

# The Boer-Mulders - Pretzelosity Asymmetry in $\pi$ p Drell-Yan at COMPASS

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# Outline

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- ④ Light-Cone Quark Model for proton and pion
- ④ Pretzelosity in SIDIS: model predictions and experimental results
- ④ Pretzelosity in Drell-Yan processes
- ④ Conclusions

# Light Cone Wave Function of the Proton

$$|P, \lambda\rangle = \sum_{\beta} \int d[1]d[2]d[3] \Psi_{\lambda, \beta}^f(x_i, \vec{k}_{\perp, i}) \frac{\epsilon^{ijk}}{\sqrt{6}} u_{i\lambda_1}^{\dagger}(1) u_{j\lambda_2}^{\dagger}(2) d_{k\lambda_3}^{\dagger}(3) |0\rangle$$

❖ classification of LCWFs in orbital angular momentum components [Ji, J.P. Ma, Yuan, 03; Burkardt, Ji, Yuan, 02]

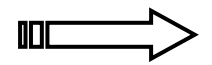
$$|P, \uparrow\rangle = |P, \uparrow\rangle_{-\frac{3}{2}}^{L_z=2} + |P, \uparrow\rangle_{-\frac{1}{2}}^{L_z=1} + |P, \uparrow\rangle_{\frac{1}{2}}^{L_z=0} + |P, \uparrow\rangle_{\frac{3}{2}}^{L_z=-1}$$

$$J_z = J_z^q + L_z^q$$

total quark helicity  $J^q$

$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$	$L_z^q = 2$
$J_z^q \rightarrow (\uparrow\uparrow\uparrow)_{LC}$	$(\uparrow\uparrow\downarrow)_{LC}$	$(\uparrow\downarrow\downarrow)_{LC}$	$(\downarrow\downarrow\downarrow)_{LC}$

parity  
time reversal  
isospin symmetry



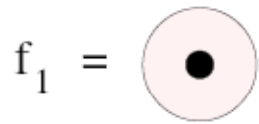
6 independent wave function amplitudes

MODEL

- ✓ momentum-space component: spherically symmetric
- ✓ **light-front boost** to convert the rest-frame spins of quarks in LF spins
- ✓ two parameters fitted to anomalous magnetic moments of proton and neutron

[B.P., Cazzaniga, Boffi, PRD78 (2008)]

# Light Cone Amplitudes Overlap Representation of TMDs



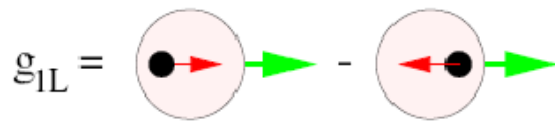
$$\Delta L_z=0$$

 $S \rightarrow S$ 
 $P \rightarrow P$ 

$$f_1^q(x, k_\perp^2) = L_z=0 \langle P \uparrow | \sum_\lambda q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=0} + L_z=1 \langle P \uparrow | \sum_\lambda q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=1}$$

 $P \rightarrow P$ 
 $D \rightarrow D$ 

$$+ L_z=-1 \langle P \uparrow | \sum_\lambda q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=-1} + L_z=2 \langle P \uparrow | \sum_\lambda q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=2}$$



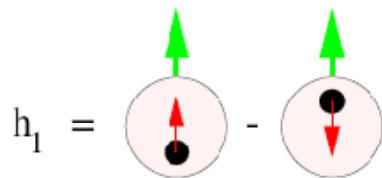
$$\Delta L_z=0$$

 $S \rightarrow S$ 
 $P \rightarrow P$ 

$$g_{1L}^q(x, k_\perp^2) = L_z=0 \langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=0} + L_z=1 \langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=1}$$

 $P \rightarrow P$ 
 $D \rightarrow D$ 

$$+ L_z=-1 \langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=-1} + L_z=2 \langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=2}$$



$$\Delta L_z=0$$

 $S \rightarrow S$ 
 $P \rightarrow P$ 

$$h_1^q(x, k_\perp^2) = \text{Re}[L_z=0 \langle P \downarrow | q_\downarrow^\dagger q_\uparrow | P \uparrow \rangle^{L_z=0}] + 2\text{Re}[L_z=-1 \langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \downarrow \rangle^{L_z=-1}]$$

$$\mathbf{g}_{1T} = \begin{array}{c} \uparrow \\ \bullet \rightarrow \end{array} - \begin{array}{c} \uparrow \\ \leftarrow \bullet \end{array} \quad \boxed{|\Delta L_z|=1}$$

$$g_{1T}^q(x, k_\perp^2) = \boxed{P \rightarrow S} \quad \boxed{P \rightarrow S}$$

$$\frac{2M}{k_\perp^2} \left( k^x \text{Re}[^{L_z=0}\langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \downarrow \rangle^{L_z=-1}] + k^y \text{Im}[^{L_z=0}\langle P \uparrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \downarrow \rangle^{L_z=-1}] \right. \\ \left. \boxed{P \rightarrow D} \quad \boxed{P \rightarrow D} \right. \\ \left. + k^x \text{Re}[^{L_z=-2}\langle P \downarrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=-1}] + k^y \text{Im}[^{L_z=-2}\langle P \downarrow | \sum_\lambda (-1)^{1/2-\lambda} q_\lambda^\dagger q_\lambda | P \uparrow \rangle^{L_z=-1}] \right)$$

$$\mathbf{h}_{1L}^\perp = \begin{array}{c} \uparrow \\ \bullet \end{array} \rightarrow - \begin{array}{c} \bullet \\ \downarrow \end{array} \rightarrow \quad \boxed{|\Delta L_z|=1}$$

$$\boxed{S \rightarrow P} \quad \boxed{S \rightarrow P}$$

$$h_{1L}^{\perp q}(x, k_\perp^2) = \frac{2M}{k_\perp^2} \left( \text{Re}[^{L_z=1}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \uparrow \rangle^{L_z=0}] - k^y \text{Im}[^{L_z=1}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \uparrow \rangle^{L_z=0}] \right.$$

$$\boxed{P \rightarrow D} \quad \boxed{P \rightarrow D}$$

$$\left. + k^x \text{Re}[^{L_z=2}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \uparrow \rangle^{L_z=1}] - k^y \text{Im}[^{L_z=2}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \uparrow \rangle^{L_z=1}] \right)$$

$$\mathbf{h}_{1T}^\perp = \begin{array}{c} \uparrow \\ \bullet \end{array} \leftarrow - \begin{array}{c} \bullet \\ \downarrow \end{array} \leftarrow \quad \boxed{|\Delta L_z|=2}$$

$$\boxed{P \rightarrow P} \quad \boxed{D \rightarrow S}$$

$$h_{1T}^{\perp q}(x, k_\perp^2) = -\text{Re}[^{L_z=1}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \downarrow \rangle^{L_z=-1}] - 2\text{Re}[^{L_z=0}\langle P \uparrow | q_\downarrow^\dagger q_\uparrow | P \downarrow \rangle^{L_z=-2}]$$

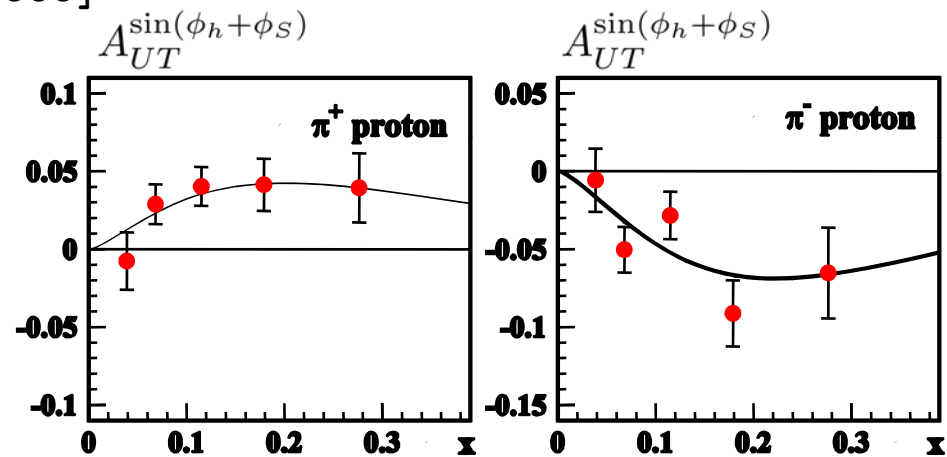
# Collins SSA

gaussian Ansatz  $\implies A_{UT}^{\sin(\phi_h+\phi_S)}(x) = \frac{\sum_a e_a^2 x h_1^a(x) \langle B_1 H_1^{\perp(1/2)a} \rangle}{\sum_a e_a^2 x f_1^a(x) \langle D_1^a \rangle}$

- $h_1(x)$  from Light-Cone CQM evolved at  $Q^2=2.5 \text{ GeV}^2$ ,  $f_1(x)$  from GRV at  $Q^2=2.5 \text{ GeV}^2$
- $H_1^{\perp(1/2)}$  from HERMES & BELLE data Efremov, Goeke, Schweitzer, PRD73 (2006); Anselmino et al., PRD75 (2007); Vogelsang, Yuan, PRD72 (2005)
- $D_1(x)$  at  $Q^2=2.5 \text{ GeV}^2$  [Kretzer, PRD62, 2000]

HERMES data:  
Diefenthaler, hep-ex/0507013

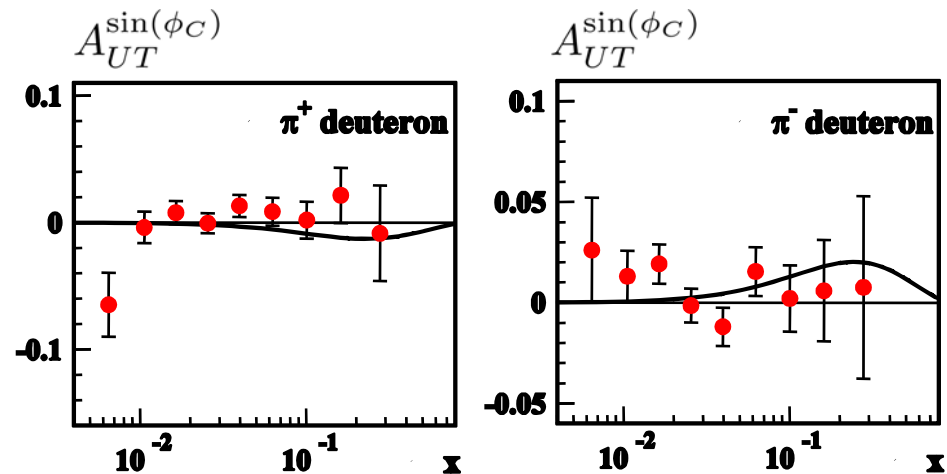
More recent HERMES and BELLE data  
 not included in the fit of Collins function



COMPASS data:  
Aleksiev et al., PLB673, (2009)

$$\phi_C = \phi_h + \phi_S + \pi$$

$$A_{UT}^{\sin(\phi_C)} = -A_{UT}^{\sin(\phi_h+\phi_S)}$$



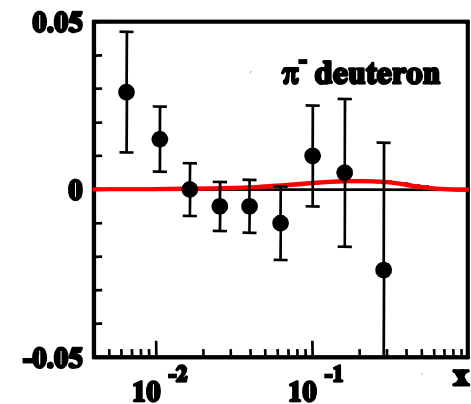
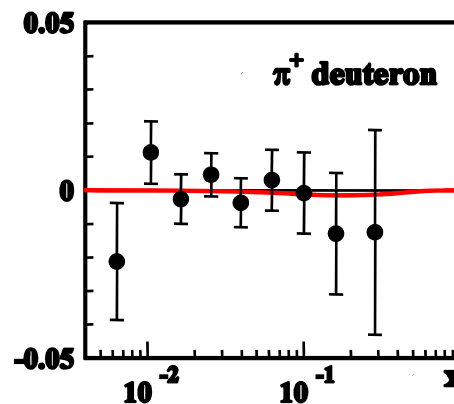
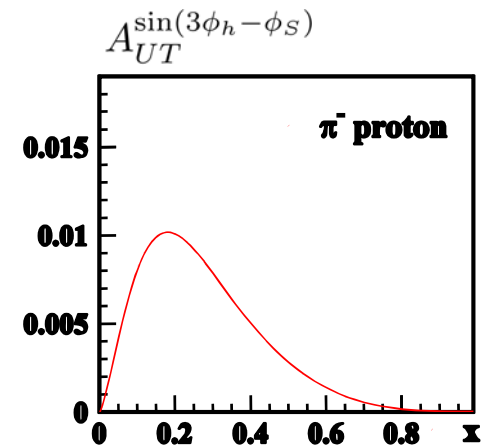
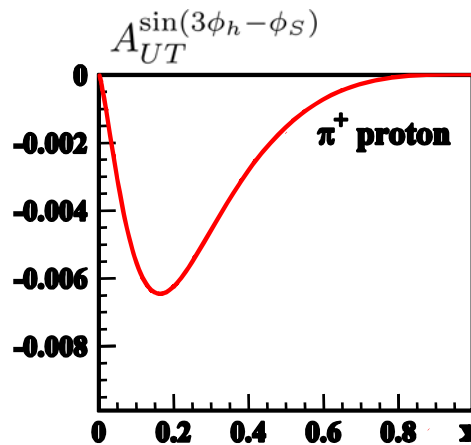
[Boffi, Efremov,  
 Pasquini, Schweitzer, PRD79, 2009]

$$A_{UT}^{\sin(3\phi_h - \phi_S)}$$

gaussian Ansatz  $\Rightarrow A_{UT}^{\sin(3\phi_h - \phi_S)} = -\frac{\sum_a e_a^2 x h_{1T}^{\perp(1)a}(x) \langle B_3 H_1^{\perp(1/2)a} \rangle}{\sum_a e_a^2 x f_1^a(x) \langle D_1^a \rangle}$

- $h_{1T}^{\perp(1)}$  from Light-Cone CQM evolved at  $Q^2=2.5 \text{ GeV}^2$ , with the evolution equations of  $h_1(x)$
- $f_1(x)$  from GRV at  $Q^2=2.5 \text{ GeV}^2$
- $H_1^{\perp(1/2)}$  from HERMES & BELLE data Efremov, Goeke, Schweitzer, PRD73 (2006); Anselmino et al., PRD75 (2007); Vogelsang, Yuan, PRD72 (2005)

✓ experiment planned at CLAS12  
(H. Avakian et al., LOI 12-06-108)



● COMPASS Coll.

Kotzinian, arXiv:0705.2402

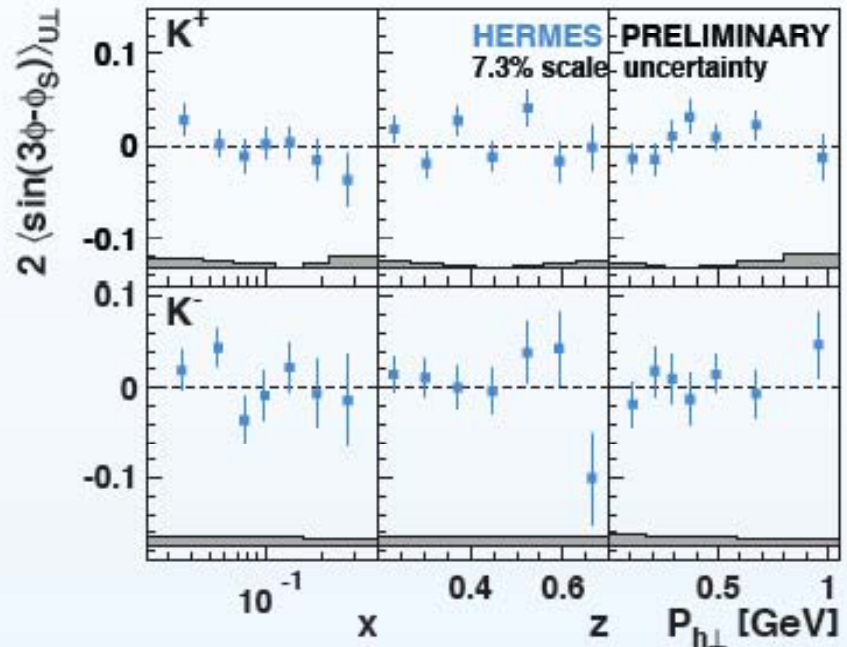
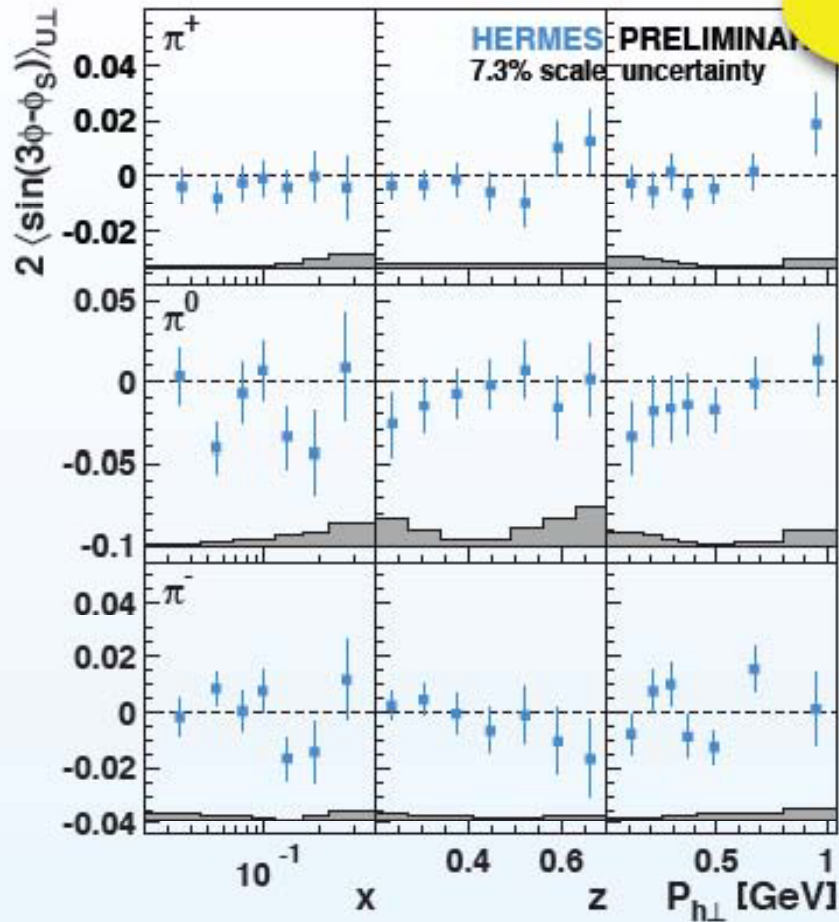
[Boffi, Efremov,  
Pasquini, Schweitzer, PRD79, 2009]

4 The  $\langle \sin(3\phi - \phi_S) \rangle_{U\perp}$  Fourier component:

**sensitive to pretzelosity!**

**zero**

suppressed w.r.t. Collins and Sivers amplitudes



**NEW**



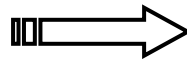
# Light Cone Wave Function of the Pion

$$|P, \pi\rangle = \sum_{\beta} \int d[1]d[2] \Psi_{\pi\beta}^f(x_i, \vec{k}_{\perp,i}) \frac{\delta^{ij}}{\sqrt{3}} u_{i\lambda_1}^{\dagger}(1) \bar{u}_{j\lambda_2}^{\dagger}(2) |0\rangle$$

$$|\pi\rangle = |\pi\rangle_{-1}^{L_z=1} + |\pi\rangle_0^{L_z=0} + |\pi\rangle_1^{L_z=-1}$$

$L_z^q = -1$	$L_z^q = 0$	$L_z^q = 1$
$J_z^q \rightarrow (\uparrow\uparrow)_{LC}$	$(\uparrow\downarrow)_{LC}$	$(\downarrow\downarrow)_{LC}$

parity  
time reversal  
isospin symmetry



2 independent wave function amplitudes

MODEL

- ✓ momentum-space component: spherically symmetric  $\Rightarrow$  gaussian shape
- ✓ Light-front boost to convert the rest-frame spins of quark and antiquark in LF spins
- ✓  $m_q$  and gaussian width fitted to exp. charge radius and pion decay constant
- ✓ Boer-Mulders function of the pion in one-gluon exchange approximation generated from S-P wave interference

[Efremov, Pasquini, Schweitzer, Yuan, in preparation]

# The BM-Pretzelosity Asymmetry in $\pi p$ Drell Yan

$$A_{TU}^{\sin(2\phi+\phi_p)} = \frac{F_{TU}^{\sin(2\phi+\phi_p)}}{F_{UU}^1}$$

$\phi$  = lepton angle in the CS frame

$\phi_p$  = proton spin angle in the CM frame

**Numerator**  $\longrightarrow$   $F_{TU}^{\sin(2\phi+\phi_p)} = \mathcal{C} [w(\vec{k}_{Tp}, \vec{K}_{T\pi}) h_{1T}^{\perp p} h_{1T}^{\perp \pi}]$

$$w(\vec{k}_{Tp}, \vec{k}_{T\pi}) = \frac{2(\vec{h} \cdot \vec{k}_{Tp})(2(\vec{h} \cdot \vec{k}_{Tp})(\vec{h} \cdot \vec{k}_{T\pi}) - \vec{k}_{Tp}^2(\vec{h} \cdot \vec{k}_{T\pi}))}{2M_p^2 m_\pi} \quad \text{with} \quad \vec{h} = \frac{\vec{q}_T}{q_T}$$

**Denominator**  $\longrightarrow$   $F_{UU}^1 = \mathcal{C} [1 f_1^p f_1^\pi]$

[Arnold, Metz, Schlegel, PRD79, (2008)]

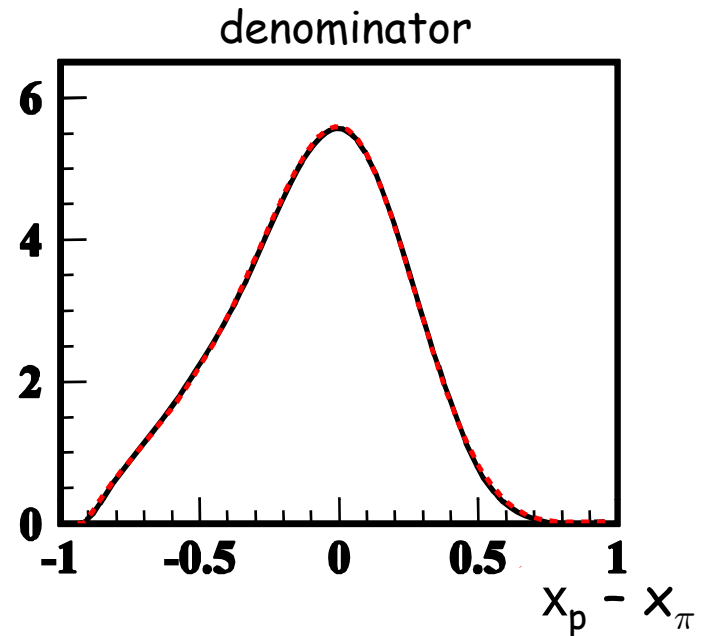
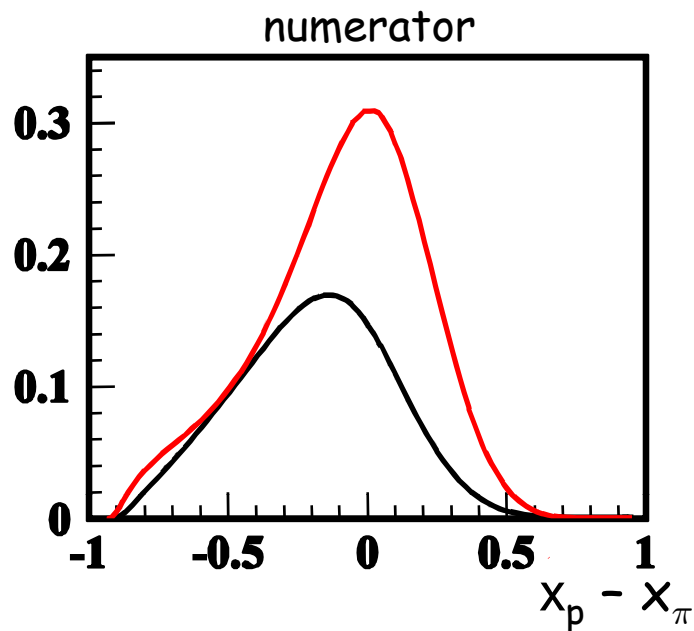
Boer, PRD60 (1999): BM-pretzelosity defined with a different angular distribution

$$A_{TU}^{\sin(3\phi-\phi_S)}$$

$\phi$  = lepton angle in the CS frame

$\phi_S$  = proton spin angle in the CS frame

# Gaussian Ansatz



COMPASS kinematics:  $x_p x_\pi = Q^2/s$  with  $Q^2 = 20 \text{ GeV}^2$  and  $s = 400 \text{ GeV}^2$

— convolution integral solved exactly, through numerical integration

— Gaussian Ansatz

$$F_{TU}^{\sin(2\phi+\phi_p)} = B_{\text{Gauss}} e_u^2 h_{1T}^{\perp(1/2)u/p}(x_p) h_1^{\perp(1)\bar{u}/\pi^-}(x_\pi) \quad F_{UU}^1 = e_u^2 f_1^{u/p}(x_p) f_1^{\bar{u}/\pi^-}(x_\pi)$$

$$B_{\text{Gauss}} = \frac{3 m_\pi}{2 M_p} \frac{1}{\left[1 + \frac{\langle p_{T\pi}^2 \rangle}{\langle p_{Tp}^2 \rangle}\right]^{3/2}}$$

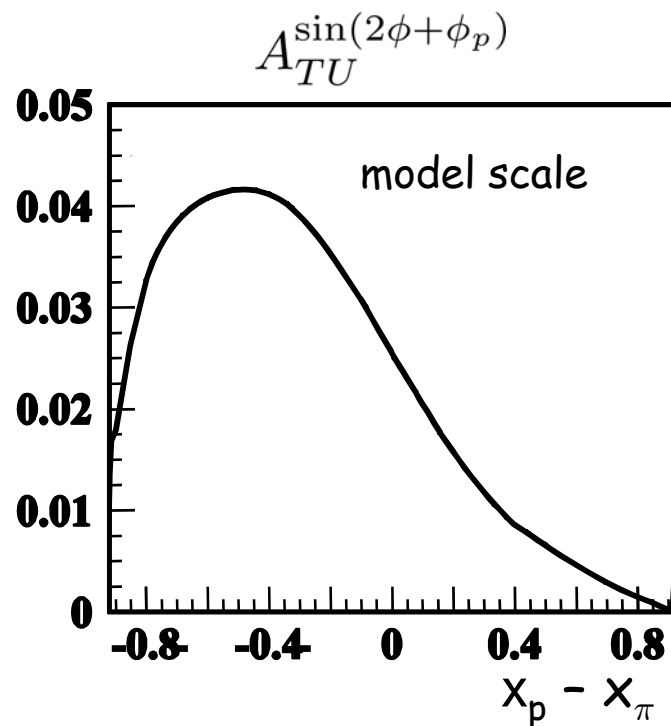
# Scale dependence

- ❖ At the hadronic scale  $\mu_0$  of the model the valence quarks carry all the momentum of the hadrons  $\rightarrow \langle x(\mu_0^2) \rangle_V = 1$
- ❖ To fix the scale  $\mu_0$  we evolve back the experimental value of  $\langle x(Q^2) \rangle_V$  at large  $Q^2$  until we match the condition  $\langle x(\mu_0^2) \rangle_V = 1$

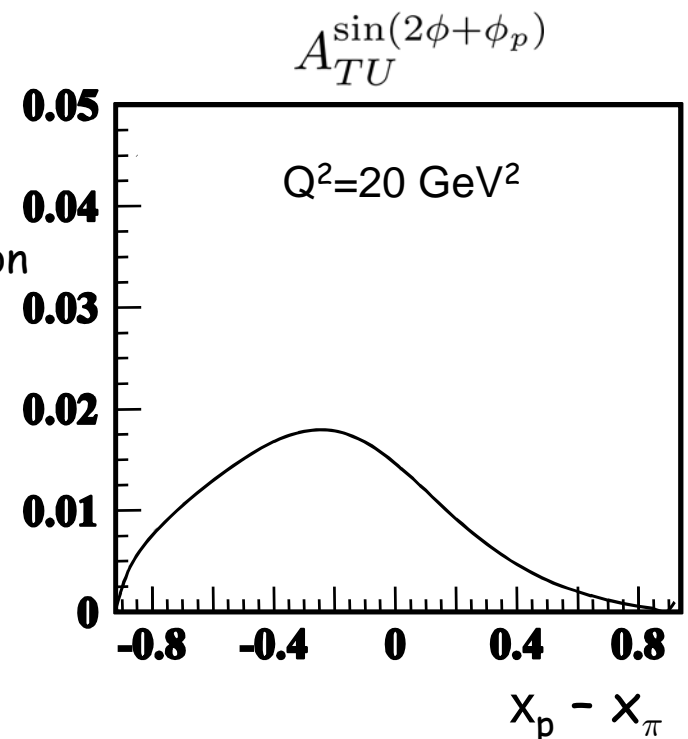
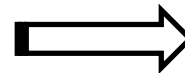
Proton:  $\mu_0 = 307 \text{ MeV}$

Pion:  $\mu_0 = 335 \text{ MeV}$

- ❖ Evolutions equations for  $h_{1T^\perp}$  and  $h_1^\perp$  are not yet known  $\rightarrow$  we include "approximate" evolution effects using the evolution equations of the transversity



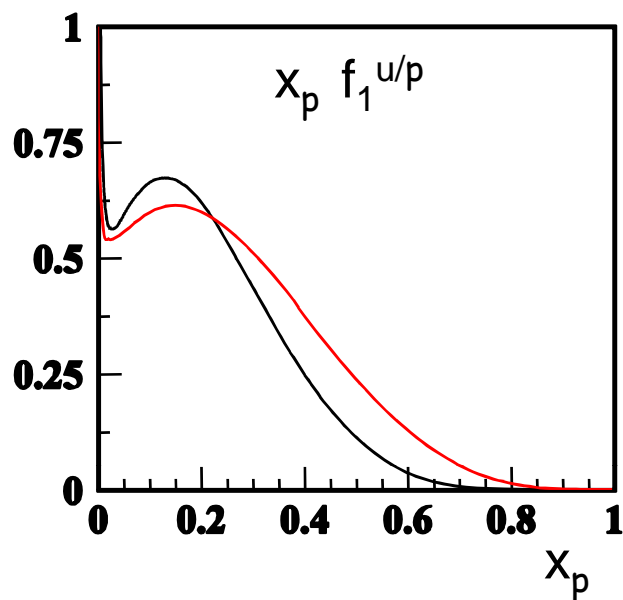
"approximate" evolution



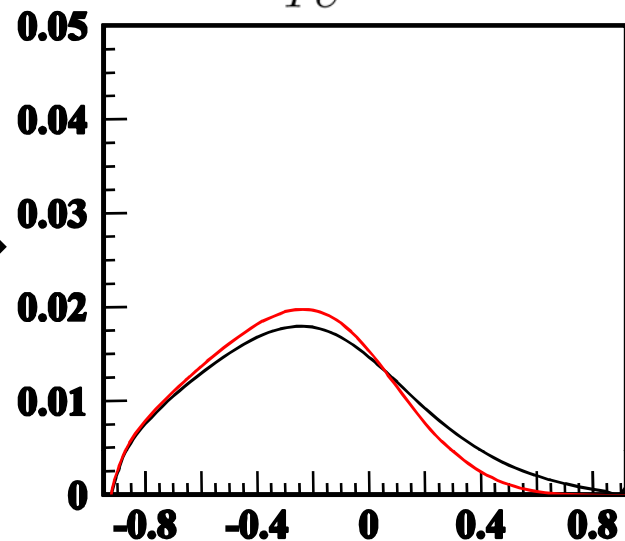
# Model dependence from $f_1^p$ and $f_1^\pi$

model

GRV



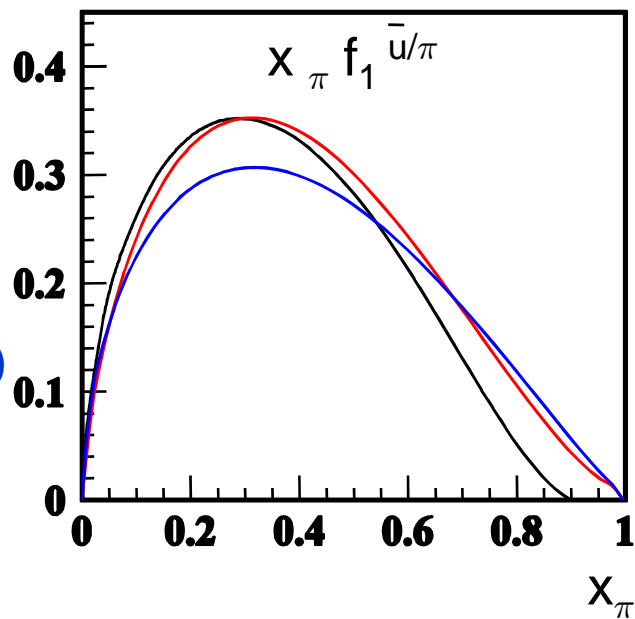
$A_{TU}^{\sin(2\phi+\phi_p)}$



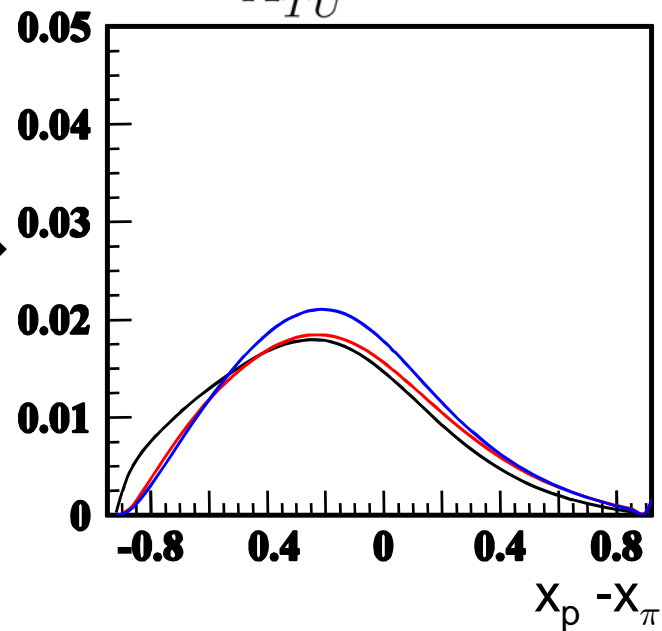
model

Wijesooriya et al.  
PRC72 (2005)

Conway et al,  
(E615 Fermilab Coll.)



$A_{TU}^{\sin(2\phi+\phi_p)}$



# Summary

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## ❖ Pretzelosity from SIDIS:

preliminary results from COMPASS and HERMES

data consistent with zero, in agreement with our model predictions

demanding to learn anything about  $h_{1T}^\perp$  from SIDIS

## ❖ Pretzelosity from Drell-Yan:

model results are encouraging!

asymmetry of the order of 2% at large negative  $x_F = x_p - x_\pi$   
where the Gaussian Ansatz works better within the model

small model dependence from  $f_1^p$  and  $f_1^\pi$

to improve the study of the effects of the evolution