular structure function, not possible for photon pair production
- TMD-factorization at low q_T: Photon pair production similar to Drell-Yan
- Sivers effect similar in Photon pair production, but higher production rate → simultaneous measurement
- Collinear factorization at larger q_T: all azimuthal modulations possible for photon pair production in contrast to lepton pair production
- Expansion to smaller q_T: Azimuthal behaviour partly recovered → photon fragmentation or Isolation needed.

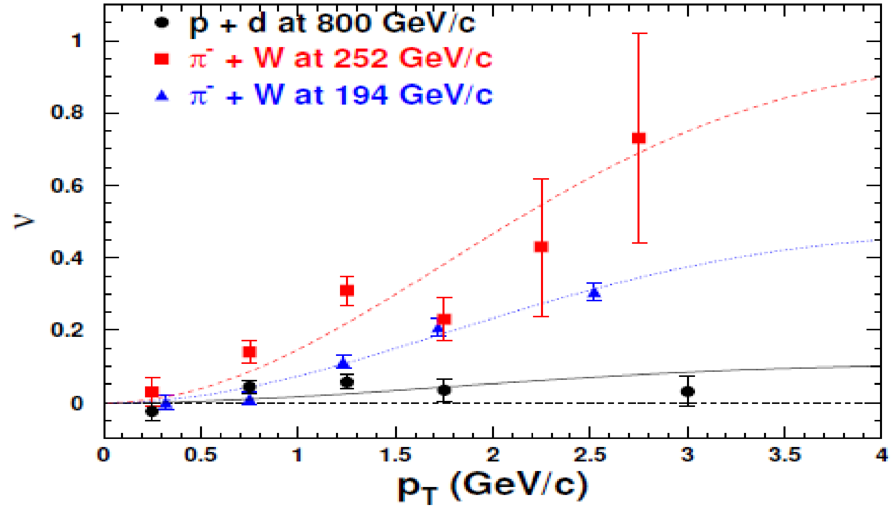
DY-data for the Lam-Tung relation

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q\,d\Omega}\right) \Big/ \left(\frac{d\sigma}{d^4q}\right) = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Lam-Tung relation: $1-\lambda-2\nu=0 \iff F_{UU}^{2} = 2F_{UU}^{\cos 2\phi}$

LT-relation even valid after resummation (Extension of coll. parton model to lower q_T)

ν has been measured in $\pi^- N \to \mu^+\mu^- X$ [NA10 Coll. ('86/'88) & E615 Coll. ('89)]



Large angular distribution violates Lam-Tung relation, far from pQCD result.

However, recent FermiLab data on proton-deuterium DY agree with LT-relation [FNAL-E866/NuSea Coll.]

Possible explanation through TMDs: *"Boer-Mulders effect"*

$$\nu(q_T \ll Q) \propto \frac{h_1^{\perp,q} \otimes h_1^{\perp,\bar{q}}}{f_1^q \otimes f_1^{\bar{q}}}$$