

# TMDs Phenomenology in SIDIS and DY

Studying the hadron structure in Drell-Yan reactions  
CERN, 25-27<sup>th</sup> April 2010



Stefano Melis

Universita' del Piemonte Orientale

INFN, Sezione di Torino & G.C. Alessandria



# Summary

- Sivers function in SIDIS from fits
- Conclusions I
- Boer-Mulders function from fit (SIDIS&DY)
- Conclusions II

Sivers function in SIDIS from fits

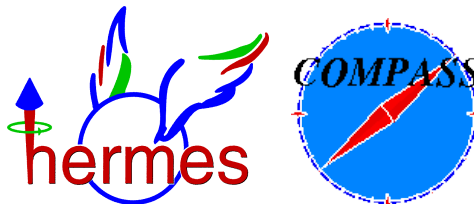
# Sivers function in SIDIS from fits

➤ Most recent fits

[1] Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

[2] Arnold, Efremov, Goeke, Schlegel, Schweitzer,  
arXiv:0805.2137 (2008)

✓ Fits of **HERMES** (2002-5) and **COMPASS** (Deuteron 2003-4)  
data on  $\pi$  and K production



The asymmetry  $A_{UT}^{\sin(\phi_h - \phi_S)}$

# Polarized SIDIS $lp^\uparrow \rightarrow l'h+X$

## Extraction of the Sivers Function

➤ The cross section can be written as:

$$d\sigma^{lp^\uparrow \rightarrow l'hX} = \sum_q f_{q/p^\uparrow}(x, \mathbf{k}_\perp, Q^2) \otimes d\sigma^{lq \rightarrow l'q} \otimes D_q^h(z, \mathbf{p}_\perp, Q^2)$$

Polarized PDF

Elementary Cross section

Fragmentation Function

# Polarized SIDIS:

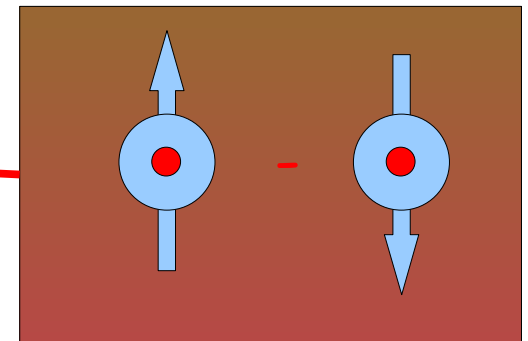
## Extraction of the Sivers Function

➤ If we consider the transverse motion and its correlation with the spin of the proton then:

$$\begin{aligned}
 f_{q/p^\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) \\
 &= \underbrace{f_{q/p}(x, \mathbf{k}_\perp)}_{\text{Unp. PDF}} - \underbrace{\frac{\mathbf{k}_\perp}{m_p} f_{1T}^\perp(x, \mathbf{k}_\perp)}_{\text{Sivers function}} \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)
 \end{aligned}$$

✓ Torino vs Amsterdam notation

$$\frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = -\frac{\mathbf{k}_\perp}{m_p} f_{1T}^{\perp q}(x, \mathbf{k}_\perp)$$



# Polarized SIDIS:

## Extraction of the Sivers Function

➤ If we consider the transverse motion and its correlation with the spin of the proton then:

$$\begin{aligned} f_{q/p^\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, \mathbf{k}_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \underbrace{\sin(\varphi - \phi_S)} \end{aligned}$$

✓ Azimuthal phase: angle between  $\mathbf{k}_\perp$  and the spin

✓ Bound:

$$\frac{|\Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp)|}{2f_{q/p}(x, \mathbf{k}_\perp)} \leq 1$$



# Polarized SIDIS:

## Extraction of the Sivers Function

- We can build an azimuthal weighted asymmetry

$$A_{UT}^{\sin(\phi_h - \phi_S)} = 2 \frac{\int d\phi_S d\phi_h [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_h - \phi_S)}{\int d\phi_S d\phi_h [d\sigma^\uparrow + d\sigma^\downarrow]}$$

- In details:


$$A_{UT}^{\sin(\phi_h - \phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2\mathbf{k}_\perp \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp) \sin(\phi_h - \phi_S)}{\sum_q \int d\phi_S d\phi_h d^2\mathbf{k}_\perp f_{q/p}(x, k_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp)}$$

[1] Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

# Polarized SIDIS:


## Extraction of the Sivers Function

- Gaussian smearing for both unpolarized PDF and FF


$$f_{q/p}(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

GRV98 set

$$\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$$


$$D_q^h(z, p_{\perp}) = D_q^h(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}$$

DSS set

$$\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$$

# Polarized SIDIS:

## Extraction of the Sivers Function

- Simple parametrization of the Sivers function

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp)$$

Unpolarized PDF

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} \leq 1$$


$$h(k_\perp) = \sqrt{2} e \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2} \leq 1$$

$N_q, \alpha_q, \beta_q$  &  $M_1$  free parameters

# Polarized SIDIS:

## Extraction of the Sivers Function

➤ **HERMES** (2002-5)   
 $(x, z, P_T)$   $\pi$ &K

➤ **COMPASS** (2004)   
 $(x, z, P_T)$   $\pi$ &K

➤ 11 free parameters:

$$\begin{array}{cccccc}
 N_u & N_d & N_{\bar{u}} & N_{\bar{d}} & N_s & N_{\bar{s}} \\
 & \alpha_u & \alpha_d & \alpha_{sea} & & \\
 & \beta & M_1 & & & 
 \end{array}$$

✓ GRV98 PDF

✓ DSS FF

✓ Gaussians:  $\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$   
 $\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$   
 (from Cahn effect)

✓ Simulated evolution (unp.-like)

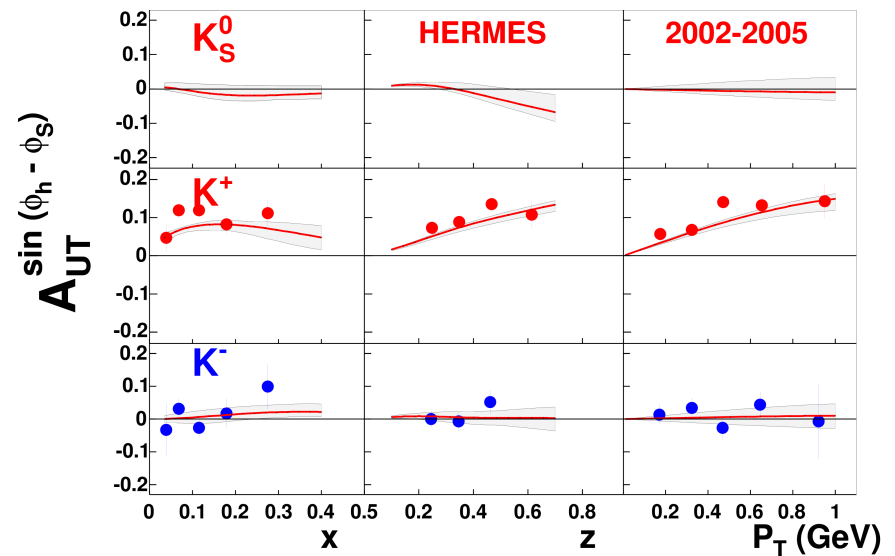
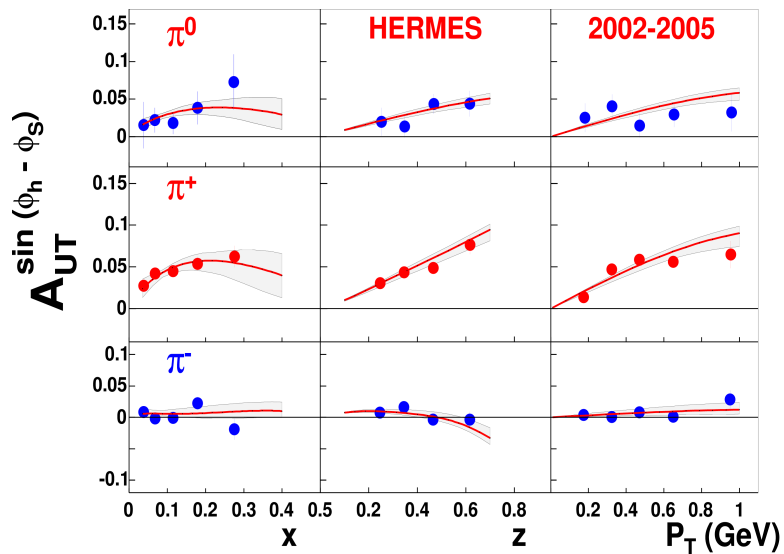
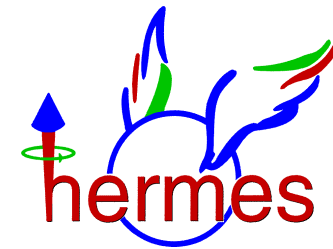
✓  $\Delta^N f_{q/p\uparrow}(x, k_{\perp}) = 2 \mathcal{N}_q(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$

✓  $\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$

✓  $h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_1} e^{-k_{\perp}^2/M_1^2}$

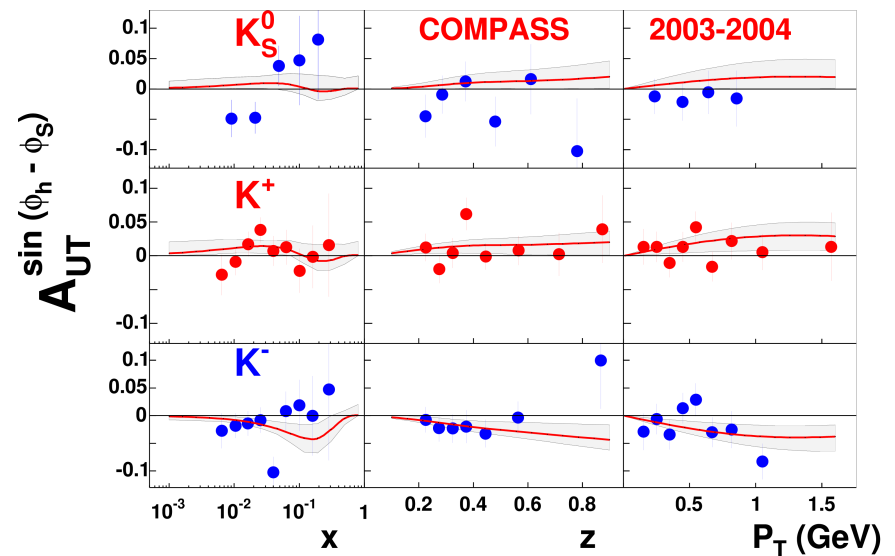
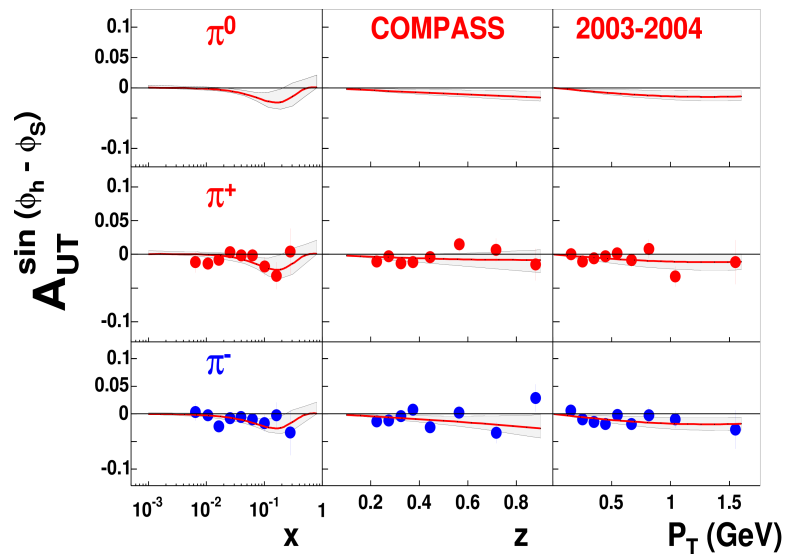
# Polarized SIDIS: Extraction of the Sivers Function

HERMES Proton Target

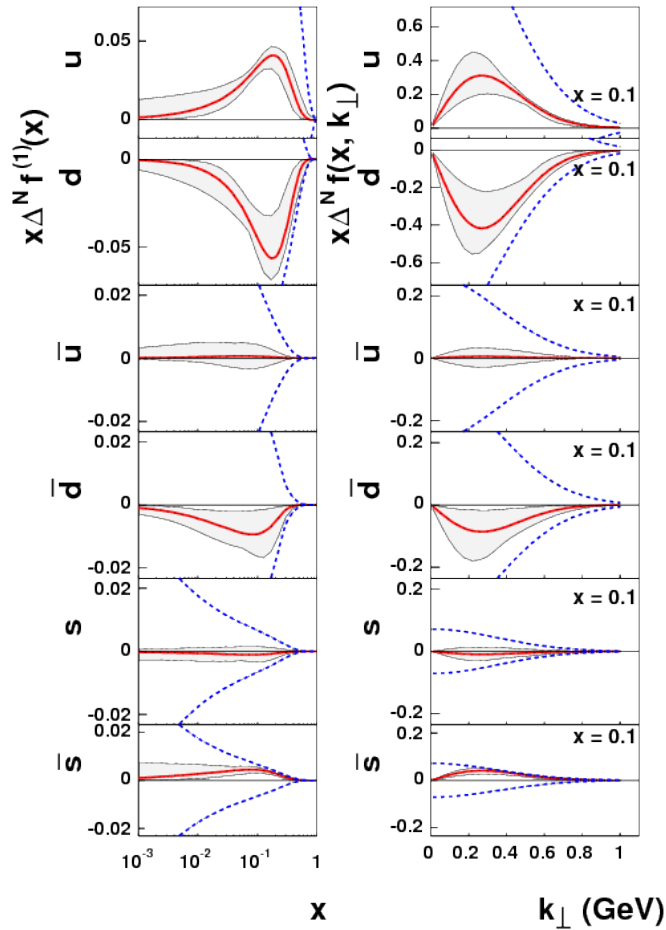


# Polarized SIDIS: Extraction of the Sivers Function

COMPASS Deuteron Target



# Polarized SIDIS: Extraction of the Sivers Function



## ✓ Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp u} < 0$

- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow f_{1T}^{\perp d} > 0$

## ✓ Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp \bar{s}} < 0$

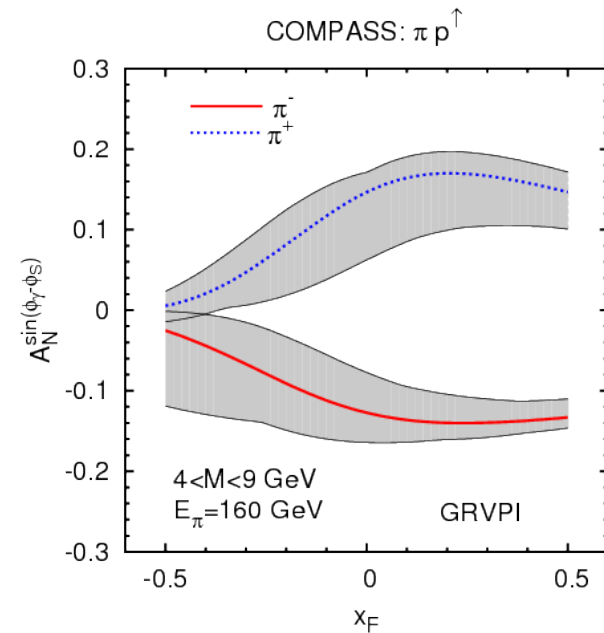
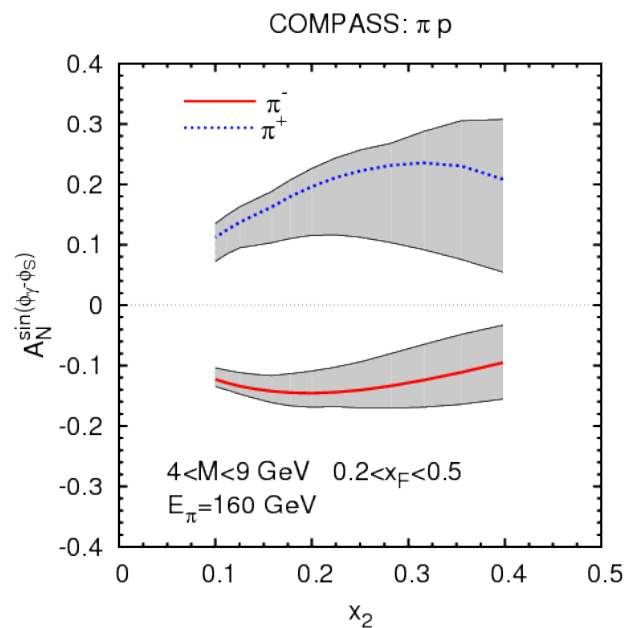
$$\rightarrow \Delta^N f_q^{(1)}(x) \equiv \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = -f_{1T}^{\perp(1)q}(x)$$

$\chi^2/d.o.f = 1$		
$N_u = 0.35^{+0.078}_{-0.079}$	$N_d = -0.9^{+0.43}_{-0.098}$	$N_s = -0.24^{+0.62}_{-0.5}$
$N_{\bar{u}} = 0.037^{+0.22}_{-0.24}$	$N_{\bar{d}} = -0.4^{+0.33}_{-0.44}$	$N_{\bar{s}} = 1^{+0}_{-0.0001}$
$\alpha_u = 0.73^{+0.72}_{-0.58}$	$\alpha_d = 1.1^{+0.82}_{-0.65}$	$\alpha_{sea} = 0.79^{+0.56}_{-0.47}$
$\beta = 3.5^{+4.9}_{-2.9}$	$M_1^2 = 0.34^{+0.3}_{-0.16} \text{ GeV}^2$	



# Predictions for COMPASS DY

- Polarized  $\text{NH}_3$
- Pion beam
- Valence region for the Sivers function




Large measurable asymmetry!!!


[2] Arnold, Efremov, Goeke, Schlegel, Schweitzer,  
arXiv:0805.2137 (2008)


# Polarized SIDIS:

## Extraction of the Sivers Function

➤ Gaussian smearing for unpolarized PDF, FF and Sivers function

  $\langle k_{\perp}^2 \rangle = 0.33 \text{ (GeV}/c)^2$  for the unpolarized PDF [\*]

  $\langle p_{\perp}^2 \rangle = 0.16 \text{ (GeV}/c)^2$  for the unpolarized FF [\*]

  $\langle k_{\perp}^2 \rangle_{\text{Siv}} = 0.2 \text{ (GeV}/c)^2$  for the Sivers function [\*]

[\*] Collins et al. Phys. Rev. D73, 014021 (2006)

Values obtained analyzing the kinematic of HERMES

# Polarized SIDIS:

## Extraction of the Sivers Function

- Parametrization of the first moment of the Sivers function

$$f_{1T}^{\perp q(1)}(x) = A_q \frac{\langle k_{\perp} \rangle}{2m_p} f_{q/p}(x)$$

where:

$$f_{1T}^{\perp q}(x) = \int d\mathbf{k}_{\perp}^2 \frac{\mathbf{k}_{\perp}^2}{2m_p} f_{1T}^{\perp q}(x, \mathbf{k}_{\perp}^2)$$


$$|A_q| < 1 \quad q = \text{all light flavours}$$

$$\langle k_{\perp} \rangle = \frac{\sqrt{\pi}}{2} \sqrt{\langle k_{\perp}^2 \rangle} = 0.5 \text{ (GeV}/c)$$

# Polarized SIDIS:

## Extraction of the Sivers Function

➤ **HERMES** (2002-5)   
(x)  $\pi$ &K

➤ **COMPASS** (2004)   
(x)  $\pi$ &K

➤ 4 free parameters:

$A_u$     $A_d$     $A_{\bar{u}}$     $A_{\bar{d}}$     $A_s=1$     $A_{\bar{s}}=-1$

✓ GRV98 PDF

✓ Kretzer FF

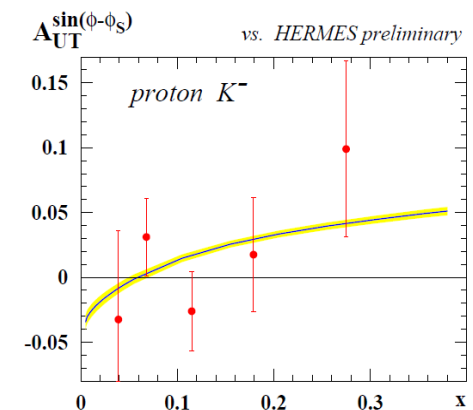
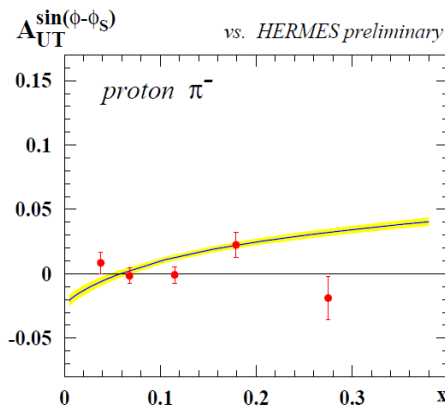
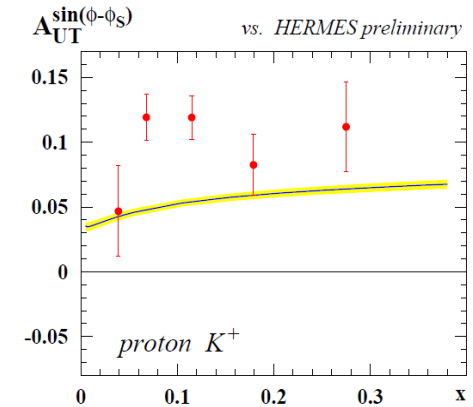
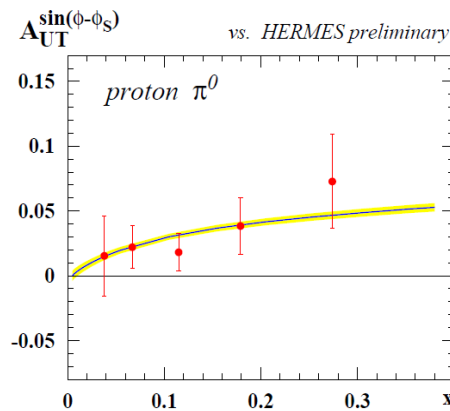
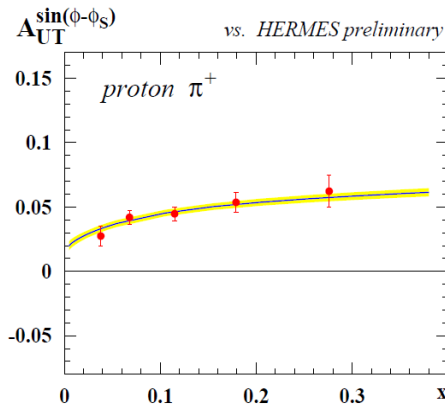
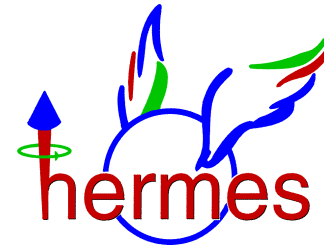
✓ Gaussians:  $\langle k_{\perp}^2 \rangle = 0.33 \text{ (GeV/c)}^2$   
 $\langle p_{\perp}^2 \rangle = 0.16 \text{ (GeV/c)}^2$   
 $\langle k_{\perp}^2 \rangle_{\text{Siv}} = 0.20 \text{ (GeV/c)}^2$

✓  $Q^2 = 2.5 \text{ (GeV/c)}^2$

$$\checkmark f_{1T}^{\perp q(1)}(x) = A_q \frac{\langle k_{\perp} \rangle}{2m_p} f_{q/p}(x)$$

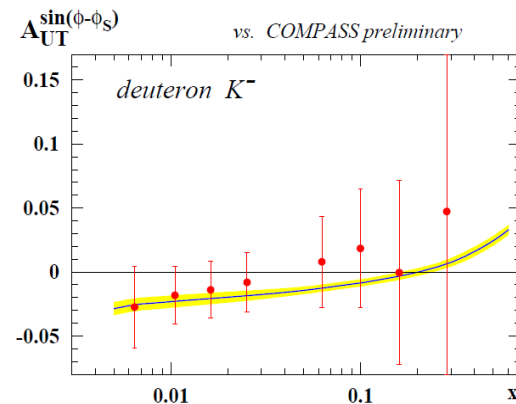
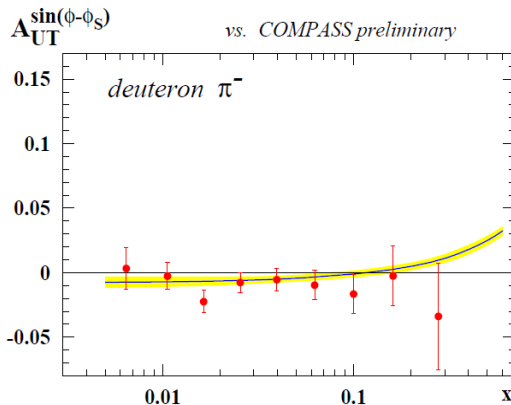
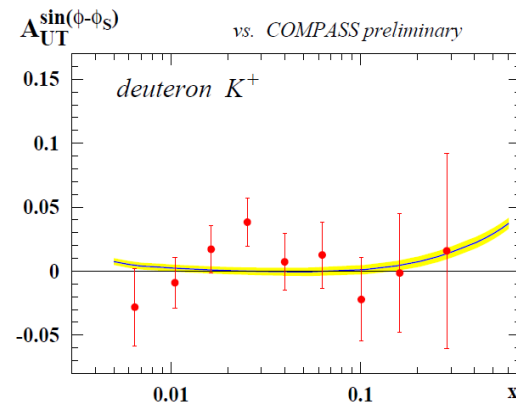
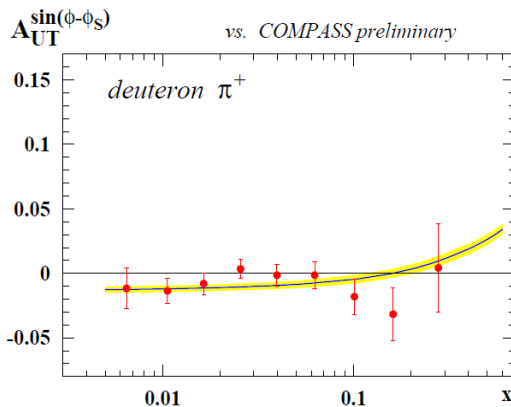
# Polarized SIDIS: Extraction of the Sivers Function

HERMES Proton Target



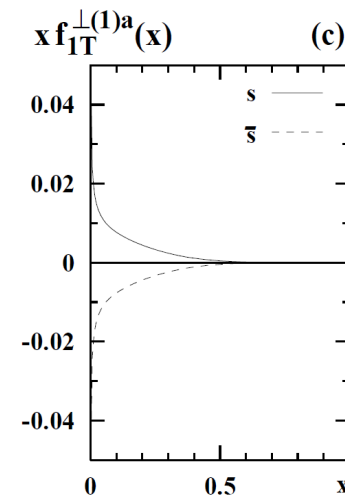
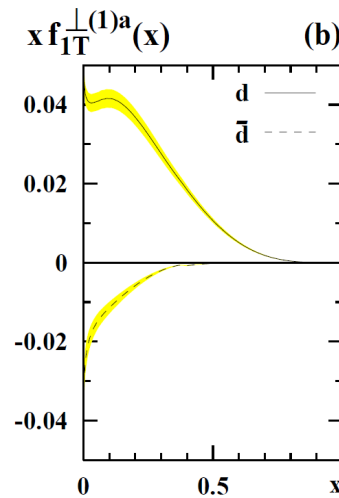
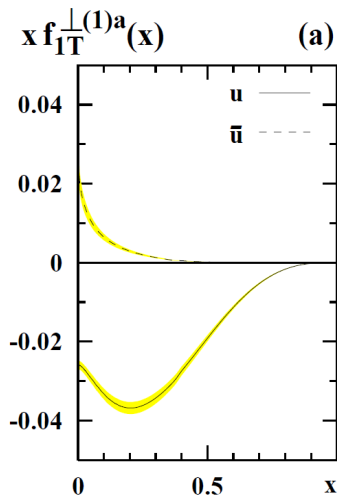
# Polarized SIDIS: Extraction of the Sivers Function

COMPASS Deuteron Target



# Polarized SIDIS:

## Extraction of the Sivers Function



✓ Valence quark

- $f_{1T}^{\perp u} < 0 \Rightarrow \Delta^N f_{u/p^\uparrow} > 0$
- $f_{1T}^{\perp d} > 0 \Rightarrow \Delta^N f_{d/p^\uparrow} < 0$

✓ Sea quarks

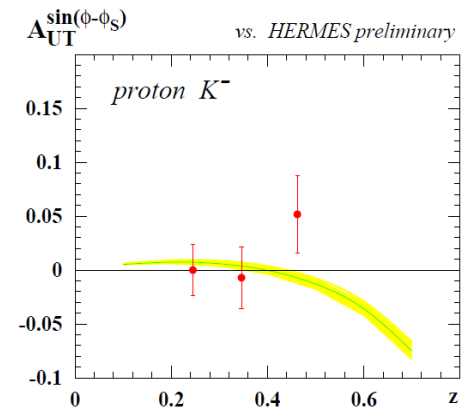
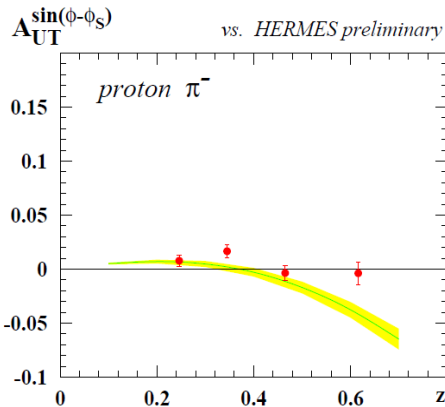
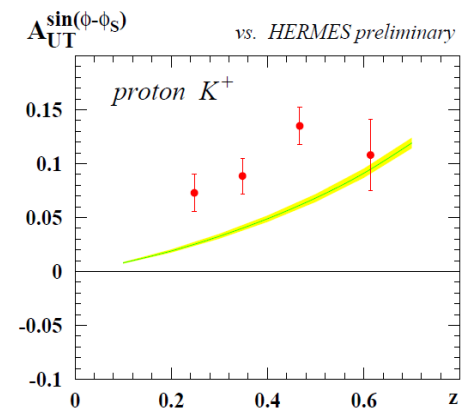
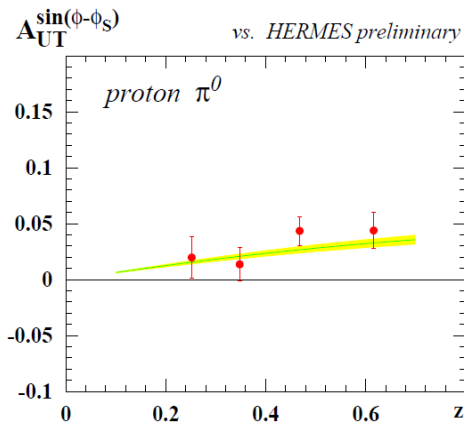
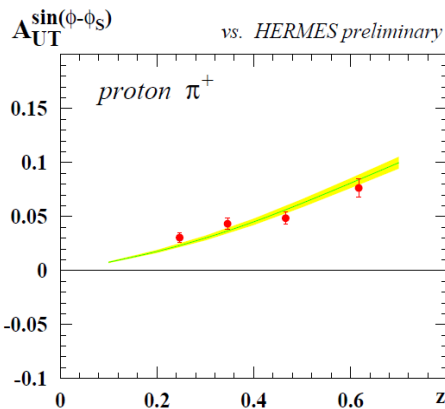
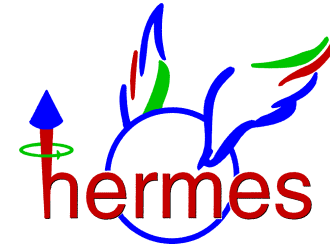
- $f_{1T}^{\perp \bar{s}} < 0 \Rightarrow \Delta^N f_{\bar{s}/p^\uparrow} > 0$

$$\chi_{\text{d.o.f.}}^2 = 1.3 \quad \begin{array}{lll} A_u = -0.21 \pm 0.01 & A_{\bar{u}} = 0.23 \pm 0.02 & A_s^{\text{fixed}} = +1 \\ A_d = 0.38 \pm 0.03 & A_{\bar{d}} = -0.28 \pm 0.04 & A_{\bar{s}}^{\text{fixed}} = -1 \end{array}$$



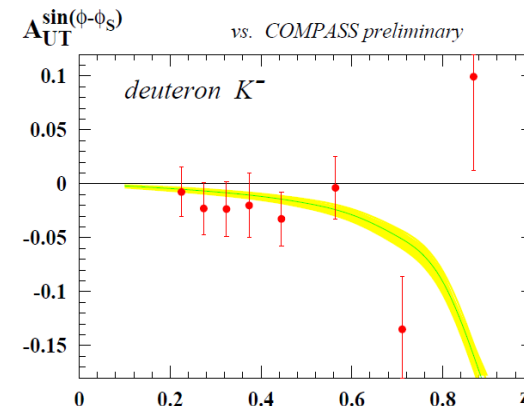
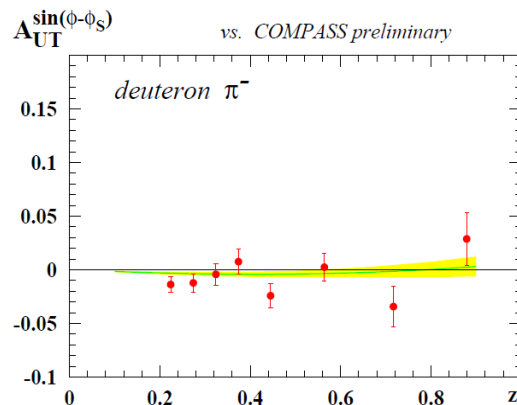
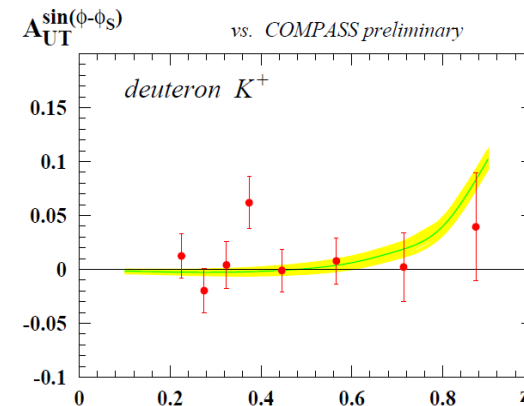
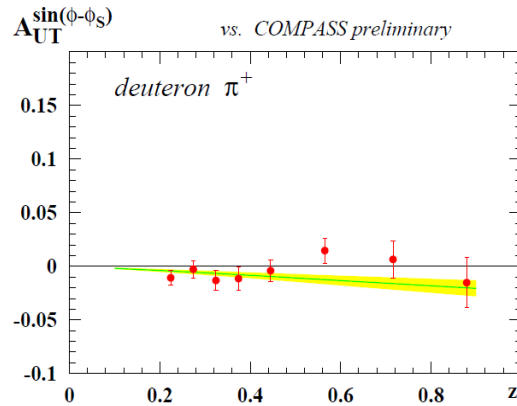
# Polarized SIDIS: Extraction of the Sivers Function

HERMES Proton Target (Predictions)



# Polarized SIDIS: Extraction of the Sivers Function

COMPASS Deuteron Target(Predictions)



# Conclusions I

- The main results of the two fits are the same:
  - u and d quark Sivers functions are opposite in sign and similar in magnitude
  - The two fits agree in the sign of the Sivers functions for u, d,  $\bar{s}$  quarks
- Open questions: Average transverse momentum  
Evolution equation....
- Analysis of new data! (COMPASS proton)
- Waiting for polarized DY!

Boer-Mulders function from fits

# Boer-Mulders function from $A^{\cos^2\phi}$ in SIDIS

➤ Most Recent fit

[3] V. Barone, S. Melis and A. Prokudin

ArXiv:0912.5194 (to be published in Phys. Rev. D)

✓ Fits of **HERMES** (proton and deuteron) and  
**COMPASS** (deuteron ) data on charged hadrons production



# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B \cos \phi + C \cos 2\phi$$

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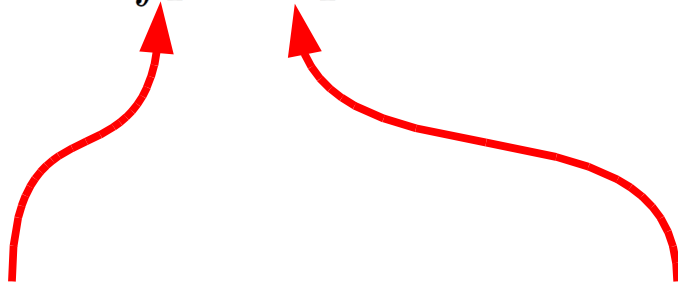
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Unpolarized PDF

Unpolarized FF





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- $B \propto \frac{1}{Q} (f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp)$  subleading Cahn+Boer-Mulders effect

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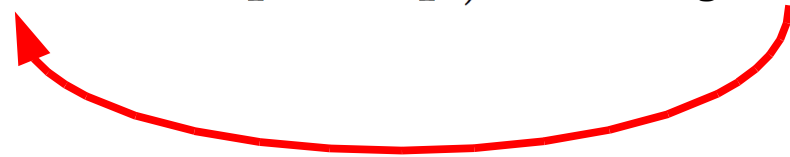
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Boer-Mulders function

Collins function

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- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$  BM effect+Twist-4 Cahn effect

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$$d\sigma = A + B \cos \phi + C \cos 2\phi$$

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- $B \propto \frac{1}{Q} (f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp)$  subleading Cahn+Boer-Mulders effect
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$  **BM effect**+Twist-4 Cahn effect



Leading Twist BM

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

➤ The angular distribution in the unpolarized SIDIS can be written as

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Twist-4 Cahn





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- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1 + \dots$  other **unknown twist-4 terms?**

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$$A^{\cos 2\phi} = 2 \frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{A}$$

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Unpolarized PDF&FF gaussian as in Anselmino et al. [1]

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Collins function as in Anselmino et. al arXiv: 0812.4366v1

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BM that we want to extract from the fit of  $A \cos 2\phi$  data

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

➤ Simple parametrization of the Boer-Mulders functions:

- $h_1^{\perp q}(x, k_{\perp}) = \lambda_q f_{1T}^{\perp q}(x, k_{\perp})$  for valence quarks

- $h_1^{\perp q}(x, k_{\perp}) = -|f_{1T}^{\perp q}(x, k_{\perp})|$  for sea quarks

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

- **HERMES** proton and deuteron target  
( $x, z, P_T$ ) charged hadrons
- **COMPASS** deuteron target  
( $x, z, P_T$ ) charged hadrons

✓ GRV98 PDF

✓ DSS FF

✓ Gaussians:  $\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$   
 $\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$   
(from Cahn-cos $\phi$  effect)

✓ Simulated evolution (unp.)

**FIT I**

- 2 free parameters:

$$\lambda_u \quad \lambda_d$$

- Sivers function as in [\*]

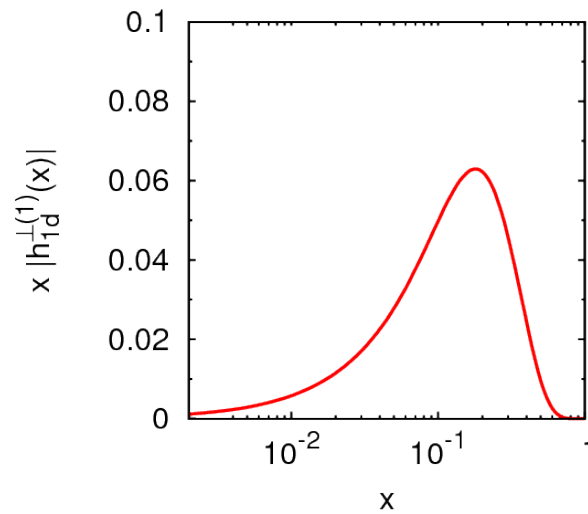
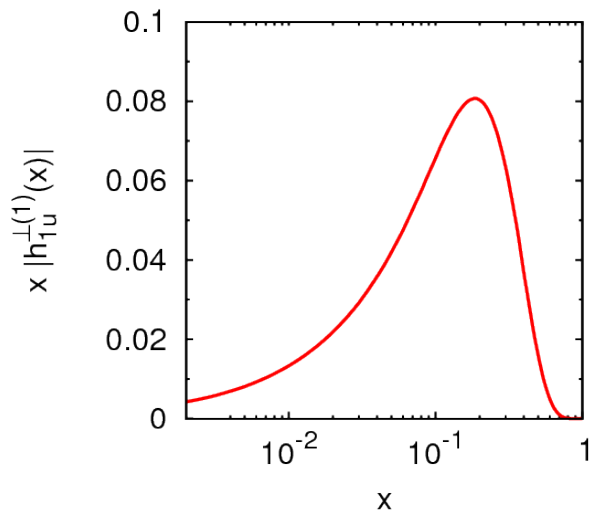
✓  $h_1^{\perp q}(x, k_{\perp}) = \lambda_q f_{1T}^{\perp q}(x, k_{\perp})$

✓  $h_1^{\perp q}(x, k_{\perp}) = -|f_{1T}^{\perp q}(x, k_{\perp})|$

• [\*] Anselmino et al. Eur. Phys. J. A39,89

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function



FIT I

$$\diamond \chi^2/d.o.f. = 3.73$$

$$\bullet \lambda_u = 2.0 \pm 0.1$$

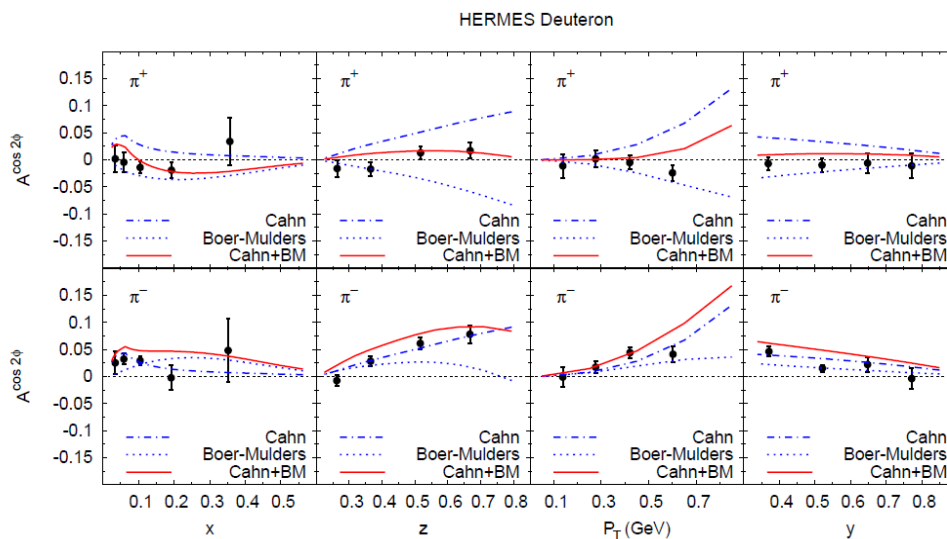
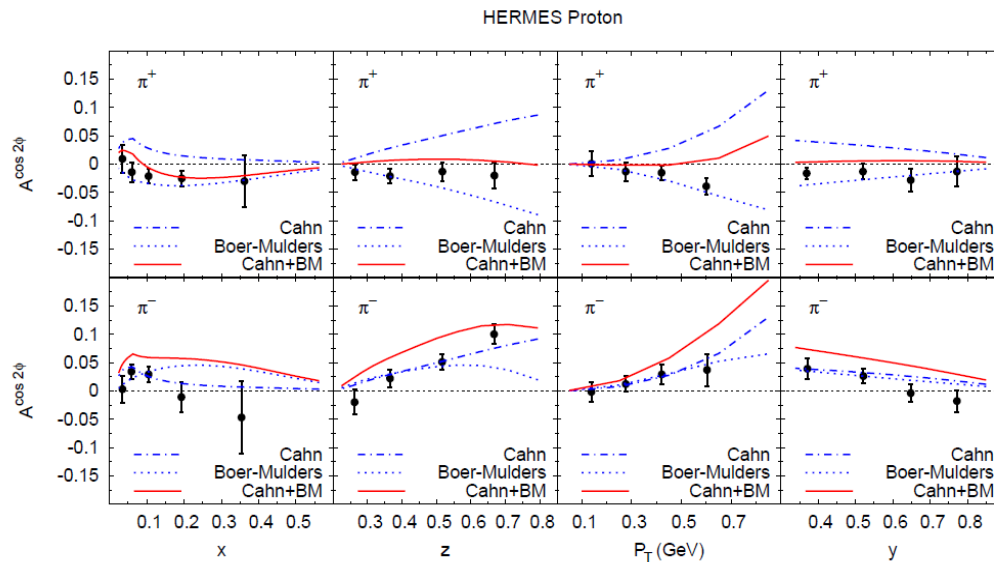
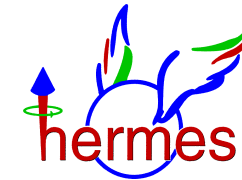
$$\bullet \lambda_d = -1.11^{+0.00}_{-0.02}$$

$\Rightarrow h_1^{\perp d}$  and  $h_1^{\perp u}$  both negative



# Unpolarized SIDIS:

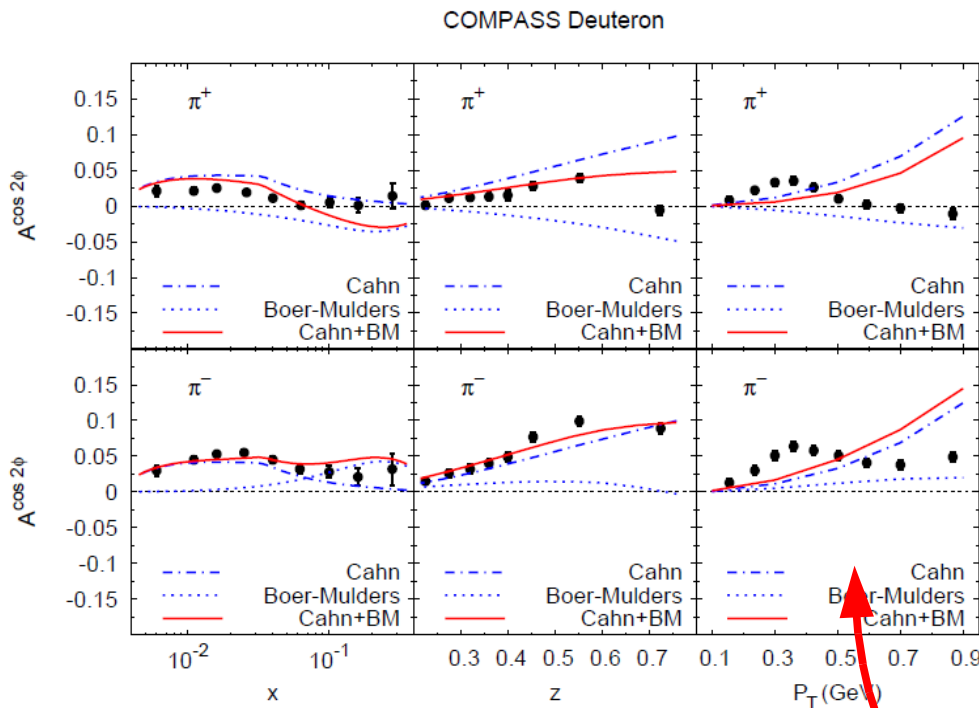
## Extraction of the Boer-Mulders Function



- ✓ Cahn effect (Twist-4) comparable to BM effect
- ✓ Same sign of Cahn contribution for positive and negative pions
- ✓ BM contribution opposite in sign for positive and negative pions

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function



- ✓ Cahn effect (Twist-4) comparable to BM effect
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- ✓ BM contribution opposite in sign for positive and negative pions

Data in  $p_T$  not included in the fit

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

➤ The Cahn effect is a crucial ingredient

✓ Gaussians:  $\langle k_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$   
 $\langle p_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$  } From Ref.[\*]: analysis of  
Cahn  $\cos\phi$  effect from EMC data

[\*] Anselmino et al. Phys. Rev. D71 074006 (2005)

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COMPASS

$$\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$$
$$\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$$

~EMC

HERMES

$$\langle k_{\perp}^2 \rangle = 0.18 \text{ (GeV/c)}^2$$
$$\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$$

~HERMES MC

[\*] Anselmino et al. Phys. Rev. D71 074006 (2005)

# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

### ► FIT II

COMPASS

$$\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$$
$$\langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2$$

~EMC

FIT II

HERMES

$$\langle k_{\perp}^2 \rangle = 0.18 \text{ (GeV/c)}^2$$
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~HERMES MC

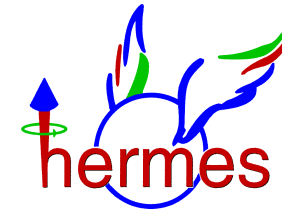
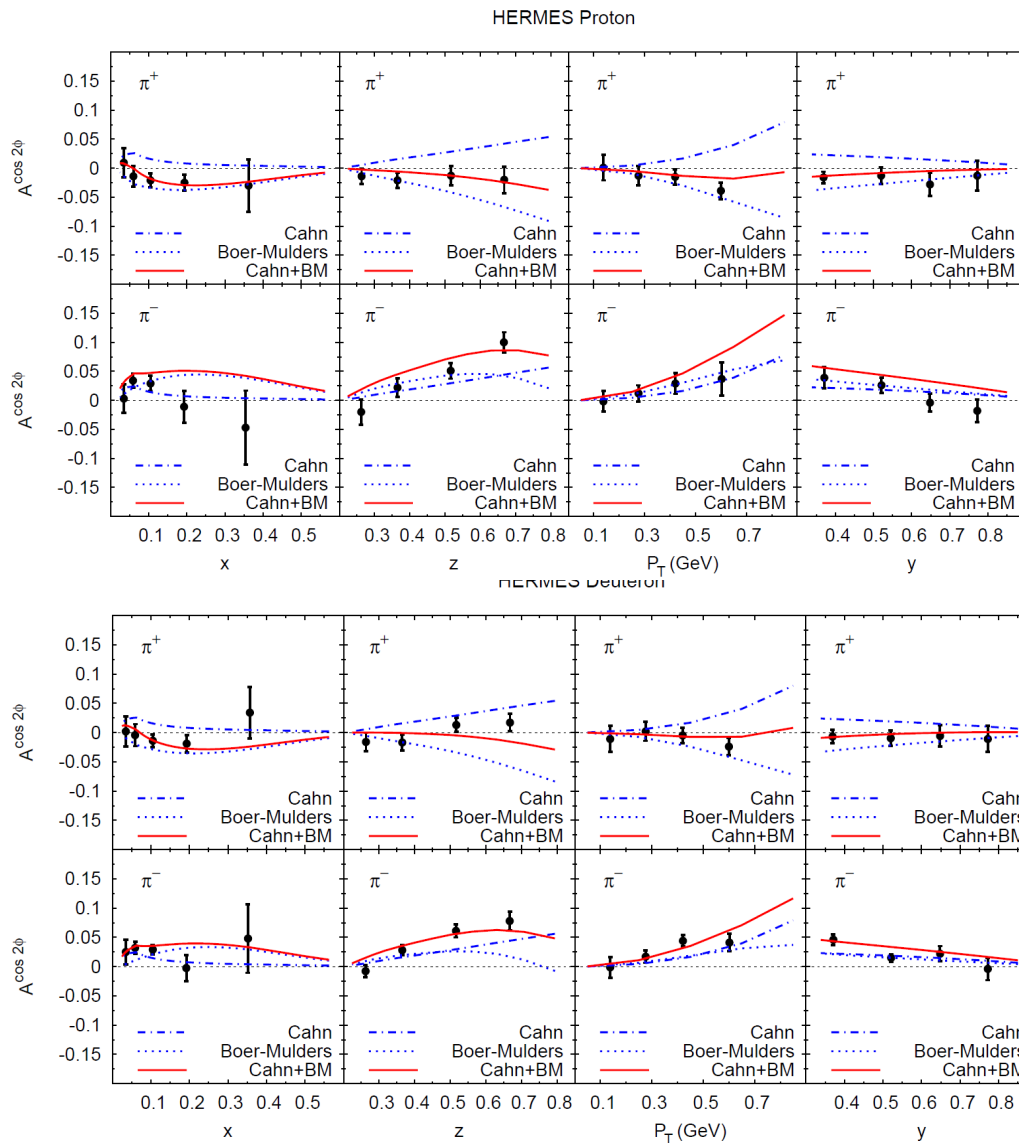
$$\diamond \chi^2/d.o.f. = 2.41$$

$$\bullet \lambda_u = 2.1 \pm 0.1$$

$$\bullet \lambda_d = -1.11^{+0.00}_{-0.02}$$

Better description of HERMES but the BM is unchanged

# Unpolarized SIDIS: Extraction of the Boer-Mulders Function



# Boer-Mulders function in DY from fits

➤ Most Recent fit

[4] Lu and Schimdt, Phys. Rev. D81, 043023 (2010)

✓ Fit of FERMILAB data, unpolarized DY, pp and pD scattering

# Boer-Mulders function in DY from fits

- General expression for the dilepton angular distributions in the dilepton rest frame:

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi(\lambda + 3)} \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + (\nu/2) \sin^2 \theta \cos 2\phi \right]$$

$$\nu \propto \frac{h_1^{\perp a} \otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$


Boer-Mulders functions


Unpolarized PDFs

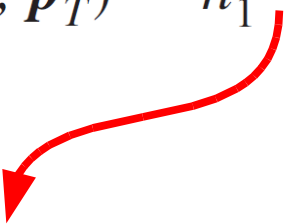


# Boer-Mulders function in DY from fits

- Parametrization of the Boer-Mulders function and of the unpolarized PDF

 Unpolarized PDFs as in Anselmino et al.

 
$$h_1^{\perp q}(x, \mathbf{p}_T^2) = h_1^{\perp q}(x) \frac{\exp(-\mathbf{p}_T^2 / p_{bm}^2)}{\pi p_{bm}^2}$$

 
$$h_1^{\perp q}(x) = H_q x^{c_q} (1-x)^b f_1^q(x)$$

$H_q, c_q, b$  and  $p_{BM}$  free parameters

# Unpolarized DY:

## Extraction of the Boer-Mulders Function

➤ 9 free parameters:

$$\begin{array}{lll} H_1 = H_u H_{\bar{u}} & c_u & b \\ H_2 = H_d H_{\bar{d}} & c_d & \\ H_3 = H_u H_{\bar{d}} & c_{\bar{u}} & p_{BM} \\ & c_{\bar{d}} & \end{array}$$

✓ MSTW 2008 LO PDF

✓ Gaussians:  $\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2$   
(as in SIDIS!)

➤ FERMILAB E866

pp and pD

data  $(x_1, x_2, x_f, M, P_T)$

$$\checkmark h_1^{\perp q}(x, \mathbf{p}_T^2) = h_1^{\perp q}(x) \frac{\exp(-\mathbf{p}_T^2 / p_{bm}^2)}{\pi p_{bm}^2}$$

$$\checkmark h_1^{\perp q}(x) = H_q x^{c^q} (1-x)^b f_1^q(x)$$

# Unpolarized DY:

## Extraction of the Boer-Mulders Function

$$\begin{array}{lll} H_1 = 0.62_{-0.29}^{+0.52}, & H_2 = 1.45_{-1.12}^{+1.30}, & H_3 = 0.61_{-0.55}^{+0.50}, \\ c_u = 0.63_{-0.21}^{+0.53}, & c_d = 0.47_{-0.39}^{+0.36}, & c_{\bar{u}} = 0.07_{-0.05}^{+0.06}, \\ c_{\bar{d}} = 0.75_{-0.52}^{+0.72}, & b_0 = 0.17_{-0.14}^{+0.15}, & p_{bm}^2 = 0.173_{-0.033}^{+0.027} \end{array}$$

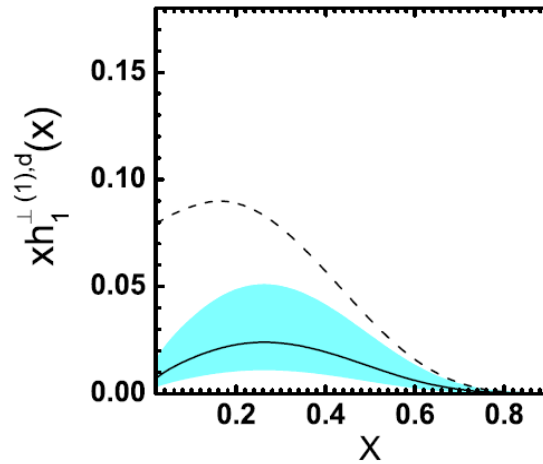
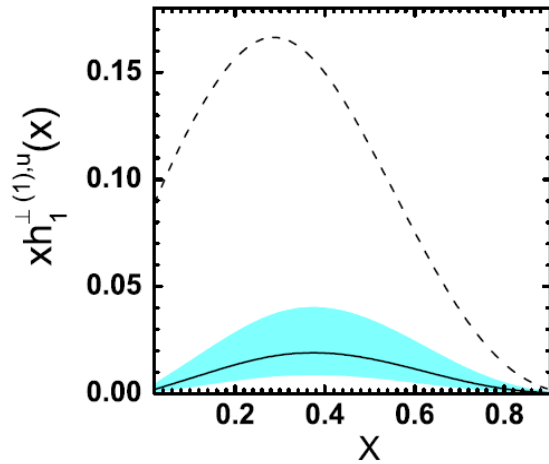
$$\chi^2/\text{d.o.f} = 0.84$$

$$\frac{|p_T h_1^{\perp q}(x, \mathbf{p}_T^2)|}{M} \leq f_1^q(x, \mathbf{p}_T^2) \Rightarrow \begin{array}{ll} H_u = 0.59_{-0.31}^{+0.64}, & H_d = 1.37_{-0.72}^{+1.53}, \\ H_{\bar{u}} = 1.10_{-0.57}^{+1.21}, & H_{\bar{d}} = 1.08_{-0.56}^{+1.18}. \end{array}$$

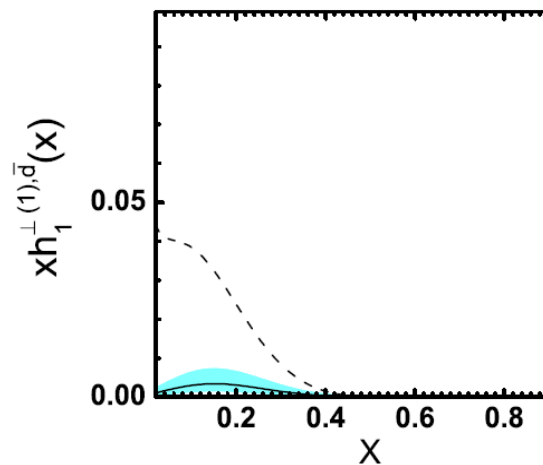
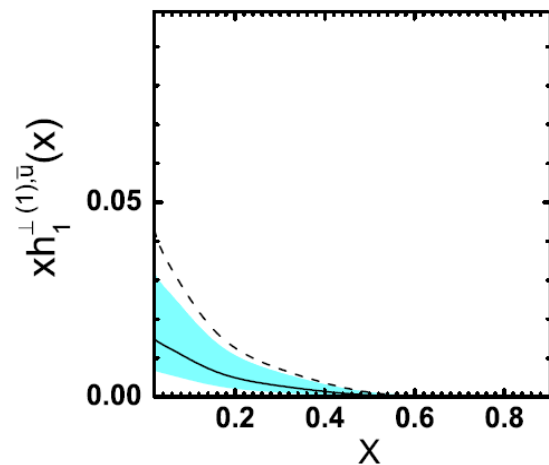
It is not the statistical error on the extraction!  
Central values are only geometrical mean values.

# Unpolarized DY:

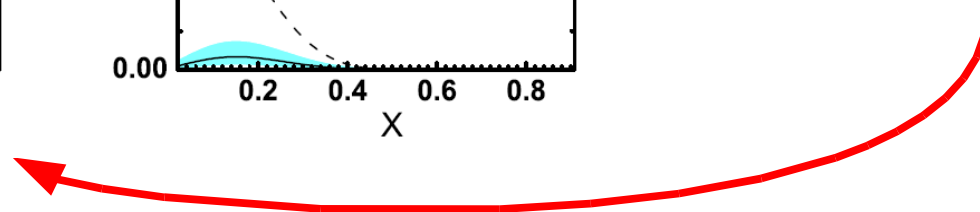
## Extraction of the Boer-Mulders Function



The dotted line is the bound

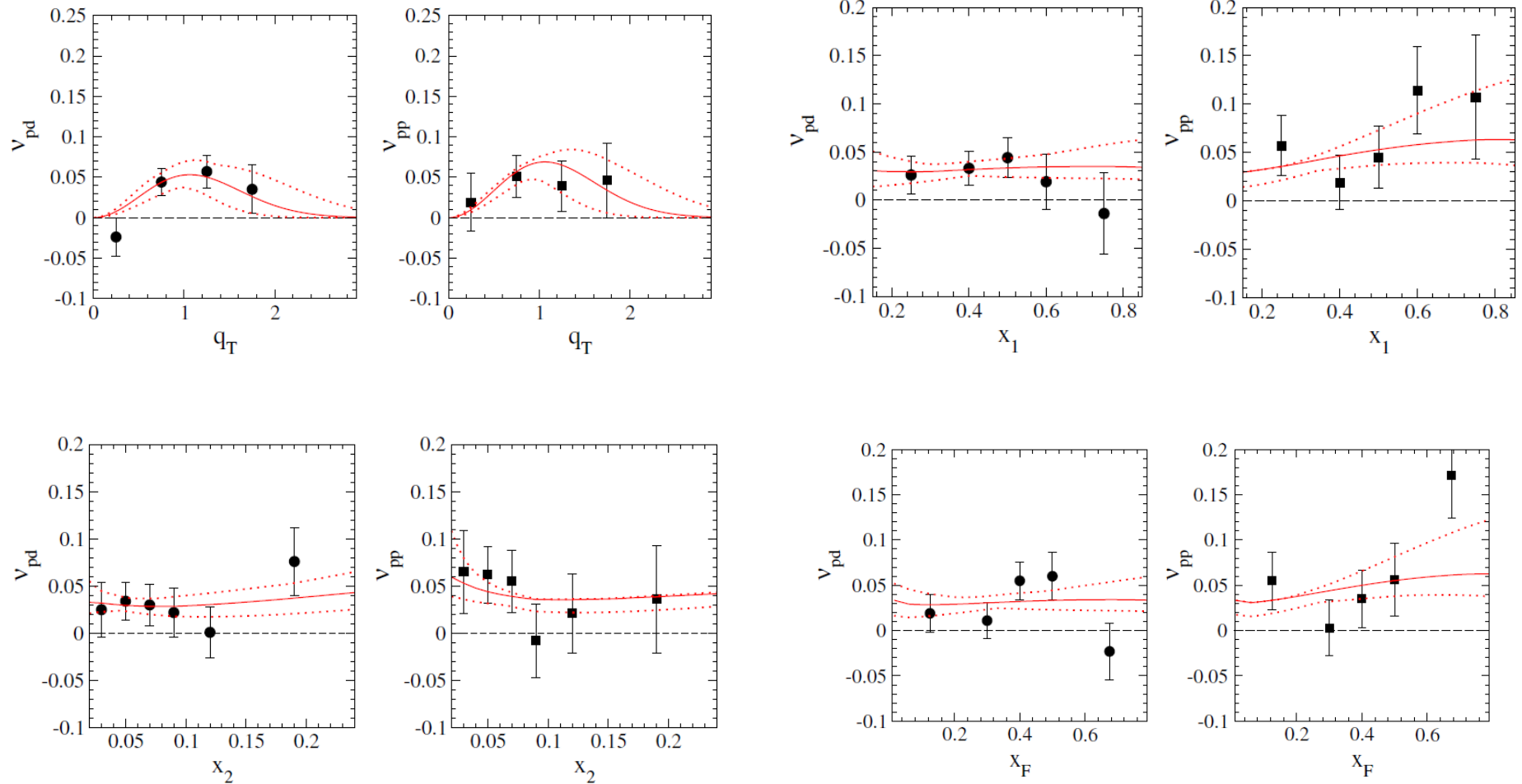


The bands do not represent the statistical error on the extraction! Central values are only geometrical mean values.



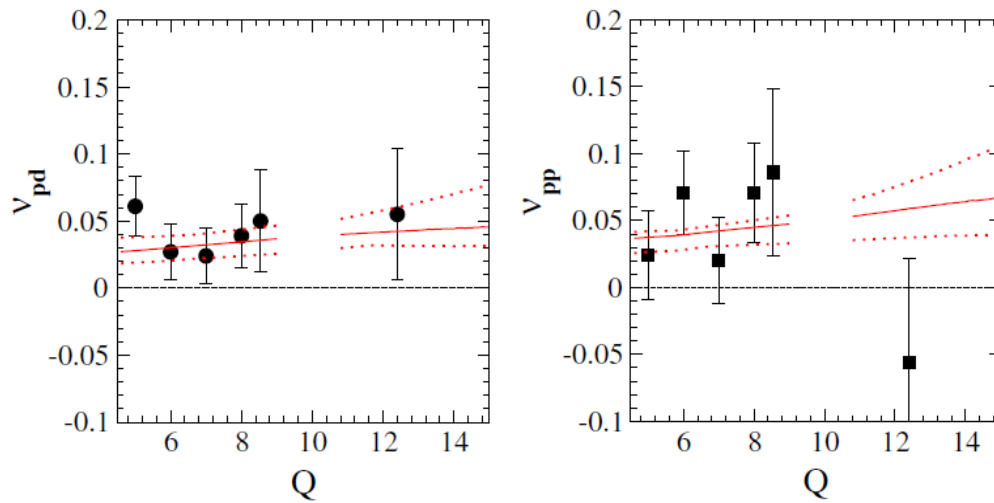
# Unpolarized DY:

## Extraction of the Boer-Mulders Function



# Unpolarized DY:

## Extraction of the Boer-Mulders Function



# Conclusions II

- The two fits agree in the relative sign of the u and d BM functions
- The two fits disagree in the magnitude of the BM functions
- Notice that DY pp/pD data are not sufficient to separate quark and antiquark  
Similarly in SIDIS we cannot extract efficiently antiquark
- Open questions: How these two different kinematics can be related?  
Average transverse momentum...  
Evolution equation....

The analysis of BMP has shown the importance of the knowledge of average transverse momentum

See also: Schweitzer, Teckentrup and Metz, ArXiv: 1003.2190

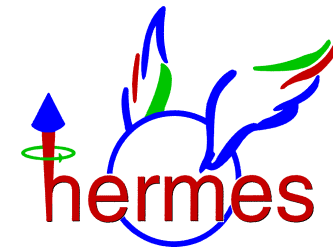
Back up



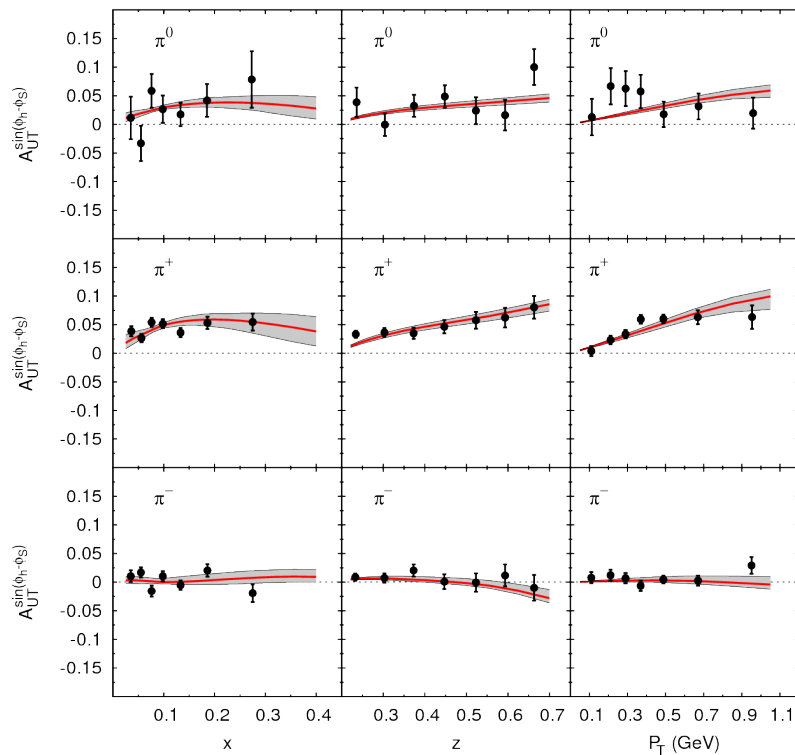


# Polarized SIDIS: Extraction of the Sivers Function

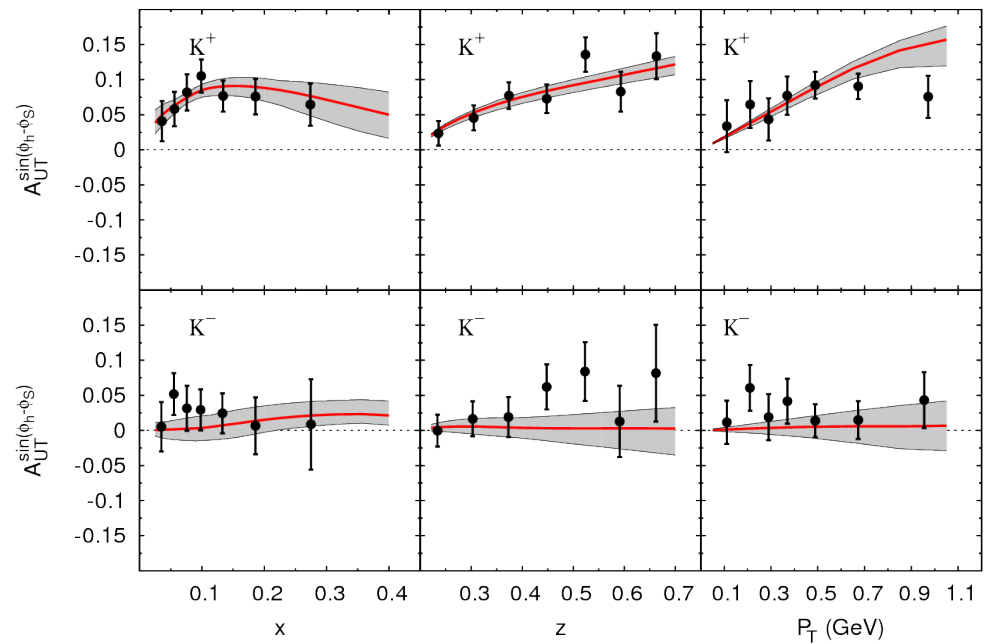
HERMES Proton Target-2009



HERMES Proton



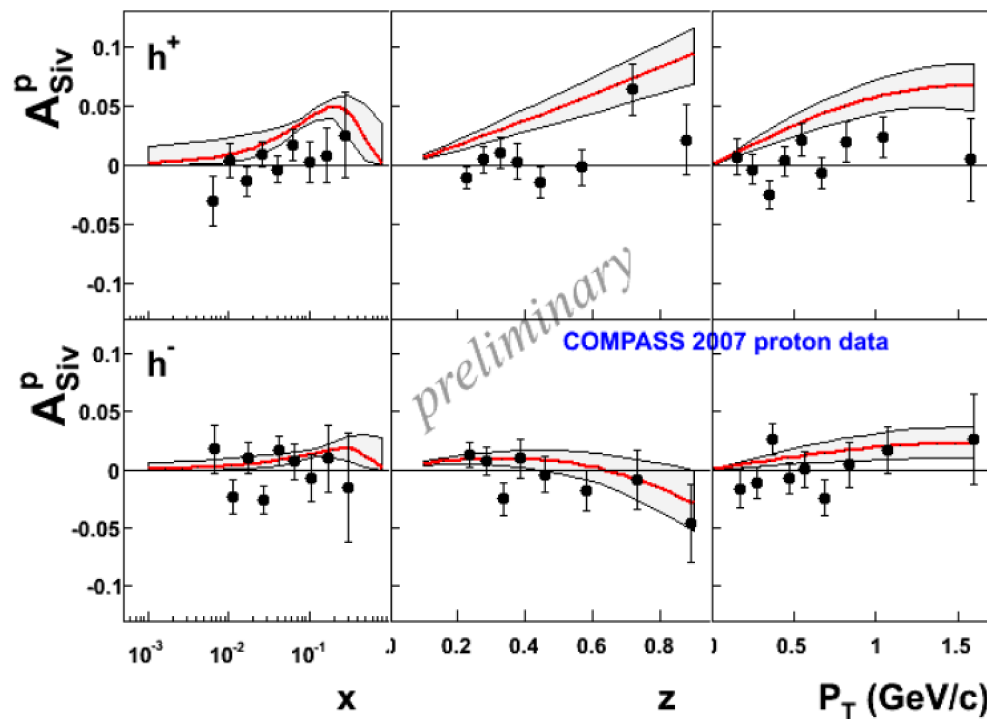
HERMES Proton



- A. Airapetian et al., Phys. Rev. Lett. 103 (2009) 152002

# Polarized SIDIS: Extraction of the Sivers Function

COMPASS Proton Target (Prediction!)



# Unpolarized SIDIS:

## Extraction of the Boer-Mulders Function

➤ Simple parametrization of the Boer-Mulders functions:

- $h_1^{\perp q}(x, k_{\perp}) = \lambda_q f_{1T}^{\perp q}(x, k_{\perp})$  for valence quarks

- $h_1^{\perp q}(x, k_{\perp}) = -|f_{1T}^{\perp q}(x, k_{\perp})|$  for sea quarks

➤ Models inspired:

$$h_1^{\perp q}(x, k_{\perp}) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_{\perp})$$

Tensor magnetic moment

Anomalous magnetic moment

Burkardt, Phys. Rev. D72, 094020 (2005)

Gockeler, Phys.Rev.Lett.98:222001,2007.

# Unpolarized SIDIS:

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➤ Models inspired:

$$h_1^{\perp q}(x, k_{\perp}) = \frac{\kappa_T^q}{\kappa^q} f_{1T}^{\perp q}(x, k_{\perp})$$

- $h_1^{\perp u}(x, k_{\perp}) \simeq 1.80 f_{1T}^{\perp u}(x, k_{\perp}) < 0$

- $h_1^{\perp d}(x, k_{\perp}) \simeq -0.94 f_{1T}^{\perp d}(x, k_{\perp}) < 0$