Basics facts about TMD factorization and universality

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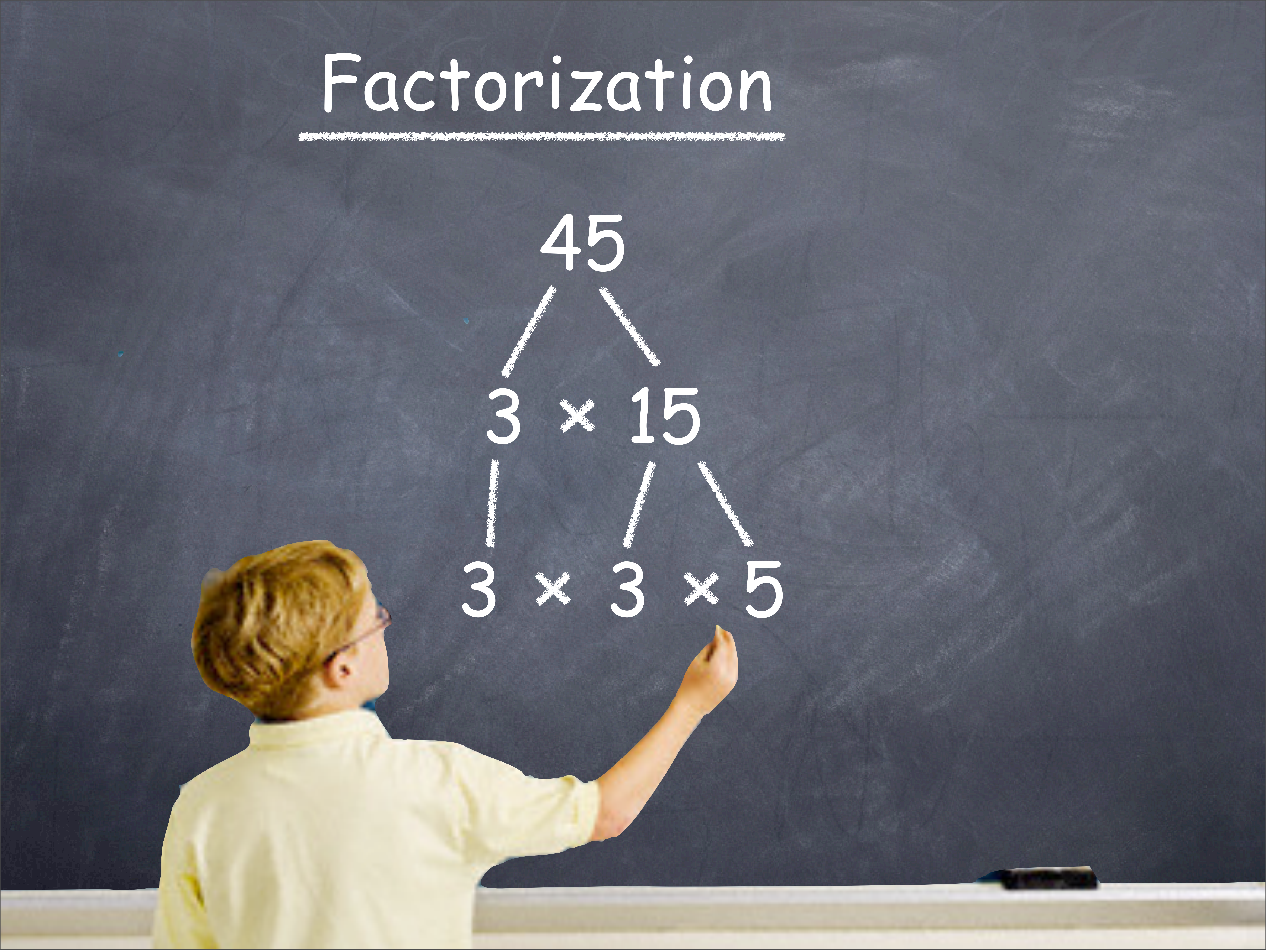
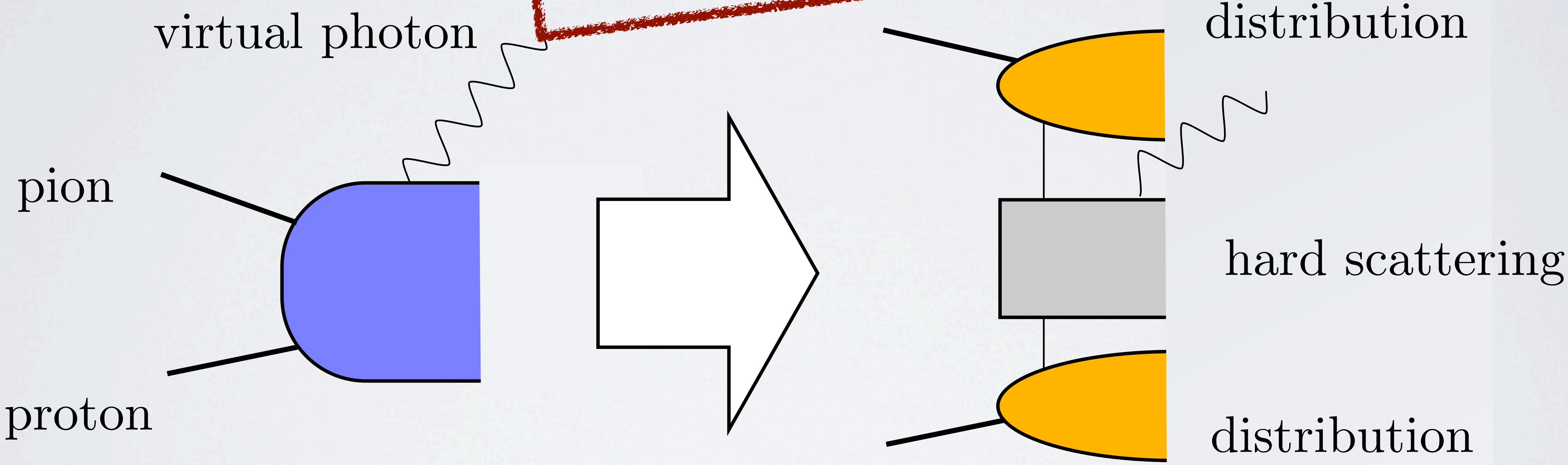

Factorization
45
3 × 15
3 × 3 × 5

Factorization

Drell-Yan

KEY RESULT OF QCD



$$F(x_a, x_b, Q^2) = \sum_{a,b} \int_{x_a}^{1} d\xi_a \int_{x_b}^{1} d\xi_b \, f^a\Big(\xi_a, \mu_F^2\Big) \, f^b\Big(\xi_b, \mu_F^2\Big) \; H_{ab}\Big(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, \ln\frac{\mu_F^2}{Q^2}\Big)$$

QCD without factorization is *almost useless**

*I added this sentence after this morning comments, so it might be too strong

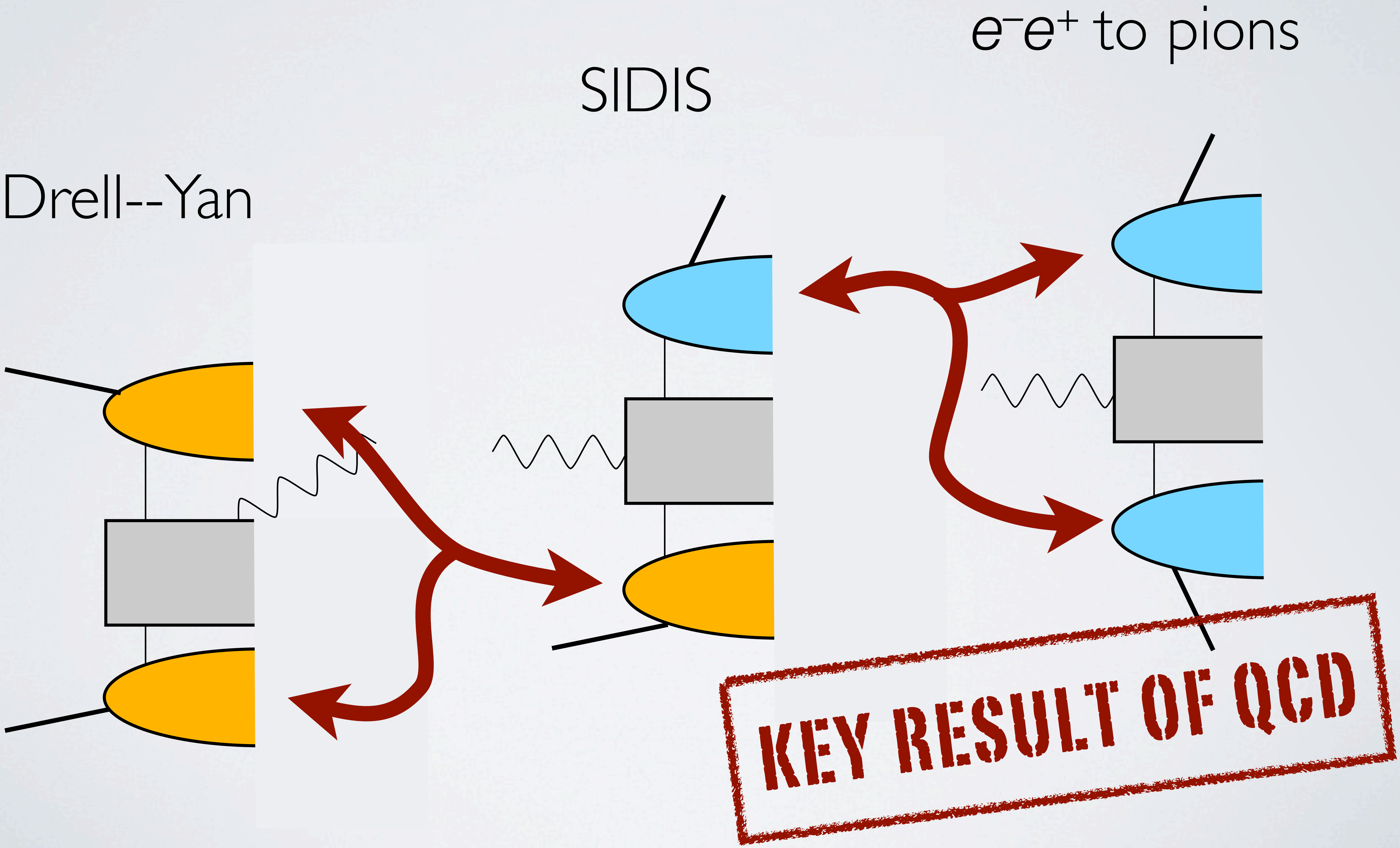
Universality

$$45 = 3 \times 3 \times 5$$

$$42 = 3 \times 2 \times 7$$

$$44 = 2 \times 2 \times 11$$

Universality



Universality

$45 = 3 \times 3 \times 5$

$42 = 3 \times 2 \times 7$

$44 = 2 \times 2 \times 11$

$195 = 3 \times 13 \times 5$

no universality!

$197 = 197$

no factorization!!

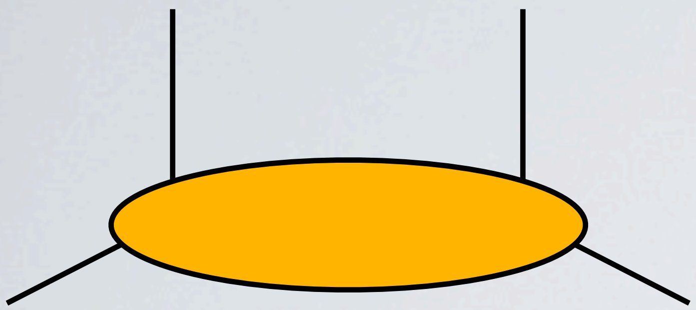
Why TMDs have these problems?

1. Gauge links

2. Light-cone divergences
   (see talk by I. Cherednikov)

Why do we need gauge links?

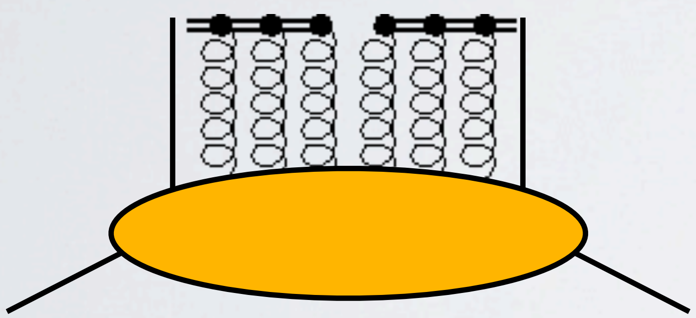
To make PDFs
(collinear and TMD)
gauge invariant



$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar\psi_j(0)\,\psi_i(\xi)\,|P,S\rangle$$

not invariant under

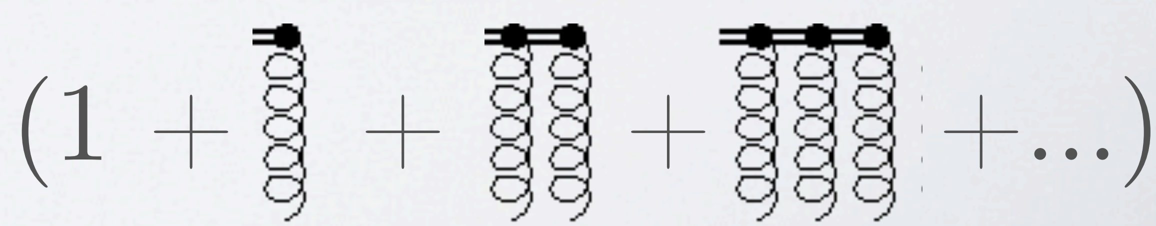
$$\psi(\xi) \to e^{i\alpha(\xi)}\,\psi(\xi)$$



$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar\psi_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle$$

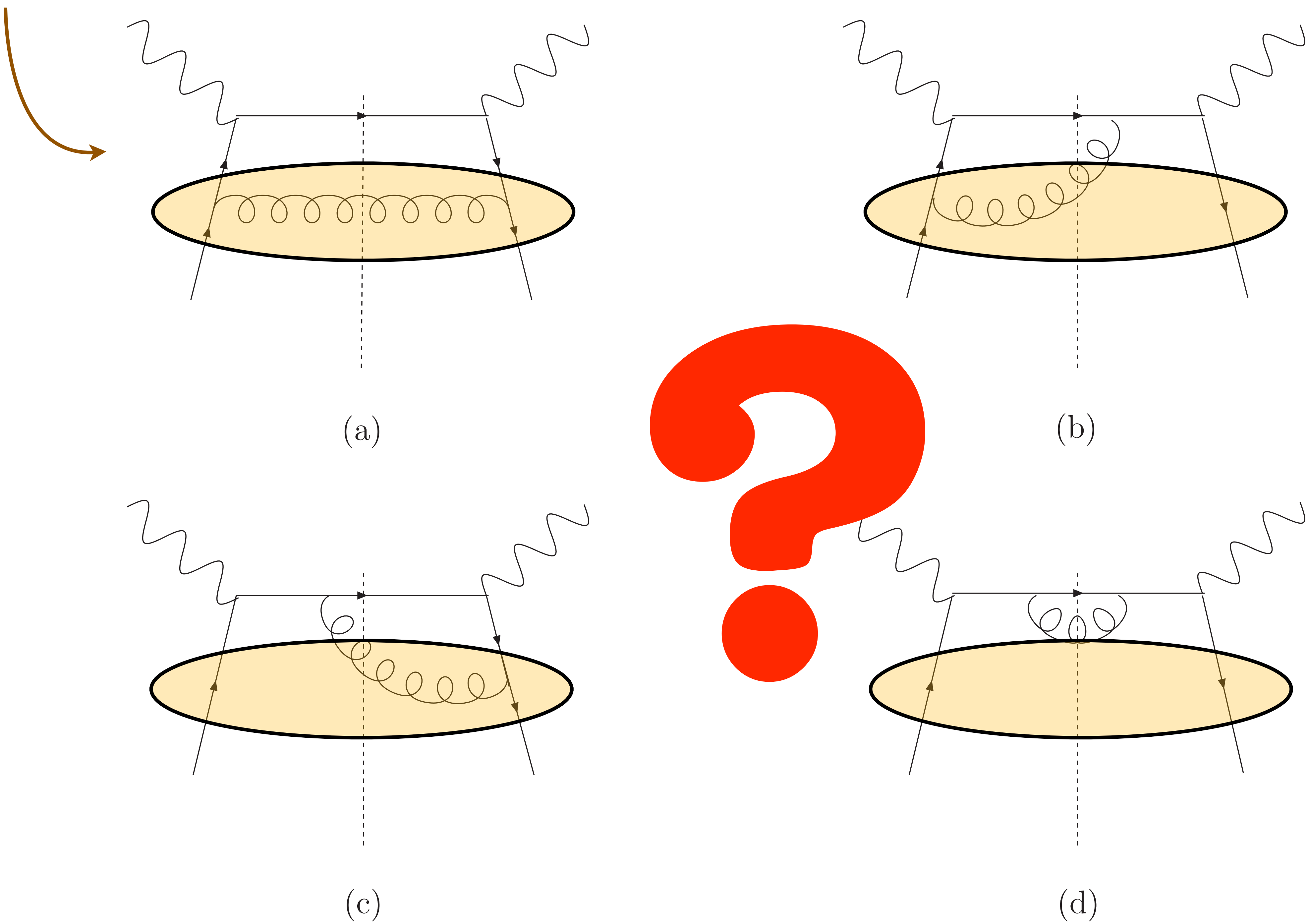
$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\,U(\xi_1,\xi_2)\,e^{-i\alpha(\xi_2)}.$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right]$$



A familiar analogy

Light-cone gauge



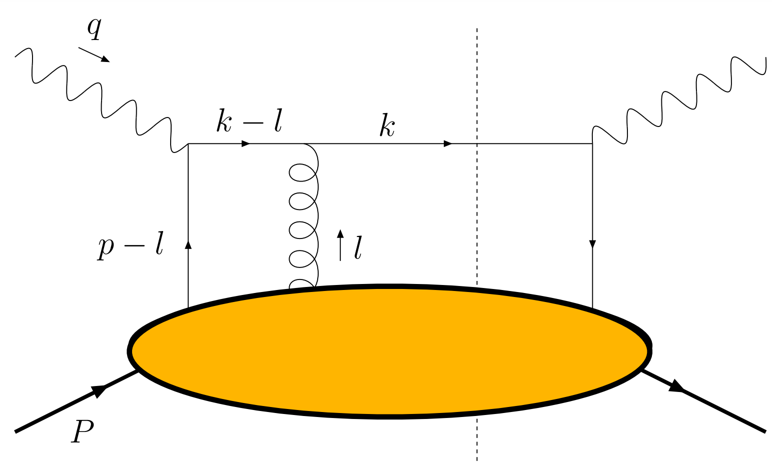
Can any path be used?

The shape of the gauge link
is fixed by the process

- Gauge links in inclusive DIS
- Gauge links in semi-inclusive DIS and Drell-Yan
- Gauge links in $\pi\, p$ to hadrons

Gauge links in inclusive DIS

Birth of the gauge link
(in Feynman gauge)



$$2MW^{(a)}_{\mu\nu} \sim \int d^4 l \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot(\eta-\xi)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+ \gamma_\alpha \frac{\not k - \not l}{(k-l)^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle$$

$$i\,\frac{\not k - \not l}{(k-l)^2+i\epsilon} \approx i\,\frac{k^-\gamma^+}{-2l^+k^- + i\epsilon} \approx \frac{i}{2}\,\frac{\gamma^+}{-l^+ + i\epsilon}$$

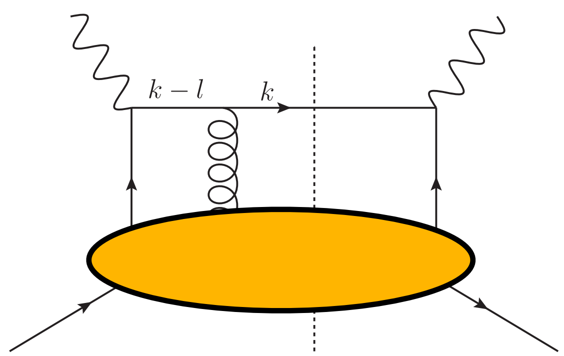
eikonal approximation

$$2MW^{(a)}_{\mu\nu} \sim \int \frac{d\eta^-}{2\pi} \int dl^+ e^{il^+(\eta^- - \xi^-)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+ \frac{\gamma^-\gamma^+}{2}\gamma_\nu (ig)\frac{A^+(\eta)}{-l^+ + i\epsilon}\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+,\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T}}$$

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\,\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T=0}}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

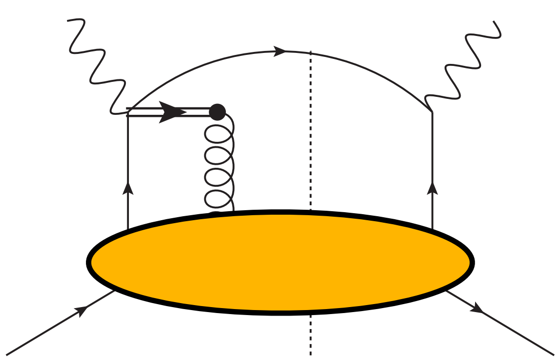
Birth of the gauge link



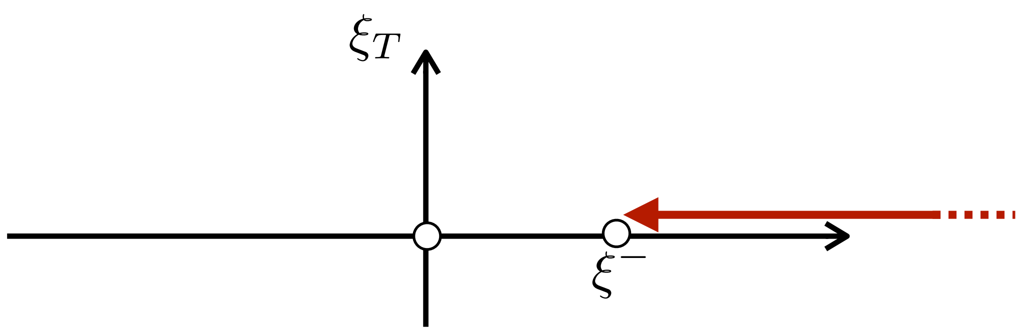
$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)|P,S\rangle$$

compare with:

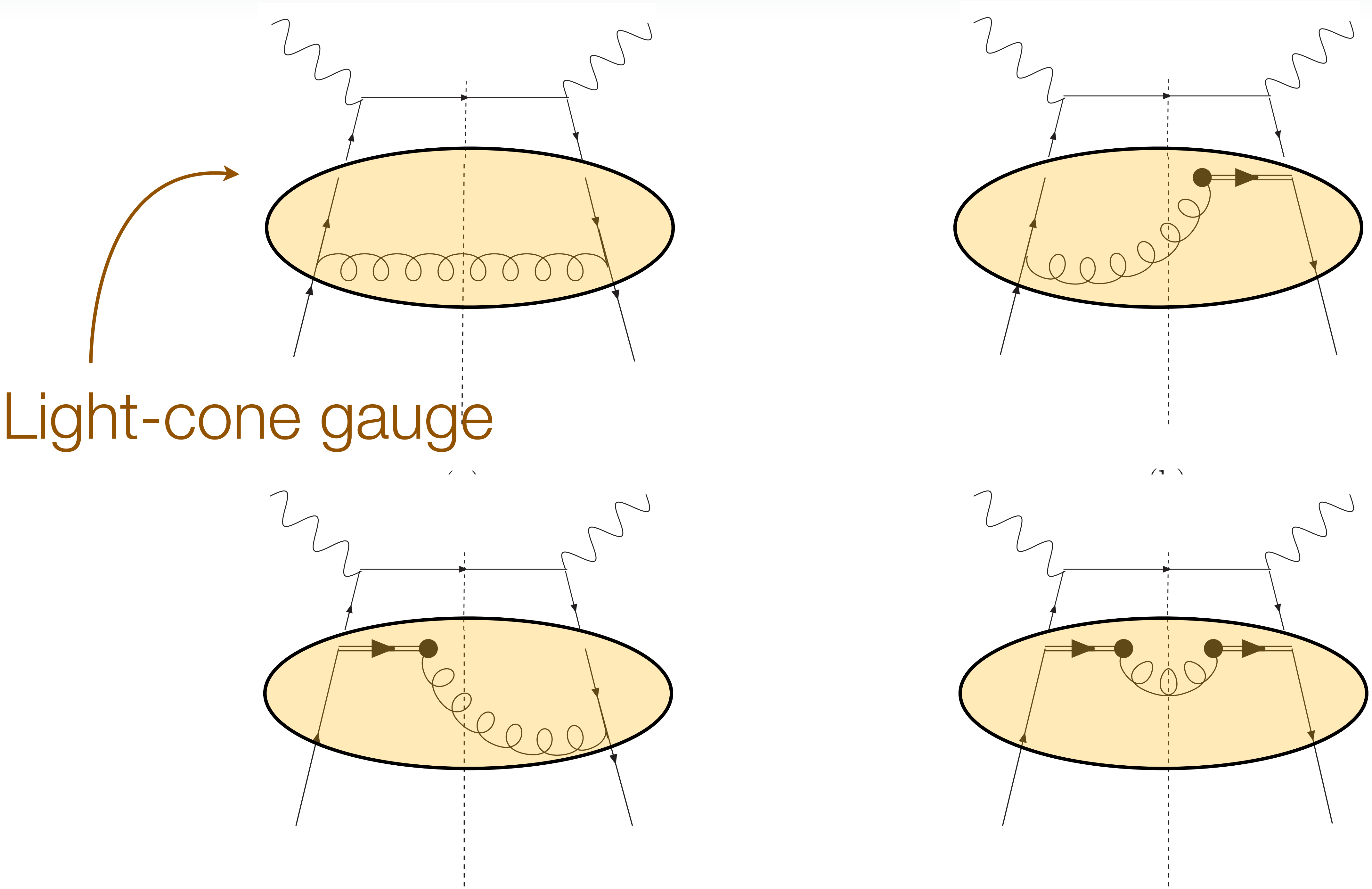
$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\,\frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$



$$\Phi^{(a)}(x,S) \sim \langle P,S|\,\bar{\psi}(0)\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)\,|P,S\rangle$$



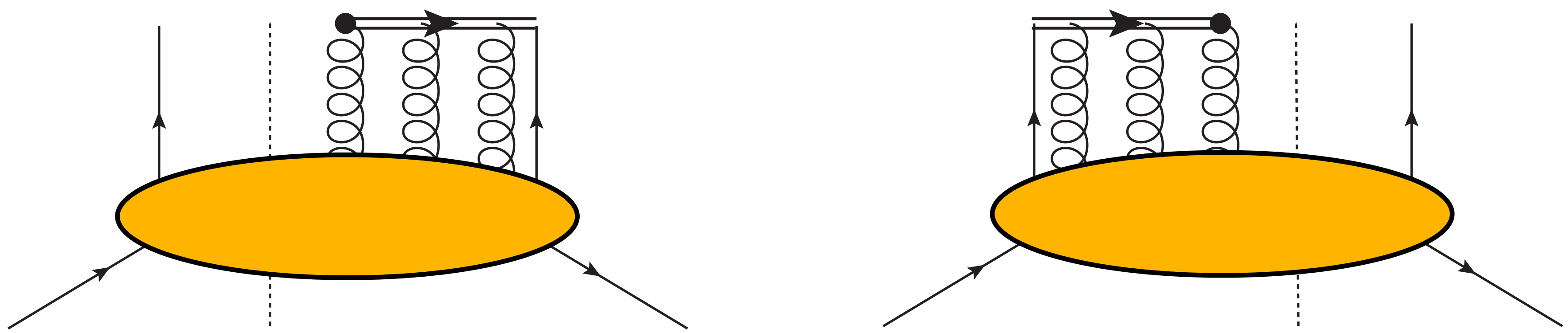
Back to familiar analogy

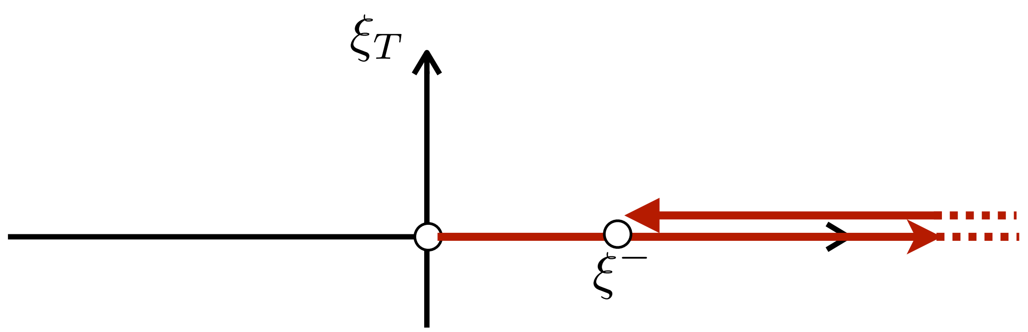


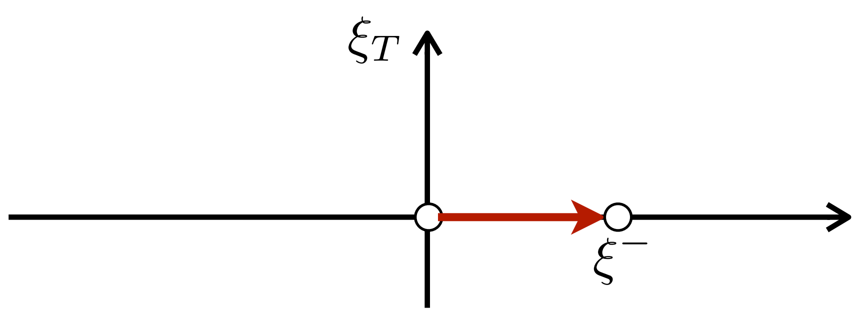
(c) (d)

Shape of the gauge link

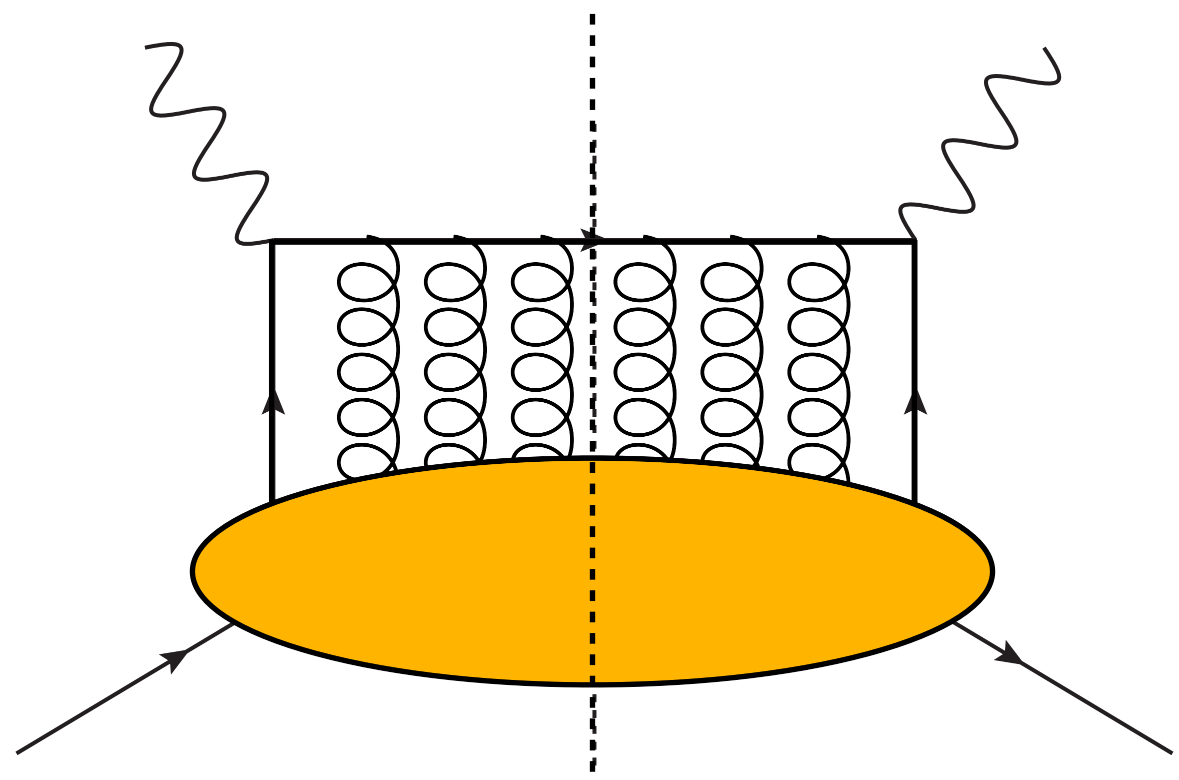
$$\Phi(x,S) \sim \langle P,S \big| \, \bar{\psi}(0) \, U_{[0,\infty^-]} \, U_{[\infty^-,\xi^-]} \psi(\xi) \, \big| P,S \rangle$$



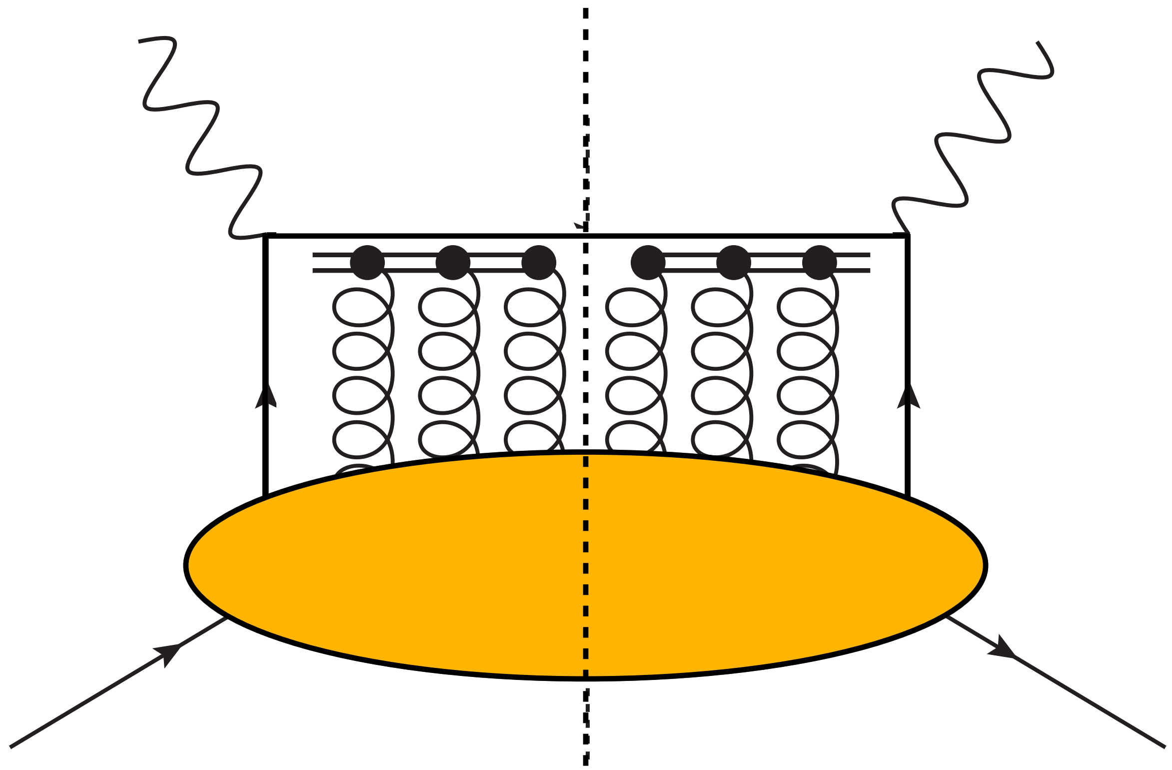




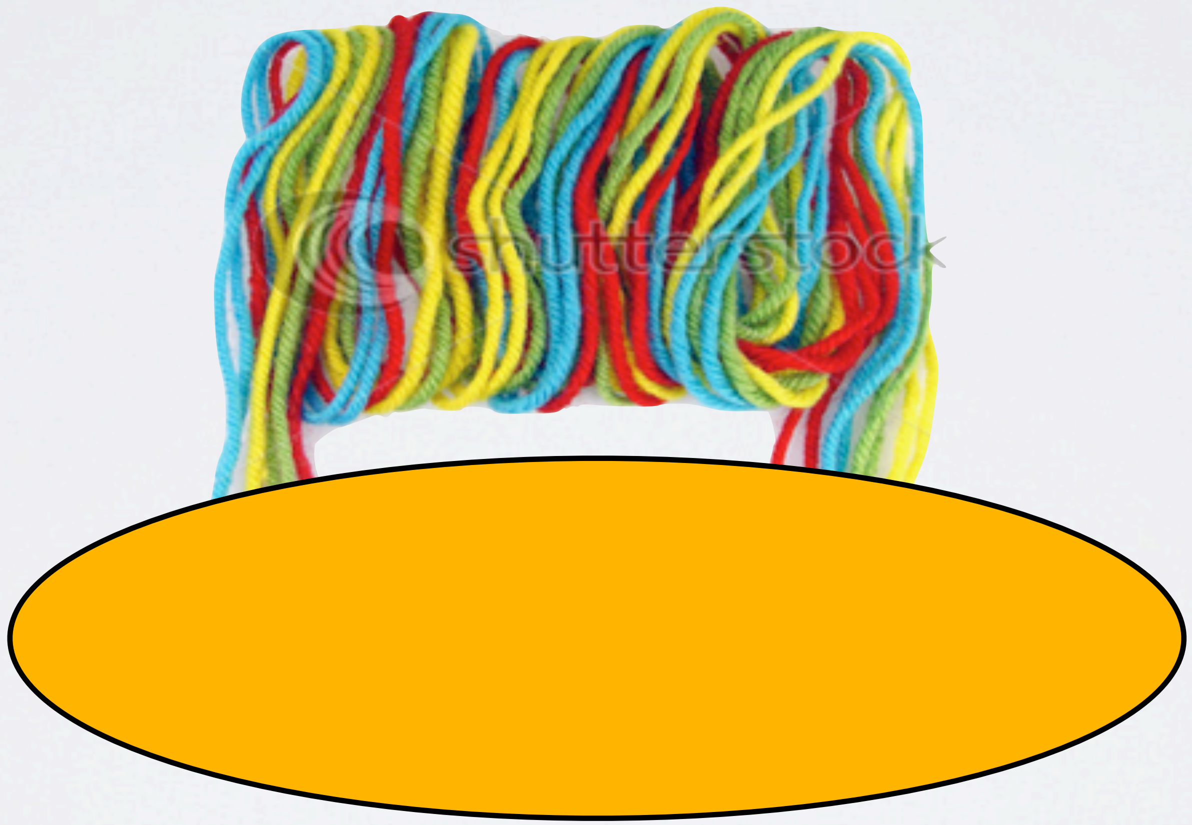
Gauge link



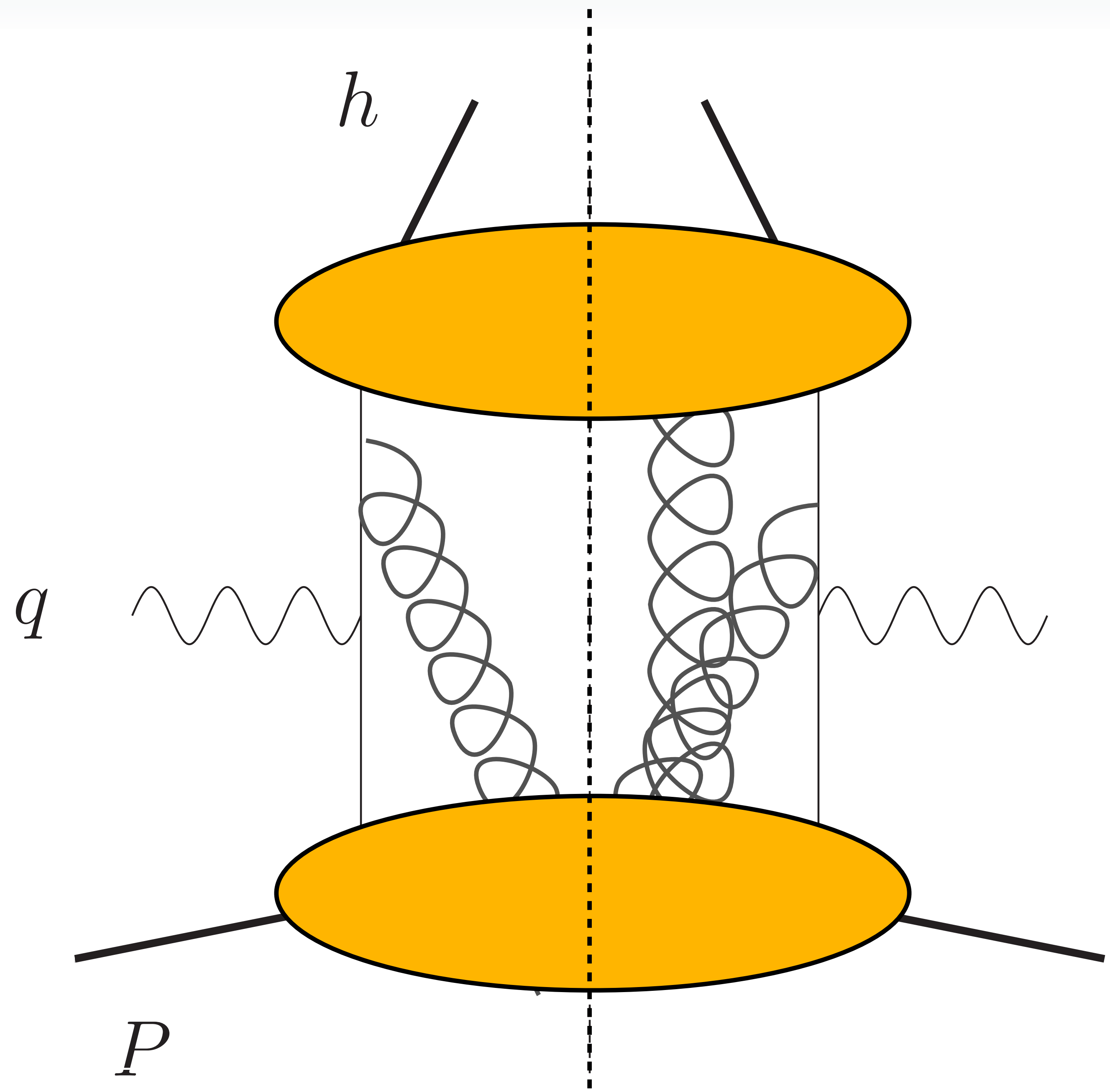
Gauge link



Gauge link

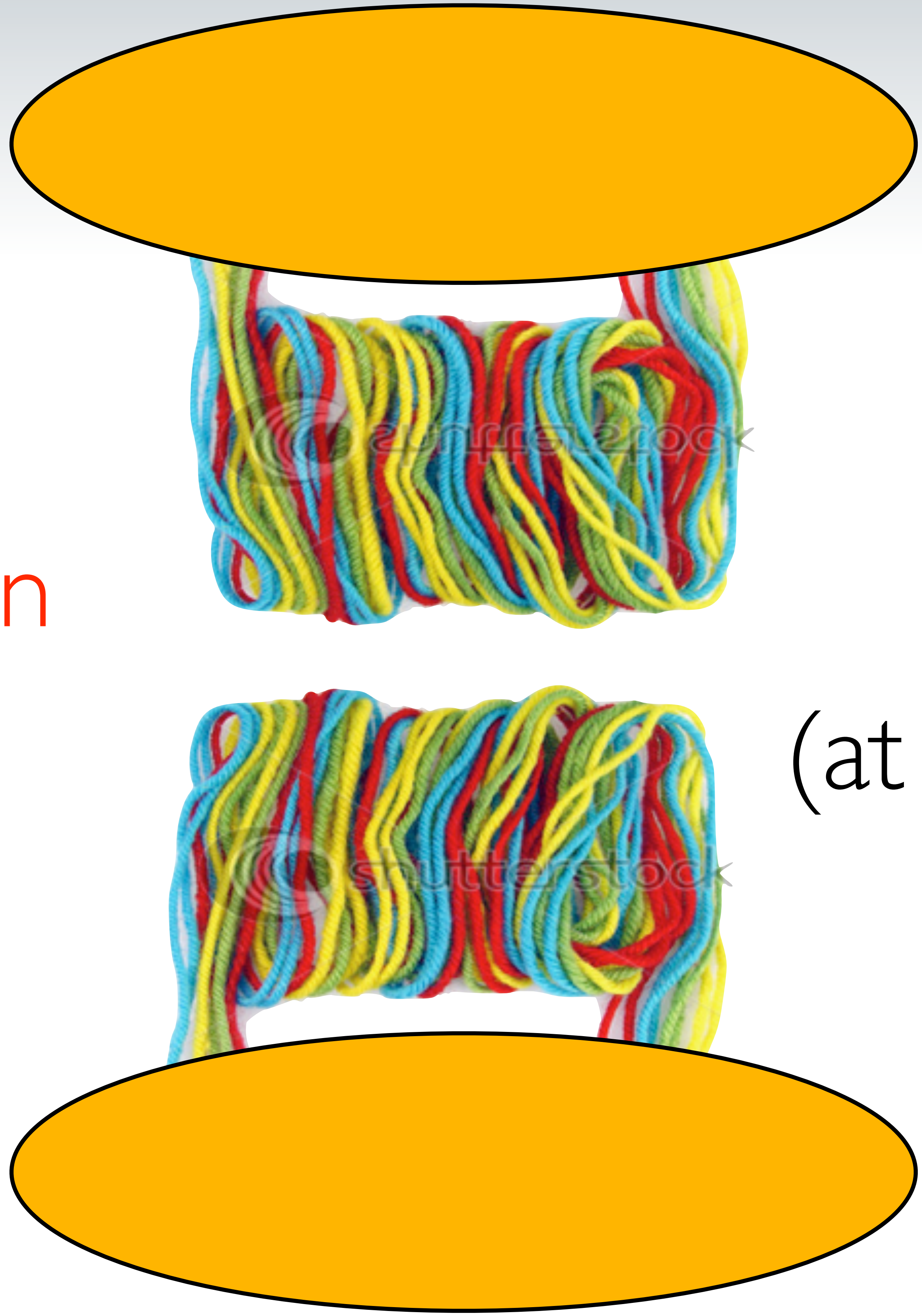


Gauge links in semi-inclusive DIS, Drell-Yan, and e^-e^+ annihilation


h
q
P

First step
to prove
factorization

First step
to prove
factorization



(at leading twist)

Inclusive DIS

$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T=0}}$$

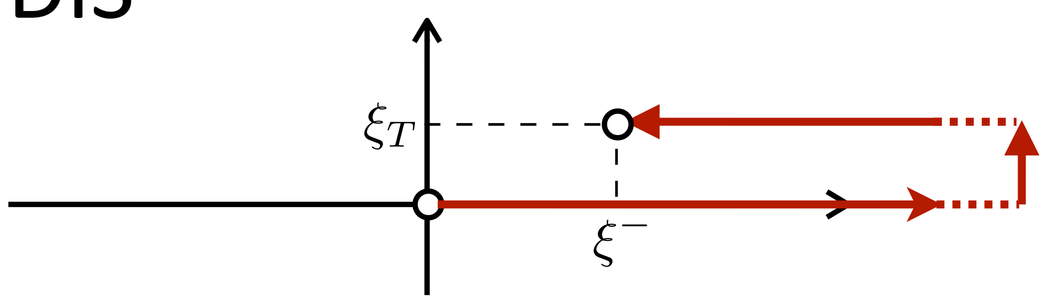
Semi-inclusive DIS

$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T}}$$

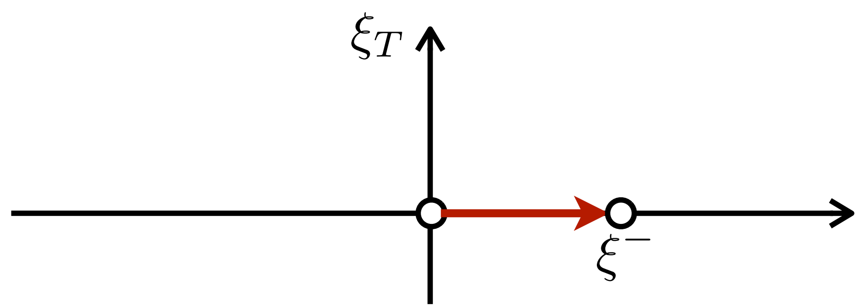
Shape of gauge links

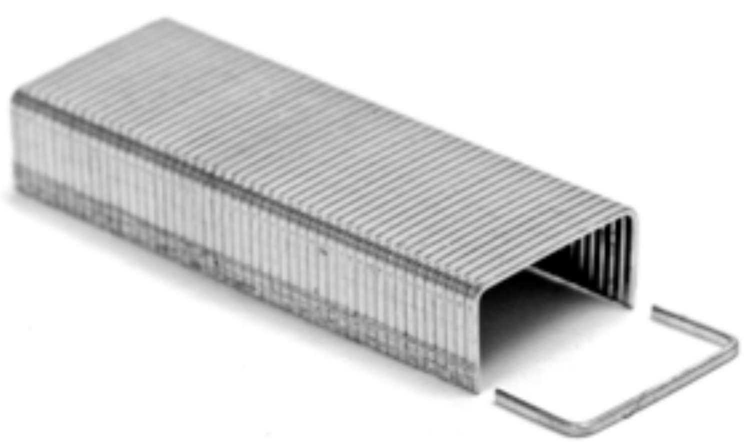
$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)|P\rangle \Big|_{\xi^+=0}$$

SIDIS



p_T integration

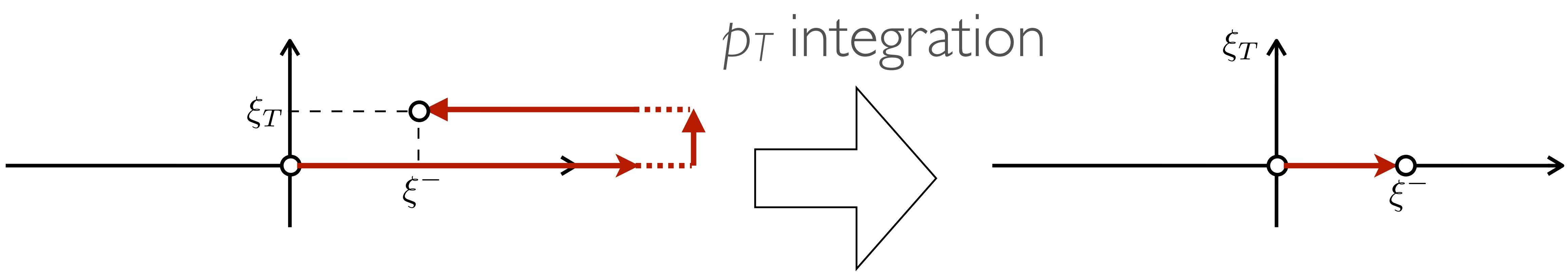


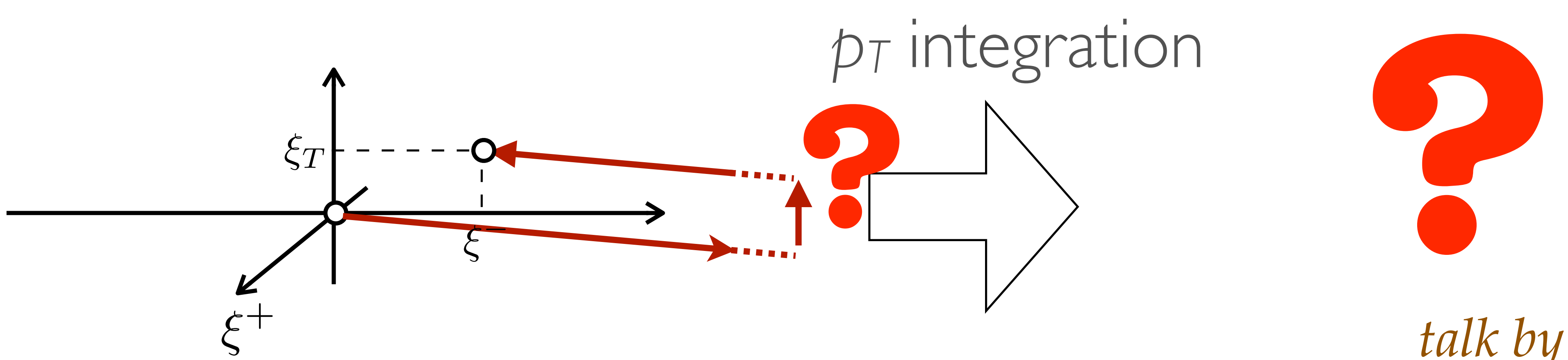


The "staple" gauge link

Light-cone divergences problems

$$f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0) U_{[0,\xi]} \gamma^+ \psi^q(\xi)|P\rangle \Big|_{\xi^+=0}$$



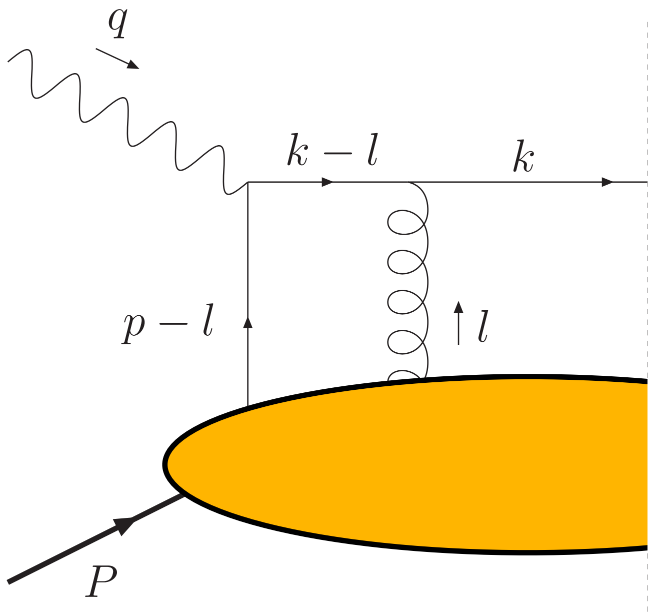


talk by I. Cherednikov

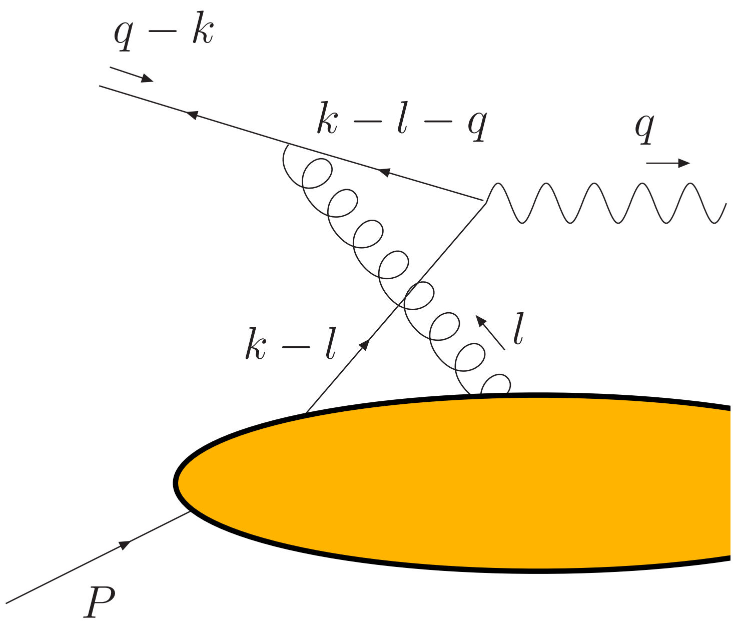
$$f_1^q(x, p_T^2, \zeta) = \int \frac{d\xi^- d^2\xi_T}{16\pi^3} e^{ip\cdot\xi} \langle P|\bar{\psi}^q(0) U^{\zeta}_{[0,\xi]} \gamma^+ \psi^q(\xi)|P\rangle \Big|_{\xi^+=0}$$

Final/initial state interactions

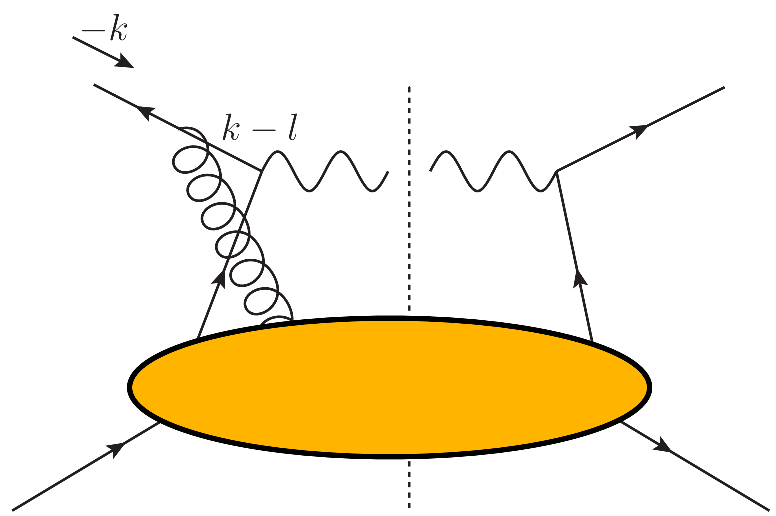
SIDIS



Drell-Yan



Gauge link in Drell-Yan



$$2MW_{\mu\nu}^{(a)} \sim \int d^4l \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot(\eta-\xi)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha \, \frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon}\,\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle$$

$$i\,\frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon} \approx i\,\frac{-(-k)^-\gamma^+}{2l^+(-k)^-+i\epsilon} \approx \frac{i}{2}\,\frac{\gamma^+}{-l^+-i\epsilon}$$

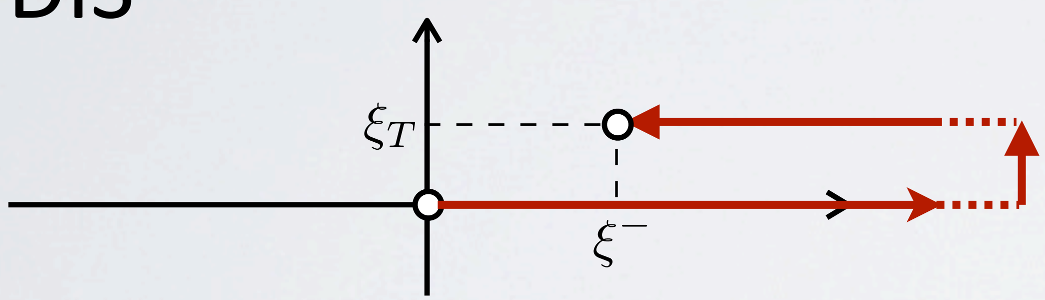
$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{-\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\;\psi(\xi)|P,S\rangle\Big|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$

Collins, PLB 536 (02)

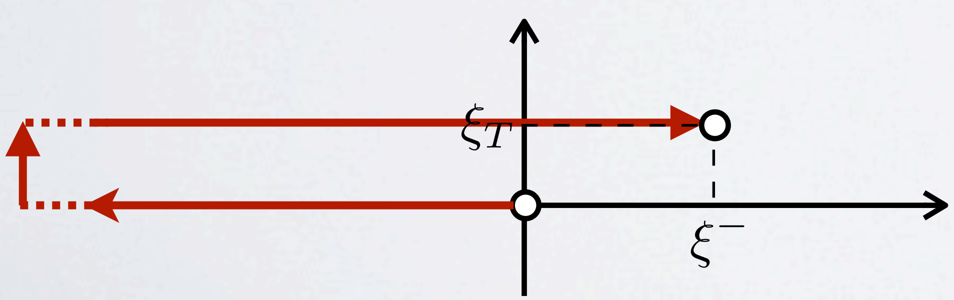
Shapes of gauge links

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3} \, e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

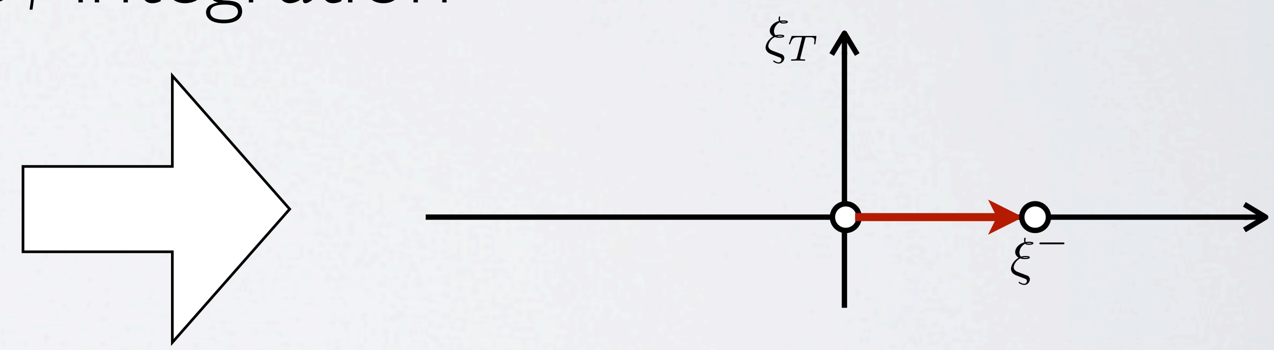
SIDIS



Drell-Yan

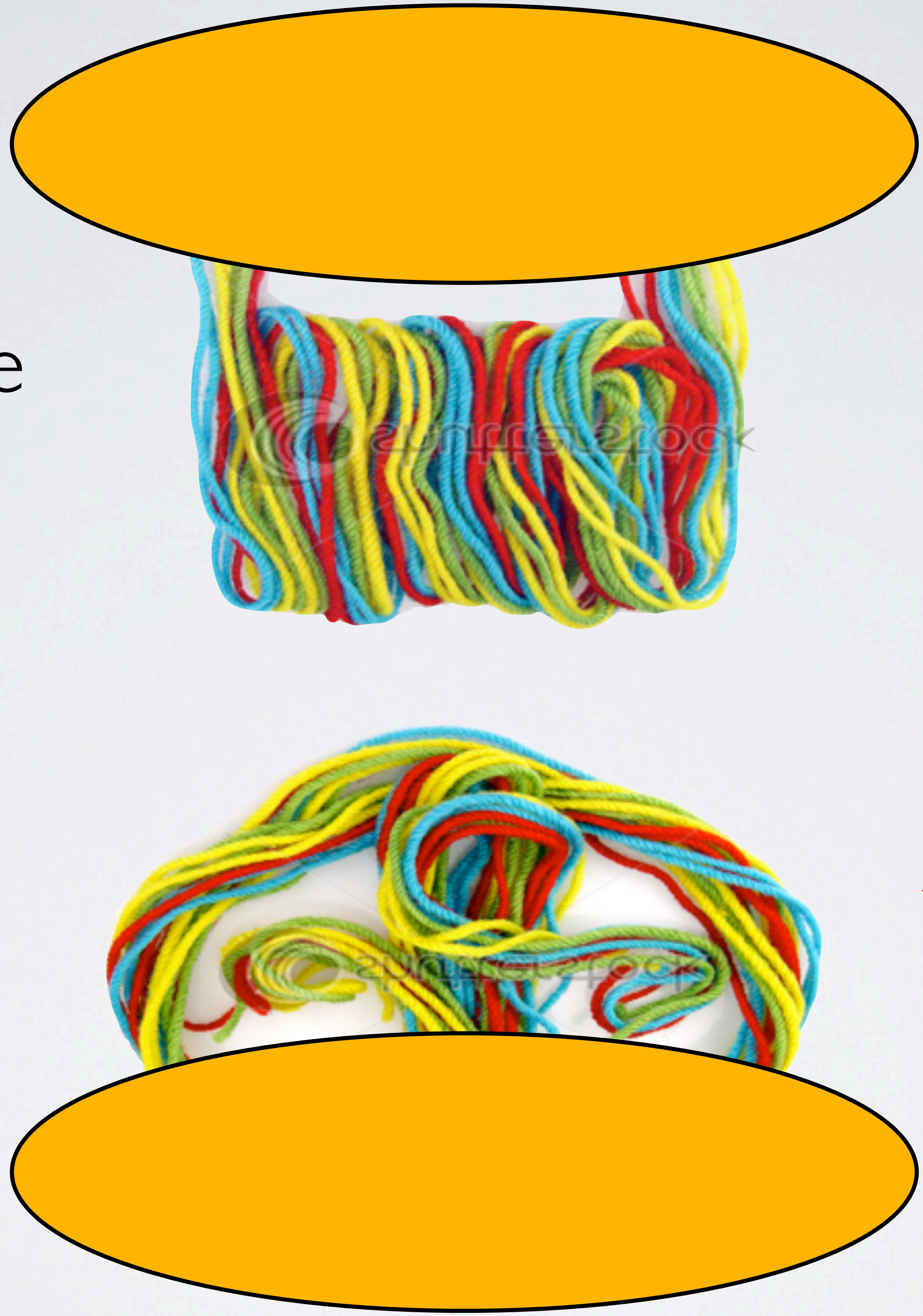


p_T integration



Collins, PLB 536 (02)

Gauge links are not always identical



Generalized Factorization (factorization without universality)

Collins, PLB 536 (02)
Bomhof, Mulders, Pijlman, PLB 596 (04)
A.B., Bomhof, Mulders, Pijlman, PRD 72 (05)
Collins, Qiu, PRD 75 (07)
Vogelsang, Yuan, PRD76 (07)

Phenomenological consequences

$$\left.\frac{1}{-l^+ + i\epsilon}\right|_{\mathrm{SIDIS}} \qquad \left.\frac{1}{-l^+ - i\epsilon}\right|_{\mathrm{DY}}$$

$$f_{1T}^{\perp}\big|_{\mathrm{SIDIS}} = -f_{1T}^{\perp}\big|_{\mathrm{DY}}$$

$$h_1^{\perp}\big|_{\mathrm{SIDIS}} = -h_1^{\perp}\big|_{\mathrm{DY}}$$

“[The experimental check of the change of sign] would crucially test the factorization approach to the description of processes sensitive to transverse parton momenta.”

Efremov, Goeke, Menzel, Metz, Schweitzer, PLB 612 (05)

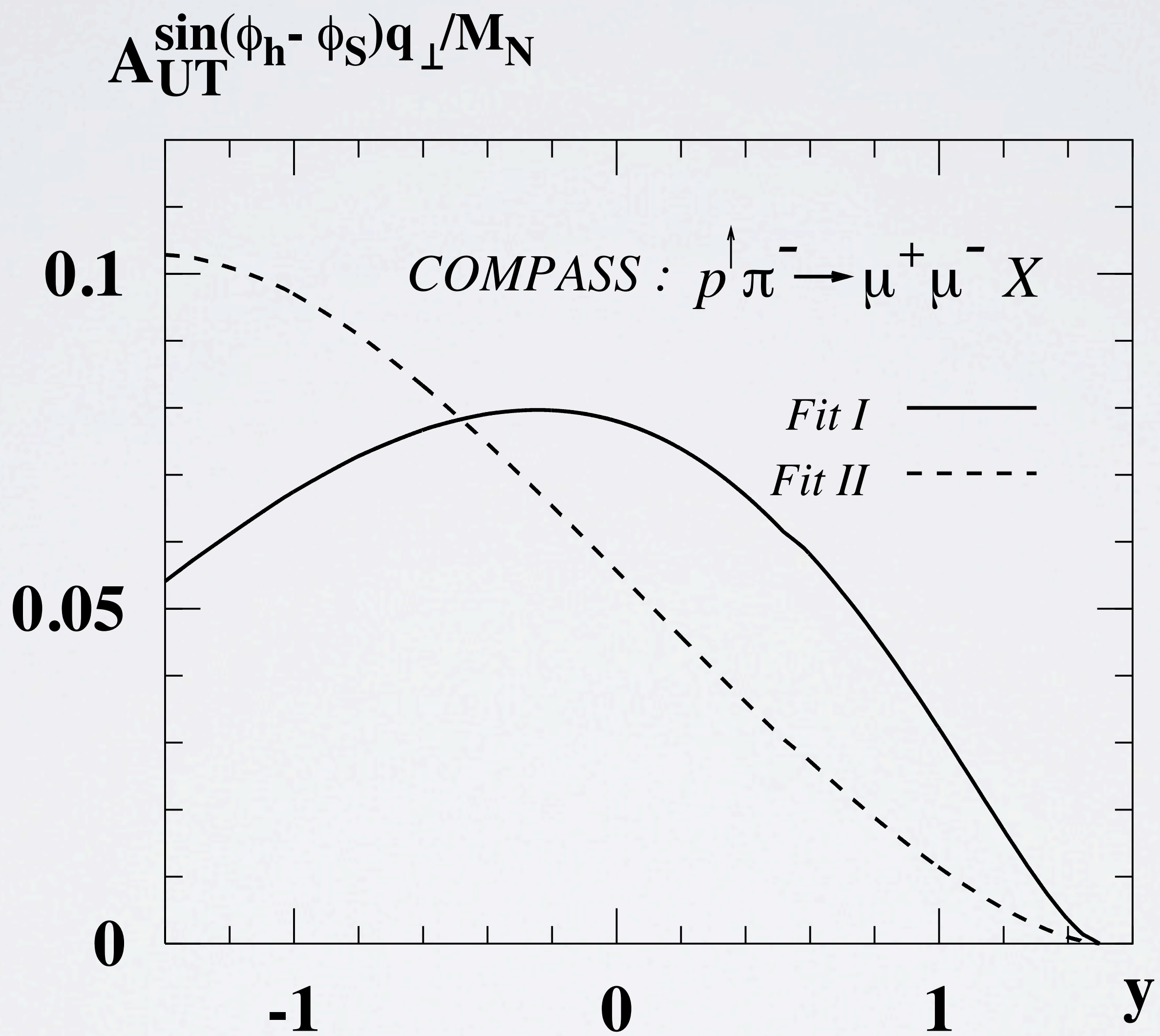
“It is a remarkable and fundamental QCD prediction that really tests all concepts we know of for analyzing hard-scattering reactions in strong interactions, and it awaits experimental verification.”

Bomhof, Mulders, Vogelsang, Yuan, PRD 75 (07)

"Its experimental verification would be crucial to confirm the validity of our present conceptual framework for analyzing hard hadronic reactions."

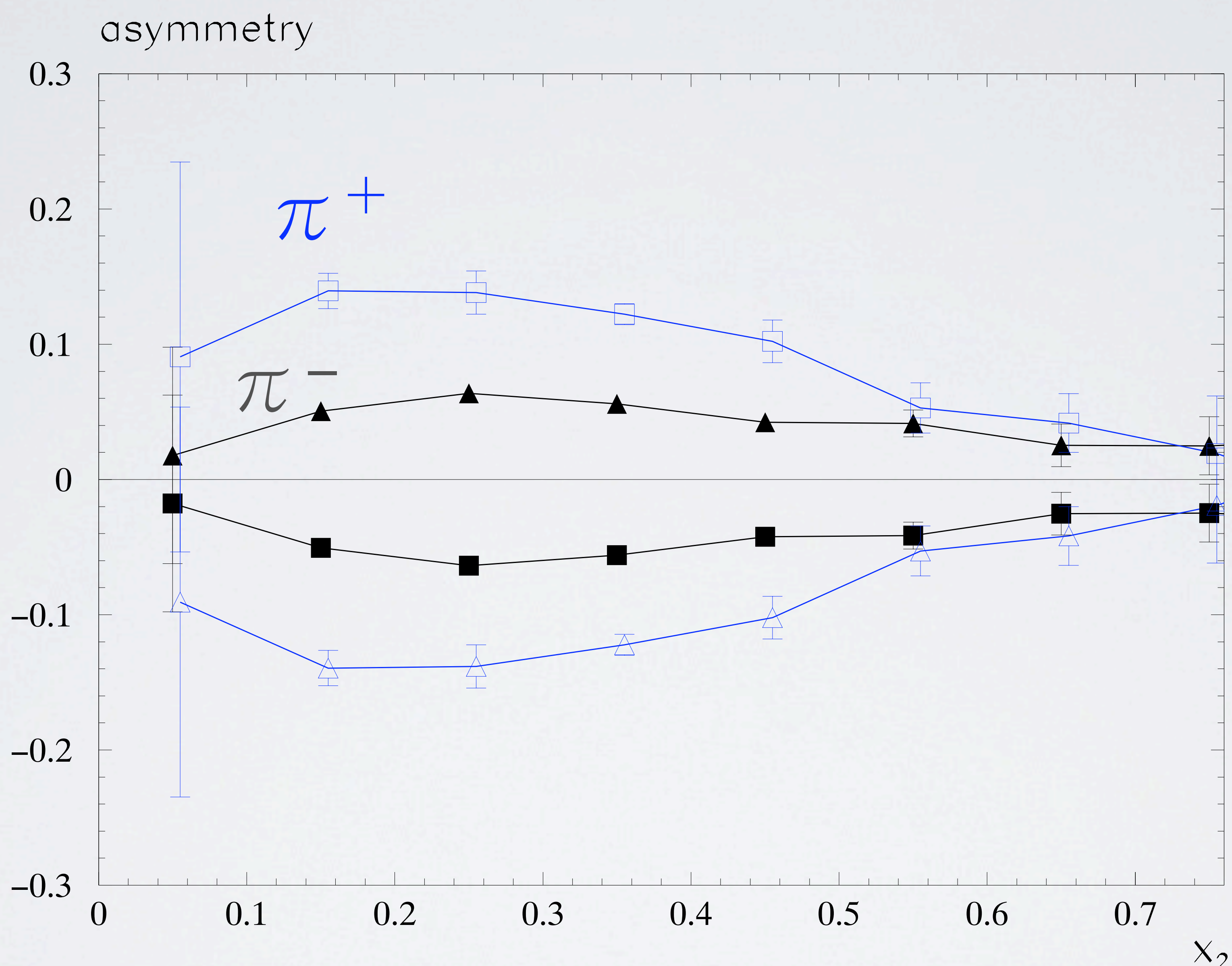
A.B., Bomhof, D'Alesio, Mulders, Murgia, PRL 99 (07)

Phenomenological consequences



Efremov, Goeke, Menzel, Metz, Schweitzer, PLB 612 (05)

Phenomenological consequences



Bianconi, Radici, PRD 73 (06)

See talks by S. Melis and Round Table

What happens if we don't find the sign change?

“Good tests kill flawed theories.
We remain alive to guess again.”

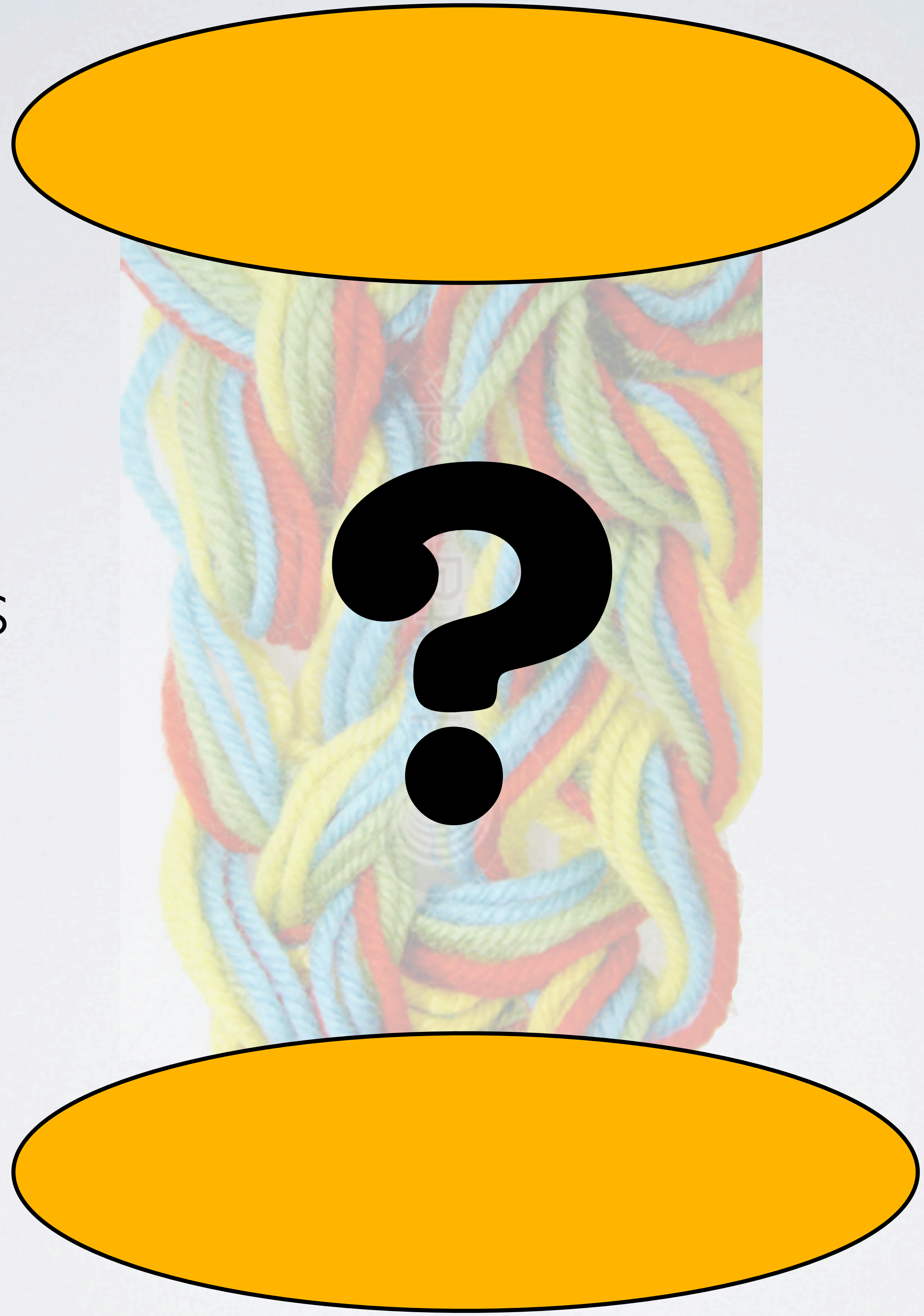
Karl Popper

- TMD factorization cannot be used (bad and far-reaching conclusion)

- SSA must have a different origin probably not factorized. Still puts serious doubts on TMD factorization.

Gauge links in $\pi\, p$ to hadrons

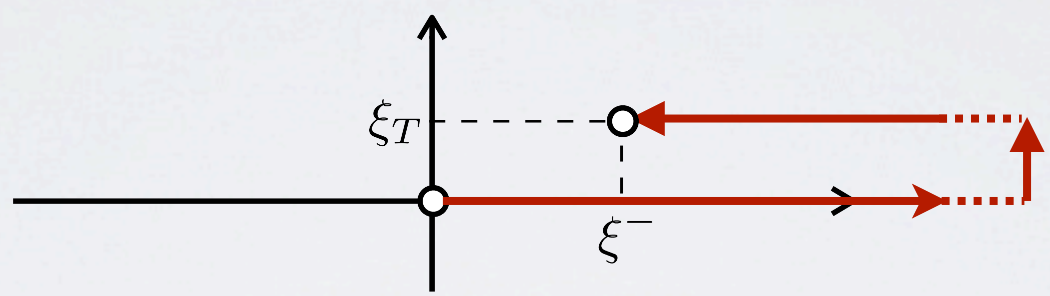
$\pi\, p$ to hadrons



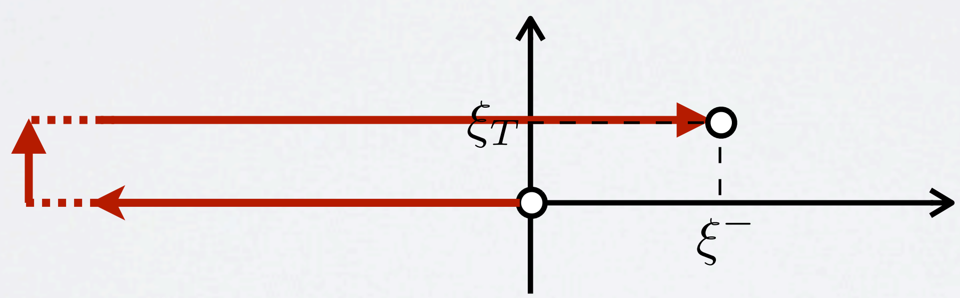
Can we have
Generalized
Factorization ?

Generalized factorization

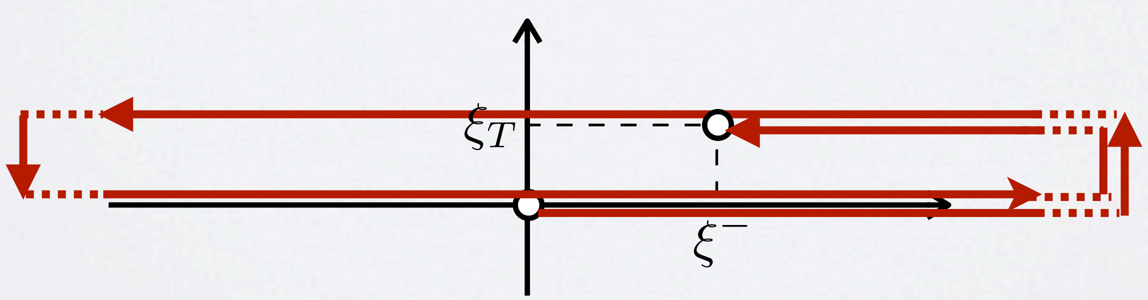
SIDIS



Drell-Yan

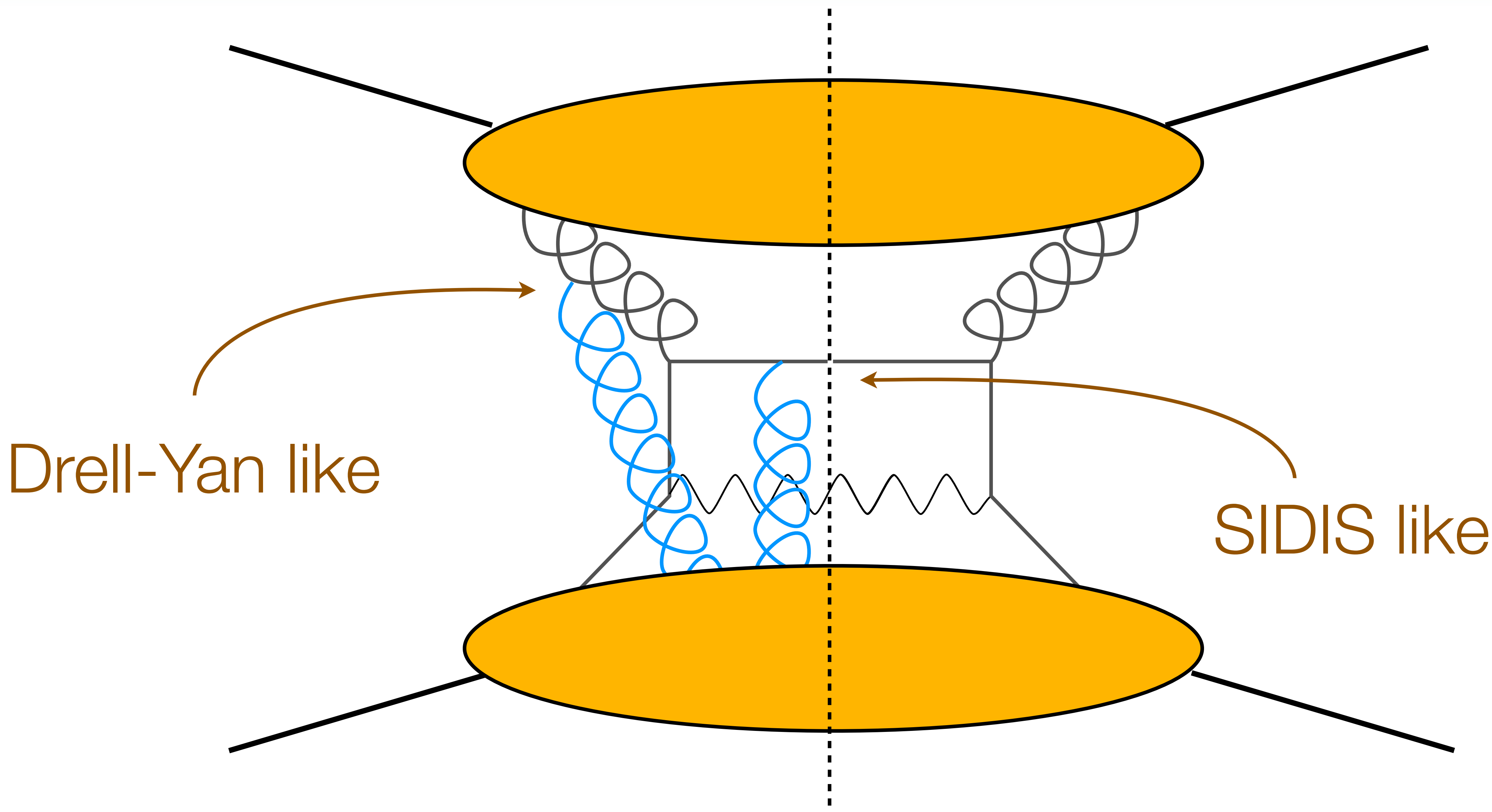


π p to hadrons



+ several others

Example of $\pi\, p$ to photon-jet

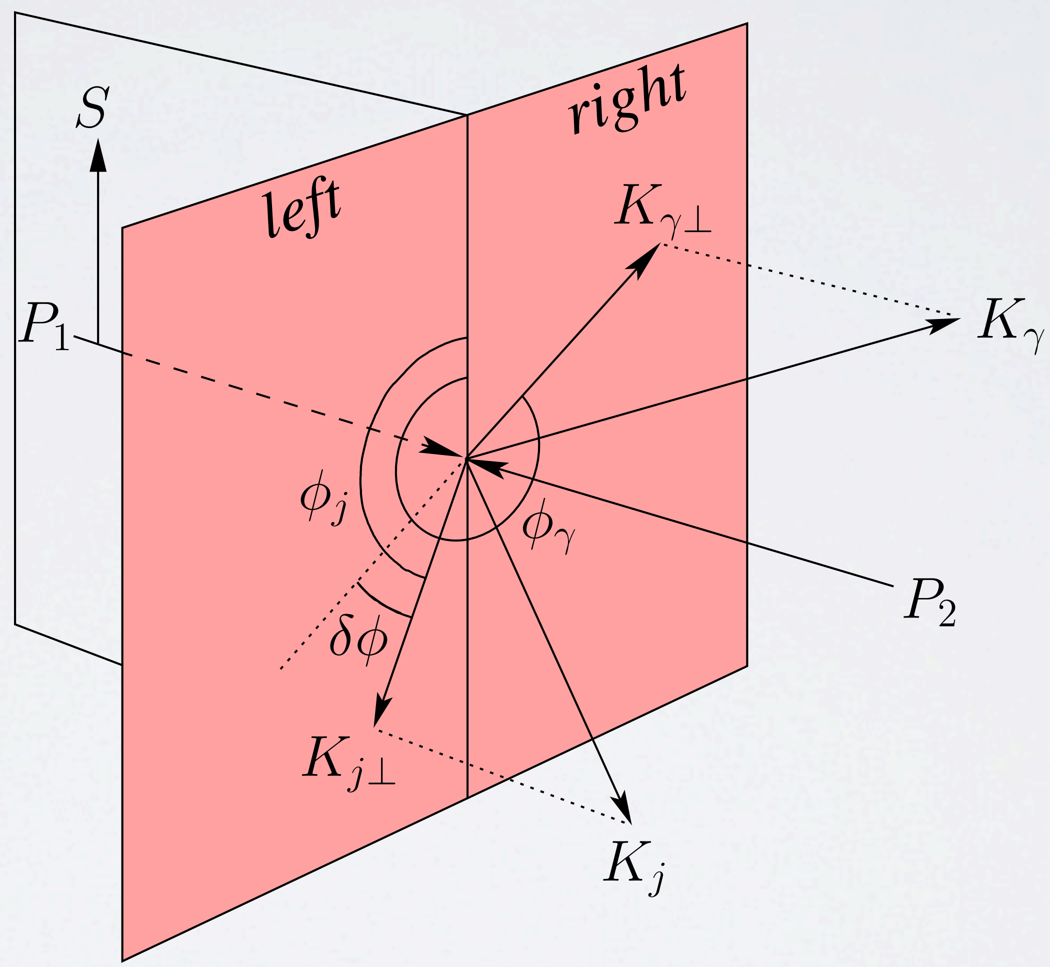


A.B., D'Alesio, Bomhof, Mulders, Murgia, PRL99 (07)
Ratcliffe, Teryaev, hep-ph/0703293
Boer, Mulders, Pisano, PLB660 (08)

Phenomenological consequences

$$f_{1T}^{\perp(1)}\Big|_{\gamma\ \mathrm{jet}} = -\frac{N_c^2+1}{N_c^2-1} f_{1T}^{\perp(1)}\Big|_{\mathrm{SIDIS}}$$

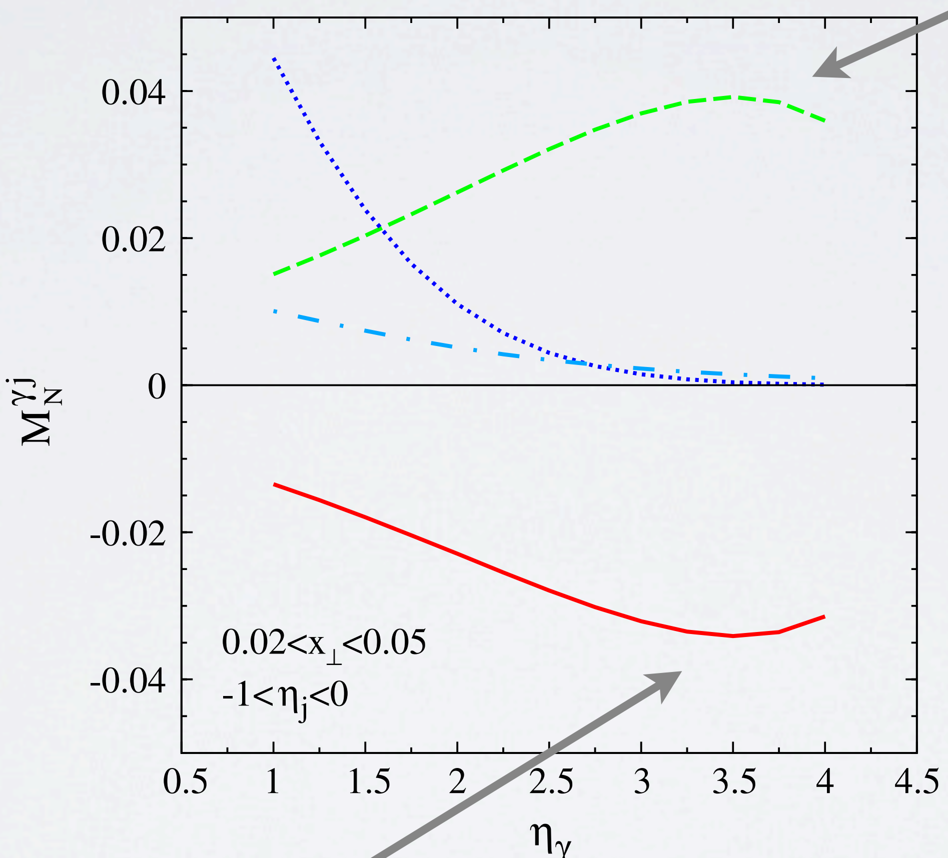
Phenomenological consequences



A.B., D'Alesio, Bomhof, Mulders, Murgia,PRL99 (07)

Phenomenological consequences

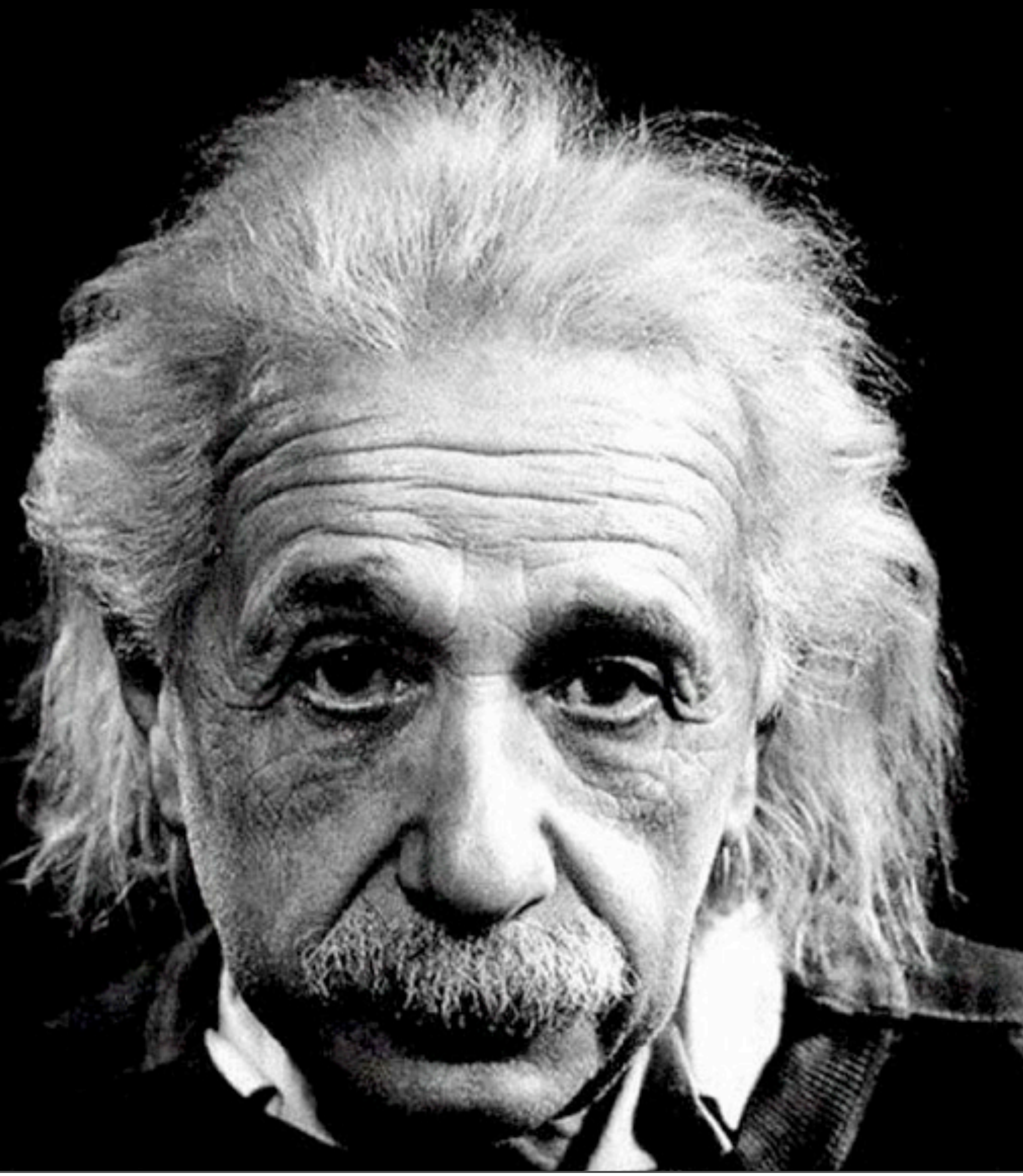
Standard universality



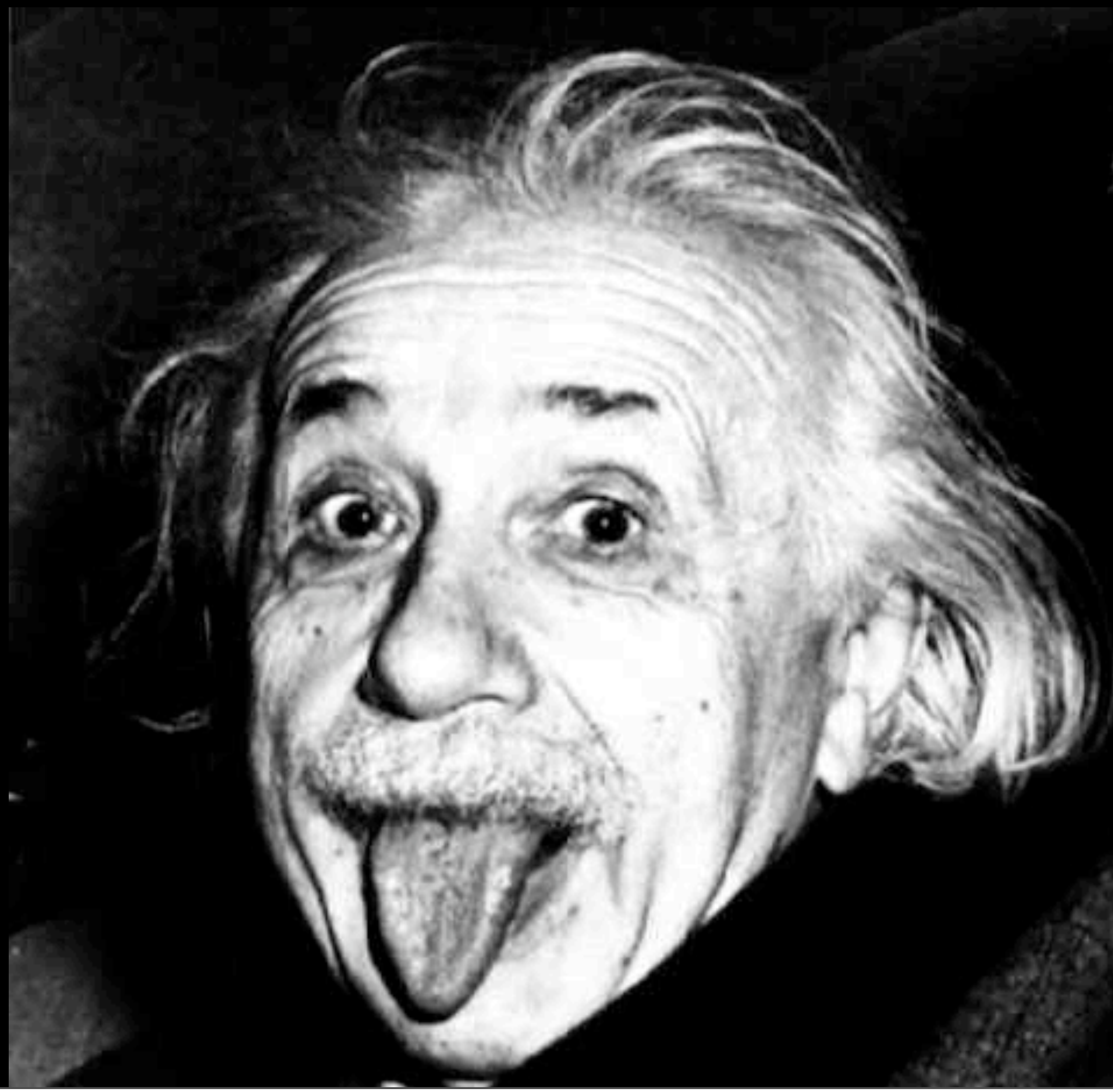
Generalized universality

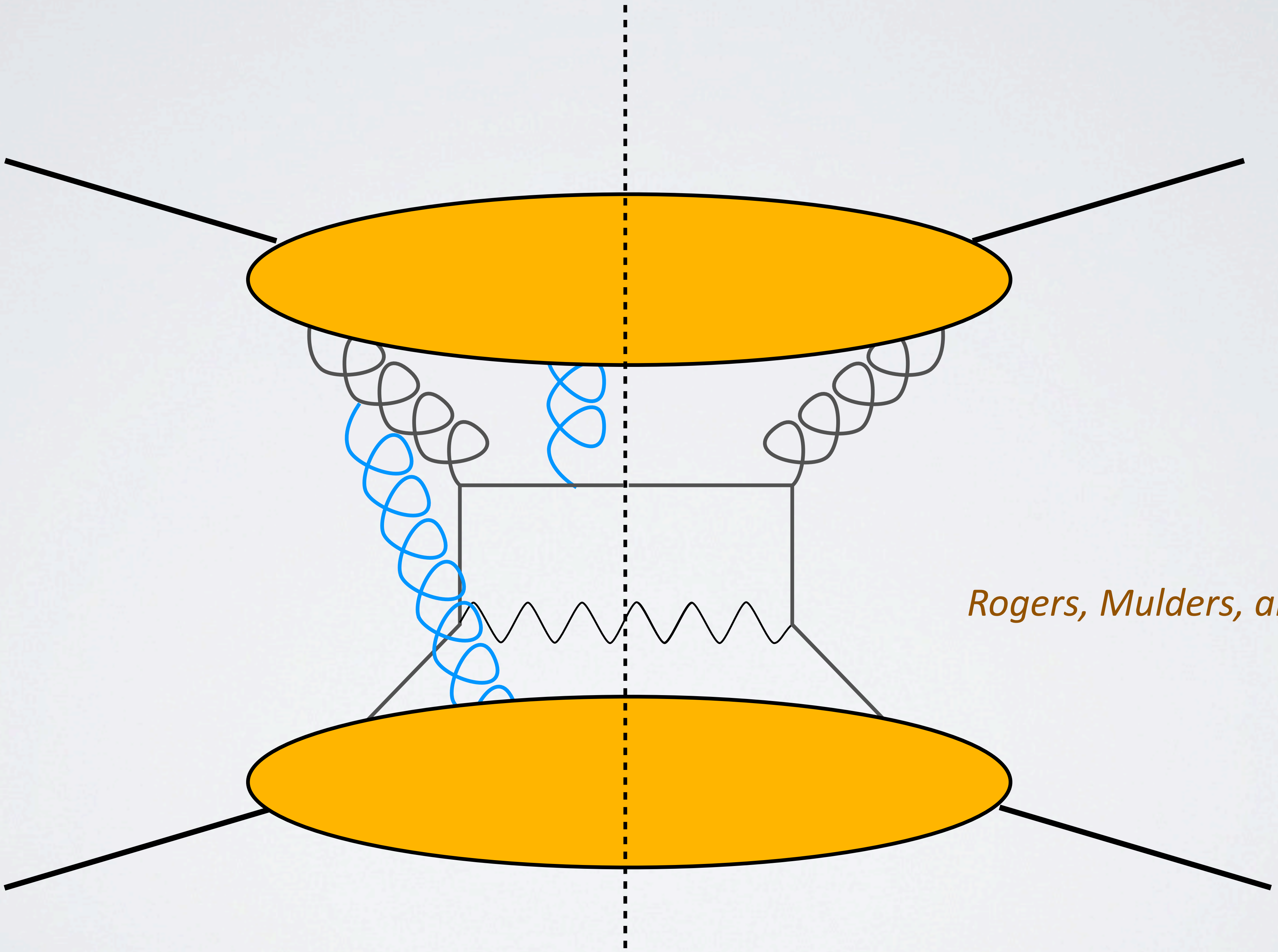
A.B., D'Alesio, Bomhof, Mulders, Murgia, PRL99 (07)

“Things should be made as simple as possible...”



“Things should be made as simple as possible, *but not any simpler*”





Rogers, Mulders, arXiv:1001.2977

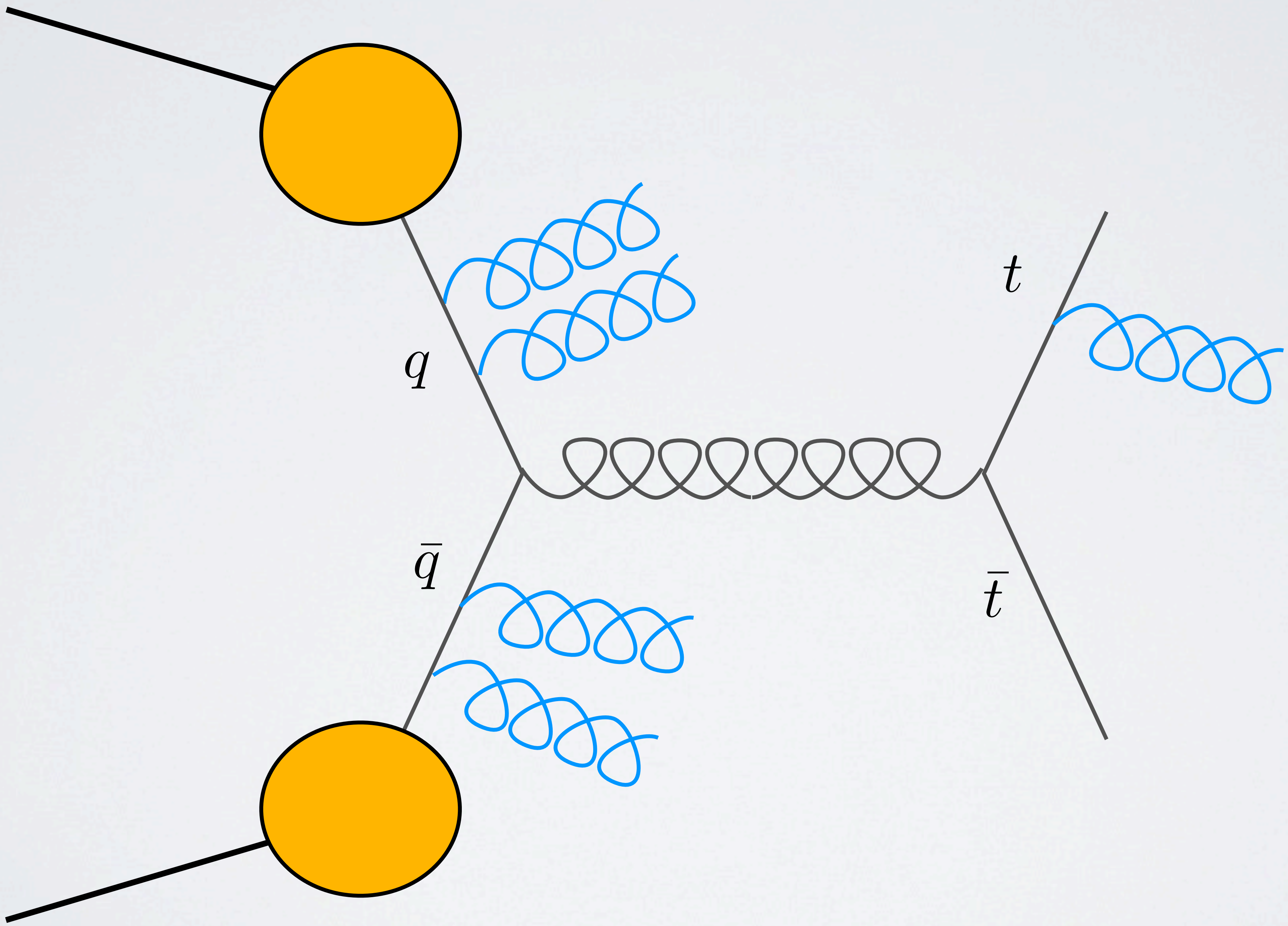
$\pi\, p$ to hadrons

No TMD factorization!

Rogers, Mulders, arXiv:1001.2977

Impact on LHC physics?

Description of *pp to jets*
by Monte Carlo event generators



Tasks for COMPASS

Check effects of
generalized factorization in Drell-Yan
(i.e., change of sign in Sivers asymmetry)
and breaking of TMD factorization
in πp to jets